GEOMETRY

Ron Larson
Laurie Boswell
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Lee Stiff

McDougal Littell
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GEOMETRY

About *Geometry*

In *Geometry*, you will develop reasoning and problem solving skills as you study topics such as congruence and similarity, and apply properties of lines, triangles, quadrilaterals, and circles. You will also develop problem solving skills by using length, perimeter, area, circumference, surface area, and volume to solve real-world problems.

In addition to its geometry content, *Geometry* includes numerous examples and exercises involving algebra, data analysis, and probability. These math topics often appear on standardized tests, so maintaining your familiarity with them is important. To help you prepare for standardized tests, *Geometry* provides instruction and practice on standardized test questions in a variety of formats—multiple choice, short response, extended response, and so on. Technology support for both learning geometry and preparing for standardized tests is available at classzone.com.

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CHAPTER

1

Unit 1
Points, Lines, and Planes

Segment Addition Postulate, p. 14
$AC = AB + BC$

Essentials of Geometry

Chapter 1 Highlights

PROBLEM SOLVING

- **Mixed Review of Problem Solving,** 23, 58
- **Multiple Representations,** 41, 55, 57
- **Multi-Step Problems,** 8, 14, 23, 46, 54, 55, 58
- **Using Alternative Methods,** 57
- **Real-World Problem Solving Examples,** 10, 15, 27, 36, 44, 51, 65

★ ASSESSMENT

- **Standardized Test Practice Examples,** 18, 50
- **Multiple Choice,** 6, 13, 20, 29, 39, 44, 46, 53
- **Short Response/Extended Response,** 7, 12, 14, 20, 22, 23, 30, 32, 40, 47, 54, 55, 58, 66
- **Writing/Open-Ended,** 5, 12, 19, 23, 28, 38, 44, 52, 58

TECHNOLOGY

At classzone.com:

- **Animated Geometry,** 1, 3, 14, 21, 25, 43, 52
- **@Home Tutor,** xxii, 7, 13, 21, 31, 40, 46, 48, 54, 60
- **Online Quiz,** 8, 14, 22, 32, 41, 47, 56
- **Animated Algebra (Algebra Review),** 65
- **State Test Practice,** 23, 58, 69

CHAPTER

2

Unit 1
Points, Lines, and Planes

Properties of Congruence, p. 115
$\overline{AB} \cong \overline{CD}$

Reasoning and Proof

Chapter 2 Highlights

PROBLEM SOLVING

- **Mixed Review of Problem Solving,** 103, 132
- **Multiple Representations,** 77, 111, 120
- **Multi-Step Problems,** 85, 102, 103, 110, 119, 130, 132
- **Using Alternative Methods,** 120
- **Real-World Problem Solving Examples,** 74, 89, 106, 115

★ ASSESSMENT

- **Standardized Test Practice Examples,** 74, 127
- **Multiple Choice,** 75, 76, 83, 90, 99, 100, 109, 116, 128
- **Short Response/Extended Response,** 76, 78, 84, 92, 101, 102, 103, 110, 117, 119, 128, 130, 132, 140
- **Writing/Open-Ended,** 75, 82, 84, 90, 99, 100, 108, 109, 116, 127, 129, 132

TECHNOLOGY

At classzone.com:

- **Animated Geometry,** 71, 72, 81, 88, 97, 106, 119, 125
- **@Home Tutor,** 70, 77, 84, 91, 101, 110, 118, 123, 129, 134
- **Online Quiz,** 78, 85, 93, 102, 111, 119, 131
- **Animated Algebra,** 139
- **State Test Practice,** 103, 132, 143

CHAPTER

3

Unit 1

Points, Lines, and Planes

Applying Slope, p. 174

Slope $= \frac{41}{80}$

Parallel and Perpendicular Lines

Chapter 3 Highlights

PROBLEM SOLVING

- **Mixed Review of Problem Solving,** 170, 200
- **Multiple Representations,** 174, 177, 188
- **Multi-Step Problems,** 166, 168, 170, 177, 186, 200
- **Using Alternative Methods,** 188
- **Real-World Problem Solving Examples,** 148, 156, 162, 164, 174, 182, 183, 193, 207

★ ASSESSMENT

- **Standardized Test Practice Example,** 173
- **Multiple Choice,** 151, 157, 158, 166, 176, 184, 185, 195, 208
- **Short Response/Extended Response,** 152, 158, 159, 166, 168, 169, 170, 176, 178, 187, 194, 196, 200
- **Writing/Open-Ended,** 150, 151, 157, 165, 170, 175, 184, 195, 200

TECHNOLOGY

At classzone.com:

- **Animated Geometry,** 145, 148, 155, 163, 174, 181
- **@Home Tutor,** 144, 151, 153, 159, 167, 176, 179, 186, 196, 202
- **Online Quiz,** 152, 160, 169, 178, 187, 197
- **Animated Algebra,** 207
- **State Test Practice,** 170, 200, 211

CHAPTER

4

Unit 2
Triangles

Indirect Measurement, p. 257
$\triangle MLK \cong \triangle MPN$

Congruent Triangles

Chapter 4 Highlights

PROBLEM SOLVING

- **Mixed Review of Problem Solving,** 248, 280
- **Multiple Representations,** 232
- **Multi-Step Problems,** 223, 231, 248, 269, 280
- **Using Alternative Methods,** 232
- **Real-World Problem Solving Examples,** 220, 226, 236, 242, 251, 257, 266, 274

★ ASSESSMENT

- **Standardized Test Practice Examples,** 235, 251
- **Multiple Choice,** 222, 223, 229, 237, 243, 246, 253, 260, 261, 268, 279, 288
- **Short Response/Extended Response,** 221, 224, 230, 231, 238, 248, 253, 254, 262, 267, 268, 270, 278, 280
- **Writing/Open-Ended,** 221, 228, 229, 230, 243, 244, 248, 252, 259, 267, 276, 277, 278, 280

TECHNOLOGY

At classzone.com:

- **Animated Geometry,** 215, 234, 242, 250, 256, 257, 274
- **@Home Tutor,** 214, 223, 230, 238, 245, 247, 254, 261, 269, 278, 282
- **Online Quiz,** 224, 231, 239, 246, 255, 263, 270, 279
- **Animated Algebra,** 287
- **State Test Practice,** 248, 280, 291

CHAPTER 5

Unit 2
Triangles

Inequalities in Triangles, p. 336
$150° > 135°$

Relationships within Triangles

Chapter 5 Highlights

PROBLEM SOLVING

- **Mixed Review of Problem Solving,** 317, 342
- **Multiple Representations,** 302
- **Multi-Step Problems,** 301, 317, 342
- **Using Alternative Methods,** 302
- **Real-World Problem Solving Examples,** 295, 305, 311, 329, 336, 349

★ ASSESSMENT

- **Standardized Test Practice Examples,** 320, 329
- **Multiple Choice,** 299, 307, 314, 322, 331, 332, 339
- **Short Response/Extended Response,** 300, 308, 315, 317, 323, 324, 332, 333, 334, 339, 340, 342, 350
- **Writing/Open-Ended,** 298, 306, 313, 317, 322, 331, 338, 342

TECHNOLOGY

At classzone.com:

- **Animated Geometry,** 293, 296, 304, 312, 321, 330, 336
- **@Home Tutor,** 292, 300, 308, 315, 324, 327, 333, 340, 344
- **Online Quiz,** 301, 309, 316, 325, 334, 341
- **Animated Algebra,** 349
- **State Test Practice,** 317, 342, 353

CHAPTER

6

Unit 2
Triangles

Applying Similar Triangles, p. 394

$$\frac{66 \text{ in.}}{7 \text{ ft}} = \frac{x \text{ in.}}{102 \text{ ft}}$$

Similarity

Chapter 6 Highlights

PROBLEM SOLVING

- **Mixed Review of Problem Solving,** 380, 416
- **Multiple Representations,** 363, 378, 404
- **Multi-Step Problems,** 362, 378, 380, 385, 394, 402, 414, 416
- **Using Alternative Methods,** 404
- **Real-World Problem Solving Examples,** 357, 359, 365, 366, 374, 390, 398, 410

★ ASSESSMENT

- **Standardized Test Practice Examples,** 383, 411
- **Multiple Choice,** 361, 368, 376, 377, 384, 385, 392, 400, 401, 412, 413
- **Short Response/Extended Response,** 361, 363, 377, 379, 380, 386, 387, 394, 402, 403, 413, 414, 415, 416, 424
- **Writing/Open-Ended,** 360, 367, 376, 380, 384, 385, 391, 394, 400, 412, 414, 416

TECHNOLOGY

At classzone.com:

- **Animated Geometry,** 355, 365, 375, 391, 394, 407, 414
- **@Home Tutor,** 354, 362, 368, 378, 386, 393, 396, 402, 414, 418
- **Online Quiz,** 363, 370, 379, 387, 395, 403, 415
- **Animated Algebra,** 423
- **State Test Practice,** 380, 416, 427

CHAPTER

7

Unit 3
Figures in the Plane

Angle of Elevation, p. 475

$\sin 21° = \frac{\text{opp.}}{\text{hyp.}}$

Right Triangles and Trigonometry

Chapter 7 Highlights

PROBLEM SOLVING

★ ASSESSMENT

TECHNOLOGY

At classzone.com:

CHAPTER

8

Unit 3
Figures in the Plane

Polygon Angle Sum, p. 512
$(n - 2) \cdot 180°$

Quadrilaterals

Chapter 8 Highlights

PROBLEM SOLVING

- **Mixed Review of Problem Solving,** 532, 558
- **Multiple Representations,** 513, 530
- **Multi-Step Problems,** 512, 532, 539, 556, 558
- **Using Alternative Methods,** 530
- **Real-World Problem Solving Examples,** 510, 517, 523, 524, 536, 543, 545

★ ASSESSMENT

- **Standardized Test Practice Examples,** 509, 517, 553
- **Multiple Choice,** 511, 518, 519, 527, 538, 546, 547, 554, 566
- **Short Response/Extended Response,** 511, 513, 519, 526, 529, 532, 538, 540, 547, 548, 556, 558
- **Writing/Open-Ended,** 510, 518, 520, 526, 537, 546, 554, 558

TECHNOLOGY

At classzone.com:

- **Animated Geometry,** 505, 509, 519, 527, 535, 545, 551, 553
- **@Home Tutor,** 504, 512, 514, 520, 528, 539, 541, 548, 556, 560
- **Online Quiz,** 513, 521, 529, 540, 549, 557
- **Animated Algebra,** 565
- **State Test Practice,** 532, 558, 569

Identifying Transformations, p. 595
$(a, b) \rightarrow (a, -b)$

CHAPTER
9
Unit 3
Figures in the Plane

Properties of Transformations

Chapter 9 Highlights

PROBLEM SOLVING

- **Mixed Review of Problem Solving,** 597, 634
- **Multiple Representations,** 606
- **Multi-Step Problems,** 577, 579, 586, 597, 605, 615, 624, 631, 634
- **Using Alternative Methods,** 606
- **Real-World Problem Solving Examples,** 575, 583, 591

★ ASSESSMENT

- **Standardized Test Practice Examples,** 601, 621
- **Multiple Choice,** 576, 584, 585, 593, 603, 613, 622, 630
- **Short Response/Extended Response,** 578, 586, 594, 596, 597, 603, 605, 614, 623, 630, 634, 642
- **Writing/Open-Ended,** 576, 584, 585, 593, 597, 602, 611, 613, 621, 623, 629, 630, 631, 634

TECHNOLOGY

At classzone.com:

- **Animated Geometry,** 571, 582, 590, 599, 602, 611, 617, 619, 626
- **@Home Tutor,** 570, 578, 586, 595, 604, 607, 613, 623, 631, 633, 636
- **Online Quiz,** 579, 587, 596, 605, 615, 624, 632
- **Animated Algebra,** 641
- **State Test Practice,** 597, 634, 645

CHAPTER

10

Unit 4

Circles and Measurement

Tangents and Secants, p. 692

$DC \cdot DB = AD^2$

Properties of Circles

Chapter 10 Highlights

PROBLEM SOLVING

- **Mixed Review of Problem Solving,** 687, 706
- **Multiple Representations,** 696
- **Multi-Step Problems,** 669, 687, 706
- **Using Alternative Methods,** 696
- **Real-World Problem Solving Examples,** 660, 665, 674, 682, 692, 701

★ ASSESSMENT

- **Standardized Test Practice Examples,** 673, 690
- **Multiple Choice,** 656, 662, 667, 677, 683, 693, 702, 703, 714
- **Short Response/Extended Response,** 657, 662, 663, 678, 684, 685, 687, 694, 695, 704, 706
- **Writing/Open-Ended,** 655, 661, 667, 668, 669, 676, 678, 683, 684, 687, 692, 702

TECHNOLOGY

At classzone.com:

- **Animated Geometry,** 649, 655, 661, 671, 682, 691, 701
- **@Home Tutor,** 648, 657, 663, 669, 677, 685, 688, 694, 703, 704, 708
- **Online Quiz,** 658, 663, 670, 679, 686, 695, 705
- **Animated Algebra,** 713
- **State Test Practice,** 687, 706, 717

CHAPTER

11

Unit 4
Circles and Measurement

Arc Length, p. 749

$2(84.39) + 2\left(\frac{1}{2} \cdot 2\pi \cdot 36.8\right)$

Measuring Length and Area

Chapter 11 Highlights

PROBLEM SOLVING

- **Mixed Review of Problem Solving,** 745, 778
- **Multiple Representations,** 744
- **Multi-Step Problems,** 726, 735, 742, 745, 778
- **Using Alternative Methods,** 744
- **Real-World Problem Solving Examples,** 722, 730, 738, 739, 747, 749, 763, 772, 773, 785

★ ASSESSMENT

- **Standardized Test Practice Examples,** 732, 738, 757
- **Multiple Choice,** 724, 733, 740, 742, 751, 759, 765, 775
- **Short Response/Extended Response,** 725, 726, 735, 736, 741, 743, 745, 751, 752, 760, 766, 768, 776, 778, 786
- **Writing/Open-Ended,** 723, 724, 733, 734, 740, 743, 745, 749, 758, 765, 774, 778

TECHNOLOGY

At classzone.com:

- **Animated Geometry,** 719, 720, 739, 749, 759, 765, 771
- **@Home Tutor,** 718, 725, 735, 742, 751, 760, 767, 769, 776, 780
- **Online Quiz,** 726, 736, 743, 752, 761, 768, 777
- **Animated Algebra,** 785
- **State Test Practice,** 745, 778, 789

CHAPTER

12

Unit 4

Circles and Measurement

Volume of Cylinders, p. 825

$V = Bh = \pi r^2 h$

Surface Area and Volume of Solids

Chapter 12 Highlights

PROBLEM SOLVING

- **Mixed Review of Problem Solving,** 818, 855
- **Multiple Representations,** 826, 835, 853
- **Multi-Step Problems,** 800, 809, 816, 818, 824, 835, 844, 852, 855
- **Using Alternative Methods,** 826
- **Real-World Problem Solving Examples,** 796, 805, 813, 822, 831, 840, 848, 849

★ ASSESSMENT

- **Standardized Test Practice Examples,** 813, 839
- **Multiple Choice,** 799, 807, 808, 815, 822, 824, 832, 833, 842, 843, 850, 851, 862
- **Short Response/Extended Response,** 800, 808, 809, 816, 818, 825, 834, 844, 853, 855
- **Writing/Open-Ended,** 798, 806, 814, 818, 822, 832, 842, 850, 852

TECHNOLOGY

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- **@Home Tutor,** 790, 800, 808, 816, 824, 834, 837, 844, 852, 857
- **Online Quiz,** 801, 809, 817, 825, 836, 845, 854
- **State Test Practice,** 818, 855, 865

Contents of Student Resources

Using Your Textbook

Your textbook contains many resources that you can use for reference when you are studying or doing your homework.

IN EVERY CHAPTER

BIG IDEAS The second page of every chapter includes a list of important ideas developed in the chapter. More information about these ideas appears in the Chapter Summary page at the end of the chapter.

POSTULATES AND THEOREMS The Postulate and Theorem notebook displays present geometric properties you will use in reasoning about figures. You may want to copy these statements into your notes.

KEY CONCEPTS The Key Concept notebook displays present main ideas of the lesson. You may want to copy these ideas into your notes.

VOCABULARY New words and review words are listed in a column on the first page of every lesson. Vocabulary terms appear highlighted and in bold print within the lesson. A list of vocabulary appears in the Chapter Review at the end of each chapter.

MIXED REVIEW Every lesson ends with Mixed Review exercises. These exercises help you review earlier lessons and include exercises to prepare you for the next lesson. Page references with the exercises point you to the lessons being reviewed.

STUDENT RESOURCES AT THE BACK OF THE BOOK

SKILLS REVIEW HANDBOOK Use the Skills Review Handbook topics on pages 869–895 to review material learned in previous courses.

EXTRA PRACTICE Use the Extra Practice on pages 896–919 for more exercises or to review a chapter before a test.

TABLES Refer to the tables on pages 920–925 for information about mathematical symbols, measures, formulas, squares, and trigonometric ratios.

POSTULATES AND THEOREMS Refer to pages 926–931 for a complete list of all postulates and theorems presented in the book.

ADDITIONAL PROOFS Refer to pages 932–938 for longer proofs of some of the theorems presented in the book.

GLOSSARY Use the English-Spanish Glossary on pages 939–980 to see definitions in English and Spanish, as well as examples illustrating vocabulary.

INDEX Look up items in the alphabetical Index on pages 981–1000 to find where a particular math topic is covered in the book.

WORKED-OUT SOLUTIONS In each lesson, exercises identified by a red circle have complete worked-out solutions starting on page WS1. These provide a model for what a full solution should include.

SELECTED ANSWERS Use the Selected Answers starting on page SA1 to check your work.

1 Essentials of Geometry

Before

In previous courses, you learned the following skills, which you'll use in Chapter 1: finding measures, evaluating expressions, and solving equations.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

1. The distance around a rectangle is called its __?__, and the distance around a circle is called its __?__.
2. The number of square units covered by a figure is called its __?__.

SKILLS AND ALGEBRA CHECK

Evaluate the expression. *(Review p. 870 for 1.2, 1.3, 1.7.)*

3. $|4 - 6|$ 4. $|3 - 11|$ 5. $|-4 + 5|$ 6. $|-8 - 10|$

Evaluate the expression when $x = 2$. *(Review p. 870 for 1.3–1.6.)*

7. $5x$ 8. $20 - 8x$ 9. $-18 + 3x$ 10. $-5x - 4 + 2x$

Solve the equation. *(Review p. 875 for 1.2–1.7.)*

11. $274 = -2z$ 12. $8x + 12 = 60$ 13. $2y - 5 + 7y = -32$
14. $6p + 11 + 3p = -7$ 15. $8m - 5 = 25 - 2m$ 16. $-2n + 18 = 5n - 24$

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 1, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 59. You will also use the key vocabulary listed below.

Big Ideas

1. Describing geometric figures
2. Measuring geometric figures
3. Understanding equality and congruence

KEY VOCABULARY

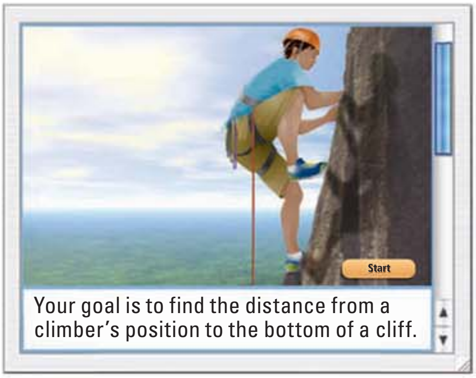
- undefined terms, *p. 2*
  point, line, plane
- defined terms, *p. 3*
- line segment, endpoints, *p. 3*
- ray, opposite rays, *p. 3*
- postulate, axiom, *p. 9*
- congruent segments, *p. 11*
- midpoint, *p. 15*
- segment bisector, *p. 15*
- acute, right, obtuse, straight angles, *p. 25*
- congruent angles, *p. 26*
- angle bisector, *p. 28*
- linear pair, *p. 37*
- vertical angles, *p. 37*
- polygon, *p. 42*
- convex, concave, *p. 42*
- *n*-gon, *p. 43*
- equilateral, equiangular, regular, *p. 43*

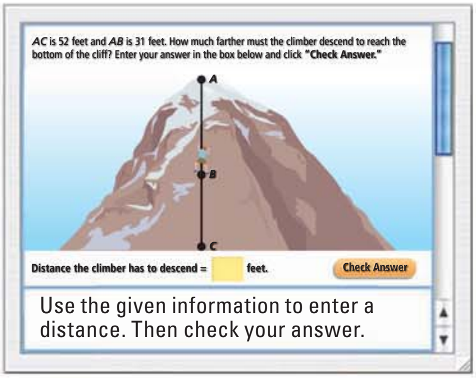
Why?

Geometric figures can be used to represent real-world situations. For example, you can show a climber's position along a stretched rope by a point on a line segment.

Animated Geometry

The animation illustrated below for Exercise 35 on page 14 helps you answer this question: How far must a climber descend to reach the bottom of a cliff?





Animated Geometry at classzone.com

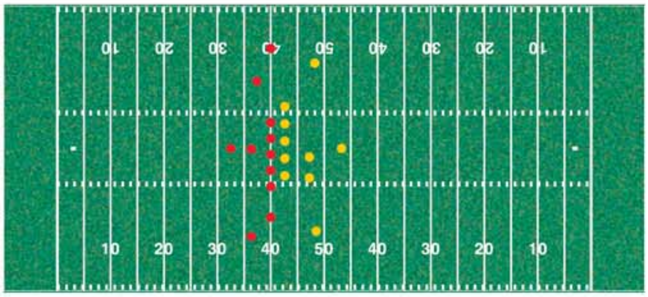
Other animations for Chapter 1: pages 3, 21, 25, 43, and 52

1.1 Identify Points, Lines, and Planes

Before	You studied basic concepts of geometry.
Now	You will name and sketch geometric figures.
Why	So you can use geometry terms in the real world, as in Ex. 13.

Key Vocabulary
- **undefined terms**
  point, line, plane
- **collinear points**
- **coplanar points**
- **defined terms**
- **line segment**
- **endpoints**
- **ray**
- **opposite rays**
- **intersection**

In the diagram of a football field, the positions of players are represented by *points*. The yard lines suggest *lines*, and the flat surface of the playing field can be thought of as a *plane*.



In geometry, the words *point*, *line*, and *plane* are **undefined terms**. These words do not have formal definitions, but there is agreement about what they mean.

TAKE NOTES
When you write new concepts and yellow-highlighted vocabulary in your notebook, be sure to copy all associated diagrams.

KEY CONCEPT — *For Your Notebook*

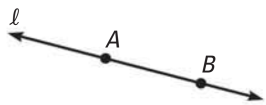
Undefined Terms

Point A **point** has no dimension. It is represented by a dot.

point *A*

Line A **line** has one dimension. It is represented by a line with two arrowheads, but it extends without end.

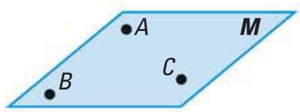
Through any two points, there is exactly one line. You can use any two points on a line to name it.



line ℓ, line *AB* ($\overleftrightarrow{AB}$), or line *BA* ($\overleftrightarrow{BA}$)

Plane A **plane** has two dimensions. It is represented by a shape that looks like a floor or a wall, but it extends without end.

Through any three points not on the same line, there is exactly one plane. You can use three points that are not all on the same line to name a plane.



plane *M* or plane *ABC*

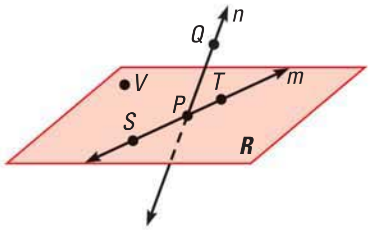
Collinear points are points that lie on the same line. **Coplanar points** are points that lie in the same plane.

EXAMPLE 1 Name points, lines, and planes

VISUAL REASONING
There is a line through points *S* and *Q* that is not shown in the diagram. Try to imagine what plane *SPQ* would look like if it were shown.

a. Give two other names for $\overleftrightarrow{PQ}$ and for plane *R*.

b. Name three points that are collinear.
Name four points that are coplanar.



Solution

a. Other names for $\overleftrightarrow{PQ}$ are $\overleftrightarrow{QP}$ and line *n*. Other names for plane *R* are plane *SVT* and plane *PTV*.

b. Points *S*, *P*, and *T* lie on the same line, so they are collinear. Points *S*, *P*, *T*, and *V* lie in the same plane, so they are coplanar.

Animated Geometry at classzone.com

✓ GUIDED PRACTICE for Example 1

1. Use the diagram in Example 1. Give two other names for $\overleftrightarrow{ST}$. Name a point that is *not* coplanar with points *Q*, *S*, and *T*.

DEFINED TERMS In geometry, terms that can be described using known words such as *point* or *line* are called **defined terms**.

KEY CONCEPT — For Your Notebook

Defined Terms: Segments and Rays

Line *AB* (written as $\overleftrightarrow{AB}$) and points *A* and *B* are used here to define the terms below.

line

Segment The **line segment** *AB*, or **segment** *AB*, (written as $\overline{AB}$) consists of the **endpoints** *A* and *B* and all points on $\overleftrightarrow{AB}$ that are between *A* and *B*. Note that $\overline{AB}$ can also be named $\overline{BA}$.

segment

Ray The **ray** *AB* (written as $\overrightarrow{AB}$) consists of the endpoint *A* and all points on $\overleftrightarrow{AB}$ that lie on the same side of *A* as *B*.

Note that $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are different rays.

ray

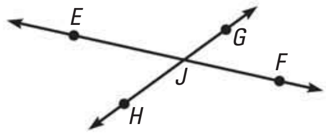
If point *C* lies on $\overleftrightarrow{AB}$ between *A* and *B*, then $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are **opposite rays**.

Segments and rays are collinear if they lie on the same line. So, opposite rays are collinear. Lines, segments, and rays are coplanar if they lie in the same plane.

EXAMPLE 2 Name segments, rays, and opposite rays

a. Give another name for $\overline{GH}$.

b. Name all rays with endpoint J. Which of these rays are opposite rays?



AVOID ERRORS

In Example 2, $\overrightarrow{JG}$ and $\overrightarrow{JF}$ have a common endpoint, but are not collinear. So they are *not* opposite rays.

Solution

a. Another name for $\overline{GH}$ is $\overline{HG}$.

b. The rays with endpoint J are $\overrightarrow{JE}$, $\overrightarrow{JG}$, $\overrightarrow{JF}$, and $\overrightarrow{JH}$. The pairs of opposite rays with endpoint J are $\overrightarrow{JE}$ and $\overrightarrow{JF}$, and $\overrightarrow{JG}$ and $\overrightarrow{JH}$.

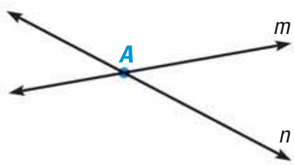
✓ GUIDED PRACTICE for Example 2

Use the diagram in Example 2.

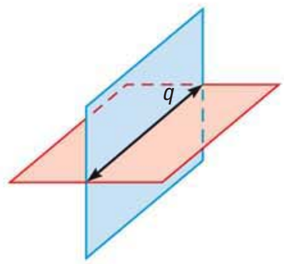
2. Give another name for $\overline{EF}$.

3. Are $\overrightarrow{HJ}$ and $\overrightarrow{JH}$ the same ray? Are $\overrightarrow{HJ}$ and $\overrightarrow{HG}$ the same ray? *Explain.*

INTERSECTIONS Two or more geometric figures *intersect* if they have one or more points in common. The **intersection** of the figures is the set of points the figures have in common. Some examples of intersections are shown below.



The intersection of two different lines is a point.



The intersection of two different planes is a line.

EXAMPLE 3 Sketch intersections of lines and planes

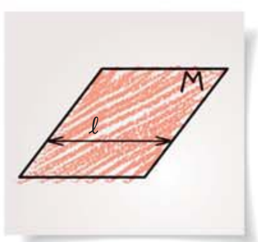
a. Sketch a plane and a line that is in the plane.

b. Sketch a plane and a line that does not intersect the plane.

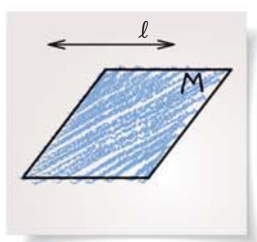
c. Sketch a plane and a line that intersects the plane at a point.

Solution

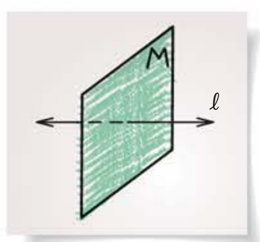
a.



b.

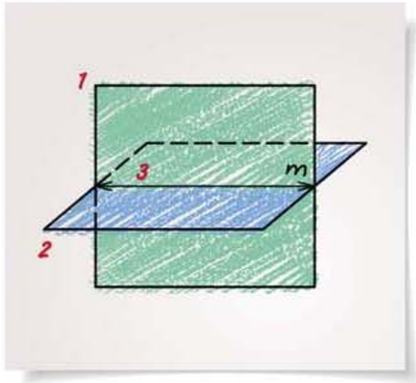


c.



EXAMPLE 4 Sketch intersections of planes

Sketch two planes that intersect in a line.



Solution

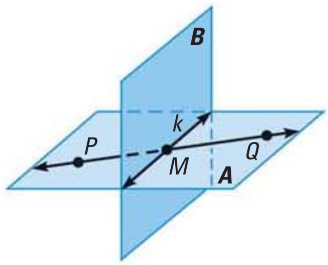
STEP 1 **Draw** a vertical plane. Shade the plane.

STEP 2 **Draw** a second plane that is horizontal. Shade this plane a different color. Use dashed lines to show where one plane is hidden.

STEP 3 **Draw** the line of intersection.

✓ GUIDED PRACTICE for Examples 3 and 4

4. Sketch two different lines that intersect a plane at the same point.



Use the diagram at the right.

5. Name the intersection of $\overleftrightarrow{PQ}$ and line k.

6. Name the intersection of plane A and plane B.

7. Name the intersection of line k and plane A.

1.1 EXERCISES

HOMEWORK KEY

WORKED-OUT SOLUTIONS on p. WS1 for Exs. 15, 19, and 43

★ **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 13, 16, and 43

SKILL PRACTICE

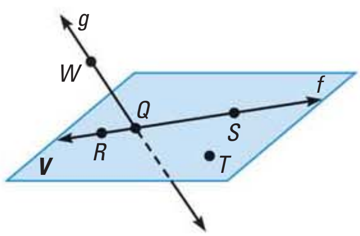
1. VOCABULARY Write in words what each of the following symbols means.

a. Q **b.** $\overline{MN}$ **c.** $\overrightarrow{ST}$ **d.** $\overleftrightarrow{FG}$

2. ★ WRITING *Compare* collinear points and coplanar points. Are collinear points also coplanar? Are coplanar points also collinear? *Explain.*

EXAMPLE 1 on p. 3 for Exs. 3–7

NAMING POINTS, LINES, AND PLANES **In Exercises 3–7, use the diagram.**



3. Give two other names for $\overleftrightarrow{WQ}$.

4. Give another name for plane V.

5. Name three points that are collinear. Then name a fourth point that is *not* collinear with these three points.

6. Name a point that is *not* coplanar with R, S, and T.

7. ★ WRITING Is point W coplanar with points Q and R? *Explain.*

EXAMPLE 2
on p. 4
for Exs. 8–13

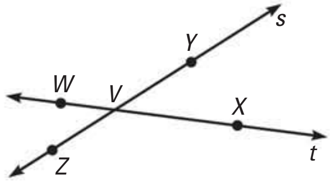
NAMING SEGMENTS AND RAYS **In Exercises 8–12, use the diagram.**

8. What is another name for $\overline{ZY}$?

9. Name all rays with endpoint V.

10. Name two pairs of opposite rays.

11. Give another name for $\overrightarrow{WV}$.



12. **ERROR ANALYSIS** A student says that $\overrightarrow{VW}$ and $\overrightarrow{VZ}$ are opposite rays because they have the same endpoint. *Describe* the error.

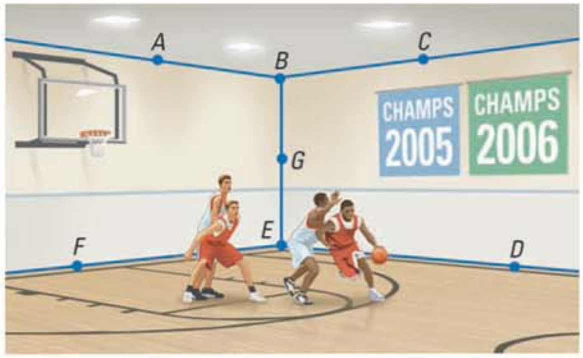
13. ★ **MULTIPLE CHOICE** Which statement about the diagram at the right is true?

Ⓐ A, B, and C are collinear.

Ⓑ C, D, E, and G are coplanar.

Ⓒ B lies on $\overrightarrow{GE}$.

Ⓓ $\overrightarrow{EF}$ and $\overrightarrow{ED}$ are opposite rays.



EXAMPLES 3 and 4
on pp. 4–5
for Exs. 14–23

SKETCHING INTERSECTIONS **Sketch the figure described.**

14. Three lines that lie in a plane and intersect at one point

15. One line that lies in a plane, and one line that does not lie in the plane

16. ★ **MULTIPLE CHOICE** Line AB and line CD intersect at point E. Which of the following are opposite rays?

Ⓐ $\overrightarrow{EC}$ and $\overrightarrow{ED}$ Ⓑ $\overrightarrow{CE}$ and $\overrightarrow{DE}$ Ⓒ $\overrightarrow{AB}$ and $\overrightarrow{BA}$ Ⓓ $\overrightarrow{AE}$ and $\overrightarrow{BE}$

READING DIAGRAMS **In Exercises 17–22, use the diagram at the right.**

17. Name the intersection of $\overleftrightarrow{PR}$ and $\overleftrightarrow{HR}$.

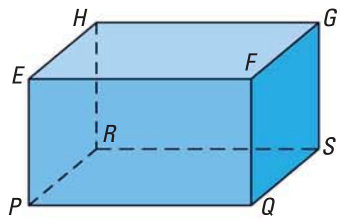
18. Name the intersection of plane EFG and plane FGS.

19. Name the intersection of plane PQS and plane HGS.

20. Are points P, Q, and F collinear? Are they coplanar?

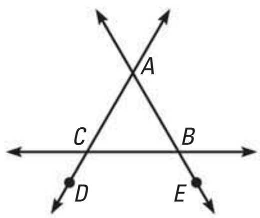
21. Are points P and G collinear? Are they coplanar?

22. Name three planes that intersect at point E.



23. **SKETCHING PLANES** Sketch plane J intersecting plane K. Then draw a line ℓ on plane J that intersects plane K at a single point.

24. **NAMING RAYS** Name 10 different rays in the diagram at the right. Then name 2 pairs of opposite rays.



25. **SKETCHING** Draw three noncollinear points J, K, and L. Sketch $\overline{JK}$ and add a point M on $\overline{JK}$. Then sketch $\overrightarrow{ML}$.

26. **SKETCHING** Draw two points P and Q. Then sketch $\overrightarrow{PQ}$. Add a point R on the ray so that Q is between P and R.

○ = **WORKED-OUT SOLUTIONS** ★ = **STANDARDIZED TEST PRACTICE**

REVIEW ALGEBRA
For help with equations of lines, see p. 878.

ALGEBRA In Exercises 27–32, you are given an equation of a line and a point. Use substitution to determine whether the point is on the line.

27. $y = x - 4$; $A(5, 1)$ **28.** $y = x + 1$; $A(1, 0)$ **29.** $y = 3x + 4$; $A(7, 1)$

30. $y = 4x + 2$; $A(1, 6)$ **31.** $y = 3x - 2$; $A(-1, -5)$ **32.** $y = -2x + 8$; $A(-4, 0)$

GRAPHING Graph the inequality on a number line. Tell whether the graph is a *segment*, a *ray* or *rays*, a *point*, or a *line*.

33. $x \leq 3$ **34.** $x \geq -4$ **35.** $-7 \leq x \leq 4$

36. $x \geq 5$ or $x \leq -2$ **37.** $x \geq -1$ or $x \leq 5$ **38.** $|x| \leq 0$

39. **CHALLENGE** Tell whether each of the following situations involving three planes is possible. If a situation is possible, make a sketch.

a. None of the three planes intersect.

b. The three planes intersect in one line.

c. The three planes intersect in one point.

d. Two planes do not intersect. The third plane intersects the other two.

e. Exactly two planes intersect. The third plane does not intersect the other two.

PROBLEM SOLVING

EXAMPLE 3
on p. 4
for Exs. 40–42

EVERYDAY INTERSECTIONS What kind of geometric intersection does the photograph suggest?

40.

41.

42.

43. ★ **SHORT RESPONSE** *Explain* why a four-legged table may rock from side to side even if the floor is level. Would a three-legged table on the same level floor rock from side to side? Why or why not?

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44. **SURVEYING** A surveying instrument is placed on a tripod. The tripod has three legs whose lengths can be adjusted.

a. When the tripod is sitting on a level surface, are the tips of the legs coplanar?

b. Suppose the tripod is used on a sloping surface. The length of each leg is adjusted so that the base of the surveying instrument is level with the horizon. Are the tips of the legs coplanar? *Explain*.

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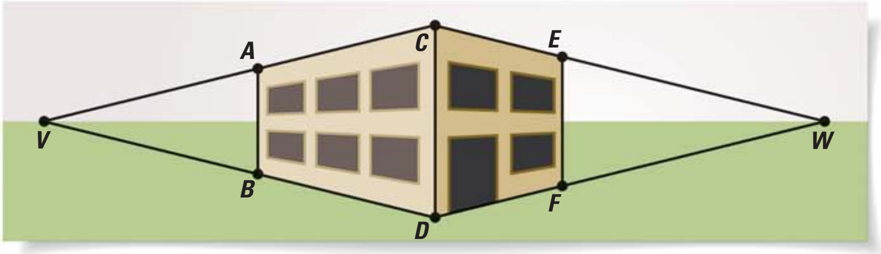
45. **MULTI-STEP PROBLEM** In a *perspective drawing*, lines that do not intersect in real life are represented by lines that appear to intersect at a point far away on the horizon. This point is called a *vanishing point*. The diagram shows a drawing of a house with two vanishing points.



a. Trace the black line segments in the drawing. Using lightly dashed lines, join points A and B to the vanishing point W. Join points E and F to the vanishing point V.

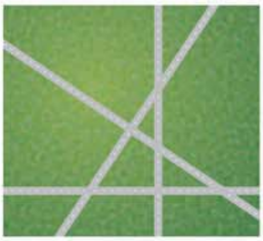
b. Label the intersection of $\overleftrightarrow{EV}$ and $\overleftrightarrow{AW}$ as G. Label the intersection of $\overleftrightarrow{FV}$ and $\overleftrightarrow{BW}$ as H.

c. Using heavy dashed lines, draw the hidden edges of the house: $\overline{AG}$, $\overline{EG}$, $\overline{BH}$, $\overline{FH}$, and $\overline{GH}$.

46. **CHALLENGE** Each street in a particular town intersects every existing street exactly one time. Only two streets pass through each intersection.

2 streets

3 streets



4 streets

a. A traffic light is needed at each intersection. How many traffic lights are needed if there are 5 streets in the town? 6 streets?

b. *Describe* a pattern you can use to find the number of additional traffic lights that are needed each time a street is added to the town.

MIXED REVIEW

PREVIEW
Prepare for Lesson 1.2 in Exs. 53–58.

Find the difference. *(p. 869)*

47. $-15 - 9$	**48.** $6 - 10$	**49.** $-25 - (-12)$
50. $13 - 20$	**51.** $16 - (-4)$	**52.** $-5 - 15$

Evaluate the expression. *(p. 870)*

53. $5 \cdot \lvert -2 + 1 \rvert$	**54.** $\lvert -8 + 7 \rvert - 6$	**55.** $-7 \cdot \lvert 8 - 10 \rvert$

Plot the point in a coordinate plane. *(p. 878)*

56. $A(2, 4)$	**57.** $B(-3, 6)$	**58.** $E(6, 7.5)$

1.2 Use Segments and Congruence

Before You learned about points, lines, and planes.

Now You will use segment postulates to identify congruent **segments**.

Why? So you can calculate flight distances, as in Ex. 33.

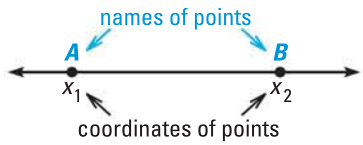
Key Vocabulary
- **postulate, axiom**
- **coordinate**
- **distance**
- **between**
- **congruent segments**

In Geometry, a rule that is accepted without proof is called a **postulate** or **axiom**. A rule that can be proved is called a *theorem*, as you will see later. Postulate 1 shows how to find the distance between two points on a line.

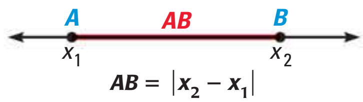
POSTULATE — *For Your Notebook*

POSTULATE 1 Ruler Postulate

The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the **coordinate** of the point.



The **distance** between points A and B, written as AB, is the absolute value of the difference of the coordinates of A and B.



In the diagrams above, the small numbers in the coordinates x_1 and x_2 are called *subscripts*. The coordinates are read as "x sub one" and "x sub two."

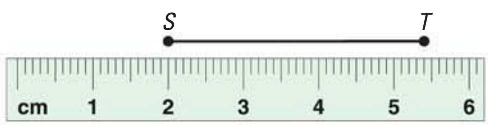
The distance between points A and B, or AB, is also called the *length* of $\overline{AB}$.

EXAMPLE 1 Apply the Ruler Postulate

Measure the length of $\overline{ST}$ to the nearest tenth of a centimeter.

Solution

Align one mark of a metric ruler with S. Then estimate the coordinate of T. For example, if you align S with 2, T appears to align with 5.4.

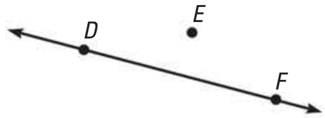


$ST = |5.4 - 2| = 3.4$ **Use Ruler Postulate.**

▶ The length of $\overline{ST}$ is about 3.4 centimeters.

ADDING SEGMENT LENGTHS When three points are collinear, you can say that one point is **between** the other two.

Point *B* is between points *A* and *C*.



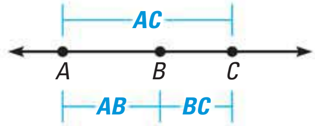
Point *E* is not between points *D* and *F*.

POSTULATE — *For Your Notebook*

POSTULATE 2 Segment Addition Postulate

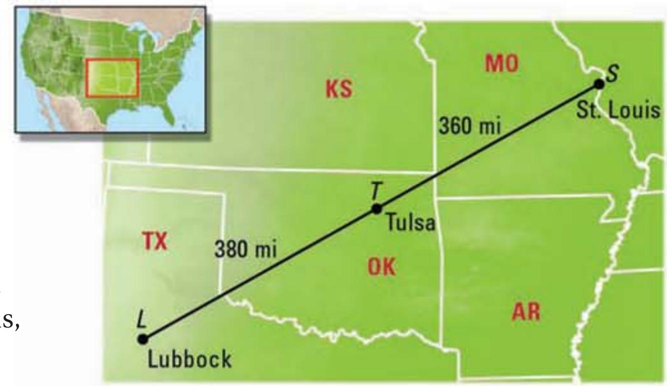
If B is between A and C, then $AB + BC = AC$.

If $AB + BC = AC$, then B is between A and C.



EXAMPLE 2 Apply the Segment Addition Postulate

MAPS The cities shown on the map lie approximately in a straight line. Use the given distances to find the distance from Lubbock, Texas, to St. Louis, Missouri.



Solution

Because Tulsa, Oklahoma, lies between Lubbock and St. Louis, you can apply the Segment Addition Postulate.

$$LS = LT + TS = 380 + 360 = 740$$

▶ The distance from Lubbock to St. Louis is about 740 miles.

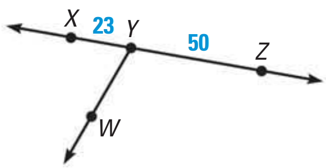
✓ GUIDED PRACTICE for Examples 1 and 2

Use a ruler to measure the length of the segment to the nearest $\frac{1}{8}$ inch.

1. 

2. 

In Exercises 3 and 4, use the diagram shown.

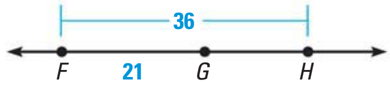


3. Use the Segment Addition Postulate to find XZ.

4. In the diagram, $WY = 30$. Can you use the Segment Addition Postulate to find the distance between points W and Z? *Explain* your reasoning.

EXAMPLE 3 Find a length

Use the diagram to find *GH*.



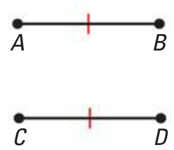
Solution

Use the Segment Addition Postulate to write an equation. Then solve the equation to find GH.

$FH = FG + GH$	**Segment Addition Postulate**
$36 = 21 + GH$	**Substitute 36 for *FH* and 21 for *FG*.**
$15 = GH$	**Subtract 21 from each side.**

CONGRUENT SEGMENTS Line segments that have the same length are called **congruent segments**. In the diagram below, you can say "the length of $\overline{AB}$ is equal to the length of $\overline{CD}$," or you can say "$\overline{AB}$ *is congruent to* $\overline{CD}$." The symbol $\cong$ means "is congruent to."

READ DIAGRAMS
In the diagram, the red tick marks indicate that $\overline{AB} \cong \overline{CD}$.



Lengths are equal.	Segments are congruent.
$AB = CD$	$\overline{AB} \cong \overline{CD}$
"is equal to"	"is congruent to"

EXAMPLE 4 Compare segments for congruence

Plot $J(-3, 4)$, $K(2, 4)$, $L(1, 3)$, and $M(1, -2)$ in a coordinate plane. Then determine whether $\overline{JK}$ and $\overline{LM}$ are congruent.

Solution

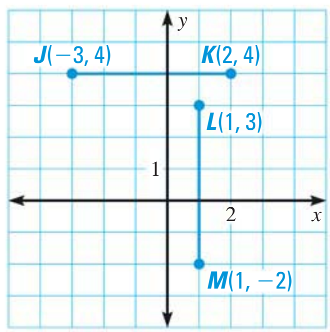
REVIEW USING A COORDINATE PLANE
For help with using a coordinate plane, see p. 878.

To find the length of a horizontal segment, find the absolute value of the difference of the x-coordinates of the endpoints.

$JK = |2 - (-3)| = 5$ **Use Ruler Postulate.**

To find the length of a vertical segment, find the absolute value of the difference of the y-coordinates of the endpoints.

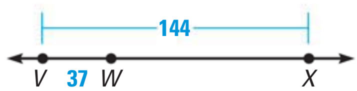
$LM = |-2 - 3| = 5$ **Use Ruler Postulate.**



▶ $\overline{JK}$ and $\overline{LM}$ have the same length. So, $\overline{JK} \cong \overline{LM}$.

✓ GUIDED PRACTICE for Examples 3 and 4

5. Use the diagram at the right to find WX.



6. Plot the points $A(-2, 4)$, $B(3, 4)$, $C(0, 2)$, and $D(0, -2)$ in a coordinate plane. Then determine whether $\overline{AB}$ and $\overline{CD}$ are congruent.

1.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 13, 17, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 20, 27, and 34

SKILL PRACTICE

In Exercises 1 and 2, use the diagram at the right.

1. **VOCABULARY** *Explain* what $\overline{MN}$ means and what MN means.

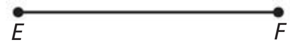
2. ★ **WRITING** *Explain* how you can find PN if you know PQ and QN. How can you find PN if you know MP and MN?

EXAMPLE 1 on p. 9 for Exs. 3–5

MEASUREMENT **Measure the length of the segment to the nearest tenth of a centimeter.**

3. 

4. 

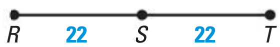
5. 

EXAMPLES 2 and 3 on pp. 10–11 for Exs. 6–12

SEGMENT ADDITION POSTULATE **Find the indicated length.**

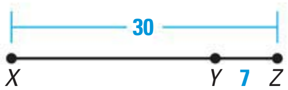
6. Find MP.

7. Find RT.

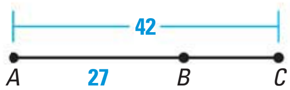


8. Find UW.

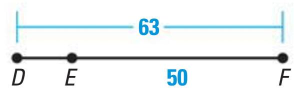
9. Find XY.



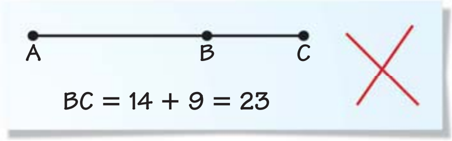
10. Find BC.



11. Find DE.



12. **ERROR ANALYSIS** In the figure at the right, $AC = 14$ and $AB = 9$. *Describe* and correct the error made in finding BC.



EXAMPLE 4 on p. 11 for Exs. 13–19

CONGRUENCE **In Exercises 13–15, plot the given points in a coordinate plane. Then determine whether the line segments named are congruent.**

13. $A(0, 1)$, $B(4, 1)$, $C(1, 2)$, $D(1, 6)$; $\overline{AB}$ and $\overline{CD}$

14. $J(-6, -8)$, $K(-6, 2)$, $L(-2, -4)$, $M(-6, -4)$; $\overline{JK}$ and $\overline{LM}$

15. $R(-200, 300)$, $S(200, 300)$, $T(300, -200)$, $U(300, 100)$; $\overline{RS}$ and $\overline{TU}$

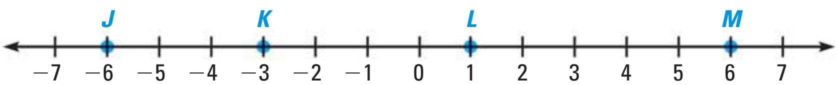
ALGEBRA **Use the number line to find the indicated distance.**

16. JK

17. JL

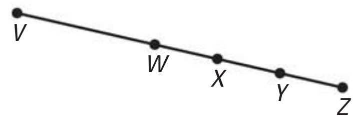
18. JM

19. KM



20. ★ **SHORT RESPONSE** Use the diagram. Is it possible to use the Segment Addition Postulate to show that $FB > CB$ or that $AC > DB$? *Explain.*

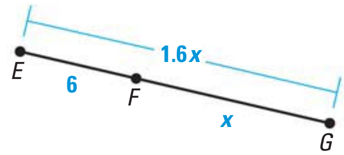
FINDING LENGTHS In the diagram, points V, W, X, Y, and Z are collinear, $VZ = 52$, $XZ = 20$, and $WX = XY = YZ$. Find the indicated length.



21. WX **22.** VW **23.** WY

24. VX **25.** WZ **26.** VY

27. ★ **MULTIPLE CHOICE** Use the diagram. What is the length of $\overline{EG}$?



Ⓐ 1 Ⓑ 4.4

Ⓒ 10 Ⓓ 16

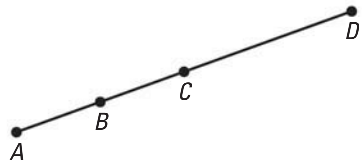
ALGEBRA Point S is between R and T on $\overline{RT}$. Use the given information to write an equation in terms of x. Solve the equation. Then find RS and ST.

28. $RS = 2x + 10$
$ST = x - 4$
$RT = 21$

29. $RS = 3x - 16$
$ST = 4x - 8$
$RT = 60$

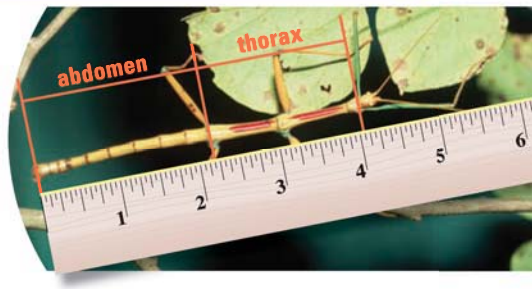
30. $RS = 2x - 8$
$ST = 3x - 10$
$RT = 17$

31. **CHALLENGE** In the diagram, $\overline{AB} \cong \overline{BC}$, $\overline{AC} \cong \overline{CD}$, and $AD = 12$. Find the lengths of all the segments in the diagram. Suppose you choose one of the segments at random. What is the probability that the measure of the segment is greater than 3? *Explain*.



PROBLEM SOLVING

32. **SCIENCE** The photograph shows an insect called a walkingstick. Use the ruler to estimate the length of the abdomen and the length of the thorax to the nearest $\frac{1}{4}$ inch. About how much longer is the walkingstick's abdomen than its thorax?



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EXAMPLE 2 on p. 10 for Ex. 33

33. **MODEL AIRPLANE** In 2003, a remote-controlled model airplane became the first ever to fly nonstop across the Atlantic Ocean. The map shows the airplane's position at three different points during its flight.

a. Find the total distance the model airplane flew.

b. The model airplane's flight lasted nearly 38 hours. Estimate the airplane's average speed in miles per hour.

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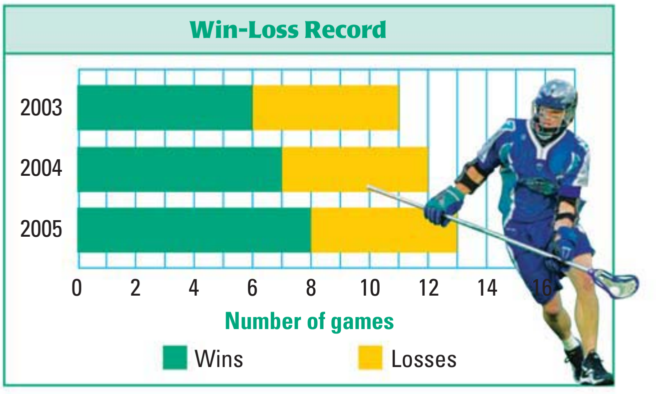
34. ★ **SHORT RESPONSE** The bar graph shows the win-loss record for a lacrosse team over a period of three years.



a. Use the scale to find the length of the yellow bar for each year. What does the length represent?

b. For each year, find the percent of games lost by the team.

c. *Explain* how you are applying the Segment Addition Postulate when you find information from a stacked bar graph like the one shown.

35. **MULTI-STEP PROBLEM** A climber uses a rope to descend a vertical cliff. Let A represent the point where the rope is secured at the top of the cliff, let B represent the climber's position, and let C represent the point where the rope is secured at the bottom of the cliff.

a. **Model** Draw and label a line segment that represents the situation.

b. **Calculate** If AC is 52 feet and AB is 31 feet, how much farther must the climber descend to reach the bottom of the cliff?

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36. **CHALLENGE** Four cities lie along a straight highway in this order: City A, City B, City C, and City D. The distance from City A to City B is 5 times the distance from City B to City C. The distance from City A to City D is 2 times the distance from City A to City B. Copy and complete the mileage chart.

	City A	City B	City C	City D
City A		?	?	?
City B	?		?	?
City C	?	?		10 mi
City D	?	?	?	

MIXED REVIEW

PREVIEW
Prepare for Lesson 1.3 in Exs. 37–42.

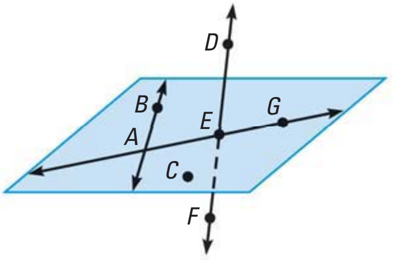
Simplify the expression. Write your answer in simplest radical form. *(p. 874)*

37. $\sqrt{45 + 99}$ 38. $\sqrt{14 + 36}$ 39. $\sqrt{42 + (-2)^2}$

Solve the equation. *(p. 875)*

40. $4m + 5 = 7 + 6m$ 41. $13 - 4h = 3h - 8$ 42. $17 + 3x = 18x - 28$

Use the diagram to decide whether the statement is *true* or *false*. *(p. 2)*



43. Points A, C, E, and G are coplanar.

44. $\overleftrightarrow{DF}$ and $\overleftrightarrow{AG}$ intersect at point E.

45. $\overrightarrow{AE}$ and $\overrightarrow{EG}$ are opposite rays.

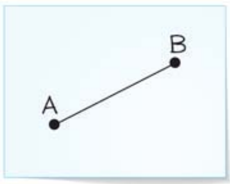
1.3 Use Midpoint and Distance Formulas

Before	You found lengths of segments.
Now	You will find lengths of segments in the coordinate plane.
Why?	So you can find an unknown length, as in Example 1.

Key Vocabulary
- **midpoint**
- **segment bisector**

ACTIVITY FOLD A SEGMENT BISECTOR

STEP 1

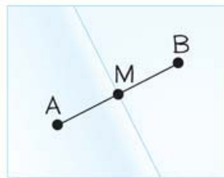


Draw $\overline{AB}$ on a piece of paper.

STEP 2

Fold the paper so that B is on top of A.

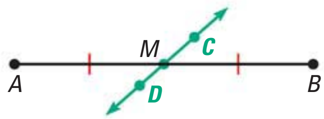
STEP 3



Label point M. Compare AM, MB, and AB.

MIDPOINTS AND BISECTORS The **midpoint** of a segment is the point that divides the segment into two congruent segments. A **segment bisector** is a point, ray, line, line segment, or plane that intersects the segment at its midpoint. A midpoint or a segment bisector *bisects* a segment.

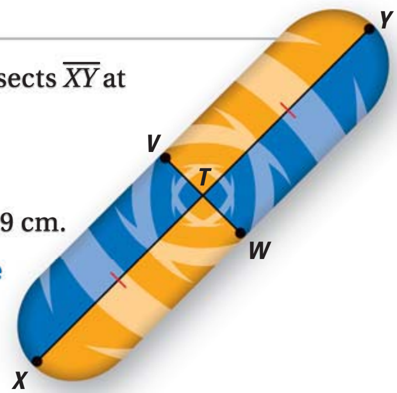
M is the midpoint of $\overline{AB}$.
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.



$\overleftrightarrow{CD}$ is a segment bisector of $\overline{AB}$.
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

EXAMPLE 1 Find segment lengths

SKATEBOARD In the skateboard design, $\overline{VW}$ bisects $\overline{XY}$ at point T, and $XT = 39.9$ cm. Find XY.



Solution

Point T is the midpoint of $\overline{XY}$. So, $XT = TY = 39.9$ cm.

$XY = XT + TY$	**Segment Addition Postulate**
$= 39.9 + 39.9$	**Substitute.**
$= 79.8$ cm	**Add.**

EXAMPLE 2 Use algebra with segment lengths

ALGEBRA Point M is the midpoint of $\overline{VW}$. Find the length of $\overline{VM}$.

Solution

STEP 1 **Write** and solve an equation. Use the fact that that $VM = MW$.

$VM = MW$	Write equation.
$4x - 1 = 3x + 3$	Substitute.
$x - 1 = 3$	Subtract $3x$ from each side.
$x = 4$	Add 1 to each side.

STEP 2 **Evaluate** the expression for VM when $x = 4$.

$VM = 4x - 1 = 4(4) - 1 = 15$

▶ So, the length of $\overline{VM}$ is 15.

CHECK Because $VM = MW$, the length of $\overline{MW}$ should be 15. If you evaluate the expression for MW, you should find that $MW = 15$.

$MW = 3x + 3 = 3(4) + 3 = 15$ ✓

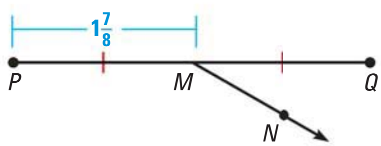
REVIEW ALGEBRA
For help with solving equations, see p. 875.

✓ GUIDED PRACTICE for Examples 1 and 2

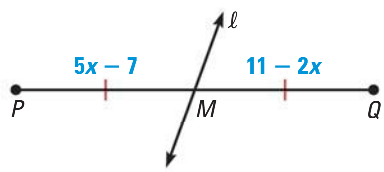
READ DIRECTIONS
Always read direction lines carefully. Notice that this direction line has two parts.

In Exercises 1 and 2, identify the segment bisector of $\overline{PQ}$. Then find PQ.

1.



2.



COORDINATE PLANE You can use the coordinates of the endpoints of a segment to find the coordinates of the midpoint.

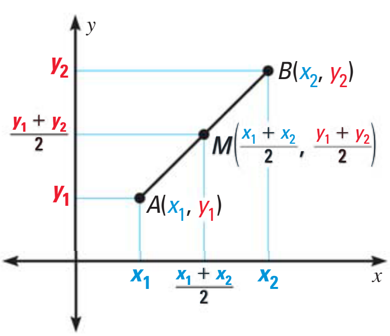
KEY CONCEPT *For Your Notebook*

The Midpoint Formula

The coordinates of the midpoint of a segment are the averages of the x-coordinates and of the y-coordinates of the endpoints.

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint M of $\overline{AB}$ has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

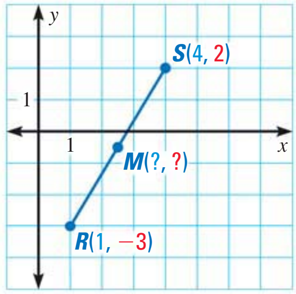


EXAMPLE 3 Use the Midpoint Formula

a. FIND MIDPOINT The endpoints of $\overline{RS}$ are $R(1, -3)$ and $S(4, 2)$. Find the coordinates of the midpoint M.

b. FIND ENDPOINT The midpoint of $\overline{JK}$ is $M(2, 1)$. One endpoint is $J(1, 4)$. Find the coordinates of endpoint K.

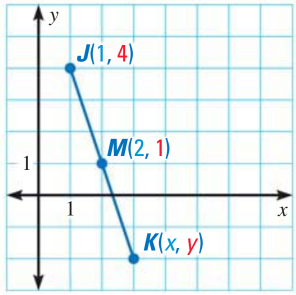
Solution



a. FIND MIDPOINT Use the Midpoint Formula.

$$M\left(\frac{1+4}{2}, \frac{-3+2}{2}\right) = M\left(\frac{5}{2}, -\frac{1}{2}\right)$$

▶ The coordinates of the midpoint M are $\left(\frac{5}{2}, -\frac{1}{2}\right)$.



b. FIND ENDPOINT Let (x, y) be the coordinates of endpoint K. Use the Midpoint Formula.

CLEAR FRACTIONS
Multiply each side of the equation by the denominator to clear the fraction.

STEP 1 Find x.	**STEP 2** Find y.
$\frac{1+x}{2} = 2$	$\frac{4+y}{2} = 1$
$1 + x = 4$	$4 + y = 2$
$x = 3$	$y = -2$

▶ The coordinates of endpoint K are $(3, -2)$.

✓ GUIDED PRACTICE for Example 3

3. The endpoints of $\overline{AB}$ are $A(1, 2)$ and $B(7, 8)$. Find the coordinates of the midpoint M.

4. The midpoint of $\overline{VW}$ is $M(-1, -2)$. One endpoint is $W(4, 4)$. Find the coordinates of endpoint V.

DISTANCE FORMULA The Distance Formula is a formula for computing the distance between two points in a coordinate plane.

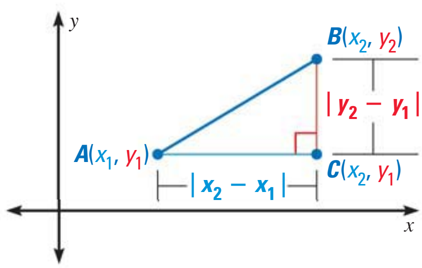
READ DIAGRAMS
The red mark at one corner of the triangle shown indicates a right triangle.

KEY CONCEPT *For Your Notebook*

The Distance Formula

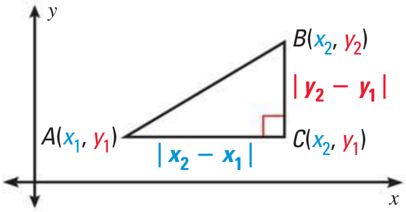
If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between A and B is

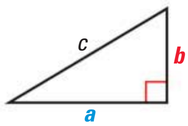
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



The Distance Formula is based on the *Pythagorean Theorem,* which you will see again when you work with right triangles in Chapter 7.

Distance Formula	**Pythagorean Theorem**
$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$	$c^2 = a^2 + b^2$





EXAMPLE 4 Standardized Test Practice

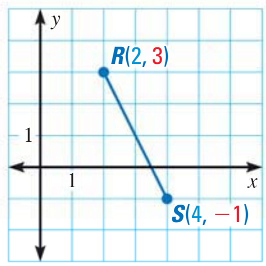
What is the approximate length of $\overline{RS}$ with endpoints $R(2, 3)$ and $S(4, -1)$?

Ⓐ 1.4 units Ⓑ 4.0 units Ⓒ 4.5 units Ⓓ 6 units

ELIMINATE CHOICES
Drawing a diagram can help you eliminate choices. You can see that choice A is not large enough to be *RS*.

Solution

Use the Distance Formula. You may find it helpful to draw a diagram.



$RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **Distance Formula**

$= \sqrt{[(4 - 2)]^2 + [(-1) - 3]^2}$ **Substitute.**

$= \sqrt{(2)^2 + (-4)^2}$ **Subtract.**

$= \sqrt{4 + 16}$ **Evaluate powers.**

$= \sqrt{20}$ **Add.**

≈ 4.47 **Use a calculator to approximate the square root.**

READ SYMBOLS
The symbol ≈ means "is approximately equal to."

▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

GUIDED PRACTICE for Example 4

5. In Example 4, does it matter which ordered pair you choose to substitute for (x_1, y_1) and which ordered pair you choose to substitute for (x_2, y_2)? *Explain.*

6. What is the approximate length of $\overline{AB}$, with endpoints $A(-3, 2)$ and $B(1, -4)$?

Ⓐ 6.1 units Ⓑ 7.2 units Ⓒ 8.5 units Ⓓ 10.0 units

1.3 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 15, 35, and 49

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 23, 34, 41, 42, and 53

SKILL PRACTICE

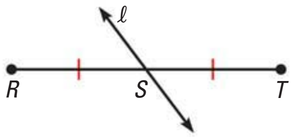
1. **VOCABULARY** Copy and complete: To find the length of $\overline{AB}$, with endpoints $A(-7, 5)$ and $B(4, -6)$, you can use the __?__.

2. ★ **WRITING** *Explain* what it means to bisect a segment. Why is it impossible to bisect a line?

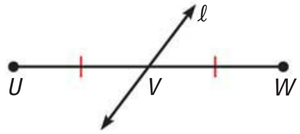
EXAMPLE 1 on p. 15 for Exs. 3–10

FINDING LENGTHS Line ℓ bisects the segment. Find the indicated length.

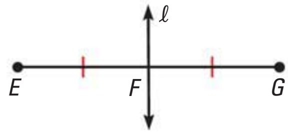
3. Find RT if $RS = 5\frac{1}{8}$ in.



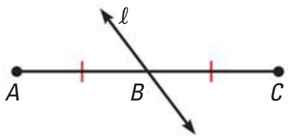
4. Find UW if $VW = \frac{5}{8}$ in.



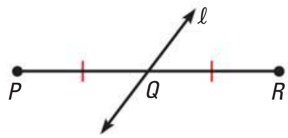
5. Find EG if $EF = 13$ cm.



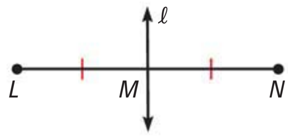
6. Find BC if $AC = 19$ cm.



7. Find QR if $PR = 9\frac{1}{2}$ in.



8. Find LM if $LN = 137$ mm.



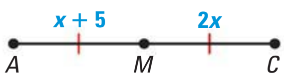
9. **SEGMENT BISECTOR** Line RS bisects $\overline{PQ}$ at point R. Find RQ if $PQ = 4\frac{3}{4}$ inches.

10. **SEGMENT BISECTOR** Point T bisects $\overline{UV}$. Find UV if $UT = 2\frac{7}{8}$ inches.

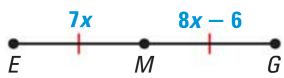
EXAMPLE 2 on p. 16 for Exs. 11–16

xy **ALGEBRA In each diagram, M is the midpoint of the segment. Find the indicated length.**

11. Find AM.

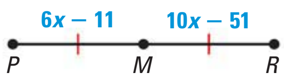


12. Find EM.

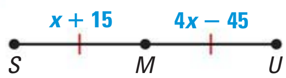


13. Find JM.

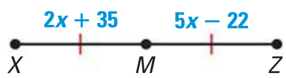
14. Find PR.



15. Find SU.



16. Find XZ.



EXAMPLE 3 on p. 17 for Exs. 17–30

FINDING MIDPOINTS Find the coordinates of the midpoint of the segment with the given endpoints.

17. $C(3, 5)$ and $D(7, 5)$

18. $E(0, 4)$ and $F(4, 3)$

19. $G(-4, 4)$ and $H(6, 4)$

20. $J(-7, -5)$ and $K(-3, 7)$

21. $P(-8, -7)$ and $Q(11, 5)$

22. $S(-3, 3)$ and $T(-8, 6)$

23. ★ **WRITING** Develop a formula for finding the midpoint of a segment with endpoints $A(0, 0)$ and $B(m, n)$. *Explain* your thinking.

24. **ERROR ANALYSIS** *Describe* the error made in finding the coordinates of the midpoint of a segment with endpoints $S(8, 3)$ and $T(2, -1)$.

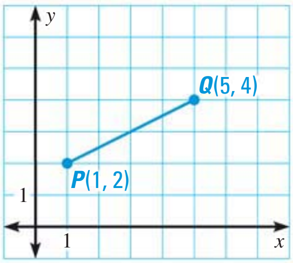
$$\left(\frac{8-2}{2}, \frac{3-(-1)}{2}\right) = (3, 2)$$

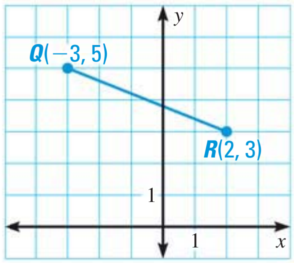
FINDING ENDPOINTS **Use the given endpoint R and midpoint M of $\overline{RS}$ to find the coordinates of the other endpoint S.**

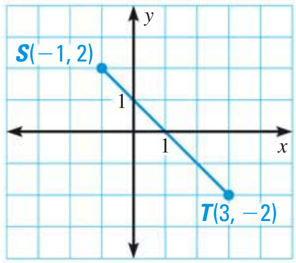
25. $R(3, 0)$, $M(0, 5)$
26. $R(5, 1)$, $M(1, 4)$
27. $R(6, -2)$, $M(5, 3)$
28. $R(-7, 11)$, $M(2, 1)$
29. $R(4, -6)$, $M(-7, 8)$
30. $R(-4, -6)$, $M(3, -4)$

EXAMPLE 4
on p. 18
for Exs. 31–34

DISTANCE FORMULA **Find the length of the segment. Round to the nearest tenth of a unit.**

31. 

32. 

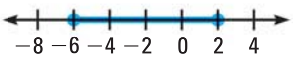
33. 

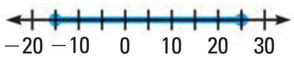
34. ★ **MULTIPLE CHOICE** The endpoints of $\overline{MN}$ are $M(-3, -9)$ and $N(4, 8)$. What is the approximate length of $\overline{MN}$?

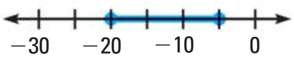
Ⓐ 1.4 units Ⓑ 7.2 units Ⓒ 13 units Ⓓ 18.4 units

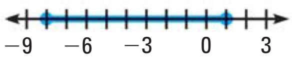
NUMBER LINE **Find the length of the segment. Then find the coordinate of the midpoint of the segment.**

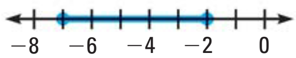
35. 

36. 

37. 

38. 

39. 

40. 

41. ★ **MULTIPLE CHOICE** The endpoints of $\overline{LF}$ are $L(-2, 2)$ and $F(3, 1)$. The endpoints of $\overline{JR}$ are $J(1, -1)$ and $R(2, -3)$. What is the approximate difference in the lengths of the two segments?

Ⓐ 2.24 Ⓑ 2.86 Ⓒ 5.10 Ⓓ 7.96

42. ★ **SHORT RESPONSE** One endpoint of $\overline{PQ}$ is $P(-2, 4)$. The midpoint of $\overline{PQ}$ is $M(1, 0)$. *Explain* how to find PQ.

COMPARING LENGTHS **The endpoints of two segments are given. Find each segment length. Tell whether the segments are congruent.**

43. $\overline{AB}$: $A(0, 2)$, $B(-3, 8)$
$\overline{CD}$: $C(-2, 2)$, $D(0, -4)$

44. $\overline{EF}$: $E(1, 4)$, $F(5, 1)$
$\overline{GH}$: $G(-3, 1)$, $H(1, 6)$

45. $\overline{JK}$: $J(-4, 0)$, $K(4, 8)$
$\overline{LM}$: $L(-4, 2)$, $M(3, -7)$

46. *xy* **ALGEBRA** Points S, T, and P lie on a number line. Their coordinates are 0, 1, and x, respectively. Given $SP = PT$, what is the value of x?

47. **CHALLENGE** M is the midpoint of $\overline{JK}$, $JM = \frac{x}{8}$, and $JK = \frac{3x}{4} - 6$. Find MK.

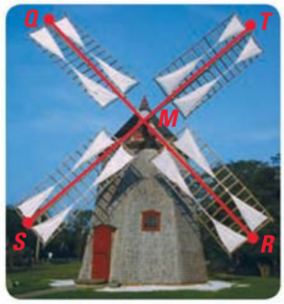
○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

Problem Solving

EXAMPLE 1 on p. 15 for Ex. 48

48. WINDMILL In the photograph of a windmill, $\overline{ST}$ bisects $\overline{QR}$ at point M. The length of $\overline{QM}$ is $18\frac{1}{2}$ feet. Find QR and MR.



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49. DISTANCES A house and a school are 5.7 kilometers apart on the same straight road. The library is on the same road, halfway between the house and the school. Draw a sketch to represent this situation. Mark the locations of the house, school, and library. How far is the library from the house?

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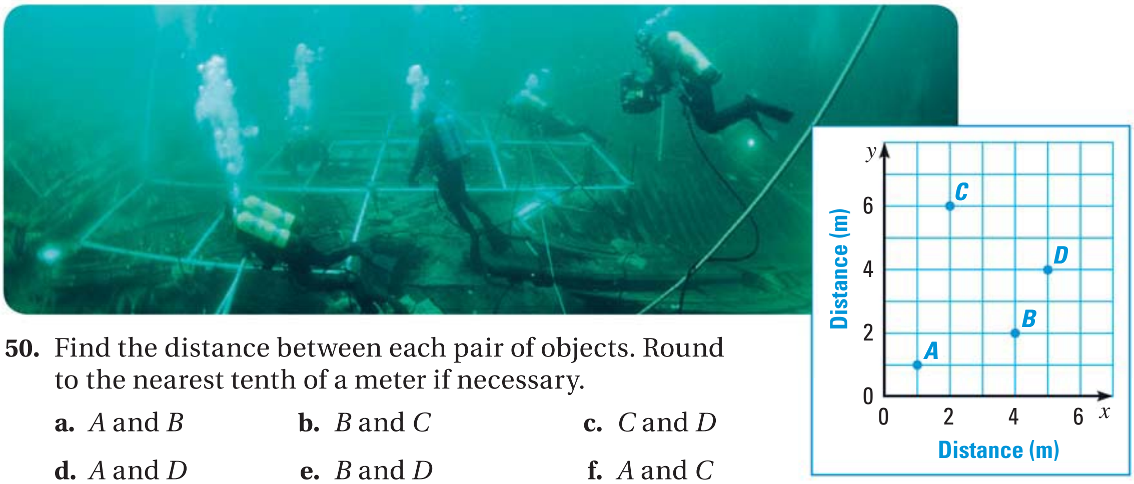
ARCHAEOLOGY The points on the diagram show the positions of objects at an underwater archaeological site. Use the diagram for Exercises 50 and 51.



50. Find the distance between each pair of objects. Round to the nearest tenth of a meter if necessary.

a. A and B **b.** B and C **c.** C and D

d. A and D **e.** B and D **f.** A and C

51. Which two objects are closest to each other? Which two are farthest apart?

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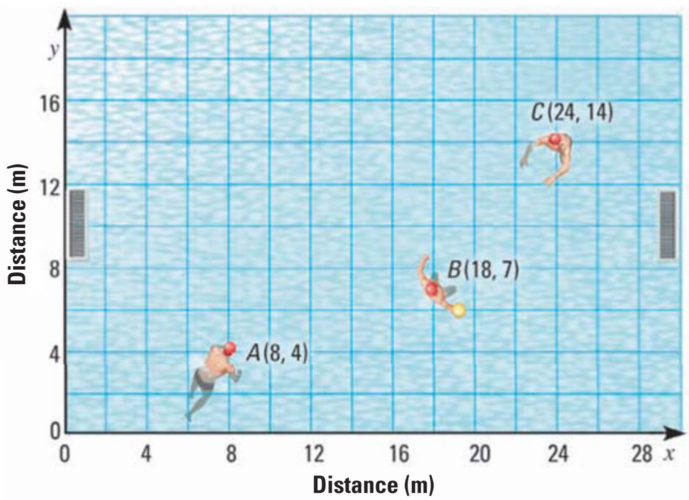
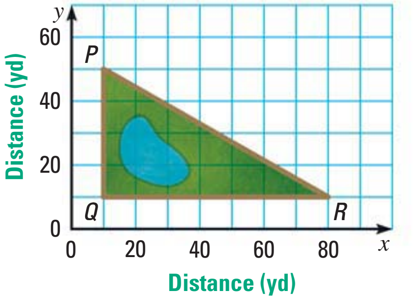
52. WATER POLO The diagram shows the positions of three players during part of a water polo match. Player A throws the ball to Player B, who then throws it to Player C. How far did Player A throw the ball? How far did Player B throw the ball? How far would Player A have thrown the ball if he had thrown it directly to Player C? Round all answers to the nearest tenth of a meter.



53. ★ **EXTENDED RESPONSE** As shown, a path goes around a triangular park.



a. Find the distance around the park to the nearest yard.

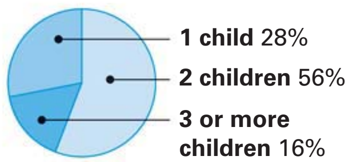
b. A new path and a bridge are constructed from point Q to the midpoint M of $\overline{PR}$. Find QM to the nearest yard.

c. A man jogs from P to Q to M to R to Q and back to P at an average speed of 150 yards per minute. About how many minutes does it take? *Explain*.

54. **CHALLENGE** $\overline{AB}$ bisects $\overline{CD}$ at point M, $\overline{CD}$ bisects $\overline{AB}$ at point M, and $AB = 4 \cdot CM$. *Describe* the relationship between AM and CD.

MIXED REVIEW

The graph shows data about the number of children in the families of students in a math class. *(p. 888)*



55. What percent of the students in the class belong to families with two or more children?

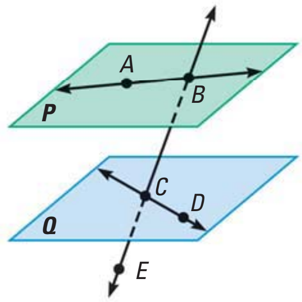
56. If there are 25 students in the class, how many students belong to families with two children?

PREVIEW
Prepare for Lesson 1.4 in Exs. 57–59.

Solve the equation. *(p. 875)*

57. $3x + 12 + x = 20$ **58.** $9x + 2x + 6 - x = 10$ **59.** $5x - 22 - 7x + 2 = 40$

In Exercises 60–64, use the diagram at the right. *(p. 2)*



60. Name all rays with endpoint B.

61. Name all the rays that contain point C.

62. Name a pair of opposite rays.

63. Name the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$.

64. Name the intersection of $\overleftrightarrow{BC}$ and plane P.

QUIZ *for Lessons 1.1–1.3*

1. Sketch two lines that intersect the same plane at two different points. The lines intersect each other at a point not in the plane. *(p. 2)*

In the diagram of collinear points, $AE = 26$, $AD = 15$, and $AB = BC = CD$. Find the indicated length. *(p. 9)*

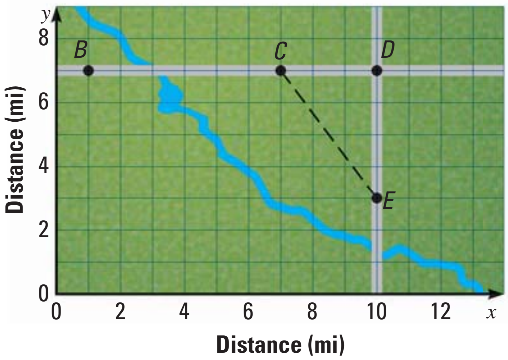
2. DE **3.** AB **4.** AC

5. BD **6.** CE **7.** BE

8. The endpoints of $\overline{RS}$ are $R(-2, -1)$ and $S(2, 3)$. Find the coordinates of the midpoint of $\overline{RS}$. Then find the distance between R and S. *(p. 15)*

Lessons 1.1–1.3

1. **MULTI-STEP PROBLEM** The diagram shows existing roads ($\overleftrightarrow{BD}$ and $\overleftrightarrow{DE}$) and a new road ($\overline{CE}$) under construction.



   a. If you drive from point B to point E on existing roads, how far do you travel?

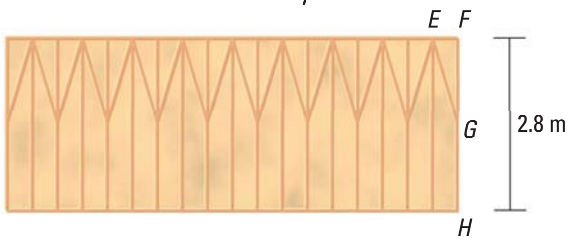
   b. If you use the new road as you drive from B to E, about how far do you travel? Round to the nearest tenth of a mile if necessary.

   c. About how much shorter is the trip from B to E if you use the new road?

2. **GRIDDED ANSWER** Point M is the midpoint of $\overline{PQ}$. If $PM = 23x + 5$ and $MQ = 25x - 4$, find the length of $\overline{PQ}$.

3. **GRIDDED ANSWER** You are hiking on a trail that lies along a straight railroad track. The total length of the trail is 5.4 kilometers. You have been hiking for 45 minutes at an average speed of 2.4 kilometers per hour. How much farther (in kilometers) do you need to hike to reach the end of the trail?

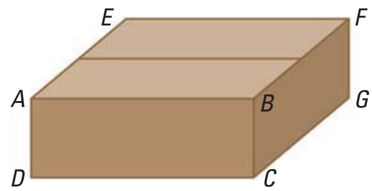
4. **SHORT RESPONSE** The diagram below shows the frame for a wall. $\overline{FH}$ represents a vertical board, and $\overline{EG}$ represents a brace. If $FG = 143$ cm, does the brace bisect $\overline{FH}$? If not, how long should $\overline{FG}$ be so that the brace does bisect $\overline{FH}$? *Explain.*



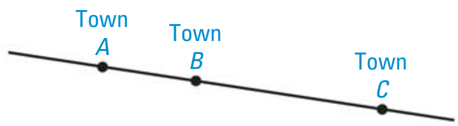
5. **SHORT RESPONSE** Point E is the midpoint of $\overline{AB}$ and the midpoint of $\overline{CD}$. The endpoints of $\overline{AB}$ are $A(-4, 5)$ and $B(6, -5)$. The coordinates of point C are $(2, 8)$. Find the coordinates of point D. *Explain* how you got your answer.

6. **OPEN-ENDED** The distance around a figure is its *perimeter*. Choose four points in a coordinate plane that can be connected to form a rectangle with a perimeter of 16 units. Then choose four other points and draw a different rectangle that has a perimeter of 16 units. Show how you determined that each rectangle has a perimeter of 16 units.

7. **SHORT RESPONSE** Use the diagram of a box. What are all the names that can be used to describe the plane that contains points B, F, and C? Name the intersection of planes ABC and BFE. *Explain.*



8. **EXTENDED RESPONSE** Jill is a salesperson who needs to visit towns A, B, and C. On the map below, $AB = 18.7$ km and $BC = 2AB$. Assume Jill travels along the road shown.



   a. Find the distance Jill travels if she starts at Town A, visits Towns B and C, and then returns to Town A.

   b. About how much time does Jill spend driving if her average driving speed is 70 kilometers per hour?

   c. Jill needs to spend 2.5 hours in each town. Can she visit all three towns and return to Town A in an 8 hour workday? *Explain.*

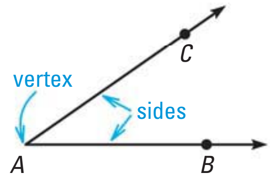
1.4 Measure and Classify Angles

Before	You named and measured line segments.
Now	You will name, measure, and classify angles.
Why?	So you can identify congruent angles, as in Example 4.

Key Vocabulary
- **angle**
  acute, right, obtuse, straight
- **sides, vertex of an angle**
- **measure of an angle**
- **congruent angles**
- **angle bisector**

An **angle** consists of two different rays with the same endpoint. The rays are the **sides** of the angle. The endpoint is the **vertex** of the angle.

The angle with sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ can be named $\angle BAC$, $\angle CAB$, or $\angle A$. Point A is the vertex of the angle.



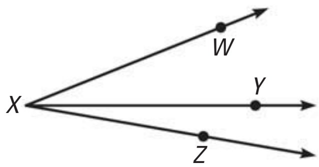
EXAMPLE 1 Name angles

Name the three angles in the diagram.

$\angle WXY$, or $\angle YXW$

$\angle YXZ$, or $\angle ZXY$

$\angle WXZ$, or $\angle ZXW$



You should not name any of these angles $\angle X$ because all three angles have X as their vertex.

MEASURING ANGLES A protractor can be used to approximate the *measure* of an angle. An angle is measured in units called *degrees* (°). For instance, the measure of $\angle WXZ$ in Example 1 above is 32°. You can write this statement in two ways.

Words The measure of $\angle WXZ$ is 32°.

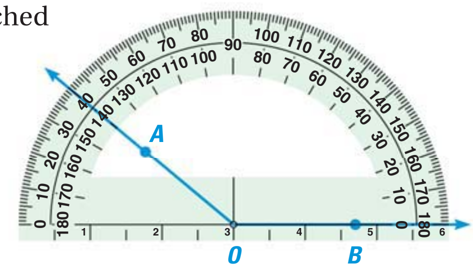
Symbols $m\angle WXZ = 32°$

POSTULATE *For Your Notebook*

POSTULATE 3 Protractor Postulate

Consider $\overrightarrow{OB}$ and a point A on one side of $\overrightarrow{OB}$. The rays of the form $\overrightarrow{OA}$ can be matched one to one with the real numbers from 0 to 180.

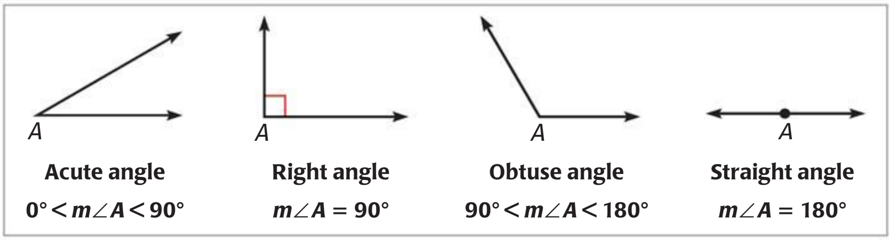
The **measure** of $\angle AOB$ is equal to the absolute value of the difference between the real numbers for $\overrightarrow{OA}$ and $\overrightarrow{OB}$.



CLASSIFYING ANGLES Angles can be classified as **acute**, **right**, **obtuse**, and **straight**, as shown below.

READ DIAGRAMS

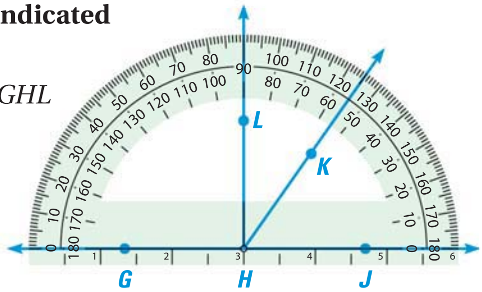
A red square inside an angle indicates that the angle is a right angle.



EXAMPLE 2 Measure and classify angles

Use the diagram to find the measure of the indicated angle. Then classify the angle.

a. $\angle KHJ$ **b.** $\angle GHK$ **c.** $\angle GHJ$ **d.** $\angle GHL$



Solution

A protractor has an inner and an outer scale. When you measure an angle, check to see which scale to use.

a. $\overrightarrow{HJ}$ is lined up with the 0° on the inner scale of the protractor. $\overrightarrow{HK}$ passes through 55° on the inner scale. So, $m\angle KHJ = 55°$. It is an acute angle.

b. $\overrightarrow{HG}$ is lined up with the 0° on the outer scale, and $\overrightarrow{HK}$ passes through 125° on the outer scale. So, $m\angle GHK = 125°$. It is an obtuse angle.

c. $m\angle GHJ = 180°$. It is a straight angle.

d. $m\angle GHL = 90°$. It is a right angle.

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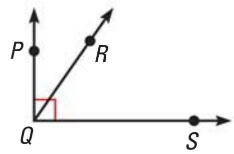
✓ GUIDED PRACTICE for Examples 1 and 2

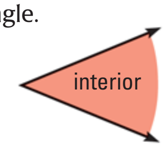


1. Name all the angles in the diagram at the right. Which angle is a right angle?

2. Draw a pair of opposite rays. What type of angle do the rays form?

READ DIAGRAMS

A point is in the *interior* of an angle if it is between points that lie on each side of the angle.

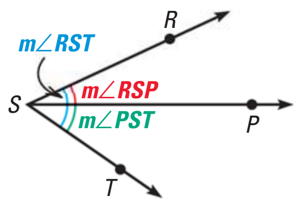


POSTULATE *For Your Notebook*

POSTULATE 4 Angle Addition Postulate

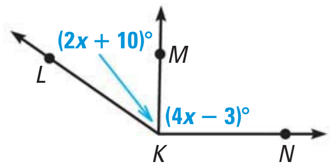
Words If P is in the interior of $\angle RST$, then the measure of $\angle RST$ is equal to the sum of the measures of $\angle RSP$ and $\angle PST$.

Symbols If P is in the interior of $\angle RST$, then $m\angle RST = m\angle RSP + m\angle PST$.



EXAMPLE 3 Find angle measures

xy ALGEBRA **Given that $m\angle LKN = 145°$, find $m\angle LKM$ and $m\angle MKN$.**



Solution

STEP 1 **Write** and solve an equation to find the value of x.

$m\angle LKN = m\angle LKM + m\angle MKN$	**Angle Addition Postulate**
$145° = (2x + 10)° + (4x - 3)°$	**Substitute angle measures.**
$145 = 6x + 7$	**Combine like terms.**
$138 = 6x$	**Subtract 7 from each side.**
$23 = x$	**Divide each side by 6.**

STEP 2 **Evaluate** the given expressions when $x = 23$.

$$m\angle LKM = (2x + 10)° = (2 \cdot 23 + 10)° = 56°$$

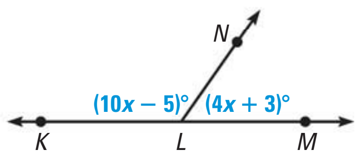
$$m\angle MKN = (4x - 3)° = (4 \cdot 23 - 3)° = 89°$$

▶ So, $m\angle LKM = 56°$ and $m\angle MKN = 89°$.

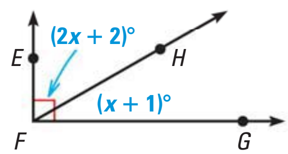
✓ GUIDED PRACTICE for Example 3

Find the indicated angle measures.

3. Given that $\angle KLM$ is a straight angle, find $m\angle KLN$ and $m\angle NLM$.

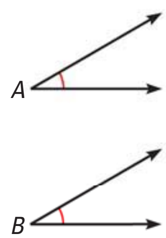


4. Given that $\angle EFG$ is a right angle, find $m\angle EFH$ and $m\angle HFG$.



CONGRUENT ANGLES Two angles are **congruent angles** if they have the same measure. In the diagram below, you can say that "the measure of angle A is equal to the measure of angle B," or you can say "angle A *is congruent to* angle B."

READ DIAGRAMS
Matching arcs are used to show that angles are congruent. If more than one pair of angles are congruent, double arcs are used, as in Example 4 on page 27.



Angle measures are equal.

$m\angle A = m\angle B$

↑

"is equal to"

Angles are congruent.

$\angle A \cong \angle B$

↑

"is congruent to"

EXAMPLE 4 Identify congruent angles

TRAPEZE The photograph shows some of the angles formed by the ropes in a trapeze apparatus. Identify the congruent angles. If $m\angle DEG = 157°$, what is $m\angle GKL$?

Solution

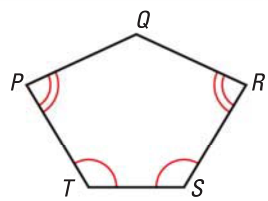
There are two pairs of congruent angles:

$\angle DEF \cong \angle JKL$ and $\angle DEG \cong \angle GKL$.

Because $\angle DEG \cong \angle GKL$, $m\angle DEG = m\angle GKL$. So, $m\angle GKL = 157°$.

✓ GUIDED PRACTICE for Example 4

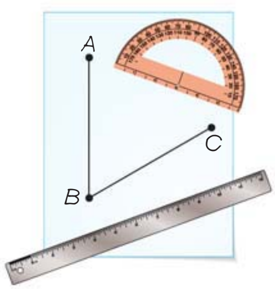
Use the diagram shown at the right.



5. Identify all pairs of congruent angles in the diagram.
6. In the diagram, $m\angle PQR = 130°$, $m\angle QRS = 84°$, and $m\angle TSR = 121°$. Find the other angle measures in the diagram.

ACTIVITY FOLD AN ANGLE BISECTOR

STEP 1

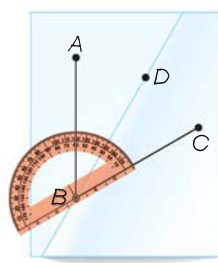


Use a straightedge to draw and label an acute angle, $\angle ABC$.

STEP 2

Fold the paper so that $\overrightarrow{BC}$ is on top of $\overrightarrow{BA}$.

STEP 3

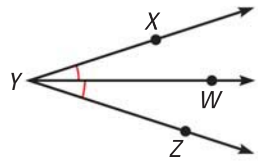


Draw a point D on the fold inside $\angle ABC$. Then measure $\angle ABD$, $\angle DBC$, and $\angle ABC$. What do you observe?

An **angle bisector** is a ray that divides an angle into two angles that are congruent. In the activity on page 27, $\overrightarrow{BD}$ bisects $\angle ABC$. So, $\angle ABD \cong \angle DBC$ and $m\angle ABD = m\angle DBC$.

EXAMPLE 5 Double an angle measure

In the diagram at the right, $\overrightarrow{YW}$ bisects $\angle XYZ$, and $m\angle XYW = 18°$. Find $m\angle XYZ$.



Solution

By the Angle Addition Postulate, $m\angle XYZ = m\angle XYW + m\angle WYZ$. Because $\overrightarrow{YW}$ bisects $\angle XYZ$, you know that $\angle XYW \cong \angle WYZ$.

So, $m\angle XYW = m\angle WYZ$, and you can write

$$m\angle XYZ = m\angle XYW + m\angle WYZ = 18° + 18° = 36°.$$

✓ GUIDED PRACTICE for Example 5

7. Angle MNP is a straight angle, and $\overrightarrow{NQ}$ bisects $\angle MNP$. Draw $\angle MNP$ and $\overrightarrow{NQ}$. Use arcs to mark the congruent angles in your diagram, and give the angle measures of these congruent angles.

1.4 EXERCISES

HOMEWORK KEY

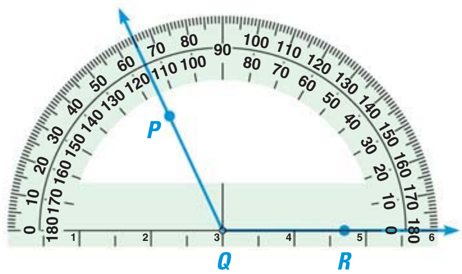
◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 15, 23, and 53

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 21, 27, 43, and 62

SKILL PRACTICE

1. **VOCABULARY** Sketch an example of each of the following types of angles: acute, obtuse, right, and straight.

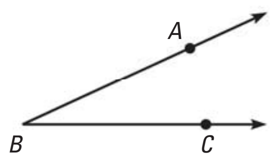
2. ★ **WRITING** *Explain* how to find the measure of $\angle PQR$, shown at the right.



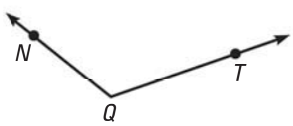
EXAMPLE 1 on p. 24 for Exs. 3–6

NAMING ANGLES AND ANGLE PARTS In Exercises 3–5, write three names for the angle shown. Then name the vertex and sides of the angle.

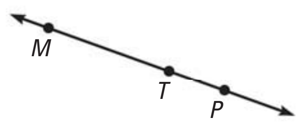
3.



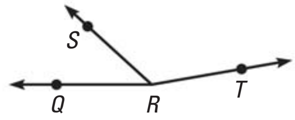
4.



5.



6. **NAMING ANGLES** Name three different angles in the diagram at the right.

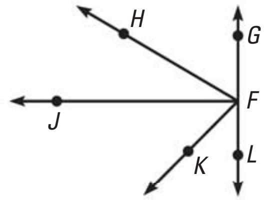


EXAMPLE 2
on p. 25
for Exs. 7–21

CLASSIFYING ANGLES Classify the angle with the given measure as *acute*, *obtuse*, *right*, or *straight*.

7. $m\angle W = 180°$ 8. $m\angle X = 30°$ 9. $m\angle Y = 90°$ 10. $m\angle Z = 95°$

MEASURING ANGLES Trace the diagram and extend the rays. Use a protractor to find the measure of the given angle. Then classify the angle as *acute*, *obtuse*, *right*, or *straight*.



11. $\angle JFL$ 12. $\angle GFH$

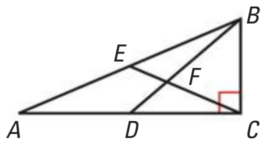
13. $\angle GFK$ 14. $\angle GFL$

NAMING AND CLASSIFYING Give another name for the angle in the diagram below. Tell whether the angle appears to be *acute*, *obtuse*, *right*, or *straight*.

15. $\angle ACB$ 16. $\angle ABC$

17. $\angle BFD$ 18. $\angle AEC$

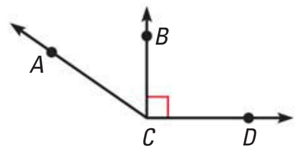
19. $\angle BDC$ 20. $\angle BEC$



21. ★ **MULTIPLE CHOICE** Which is a correct name for the obtuse angle in the diagram?

(A) $\angle ACB$ (B) $\angle ACD$

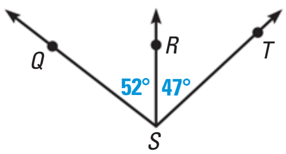
(C) $\angle BCD$ (D) $\angle C$

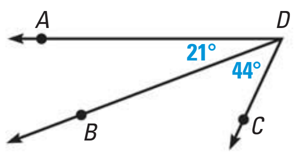


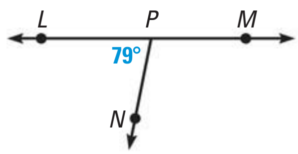
EXAMPLE 3
on p. 26
for Exs. 22–27

ANGLE ADDITION POSTULATE Find the indicated angle measure.

22. $m\angle QST = \underline{\ ?\ }$ 23. $m\angle ADC = \underline{\ ?\ }$ 24. $m\angle NPM = \underline{\ ?\ }$

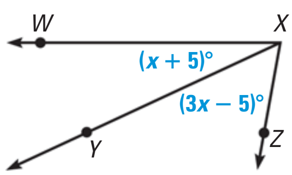




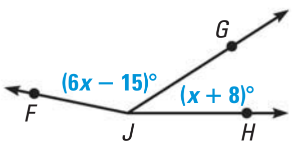


ALGEBRA Use the given information to find the indicated angle measure.

25. Given $m\angle WXZ = 80°$, find $m\angle YXZ$.



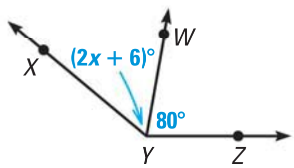
26. Given $m\angle FJH = 168°$, find $m\angle FJG$.



27. ★ **MULTIPLE CHOICE** In the diagram, the measure of $\angle XYZ$ is 140°. What is the value of x?

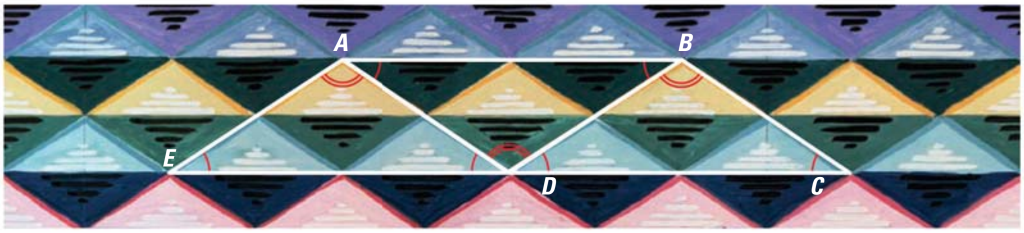
(A) 27 (B) 33

(C) 67 (D) 73



EXAMPLE 4
on p. 27
for Ex. 28

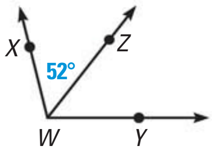
28. **CONGRUENT ANGLES** In the photograph below, $m\angle AED = 34°$ and $m\angle EAD = 112°$. Identify the congruent angles in the diagram. Then find $m\angle BDC$ and $m\angle ADB$.



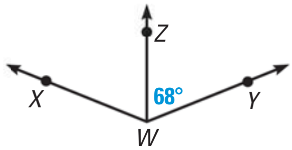
EXAMPLE 5
on p. 28
for Exs. 29–32

ANGLE BISECTORS **Given that $\overrightarrow{WZ}$ bisects $\angle XWY$, find the two angle measures not given in the diagram.**

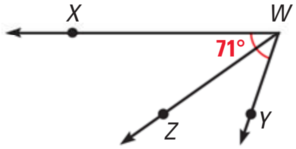
29.



30.



31.



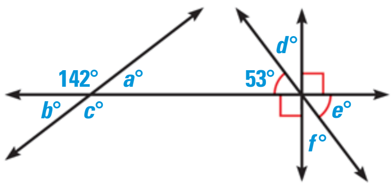
32. **ERROR ANALYSIS** $\overrightarrow{KM}$ bisects $\angle JKL$ and $m\angle JKM = 30°$. *Describe* and correct the error made in stating that $m\angle JKL = 15°$. Draw a sketch to support your answer.

FINDING ANGLE MEASURES **Find the indicated angle measure.**

33. $a°$ **34.** $b°$

35. $c°$ **36.** $d°$

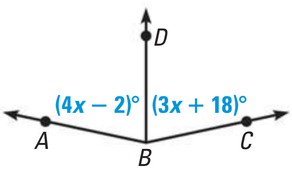
37. $e°$ **38.** $f°$



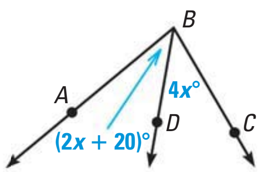
39. **ERROR ANALYSIS** A student states that $\overrightarrow{AD}$ can bisect $\angle AGC$. *Describe* and correct the student's error. Draw a sketch to support your answer.

ALGEBRA **In each diagram, $\overrightarrow{BD}$ bisects $\angle ABC$. Find $m\angle ABC$.**

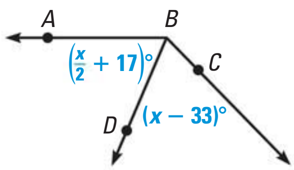
40.



41.



42.



43. ★ **SHORT RESPONSE** You are measuring $\angle PQR$ with a protractor. When you line up $\overrightarrow{QR}$ with the 20° mark, $\overrightarrow{QP}$ lines up with the 80° mark. Then you move the protractor so that $\overrightarrow{QR}$ lines up with the 15° mark. What mark does $\overrightarrow{QP}$ line up with? *Explain.*

ALGEBRA **Plot the points in a coordinate plane and draw $\angle ABC$. Classify the angle. Then give the coordinates of a point that lies in the interior of the angle.**

44. $A(3, 3)$, $B(0, 0)$, $C(3, 0)$

45. $A(-5, 4)$, $B(1, 4)$, $C(-2, -2)$

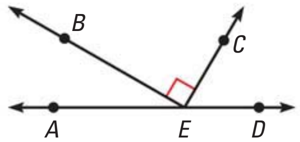
46. $A(-5, 2)$, $B(-2, -2)$, $C(4, -3)$

47. $A(-3, -1)$, $B(2, 1)$, $C(6, -2)$

48. xy **ALGEBRA** Let $(2x - 12)°$ represent the measure of an acute angle. What are the possible values of x?

49. **CHALLENGE** $\overrightarrow{SQ}$ bisects $\angle RST$, $\overrightarrow{SP}$ bisects $\angle RSQ$, and $\overrightarrow{SV}$ bisects $\angle RSP$. The measure of $\angle VSP$ is 17°. Find $m\angle TSQ$. *Explain.*

50. **FINDING MEASURES** In the diagram, $m\angle AEB = \frac{1}{2} \cdot m\angle CED$, and $\angle AED$ is a straight angle. Find $m\angle AEB$ and $m\angle CED$.



PROBLEM SOLVING

51. **SCULPTURE** In the sculpture shown in the photograph, suppose the measure of $\angle LMN$ is 79° and the measure of $\angle PMN$ is 47°. What is the measure of $\angle LMP$?

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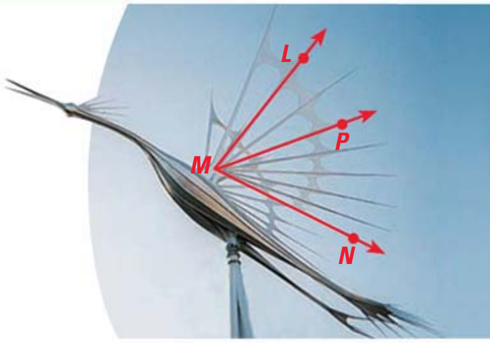
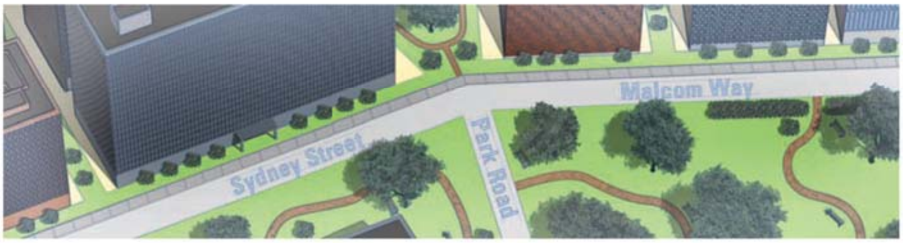


52. **MAP** The map shows the intersection of three roads. Malcom Way intersects Sydney Street at an angle of 162°. Park Road intersects Sydney Street at an angle of 87°. Find the angle at which Malcom Way intersects Park Road.



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EXAMPLES 4 and 5 on pp. 27–28 for Exs. 53–55

CONSTRUCTION **In Exercises 53–55, use the photograph of a roof truss.**

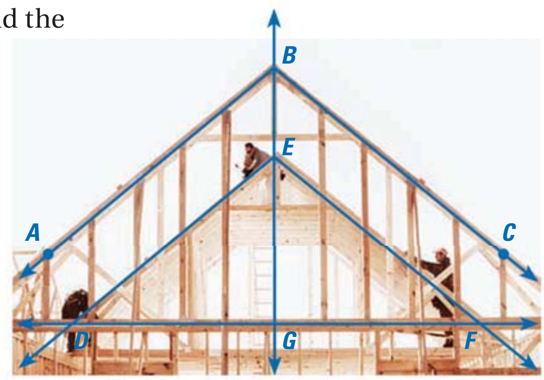
53. In the roof truss, $\overrightarrow{BG}$ bisects $\angle ABC$ and $\angle DEF$, $m\angle ABC = 112°$, and $\angle ABC \cong \angle DEF$. Find the measure of the following angles.

a. $m\angle DEF$ **b.** $m\angle ABG$

c. $m\angle CBG$ **d.** $m\angle DEG$

54. In the roof truss, $\overrightarrow{GB}$ bisects $\angle DGF$. Find $m\angle DGE$ and $m\angle FGE$.

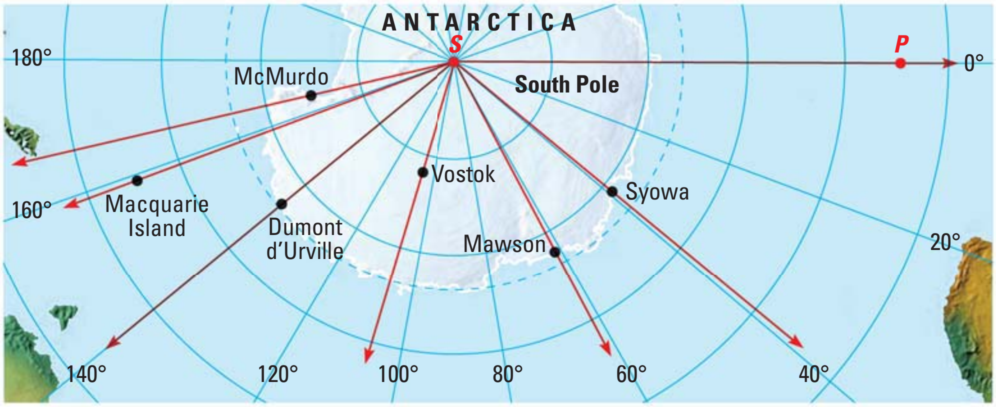
55. Name an example of each of the following types of angles: *acute*, *obtuse*, *right*, and *straight*.



GEOGRAPHY **For the given location on the map, estimate the measure of $\angle PSL$, where P is on the Prime Meridian (0° longitude), S is the South Pole, and L is the location of the indicated research station.**

56. Macquarie Island **57.** Dumont d'Urville **58.** McMurdo

59. Mawson **60.** Syowa **61.** Vostok

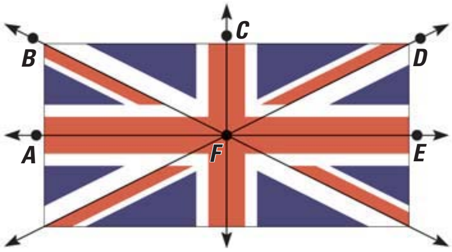


62. ★ **EXTENDED RESPONSE** In the flag shown, $\angle AFE$ is a straight angle and $\overrightarrow{FC}$ bisects $\angle AFE$ and $\angle BFD$.

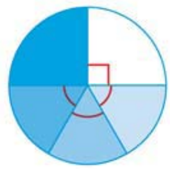
a. Which angles are acute? obtuse? right?

b. Identify the congruent angles.

c. If $m\angle AFB = 26°$, find $m\angle DFE$, $m\angle BFC$, $m\angle CFD$, $m\angle AFC$, $m\angle AFD$, and $m\angle BFD$. *Explain.*



63. **CHALLENGE** Create a set of data that could be represented by the circle graph at the right. *Explain* your reasoning.



MIXED REVIEW

PREVIEW
Prepare for Lesson 1.5 in Ex. 64.

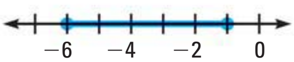
64. You and a friend go out to dinner and each pay for your own meal. The total cost of the two meals is \$25. Your meal cost \$4 more than your friend's meal. How much does each meal cost? *(p. 894)*

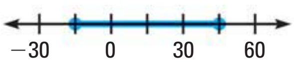
Graph the inequality on a number line. Tell whether the graph is a *segment*, a *ray* or *rays*, a *point*, or a *line*. *(p. 2)*

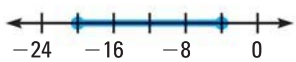
65. $x \le -8$ **66.** $x \ge 6$ **67.** $-3 \le x \le 5$

68. $x \ge -7$ and $x \le -1$ **69.** $x \ge -2$ or $x \le 4$ **70.** $|x| \ge 0$

Find the coordinate of the midpoint of the segment. *(p. 15)*

71. 

72. 

73. 

Investigating Geometry **CONSTRUCTION** *Use after Lesson 1.4*

1.4 Copy and Bisect Segments and Angles

MATERIALS • compass • straightedge

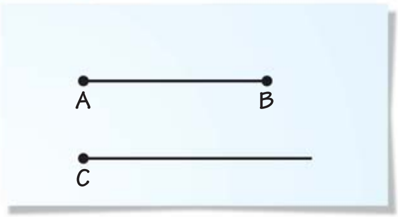
QUESTION **How can you copy and bisect segments and angles?**

A **construction** is a geometric drawing that uses a limited set of tools, usually a *compass* and *straightedge*. You can use a compass and straightedge (a ruler without marks) to construct a segment that is congruent to a given segment, and an angle that is congruent to a given angle.

EXPLORE 1 Copy a segment

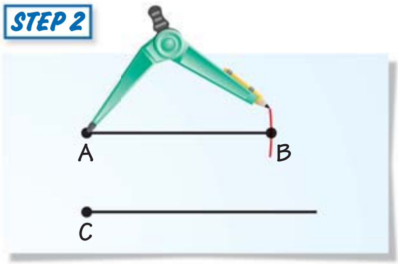
Use the following steps to construct a segment that is congruent to $\overline{AB}$.

STEP 1



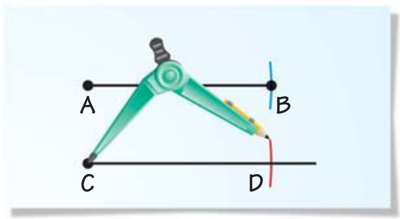
Draw a segment Use a straightedge to draw a segment longer than $\overline{AB}$. Label point C on the new segment.

STEP 2



Measure length Set your compass at the length of $\overline{AB}$.

STEP 3

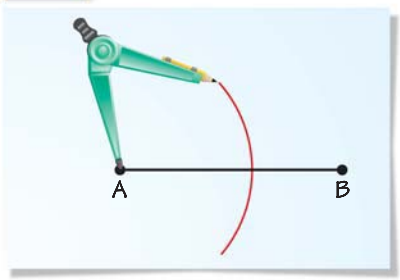


Copy length Place the compass at C. Mark point D on the new segment. $\overline{CD} \cong \overline{AB}$.

EXPLORE 2 Bisect a segment

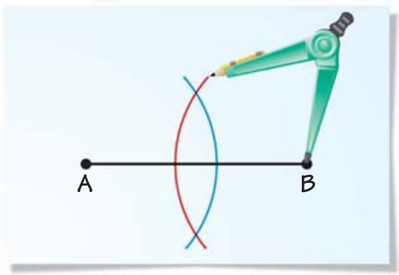
Use the following steps to construct a bisector of $\overline{AB}$ and to find the midpoint M of $\overline{AB}$.

STEP 1



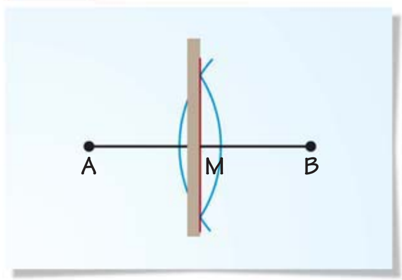
Draw an arc Place the compass at A. Use a compass setting that is greater than half the length of $\overline{AB}$. Draw an arc.

STEP 2



Draw a second arc Keep the same compass setting. Place the compass at B. Draw an arc. It should intersect the other arc at two points.

STEP 3

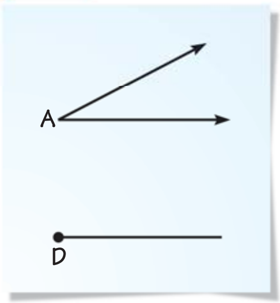


Bisect segment Draw a segment through the two points of intersection. This segment bisects $\overline{AB}$ at M, the midpoint of $\overline{AB}$.

EXPLORE 3 Copy an angle

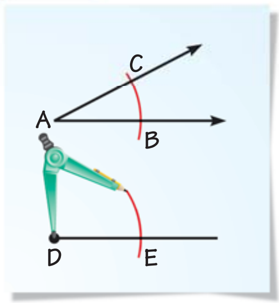
Use the following steps to construct an angle that is congruent to $\angle A$. In this construction, the *radius* of an arc is the distance from the point where the compass point rests (the *center* of the arc) to a point on the arc drawn by the compass.

STEP 1



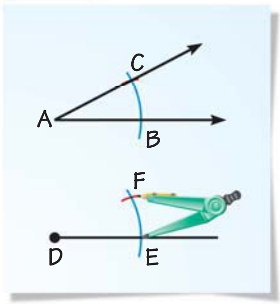
Draw a segment Draw a segment. Label a point D on the segment.

STEP 2



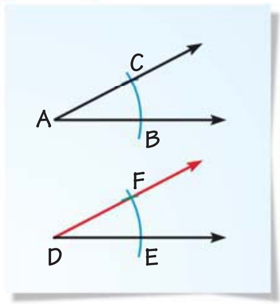
Draw arcs Draw an arc with center A. Using the same radius, draw an arc with center D.

STEP 3



Draw arcs Label B, C, and E. Draw an arc with radius BC and center E. Label the intersection F.

STEP 4

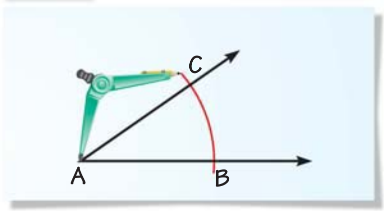


Draw a ray Draw $\overrightarrow{DF}$. $\angle EDF \cong \angle BAC$.

EXPLORE 4 Bisect an angle

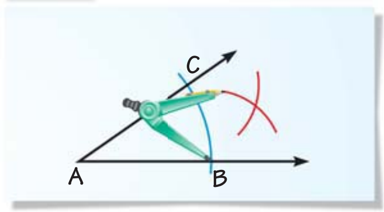
Use the following steps to construct an angle bisector of $\angle A$.

STEP 1



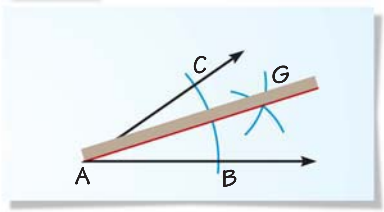
Draw an arc Place the compass at A. Draw an arc that intersects both sides of the angle. Label the intersections C and B.

STEP 2



Draw arcs Place the compass at C. Draw an arc. Then place the compass point at B. Using the same radius, draw another arc.

STEP 3



Draw a ray Label the intersection G. Use a straightedge to draw a ray through A and G. $\overrightarrow{AG}$ bisects $\angle A$.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. *Describe* how you could use a compass and a straightedge to draw a segment that is twice as long as a given segment.

2. Draw an obtuse angle. Copy the angle using a compass and a straightedge. Then bisect the angle using a compass and straightedge.

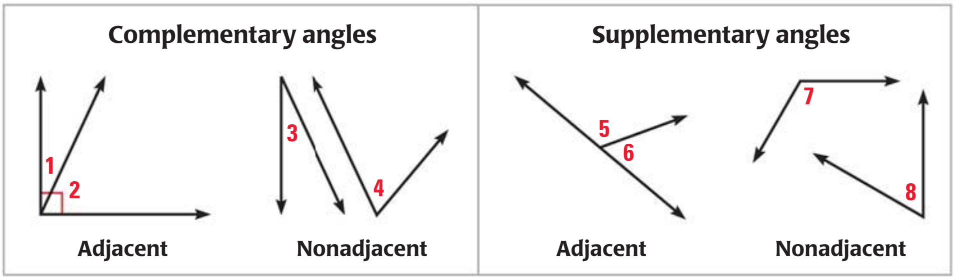
1.5 Describe Angle Pair Relationships

Before	You used angle postulates to measure and classify angles.
Now	You will use special angle relationships to find angle measures.
Why?	So you can find measures in a building, as in Ex. 53.

Key Vocabulary
- **complementary angles**
- **supplementary angles**
- **adjacent angles**
- **linear pair**
- **vertical angles**

Two angles are **complementary angles** if the sum of their measures is 90°. Each angle is the *complement* of the other. Two angles are **supplementary angles** if the sum of their measures is 180°. Each angle is the *supplement* of the other.

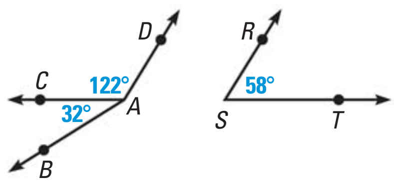
Complementary angles and supplementary angles can be *adjacent angles* or *nonadjacent angles*. **Adjacent angles** are two angles that share a common vertex and side, but have no common interior points.



EXAMPLE 1 Identify complements and supplements

AVOID ERRORS
In Example 1, $\angle DAC$ and $\angle DAB$ share a common vertex. But they share common interior points, so they are *not* adjacent angles.

In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.



Solution

Because $32° + 58° = 90°$, $\angle BAC$ and $\angle RST$ are complementary angles.

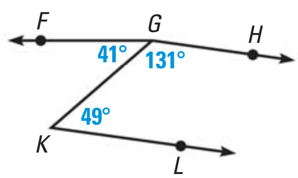
Because $122° + 58° = 180°$, $\angle CAD$ and $\angle RST$ are supplementary angles.

Because $\angle BAC$ and $\angle CAD$ share a common vertex and side, they are adjacent.

✓ GUIDED PRACTICE for Example 1

1. In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.

2. Are $\angle KGH$ and $\angle LKG$ adjacent angles? Are $\angle FGK$ and $\angle FGH$ adjacent angles? *Explain.*



EXAMPLE 2 Find measures of a complement and a supplement

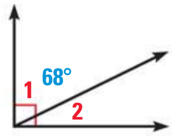
READ DIAGRAMS
Angles are sometimes named with numbers. An angle measure in a diagram has a degree symbol. An angle name does not.

a. Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 1 = 68°$, find $m\angle 2$.

b. Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 4 = 56°$, find $m\angle 3$.

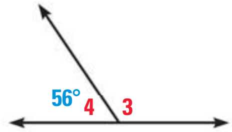
Solution

a. You can draw a diagram with complementary adjacent angles to illustrate the relationship.



$m\angle 2 = 90° - m\angle 1 = 90° - 68° = 22°$

b. You can draw a diagram with supplementary adjacent angles to illustrate the relationship.

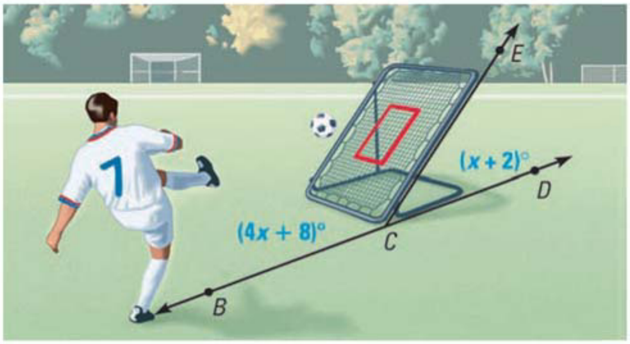


$m\angle 3 = 180° - m\angle 4 = 180° - 56° = 124°$

EXAMPLE 3 Find angle measures

READ DIAGRAMS
In a diagram, you can assume that a line that looks straight *is* straight. In Example 3, *B*, *C*, and *D* lie on $\overleftrightarrow{BD}$. So, $\angle BCD$ is a straight angle.

SPORTS When viewed from the side, the frame of a ball-return net forms a pair of supplementary angles with the ground. Find $m\angle BCE$ and $m\angle ECD$.



Solution

STEP 1 **Use** the fact that the sum of the measures of supplementary angles is 180°.

$m\angle BCE + m\angle ECD = 180°$ **Write equation.**

$(4x + 8)° + (x + 2)° = 180°$ **Substitute.**

$5x + 10 = 180$ **Combine like terms.**

$5x = 170$ **Subtract 10 from each side.**

$x = 34$ **Divide each side by 5.**

STEP 2 **Evaluate** the original expressions when $x = 34$.

$m\angle BCE = (4x + 8)° = (4 \cdot 34 + 8)° = 144°$

$m\angle ECD = (x + 2)° = (34 + 2)° = 36°$

▸ The angle measures are 144° and 36°.

✓ GUIDED PRACTICE for Examples 2 and 3

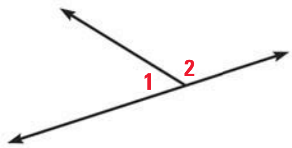
3. Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 2 = 8°$, find $m\angle 1$.

4. Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 3 = 117°$, find $m\angle 4$.

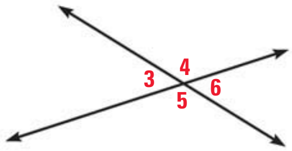
5. $\angle LMN$ and $\angle PQR$ are complementary angles. Find the measures of the angles if $m\angle LMN = (4x - 2)°$ and $m\angle PQR = (9x + 1)°$.

ANGLE PAIRS Two adjacent angles are a **linear pair** if their noncommon sides are opposite rays. The angles in a linear pair are supplementary angles.

Two angles are **vertical angles** if their sides form two pairs of opposite rays.



∠1 and ∠2 are a linear pair.

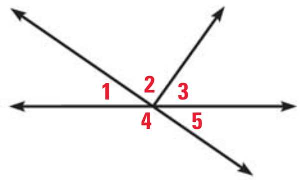


∠3 and ∠6 are vertical angles.
∠4 and ∠5 are vertical angles.

EXAMPLE 4 Identify angle pairs

AVOID ERRORS
In the diagram, one side of ∠1 and one side of ∠3 are opposite rays. But the angles are not a linear pair because they are not adjacent.

Identify all of the linear pairs and all of the vertical angles in the figure at the right.



Solution

To find vertical angles, look for angles formed by intersecting lines.

▶ ∠1 and ∠5 are vertical angles.

To find linear pairs, look for adjacent angles whose noncommon sides are opposite rays.

▶ ∠1 and ∠4 are a linear pair. ∠4 and ∠5 are also a linear pair.

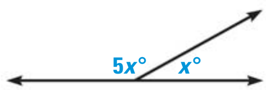
EXAMPLE 5 Find angle measures in a linear pair

xy ALGEBRA **Two angles form a linear pair. The measure of one angle is 5 times the measure of the other. Find the measure of each angle.**

DRAW DIAGRAMS
You may find it useful to draw a diagram to represent a word problem like the one in Example 5.

Solution

Let $x°$ be the measure of one angle. The measure of the other angle is $5x°$. Then use the fact that the angles of a linear pair are supplementary to write an equation.



$x° + 5x° = 180°$ **Write an equation.**

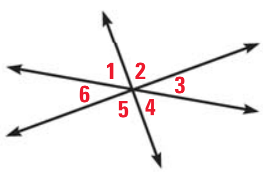
$6x = 180$ **Combine like terms.**

$x = 30$ **Divide each side by 6.**

▶ The measures of the angles are $30°$ and $5(30°) = 150°$.

✓ GUIDED PRACTICE for Examples 4 and 5

6. Do any of the numbered angles in the diagram at the right form a linear pair? Which angles are vertical angles? *Explain*.

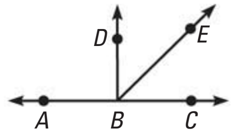


7. The measure of an angle is twice the measure of its complement. Find the measure of each angle.

CONCEPT SUMMARY — *For Your Notebook*

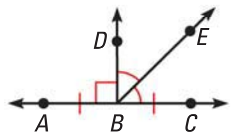
Interpreting a Diagram

There are some things you can conclude from a diagram, and some you cannot. For example, here are some things that you ***can* conclude** from the diagram at the right:



- All points shown are coplanar.
- Points *A, B,* and *C* are collinear, and *B* is between *A* and *C.*
- $\overleftrightarrow{AC}$, $\overrightarrow{BD}$, and $\overrightarrow{BE}$ intersect at point *B.*
- $\angle DBE$ and $\angle EBC$ are adjacent angles, and $\angle ABC$ is a straight angle.
- Point *E* lies in the interior of $\angle DBC$.

In the diagram above, you ***cannot* conclude** that $\overline{AB} \cong \overline{BC}$, that $\angle DBE \cong \angle EBC$, or that $\angle ABD$ is a right angle. This information must be indicated, as shown at the right.



1.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 21, and 47

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 30, and 53

◆ = **MULTIPLE REPRESENTATIONS** Ex. 55

SKILL PRACTICE

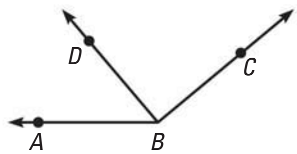
1. **VOCABULARY** Sketch an example of adjacent angles that are complementary. Are all complementary angles adjacent angles? *Explain*.

2. ★ **WRITING** Are all linear pairs supplementary angles? Are all supplementary angles linear pairs? *Explain*.

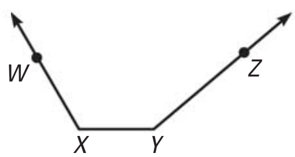
EXAMPLE 1 on p. 35 for Exs. 3–7

IDENTIFYING ANGLES **Tell whether the indicated angles are adjacent.**

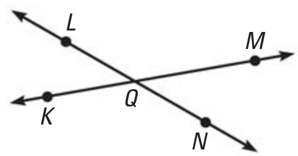
3. $\angle ABD$ and $\angle DBC$



4. $\angle WXY$ and $\angle XYZ$

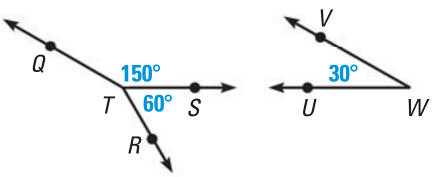


5. $\angle LQM$ and $\angle NQM$

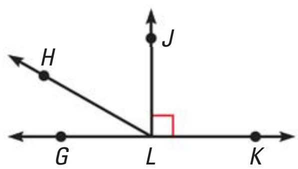


IDENTIFYING ANGLES **Name a pair of complementary angles and a pair of supplementary angles.**

6.



7.



EXAMPLE 2
on p. 36
for Exs. 8–16

COMPLEMENTARY ANGLES $\angle 1$ **and** $\angle 2$ **are complementary angles. Given the measure of** $\angle 1$**, find** $m\angle 2$**.**

8. $m\angle 1 = 43°$ **9.** $m\angle 1 = 21°$ **10.** $m\angle 1 = 89°$ **11.** $m\angle 1 = 5°$

SUPPLEMENTARY ANGLES $\angle 1$ **and** $\angle 2$ **are supplementary angles. Given the measure of** $\angle 1$**, find** $m\angle 2$**.**

12. $m\angle 1 = 60°$ **13.** $m\angle 1 = 155°$ **14.** $m\angle 1 = 130°$ **15.** $m\angle 1 = 27°$

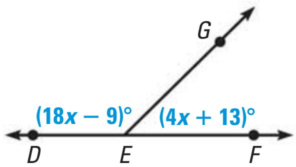
16. ★ **MULTIPLE CHOICE** The arm of a crossing gate moves 37° from vertical. How many more degrees does the arm have to move so that it is horizontal?

Ⓐ 37°
Ⓑ 53°
Ⓒ 90°
Ⓓ 143°

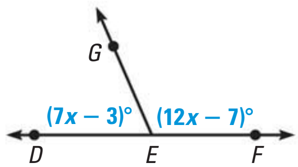
EXAMPLE 3
on p. 36
for Exs. 17–19

ALGEBRA Find $m\angle DEG$ **and** $m\angle GEF$**.**

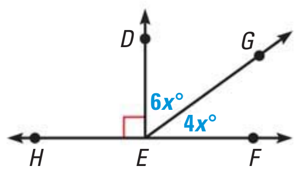
17.



18.



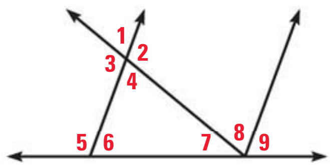
19.



EXAMPLE 4
on p. 37
for Exs. 20–27

IDENTIFYING ANGLE PAIRS Use the diagram below. Tell whether the angles are ***vertical angles*****, a** ***linear pair*****, or** ***neither*****.**

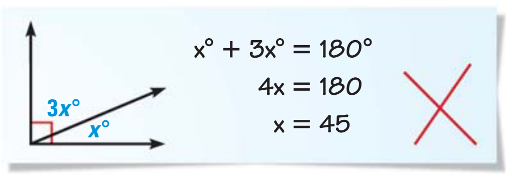
20. $\angle 1$ and $\angle 4$
21. $\angle 1$ and $\angle 2$
22. $\angle 3$ and $\angle 5$
23. $\angle 2$ and $\angle 3$
24. $\angle 7$, $\angle 8$, and $\angle 9$
25. $\angle 5$ and $\angle 6$
26. $\angle 6$ and $\angle 7$
27. $\angle 5$ and $\angle 9$



EXAMPLE 5
on p. 37
for Exs. 28–30

28. **ALGEBRA** Two angles form a linear pair. The measure of one angle is 4 times the measure of the other angle. Find the measure of each angle.

29. **ERROR ANALYSIS** *Describe* and correct the error made in finding the value of x.

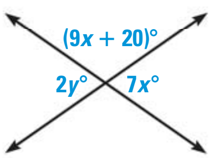


30. ★ **MULTIPLE CHOICE** The measure of one angle is 24° greater than the measure of its complement. What are the measures of the angles?

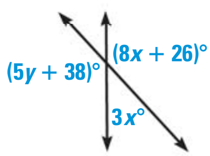
Ⓐ 24° and 66° Ⓑ 24° and 156° Ⓒ 33° and 57° Ⓓ 78° and 102°

ALGEBRA Find the values of x **and** y**.**

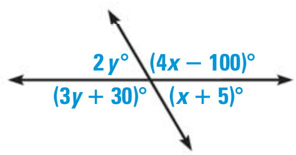
31.



32.



33.



REASONING **Tell whether the statement is *always*, *sometimes*, or *never* true. *Explain* your reasoning.**

34. An obtuse angle has a complement.

35. A straight angle has a complement.

36. An angle has a supplement.

37. The complement of an acute angle is an acute angle.

38. The supplement of an acute angle is an obtuse angle.

FINDING ANGLES **$\angle A$ and $\angle B$ are complementary. Find $m\angle A$ and $m\angle B$.**

39. $m\angle A = (3x + 2)°$
$m\angle B = (x - 4)°$

40. $m\angle A = (15x + 3)°$
$m\angle B = (5x - 13)°$

41. $m\angle A = (11x + 24)°$
$m\angle B = (x + 18)°$

FINDING ANGLES **$\angle A$ and $\angle B$ are supplementary. Find $m\angle A$ and $m\angle B$.**

42. $m\angle A = (8x + 100)°$
$m\angle B = (2x + 50)°$

43. $m\angle A = (2x - 20)°$
$m\angle B = (3x + 5)°$

44. $m\angle A = (6x + 72)°$
$m\angle B = (2x + 28)°$

45. **CHALLENGE** You are given that $\angle GHJ$ is a complement of $\angle RST$ and $\angle RST$ is a supplement of $\angle ABC$. Let $m\angle GHJ$ be $x°$. What is the measure of $\angle ABC$? *Explain* your reasoning.

PROBLEM SOLVING

IDENTIFYING ANGLES **Tell whether the two angles shown are *complementary*, *supplementary*, or *neither*.**

46. 

47. 

48. 

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ARCHITECTURE **The photograph shows the Rock and Roll Hall of Fame in Cleveland, Ohio. Use the photograph to identify an example type of the indicated type of angle pair.**

49. Supplementary angles

50. Vertical angles

51. Linear pair

52. Adjacent angles

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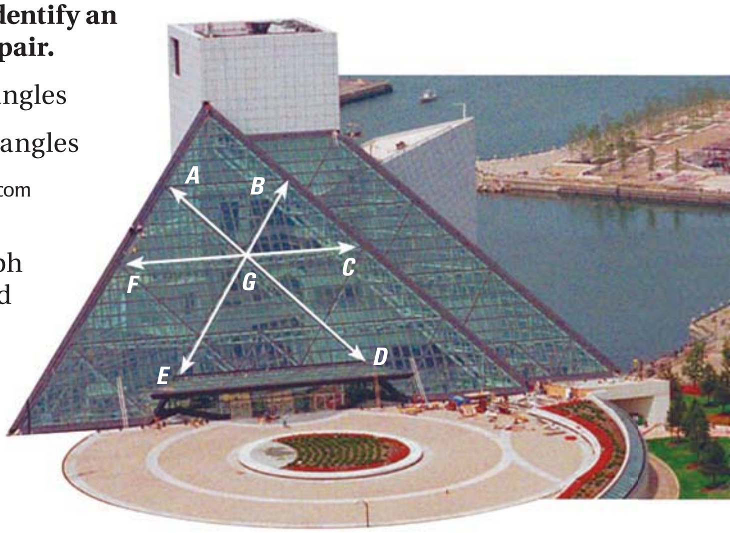
53. ★ **SHORT RESPONSE** Use the photograph shown at the right. Given that $\angle FGB$ and $\angle BGC$ are supplementary angles, and $m\angle FGB = 120°$, *explain* how to find the measure of the complement of $\angle BGC$.



◯ = WORKED-OUT SOLUTIONS on p. WS1
★ = STANDARDIZED TEST PRACTICE
◆ = MULTIPLE REPRESENTATIONS

54. **SHADOWS** The length of a shadow changes as the sun rises. In the diagram below, the length of $\overline{CB}$ is the length of a shadow. The end of the shadow is the vertex of $\angle ABC$, which is formed by the ground and the sun's rays. *Describe* how the shadow and angle change as the sun rises.

55. **MULTIPLE REPRESENTATIONS** Let $x°$ be an angle measure. Let $y_1°$ be the measure of a complement of the angle and let $y_2°$ be the measure of a supplement of the angle.

   a. **Writing an Equation** Write equations for y_1 as a function of x, and for y_2 as a function of x. What is the domain of each function? *Explain*.

   b. **Drawing a Graph** Graph each function and *describe* its range.

56. **CHALLENGE** The sum of the measures of two complementary angles exceeds the difference of their measures by 86°. Find the measure of each angle. *Explain* how you found the angle measures.

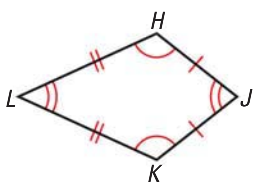
MIXED REVIEW

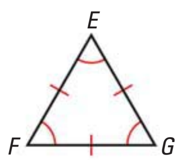
Make a table of values and graph the function. *(p. 884)*

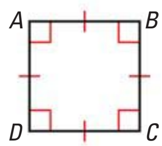
57. $y = 5 - x$ 58. $y = 3x$ 59. $y = x^2 - 1$ 60. $y = -2x^2$

PREVIEW
Prepare for Lesson 1.6 in Exs. 61–63.

In each figure, name the congruent sides and congruent angles. *(pp. 9, 24)*

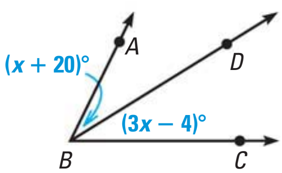
61.


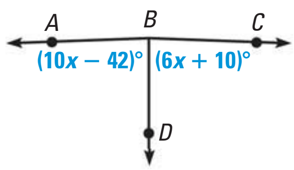
62.


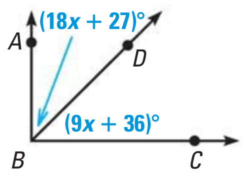
63.


QUIZ for Lessons 1.4–1.5

In each diagram, $\overrightarrow{BD}$ bisects $\angle ABC$. Find $m\angle ABD$ and $m\angle DBC$. *(p. 24)*

1.


2.


3.


Find the measure of (a) the complement and (b) the supplement of $\angle 1$. *(p. 35)*

4. $m\angle 1 = 47°$ 5. $m\angle 1 = 19°$ 6. $m\angle 1 = 75°$ 7. $m\angle 1 = 2°$

1.6 Classify Polygons

Before You classified angles.

Now You will classify polygons.

Why? So you can find lengths in a floor plan, as in Ex. 32.

Key Vocabulary
- **polygon**
  side, vertex
- **convex**
- **concave**
- ***n*-gon**
- **equilateral**
- **equiangular**
- **regular**

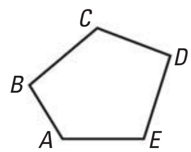
KEY CONCEPT *For Your Notebook*

Identifying Polygons

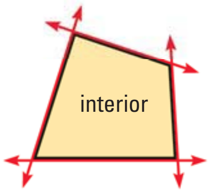
In geometry, a figure that lies in a plane is called a *plane figure*. A **polygon** is a closed plane figure with the following properties.

1. It is formed by three or more line segments called **sides**.
2. Each side intersects exactly two sides, one at each endpoint, so that no two sides with a common endpoint are collinear.

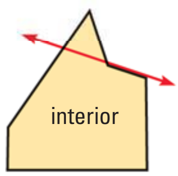
Each endpoint of a side is a **vertex** of the polygon. The plural of vertex is *vertices*. A polygon can be named by listing the vertices in consecutive order. For example, *ABCDE* and *CDEAB* are both correct names for the polygon at the right.



A polygon is **convex** if no line that contains a side of the polygon contains a point in the interior of the polygon. A polygon that is not convex is called *nonconvex* or **concave**.



convex polygon



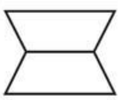
concave polygon

EXAMPLE 1 Identify polygons

READ VOCABULARY
A *plane figure* is two-dimensional. Later, you will study three-dimensional *space figures* such as prisms and cylinders.

Tell whether the figure is a polygon and whether it is *convex* or *concave*.

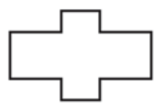
a.



b.

c.

d.



Solution

a. Some segments intersect more than two segments, so it is not a polygon.

b. The figure is a convex polygon.

c. Part of the figure is not a segment, so it is not a polygon.

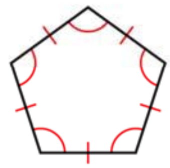
d. The figure is a concave polygon.

CLASSIFYING POLYGONS A polygon is named by the number of its sides.

Number of sides	Type of polygon	Number of sides	Type of polygon
3	Triangle	8	Octagon
4	Quadrilateral	9	Nonagon
5	Pentagon	10	Decagon
6	Hexagon	12	Dodecagon
7	Heptagon	n	n-gon

The term ***n*-gon**, where n is the number of a polygon's sides, can also be used to name a polygon. For example, a polygon with 14 sides is a 14-gon.

In an **equilateral** polygon, all sides are congruent. In an **equiangular** polygon, all angles in the interior of the polygon are congruent. A **regular** polygon is a convex polygon that is both equilateral and equiangular.



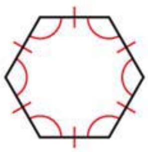
regular pentagon

EXAMPLE 2 Classify polygons

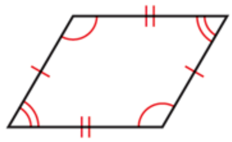
READ DIAGRAMS
Double marks are used in part (b) of Example 2 to show that more than one pair of sides are congruent and more than one pair of angles are congruent.

Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. Explain your reasoning.

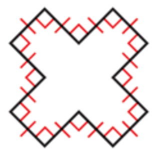
a.



b.



c.



Solution

a. The polygon has 6 sides. It is equilateral and equiangular, so it is a regular hexagon.

b. The polygon has 4 sides, so it is a quadrilateral. It is not equilateral or equiangular, so it is not regular.

c. The polygon has 12 sides, so it is a dodecagon. The sides are congruent, so it is equilateral. The polygon is not convex, so it is not regular.

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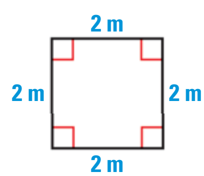
✓ GUIDED PRACTICE for Examples 1 and 2

1. Sketch an example of a convex heptagon and an example of a concave heptagon.

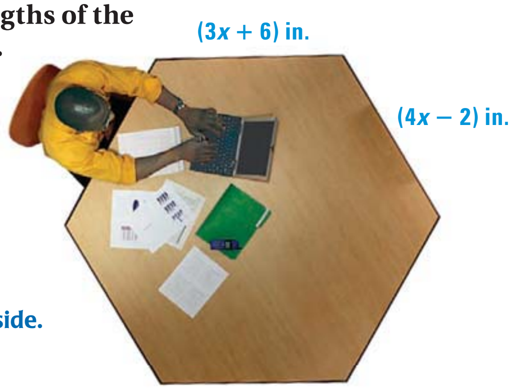
2. Classify the polygon shown at the right by the number of sides. *Explain* how you know that the sides of the polygon are congruent and that the angles of the polygon are congruent.



EXAMPLE 3 Find side lengths

READ VOCABULARY
Hexagonal means "shaped like a hexagon."

ALGEBRA **A table is shaped like a regular hexagon. The expressions shown represent side lengths of the hexagonal table. Find the length of a side.**



Solution

First, write and solve an equation to find the value of x. Use the fact that the sides of a regular hexagon are congruent.

$3x + 6 = 4x - 2$ **Write equation.**

$6 = x - 2$ **Subtract $3x$ from each side.**

$8 = x$ **Add 2 to each side.**

Then find a side length. Evaluate one of the expressions when $x = 8$.

$3x + 6 = 3(8) + 6 = 30$

▶ The length of a side of the table is 30 inches.

✓ GUIDED PRACTICE for Example 3

3. The expressions $8y°$ and $(9y - 15)°$ represent the measures of two of the angles in the table in Example 3. Find the measure of an angle.

1.6 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 13, 19, and 33
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 37, 39, and 40

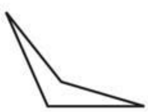
SKILL PRACTICE

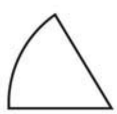
1. VOCABULARY *Explain* what is meant by the term n-gon.

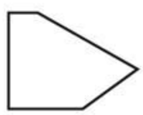
2. ★ **WRITING** Imagine that you can tie a string tightly around a polygon. If the polygon is convex, will the length of the string be equal to the distance around the polygon? What if the polygon is concave? *Explain.*

EXAMPLE 1
on p. 42
for Exs. 3–7

IDENTIFYING POLYGONS **Tell whether the figure is a polygon. If it is not, *explain* why. If it is a polygon, tell whether it is *convex* or *concave*.**

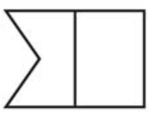
3. 

4. 

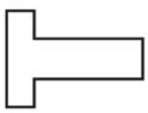
5. 

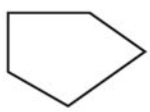
6. 

7. ★ **MULTIPLE CHOICE** Which of the figures is a concave polygon?

Ⓐ 

Ⓑ 

Ⓒ 

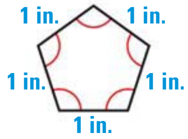
Ⓓ 

EXAMPLE 2
on p. 43
for Exs. 8–14

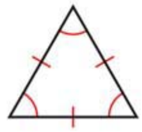
CLASSIFYING Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. *Explain* your reasoning.

8.

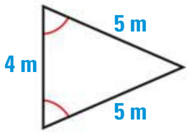
9.



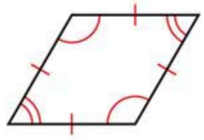
10.



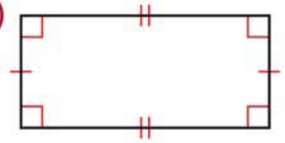
11.



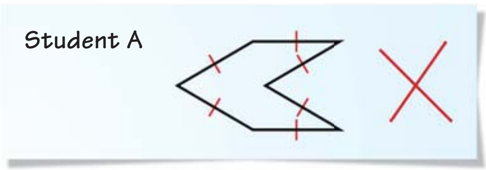
12.

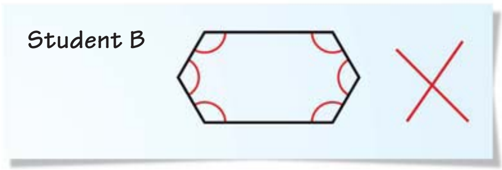


13.



14. ERROR ANALYSIS Two students were asked to draw a regular hexagon, as shown below. *Describe* the error made by each student.





EXAMPLE 3
on p. 44
for Exs. 15–17

15. ALGEBRA The lengths (in inches) of two sides of a regular pentagon are represented by the expressions $5x - 27$ and $2x - 6$. Find the length of a side of the pentagon.

16. ALGEBRA The expressions $(9x + 5)°$ and $(11x - 25)°$ represent the measures of two angles of a regular nonagon. Find the measure of an angle of the nonagon.

17. ALGEBRA The expressions $3x - 9$ and $23 - 5x$ represent the lengths (in feet) of two sides of an equilateral triangle. Find the length of a side.

USING PROPERTIES Tell whether the statement is *always*, *sometimes*, or *never* true.

18. A triangle is convex.

19. A decagon is regular.

20. A regular polygon is equiangular.

21. A circle is a polygon.

22. A polygon is a plane figure.

23. A concave polygon is regular.

DRAWING Draw a figure that fits the description.

24. A triangle that is not regular

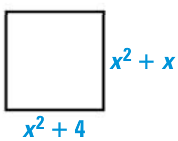
25. A concave quadrilateral

26. A pentagon that is equilateral but not equiangular

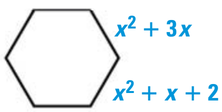
27. An octagon that is equiangular but not equilateral

ALGEBRA Each figure is a regular polygon. Expressions are given for two side lengths. Find the value of *x*.

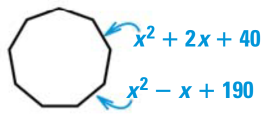
28.



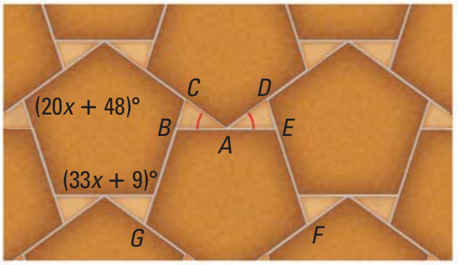
29.



30.



31. **CHALLENGE** Regular pentagonal tiles and triangular tiles are arranged in the pattern shown. The pentagonal tiles are all the same size and shape and the triangular tiles are all the same size and shape. Find the angle measures of the triangular tiles. *Explain* your reasoning.



PROBLEM SOLVING

32. **ARCHITECTURE** Longwood House, shown in the photograph on page 42, is located in Natchez, Mississippi. The diagram at the right shows the floor plan of a part of the house.

 a. Tell whether the red polygon in the diagram is *convex* or *concave*.

 b. Classify the red polygon and tell whether it appears to be regular.

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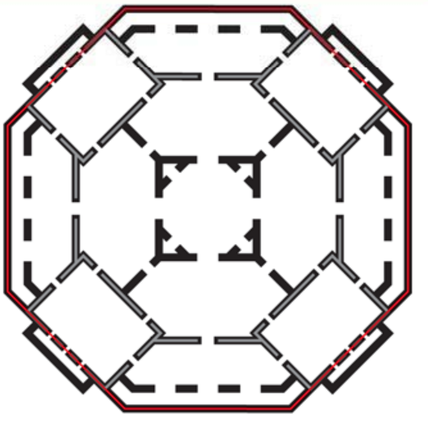

EXAMPLE 2 on p. 43 for Exs. 33–36

SIGNS Each sign suggests a polygon. Classify the polygon by the number of sides. Tell whether it appears to be *equilateral*, *equiangular*, or *regular*.

33. 

34. 

35. 

36. 

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37. ★ **MULTIPLE CHOICE** Two vertices of a regular quadrilateral are $A(0, 4)$ and $B(0, -4)$. Which of the following could be the other two vertices?

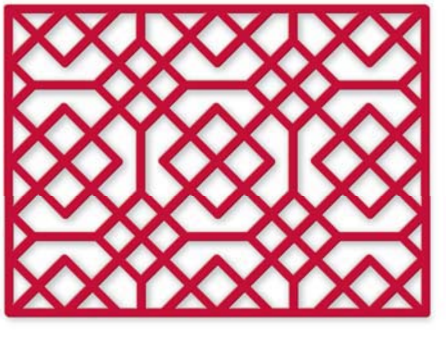
 Ⓐ $C(4, 4)$ and $D(4, -4)$ Ⓑ $C(-4, 4)$ and $D(-4, -4)$

 Ⓒ $C(8, -4)$ and $D(8, 4)$ Ⓓ $C(0, 8)$ and $D(0, -8)$

38. **MULTI-STEP PROBLEM** The diagram shows the design of a lattice made in China in 1850.

 a. Sketch five different polygons you see in the diagram. Classify each polygon by the number of sides.

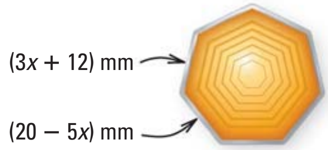
 b. Tell whether each polygon you sketched is concave or convex, and whether the polygon appears to be equilateral, equiangular, or regular.



○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE

EXAMPLE 3
on p. 44
for Ex. 39

39. ★ **SHORT RESPONSE** The shape of the button shown is a regular polygon. The button has a border made of silver wire. How many millimeters of silver wire are needed for this border? *Explain*.



40. ★ **EXTENDED RESPONSE** A segment that joins two nonconsecutive vertices of a polygon is called a *diagonal*. For example, a quadrilateral has two diagonals, as shown below.

Type of polygon	Diagram	Number of sides	Number of diagonals
Quadrilateral		4	2
Pentagon	?	?	?
Hexagon	?	?	?
Heptagon	?	?	?

a. Copy and complete the table. *Describe* any patterns you see.

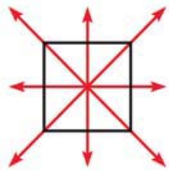
b. How many diagonals does an octagon have? a nonagon? *Explain*.

c. The expression $\frac{n(n-3)}{2}$ can be used to find the number of diagonals in an n-gon. Find the number of diagonals in a 60-gon.

41. **LINE SYMMETRY** A figure has *line symmetry* if it can be folded over exactly onto itself. The fold line is called the *line of symmetry*. A regular quadrilateral has four lines of symmetry, as shown. Find the number of lines of symmetry in each polygon.

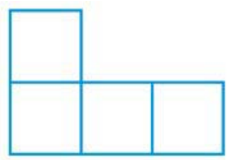
a. A regular triangle **b.** A regular pentagon

c. A regular hexagon **d.** A regular octagon



regular quadrilateral
4 lines of symmetry

42. **CHALLENGE** The diagram shows four identical squares lying edge-to-edge. Sketch all the different ways you can arrange four squares edge-to-edge. Sketch all the different ways you can arrange five identical squares edge-to-edge.



MIXED REVIEW

PREVIEW
Prepare for Lesson 1.7 in Exs. 43–51.

Solve the equation.

43. $\frac{1}{2}(35)b = 140$ *(p. 875)* **44.** $x^2 = 144$ *(p. 882)* **45.** $3.14r^2 = 314$ *(p. 882)*

Copy and complete the statement. *(p. 886)*

46. 500 m = __?__ cm **47.** 12 mi = __?__ ft **48.** 672 in. = __?__ yd

49. 1200 km = __?__ m **50.** $4\frac{1}{2}$ ft = __?__ yd **51.** 3800 m = __?__ km

Find the distance between the two points. *(p. 15)*

52. $D(-13, 13), E(0, -12)$ **53.** $F(-9, -8), G(-9, 7)$ **54.** $H(10, 5), J(-2, -2)$

@HomeTutor **classzone.com** Keystrokes

1.7 Investigate Perimeter and Area

MATERIALS • graph paper • graphing calculator

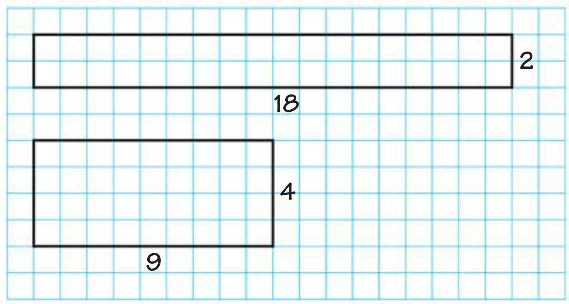
QUESTION **How can you use a graphing calculator to find the smallest possible perimeter for a rectangle with a given area?**

You can use the formulas below to find the perimeter P and the area A of a rectangle with length ℓ and width w.

$$P = 2\ell + 2w \qquad\qquad A = \ell w$$

EXPLORE **Find perimeters of rectangles with fixed areas**

STEP 1 ***Draw rectangles*** Draw different rectangles, each with an area of 36 square units. Use lengths of 2, 4, 6, 8, 10, 12, 14, 16, and 18 units.



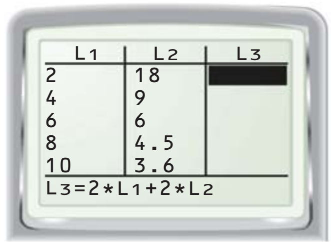
STEP 2 ***Enter data*** Use the STATISTICS menu on a graphing calculator. Enter the rectangle lengths in List 1. Use the keystrokes below to calculate and enter the rectangle widths and perimeters in Lists 2 and 3.

Keystrokes for entering widths in List 2:

36 ÷ 2nd [L1] ENTER

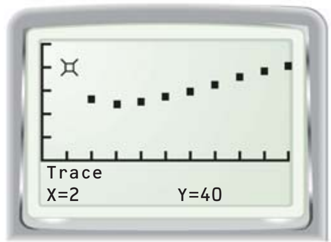
Keystrokes for entering perimeters in List 3:

2 × 2nd [L1] + 2nd 2 × [L2] ENTER



STEP 3 ***Make a scatter plot*** Make a scatter plot using the lengths from List 1 as the x-values and the perimeters from List 3 as the y-values. Choose an appropriate viewing window. Then use the *trace* feature to see the coordinates of each point.

How does the graph show which of your rectangles from Step 1 has the smallest perimeter?



DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Repeat the steps above for rectangles with areas of 64 square units.
2. Based on the Explore and your results from Exercise 1, what do you notice about the shape of the rectangle with the smallest perimeter?

1.7 Find Perimeter, Circumference, and Area

Before You classified polygons.

Now You will find dimensions of polygons.

Why? So you can use measures in science, as in Ex. 46.

Key Vocabulary
- **perimeter,** *p. 923*
- **circumference,** *p. 923*
- **area,** *p. 923*
- **diameter,** *p. 923*
- **radius,** *p. 923*

Recall that *perimeter* is the distance around a figure, *circumference* is the distance around a circle, and *area* is the amount of surface covered by a figure. Perimeter and circumference are measured in units of length, such as meters (m) and feet (ft). Area is measured in square units, such as square meters (m^2) and square feet (ft^2).

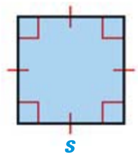
KEY CONCEPT — *For Your Notebook*

Formulas for Perimeter *P*, Area *A*, and Circumference *C*

Square
side length s

$P = 4s$

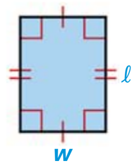
$A = s^2$



Rectangle
length ℓ and width w

$P = 2\ell + 2w$

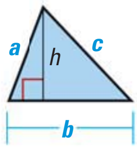
$A = \ell w$



Triangle
side lengths a, b, and c, base b, and height h

$P = a + b + c$

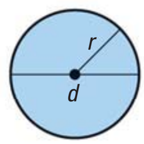
$A = \frac{1}{2}bh$



Circle
diameter d and radius r

$C = \pi d = 2\pi r$

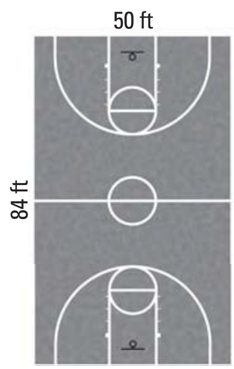
$A = \pi r^2$



Pi (π) is the ratio of a circle's circumference to its diameter.

EXAMPLE 1 Find the perimeter and area of a rectangle

BASKETBALL Find the perimeter and area of the rectangular basketball court shown.



Perimeter

$P = 2\ell + 2w$

$= 2(84) + 2(50)$

$= 268$

Area

$A = \ell w$

$= 84(50)$

$= 4200$

▶ The perimeter is 268 feet and the area is 4200 square feet.

EXAMPLE 2 Find the circumference and area of a circle

TEAM PATCH You are ordering circular cloth patches for your soccer team's uniforms. Find the approximate circumference and area of the patch shown.

Solution

First find the radius. The diameter is 9 centimeters, so the radius is $\frac{1}{2}(9) = 4.5$ centimeters.

Then find the circumference and area.
Use 3.14 to approximate the value of π.

$$C = 2\pi r \approx 2(3.14)(4.5) = 28.26$$

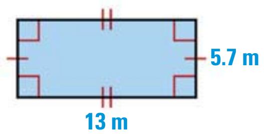
$$A = \pi r^2 \approx 3.14(4.5)^2 = 63.585$$

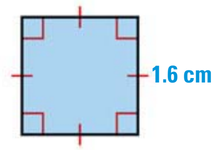
▶ The circumference is about 28.3 cm. The area is about 63.6 cm^2.

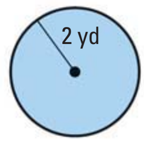
APPROXIMATE π
The approximations 3.14 and $\frac{22}{7}$ are commonly used as approximations for the irrational number π. Unless told otherwise, use 3.14 for π.

✓ GUIDED PRACTICE for Examples 1 and 2

Find the area and perimeter (or circumference) of the figure. If necessary, round to the nearest tenth.

1.


2.


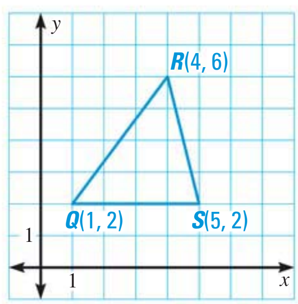
3.


★ EXAMPLE 3 Standardized Test Practice

Triangle *QRS* has vertices *Q*(1, 2), *R*(4, 6), and *S*(5, 2). What is the approximate perimeter of triangle *QRS*?

Ⓐ 8 units Ⓑ 8.3 units Ⓒ 13.1 units Ⓓ 25.4 units

Solution



First draw triangle *QRS* in a coordinate plane. Find the side lengths. Use the Distance Formula to find *QR* and *RS*.

$$QS = |5 - 1| = 4 \text{ units}$$

$$QR = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{25} = 5 \text{ units}$$

$$RS = \sqrt{(5-4)^2 + (2-6)^2} = \sqrt{17} \approx 4.1 \text{ units}$$

Then find the perimeter.

$$P = QS + QR + RS \approx 4 + 5 + 4.1 = 13.1 \text{ units}$$

▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

AVOID ERRORS
Write down your calculations to make sure you do not make a mistake substituting values in the Distance Formula.

EXAMPLE 4 Solve a multi-step problem

SKATING RINK An ice-resurfacing machine is used to smooth the surface of the ice at a skating rink. The machine can resurface about 270 square yards of ice in one minute.

About how many minutes does it take the machine to resurface a rectangular skating rink that is 200 feet long and 90 feet wide?

ANOTHER WAY

For an alternative method for solving the problem in Example 4, turn to page 57 for the **Problem Solving Workshop**.

Solution

The machine can resurface the ice at a rate of 270 square yards per minute. So, the amount of time it takes to resurface the skating rink depends on its area.

STEP 1 **Find** the area of the rectangular skating rink.

Area $= \ell w = 200(90) = 18{,}000 \text{ ft}^2$

The resurfacing rate is in square yards per minute. Rewrite the area of the rink in square yards. There are 3 feet in 1 yard, and $3^2 = 9$ square feet in 1 square yard.

$18{,}000 \cancel{\text{ft}^2} \cdot \dfrac{1 \text{ yd}^2}{9 \cancel{\text{ft}^2}} = 2000 \text{ yd}^2$ **Use unit analysis.**

STEP 2 **Write** a verbal model to represent the situation. Then write and solve an equation based on the verbal model.

Let t represent the total time (in minutes) needed to resurface the skating rink.

Area of rink (yd²) = Resurfacing rate (yd² per min) × Total time (min)

$2000 = 270 \cdot t$ **Substitute.**

$7.4 \approx t$ **Divide each side by 270.**

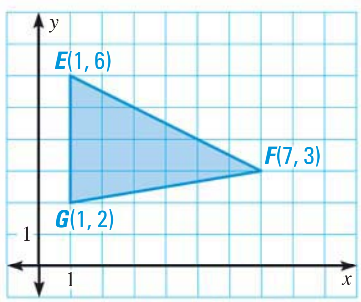
▶ It takes the ice-resurfacing machine about 7 minutes to resurface the skating rink.

✓ GUIDED PRACTICE for Examples 3 and 4

4. Describe how to find the height from F to $\overline{EG}$ in the triangle at the right.

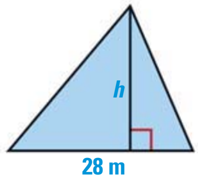
5. Find the perimeter and the area of the triangle shown at the right.

6. **WHAT IF?** In Example 4, suppose the skating rink is twice as long and twice as wide. Will it take an ice-resurfacing machine twice as long to resurface the skating rink? *Explain* your reasoning.



EXAMPLE 5 Find unknown length

The base of a triangle is 28 meters. Its area is 308 square meters. Find the height of the triangle.



Solution

$A = \frac{1}{2}bh$ Write formula for the area of a triangle.

$308 = \frac{1}{2}(28)h$ Substitute 308 for *A* and 28 for *b*.

$22 = h$ Solve for *h*.

▶ The height is 22 meters.

✓ GUIDED PRACTICE for Example 5

7. The area of a triangle is 64 square meters, and its height is 16 meters. Find the length of its base.

1.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 21, and 41

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 19, 26, 38, and 45

◆ = **MULTIPLE REPRESENTATIONS** Ex. 44

SKILL PRACTICE

1. VOCABULARY How are the diameter and radius of a circle related?

2. ★ **WRITING** *Describe* a real-world situation in which you would need to find a perimeter, and a situation in which you would need to find an area. What measurement units would you use in each situation?

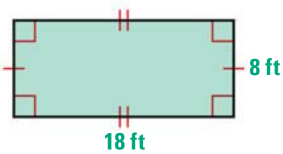
EXAMPLE 1 on p. 49 for Exs. 3–10

3. ERROR ANALYSIS *Describe* and correct the error made in finding the area of a triangle with a height of 9 feet and a base of 52 feet.

$A = 52(9) = 468\ ft^2$ ✗

PERIMETER AND AREA **Find the perimeter and area of the shaded figure.**

4.



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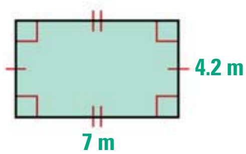

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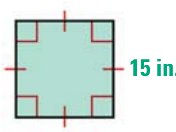

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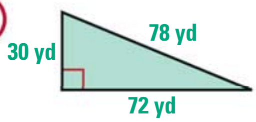

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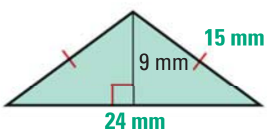

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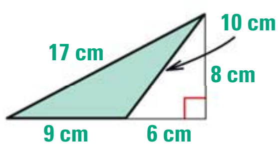


Animated Geometry at classzone.com

10. **DRAWING A DIAGRAM** The base of a triangle is 32 feet. Its height is $16\frac{1}{2}$ feet. Sketch the triangle and find its area.

EXAMPLE 2
on p. 50
for Exs. 11–15

CIRCUMFERENCE AND AREA **Use the given diameter d or radius r to find the circumference and area of the circle. Round to the nearest tenth.**

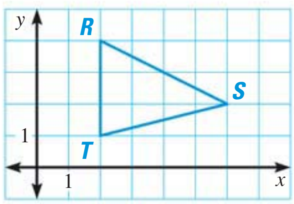
11. $d = 27$ cm
12. $d = 5$ in.
13. $r = 12.1$ cm
14. $r = 3.9$ cm

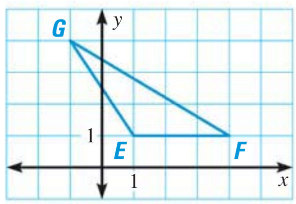


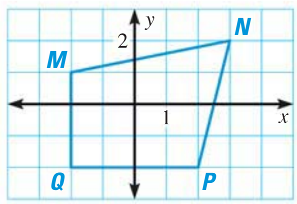
15. **DRAWING A DIAGRAM** The diameter of a circle is 18.9 centimeters. Sketch the circle and find its circumference and area. Round your answers to the nearest tenth.

EXAMPLE 3
on p. 50
for Exs. 16–19

DISTANCE FORMULA **Find the perimeter of the figure. Round to the nearest tenth of a unit.**

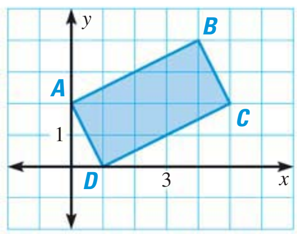
16.


17.


18.


19. ★ **MULTIPLE CHOICE** What is the approximate area (in square units) of the rectangle shown at the right?

Ⓐ 6.7 Ⓑ 8.0
Ⓒ 9.0 Ⓓ 10.0



EXAMPLE 4
on p. 51
for Exs. 20–26

CONVERTING UNITS **Copy and complete the statement.**

20. 187 cm^2 = __?__ m^2
21. 13 ft^2 = __?__ yd^2
22. 18 $in.^2$ = __?__ ft^2
23. 8 km^2 = __?__ m^2
24. 12 yd^2 = __?__ ft^2
25. 24 ft^2 = __?__ $in.^2$

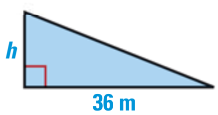
26. ★ **MULTIPLE CHOICE** A triangle has an area of 2.25 square feet. What is the area of the triangle in square inches?

Ⓐ 27 $in.^2$ Ⓑ 54 $in.^2$ Ⓒ 144 $in.^2$ Ⓓ 324 $in.^2$

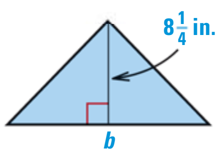
EXAMPLE 5
on p. 52
for Exs. 27–30

UNKNOWN MEASURES **Use the information about the figure to find the indicated measure.**

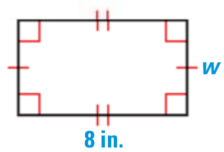
27. Area = 261 m^2
Find the height h.



28. Area = 66 $in.^2$
Find the base b.



29. Perimeter = 25 in.
Find the width w.



30. **UNKNOWN MEASURE** The width of a rectangle is 17 inches. Its perimeter is 102 inches. Find the length of the rectangle.

31. **xy ALGEBRA** The area of a rectangle is 18 square inches. The length of the rectangle is twice its width. Find the length and width of the rectangle.

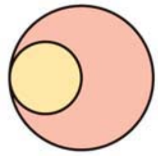
32. **xy ALGEBRA** The area of a triangle is 27 square feet. Its height is three times the length of its base. Find the height and base of the triangle.

33. **xy ALGEBRA** Let x represent the side length of a square. Find a regular polygon with side length x whose perimeter is twice the perimeter of the square. Find a regular polygon with side length x whose perimeter is three times the length of the square. *Explain* your thinking.

FINDING SIDE LENGTHS Find the side length of the square with the given area. Write your answer as a radical in simplest form.

34. $A = 184\ \text{cm}^2$ 35. $A = 346\ \text{in.}^2$ 36. $A = 1008\ \text{mi}^2$ 37. $A = 1050\ \text{km}^2$

38. ★ **SHORT RESPONSE** In the diagram, the diameter of the yellow circle is half the diameter of the red circle. What fraction of the area of the red circle is *not* covered by the yellow circle? *Explain*.



39. **CHALLENGE** The area of a rectangle is $30\ \text{cm}^2$ and its perimeter is 26 cm. Find the length and width of the rectangle.

PROBLEM SOLVING

EXAMPLES 1 and 2 on pp. 49–50 for Exs. 40–41

40. **WATER LILIES** The giant Amazon water lily has a lily pad that is shaped like a circle. Find the circumference and area of a lily pad with a diameter of 60 inches. Round your answers to the nearest tenth.

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41. **LAND** You are planting grass on a rectangular plot of land. You are also building a fence around the edge of the plot. The plot is 45 yards long and 30 yards wide. How much area do you need to cover with grass seed? How many feet of fencing do you need?

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EXAMPLE 4 on p. 51 for Ex. 42

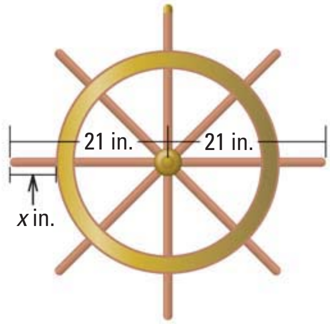
42. **MULTI-STEP PROBLEM** Chris is installing a solar panel. The maximum amount of power the solar panel can generate in a day depends in part on its area. On a sunny day in the city where Chris lives, each square meter of the panel can generate up to 125 watts of power. The flat rectangular panel is 84 centimeters long and 54 centimeters wide.

a. Find the area of the solar panel in square meters.

b. What is the maximum amount of power (in watts) that the panel could generate if its area was 1 square meter? 2 square meters? *Explain*.

c. Estimate the maximum amount of power Chris's solar panel can generate. *Explain* your reasoning.

43. **MULTI-STEP PROBLEM** The eight spokes of a ship's wheel are joined at the wheel's center and pass through a large wooden circle, forming handles on the outside of the circle. From the wheel's center to the tip of the handle, each spoke is 21 inches long.



a. The circumference of the outer edge of the large wooden circle is 94 inches. Find the radius of the outer edge of the circle to the nearest inch.

b. Find the length x of a handle on the wheel. *Explain*.

44. **MULTIPLE REPRESENTATIONS** Let x represent the length of a side of a square. Let y_1 and y_2 represent the perimeter and area of that square.

a. **Making a Table** Copy and complete the table.

Length, x	1	2	5	10	25
Perimeter, y_1	?	?	?	?	?
Area, y_2	?	?	?	?	?

b. **Making a Graph** Use the completed table to write two sets of ordered pairs: (x, y_1) and (x, y_2). Graph each set of ordered pairs.

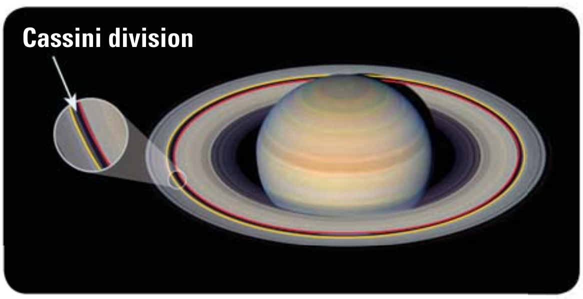
c. **Analyzing Data** *Describe* any patterns you see in the table from part (a) and in the graphs from part (b).

45. ★ **EXTENDED RESPONSE** The photograph at the right shows the Crown Fountain in Chicago, Illinois. At this fountain, images of faces appear on a large screen. The images are created by light-emitting diodes (LEDs) that are clustered in groups called modules. The LED modules are arranged in a rectangular grid.

a. The rectangular grid is approximately 7 meters wide and 15.2 meters high. Find the area of the grid.

b. Suppose an LED module is a square with a side length of 4 centimeters. How many rows and how many columns of LED modules would be needed to make the Crown Fountain screen? *Explain* your reasoning.

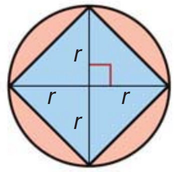
46. **ASTRONOMY** The diagram shows a gap in Saturn's circular rings. This gap is known as the *Cassini division*. In the diagram, the red circle represents the ring that borders the inside of the Cassini division. The yellow circle represents the ring that borders the outside of the division.



a. The radius of the red ring is 115,800 kilometers. The radius of the yellow ring is 120,600 kilometers. Find the circumference of the red ring and the circumference of the yellow ring. Round your answers to the nearest hundred kilometers.

b. Compare the circumferences of the two rings. About how many kilometers greater is the yellow ring's circumference than the red ring's circumference?

47. **CHALLENGE** In the diagram at the right, how many times as great is the area of the circle as the area of the square? *Explain* your reasoning.



48. **ALGEBRA** You have 30 yards of fencing with which to make a rectangular pen. Let x be the length of the pen.

  a. Write an expression for the width of the pen in terms of x. Then write a formula for the area y of the pen in terms of x.

  b. You want the pen to have the greatest possible area. What length and width should you use? *Explain* your reasoning.

MIXED REVIEW

PREVIEW
Prepare for Lesson 2.1 in Exs. 49–50.

49. Use the equation $y = 2x + 1$ to copy and complete the table of values. *(p. 884)*

x	1	2	3	4	5
y	?	?	?	?	?

50. Each number in a pattern is 6 less than the previous number. The first number in the pattern is 100. Write the next three numbers. *(p. 894)*

In Exercises 51 and 52, draw a diagram to represent the problem. Then find the indicated measure. *(p. 42)*

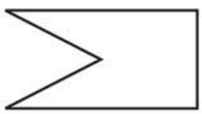
51. The lengths (in inches) of two sides of a regular triangle are given by the expressions $5x + 40$ and $8x - 13$. Find the length of a side of the triangle.

52. The measures of two angles of an equiangular hexagon are $12x°$ and $(10x + 20)°$. Find the measure of an angle of the hexagon.

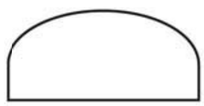
QUIZ *for Lessons 1.6–1.7*

Tell whether the figure is a polygon. If it is not, *explain* why. If it is a polygon, tell whether it is *convex* or *concave*. *(p. 42)*

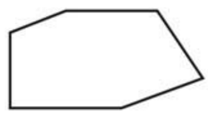
1.



2.

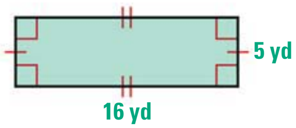


3.

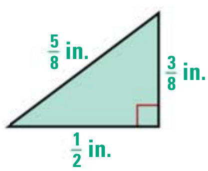


Find the perimeter and area of the shaded figure. *(p. 49)*

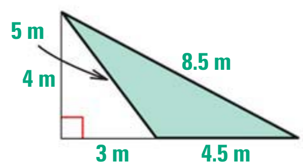
4.



5.



6.



7. **GARDENING** You are spreading wood chips on a rectangular garden. The garden is $3\frac{1}{2}$ yards long and $2\frac{1}{2}$ yards wide. One bag of wood chips covers 10 square feet. How many bags of wood chips do you need? *(p. 49)*

PROBLEM SOLVING WORKSHOP
LESSON 1.7

Using ALTERNATIVE METHODS

Another Way to Solve Example 4, page 51

MULTIPLE REPRESENTATIONS In Example 4 on page 51, you saw how to use an equation to solve a problem about a skating rink. *Looking for a pattern* can help you write an equation.

PROBLEM

SKATING RINK An ice-resurfacing machine is used to smooth the surface of the ice at a skating rink. The machine can resurface about 270 square yards of ice in one minute. About how many minutes does it take the machine to resurface a rectangular skating rink that is 200 feet long and 90 feet wide?

METHOD

Using a Pattern You can use a table to look for a pattern.

STEP 1 **Find** the area of the rink in square yards. In Example 4 on page 51, you found that the area was 2000 square yards.

STEP 2 **Make** a table that shows the relationship between the time spent resurfacing the ice and the area resurfaced. Look for a pattern.

Time (min)	Area resurfaced (yd^2)
1	$1 \cdot 270 = 270$
2	$2 \cdot 270 = 540$
t	$t \cdot 270 = A$

Use the pattern to write an equation for the area A that has been resurfaced after t minutes.

STEP 3 **Use** the equation to find the time t (in minutes) that it takes the machine to resurface 2000 square yards of ice.

$$270t = A$$
$$270t = 2000$$
$$t \approx 7.4$$

▶ It takes about 7 minutes.

PRACTICE

1. **PLOWING** A square field is $\frac{1}{8}$ mile long on each side. A tractor can plow about 180,000 square feet per hour. To the nearest tenth of an hour, about how long does it take to plow the field? (1 mi = 5280 ft.)

2. **ERROR ANALYSIS** To solve Exercise 1 above, a student writes the equation $660 = 180{,}000t$, where t is the number of hours spent plowing. *Describe* and correct the error in the equation.

3. **PARKING LOT** A rectangular parking lot is 110 yards long and 45 yards wide. It costs about \$.60 to pave each square foot of the parking lot with asphalt. About how much will it cost to pave the parking lot?

4. **WALKING** A circular path has a diameter of 120 meters. Your average walking speed is 4 kilometers per hour. About how many minutes will it take you to walk around the path 3 times?

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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Lessons 1.4–1.7

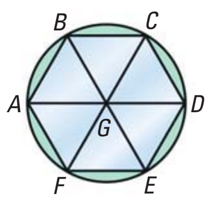
1. **MULTI-STEP PROBLEM** You are covering the rectangular roof of a shed with shingles. The roof is a rectangle that is 4 yards long and 3 yards wide. Asphalt shingles cost \$.75 per square foot and wood shingles cost \$1.15 per square foot.

   a. Find the area of the roof in square feet.

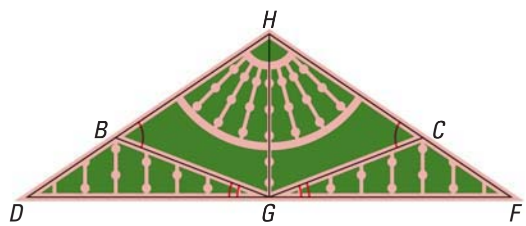
   b. Find the cost of using asphalt shingles and the cost of using wood shingles.

   c. About how much more will you pay to use wood shingles for the roof?

2. **OPEN-ENDED** In the window below, name a convex polygon and a concave polygon. Classify each of your polygons by the number of sides.



3. **EXTENDED RESPONSE** The diagram shows a decoration on a house. In the diagram, $\angle HGD$ and $\angle HGF$ are right angles, $m\angle DGB = 21°$, $m\angle HBG = 55°$, $\angle DGB \cong \angle FGC$, and $\angle HBG \cong \angle HCG$.



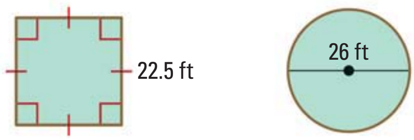
   a. List two pairs of complementary angles and five pairs of supplementary angles.

   b. Find $m\angle FGC$, $m\angle BGH$, and $m\angle HGC$. *Explain* your reasoning.

   c. Find $m\angle HCG$, $m\angle DBG$, and $m\angle FCG$. *Explain* your reasoning.

4. **GRIDDED ANSWER** $\angle 1$ and $\angle 2$ are supplementary angles, and $\angle 1$ and $\angle 3$ are complementary angles. Given $m\angle 1$ is 28° less than $m\angle 2$, find $m\angle 3$ in degrees.

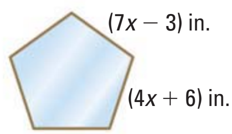
5. **EXTENDED RESPONSE** You use bricks to outline the borders of the two gardens shown below. Each brick is 10 inches long.



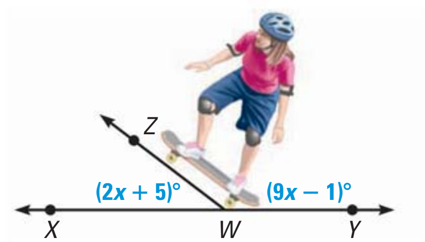
   a. You lay the bricks end-to-end around the border of each garden. How many bricks do you need for each garden? *Explain*.

   b. The bricks are sold in bundles of 100. How many bundles should you buy? *Explain*.

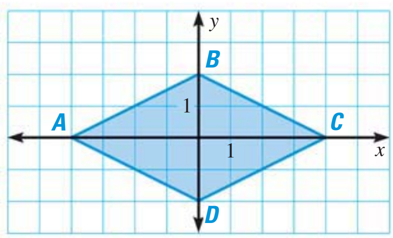
6. **SHORT RESPONSE** The frame of a mirror is a regular pentagon made from pieces of bamboo. Use the diagram to find how many feet of bamboo are used in the frame.



7. **GRIDDED ANSWER** As shown in the diagram, a skateboarder tilts one end of a skateboard. Find $m\angle ZWX$ in degrees.



8. **SHORT RESPONSE** Use the diagram below.



   a. Find the perimeter of quadrilateral $ABCD$.

   b. Find the area of triangle ABC and the area of triangle ADC. What is the area of quadrilateral $ABCD$? *Explain*.

1 CHAPTER SUMMARY

BIG IDEAS *For Your Notebook*

Big Idea 1

Describing Geometric Figures

You learned to identify and classify geometric figures.

Point A

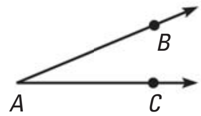
Line *AB* ($\overleftrightarrow{AB}$)

Plane *M*

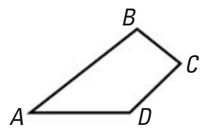
Segment *AB* ($\overline{AB}$)

Ray *AB* ($\overrightarrow{AB}$)

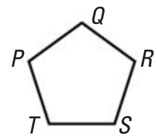
Angle *A*
($\angle A$, $\angle BAC$, or $\angle CAB$)



Polygon



Quadrilateral *ABCD*



Pentagon *PQRST*

Big Idea 2

Measuring Geometric Figures

SEGMENTS You measured segments in the coordinate plane.

Distance Formula

Distance between $A(x_1, y_1)$ and $B(x_2, y_2)$:

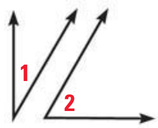
$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Midpoint Formula

Coordinates of midpoint M of $\overline{AB}$, with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$:

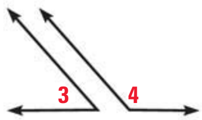
$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

ANGLES You classified angles and found their measures.



Complementary angles

$m\angle 1 + m\angle 2 = 90°$



Supplementary angles

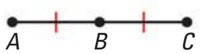
$m\angle 3 + m\angle 4 = 180°$

FORMULAS Perimeter and area formulas are reviewed on page 49.

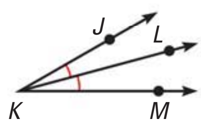
Big Idea 3

Understanding Equality and Congruence

Congruent segments have equal lengths. Congruent angles have equal measures.



$\overline{AB} \cong \overline{BC}$ **and** $AB = BC$



$\angle JKL \cong \angle LKM$ **and** $m\angle JKL = m\angle LKM$

1 CHAPTER REVIEW

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• Multi-Language Glossary
• Vocabulary Practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- undefined terms, *p. 2*
  point, line, plane
- collinear, coplanar points, *p. 2*
- defined terms, *p. 3*
- line segment, endpoints, *p. 3*
- ray, opposite rays, *p. 3*
- intersection, *p. 4*
- postulate, axiom, *p. 9*
- coordinate, *p. 9*
- distance, *p. 9*
- between, *p. 10*
- congruent segments, *p. 11*
- midpoint, *p. 15*
- segment bisector, *p. 15*
- angle, *p. 24*
  sides, vertex, measure
- acute, right, obtuse, straight, *p. 25*
- congruent angles, *p. 26*
- angle bisector, *p. 28*
- construction, *p. 33*
- complementary angles, *p. 35*
- supplementary angles, *p. 35*
- adjacent angles, *p. 35*
- linear pair, *p. 37*
- vertical angles, *p. 37*
- polygon, *p. 42*
  side, vertex
- convex, concave, *p. 42*
- *n*-gon, *p. 43*
- equilateral, equiangular, regular, *p. 43*

VOCABULARY EXERCISES

1. Copy and complete: Points A and B are the __?__ of $\overline{AB}$.

2. Draw an example of a *linear pair*.

3. If Q is between points P and R on $\overleftrightarrow{PR}$, and $PQ = QR$, then Q is the __?__ of $\overline{PR}$.

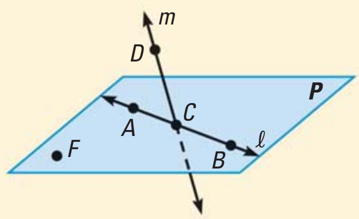
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 1.

1.1 Identify Points, Lines, and Planes *pp. 2–8*

EXAMPLE

Use the diagram shown at the right.



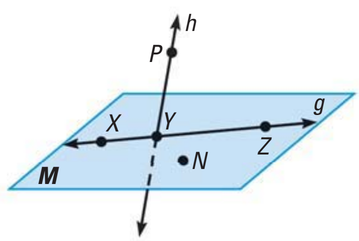
Another name for $\overleftrightarrow{CD}$ is line m.

Points A, B, and C are collinear.

Points A, B, C, and F are coplanar.

EXERCISES

EXAMPLES 1, 2, and 3 on pp. 3–4 for Exs. 4–8



4. Give another name for line g.

5. Name three points that are *not* collinear.

6. Name four points that are coplanar.

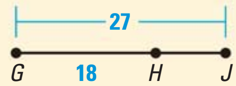
7. Name a pair of opposite rays.

8. Name the intersection of line h and plane M.

1.2 Use Segments and Congruence *pp. 9–14*

EXAMPLE

Find the length of $\overline{HJ}$.



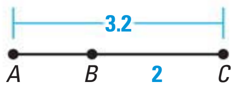
$GJ = GH + HJ$	**Segment Addition Postulate**
$27 = 18 + HJ$	**Substitute 27 for *GJ* and 18 for *GH*.**
$9 = HJ$	**Subtract 18 from each side.**

EXERCISES

EXAMPLES 2, 3, and 4 on pp. 10–11 for Exs. 9–12

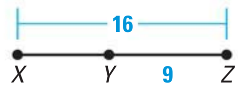
Find the indicated length.

9. Find AB.



10. Find NP.

11. Find XY.



12. The endpoints of $\overline{DE}$ are $D(-4, 11)$ and $E(-4, -13)$. The endpoints of $\overline{GH}$ are $G(-14, 5)$ and $H(-9, 5)$. Are $\overline{DE}$ and $\overline{GH}$ congruent? *Explain.*

1.3 Use Midpoint and Distance Formulas *pp. 15–22*

EXAMPLE

$\overline{EF}$ has endpoints $E(1, 4)$ and $F(3, 2)$. Find (a) the length of $\overline{EF}$ rounded to the nearest tenth of a unit, and (b) the coordinates of the midpoint M of $\overline{EF}$.

a. Use the Distance Formula.

$$EF = \sqrt{(3-1)^2 + (2-4)^2} = \sqrt{2^2 + (-2)^2} = \sqrt{8} \approx 2.8 \text{ units}$$

b. Use the Midpoint Formula.

$$M\left(\frac{1+3}{2}, \frac{4+2}{2}\right) = M(2, 3)$$

EXERCISES

EXAMPLES 2, 3, and 4 on pp. 16–18 for Exs. 13–19

13. Point M is the midpoint of $\overline{JK}$. Find JK when $JM = 6x - 7$ and $MK = 2x + 3$.

In Exercises 14–17, the endpoints of a segment are given. Find the length of the segment rounded to the nearest tenth. Then find the coordinates of the midpoint of the segment.

14. $A(2, 5)$ and $B(4, 3)$

15. $F(1, 7)$ and $G(6, 0)$

16. $H(-3, 9)$ and $J(5, 4)$

17. $K(10, 6)$ and $L(0, -7)$

18. Point $C(3, 8)$ is the midpoint of $\overline{AB}$. One endpoint is $A(-1, 5)$. Find the coordinates of endpoint B.

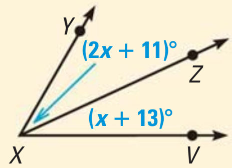
19. The endpoints of $\overline{EF}$ are $E(2, 3)$ and $F(8, 11)$. The midpoint of $\overline{EF}$ is M. Find the length of $\overline{EM}$.

1 CHAPTER REVIEW

1.4 Measure and Classify Angles
pp. 24–32

EXAMPLE

Given that $m\angle YXV$ is 60°, find $m\angle YXZ$ and $m\angle ZXV$.



STEP 1 **Find** the value of x.

$m\angle YXV = m\angle YXZ + m\angle ZXV$ — **Angle Addition Postulate**

$60° = (2x + 11)° + (x + 13)°$ — **Substitute angle measures.**

$x = 12$ — **Solve for x.**

STEP 2 **Evaluate** the given expressions when $x = 12$.

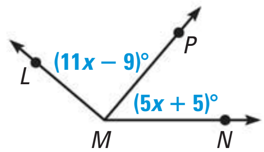
$m\angle YXZ = (2x + 11)° = (2 \cdot 12 + 11)° = 35°$

$m\angle ZXV = (x + 13)° = (12 + 13)° = 25°$

EXERCISES

EXAMPLES 3 and 5 on pp. 26, 28 for Exs. 20–21

20. In the diagram shown at the right, $m\angle LMN = 140°$. Find $m\angle PMN$.



21. $\overrightarrow{VZ}$ bisects $\angle UVW$, and $m\angle UVZ = 81°$. Find $m\angle UVW$. Then classify $\angle UVW$ by its angle measure.

1.5 Describe Angle Pair Relationships
pp. 35–41

EXAMPLE

a. **$\angle 1$ and $\angle 2$ are complementary angles. Given that $m\angle 1 = 37°$, find $m\angle 2$.**

$m\angle 2 = 90° - m\angle 1 = 90° - 37° = 53°$

b. **$\angle 3$ and $\angle 4$ are supplementary angles. Given that $m\angle 3 = 106°$, find $m\angle 4$.**

$m\angle 4 = 180° - m\angle 3 = 180° - 106° = 74°$

EXERCISES

EXAMPLES 2 and 3 on p. 36 for Exs. 22–31

$\angle 1$ and $\angle 2$ are complementary angles. Given the measure of $\angle 1$, find $m\angle 2$.

22. $m\angle 1 = 12°$ **23.** $m\angle 1 = 83°$ **24.** $m\angle 1 = 46°$ **25.** $m\angle 1 = 2°$

$\angle 3$ and $\angle 4$ are supplementary angles. Given the measure of $\angle 3$, find $m\angle 4$.

26. $m\angle 3 = 116°$ **27.** $m\angle 3 = 56°$ **28.** $m\angle 3 = 89°$ **29.** $m\angle 3 = 12°$

30. $\angle 1$ and $\angle 2$ are complementary angles. Find the measures of the angles when $m\angle 1 = (x - 10)°$ and $m\angle 2 = (2x + 40)°$.

31. $\angle 1$ and $\angle 2$ are supplementary angles. Find the measures of the angles when $m\angle 1 = (3x + 50)°$ and $m\angle 2 = (4x + 32)°$. Then classify $\angle 1$ by its angle measure.

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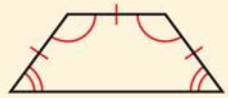
Chapter Review Practice

1.6 Classify Polygons

pp. 42–47

EXAMPLE

Classify the polygon by the number of sides. Tell whether it is equilateral, equiangular, or regular. *Explain*.



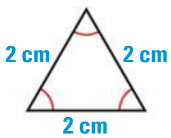
The polygon has four sides, so it is a quadrilateral. It is not equiangular or equilateral, so it is not regular.

EXERCISES

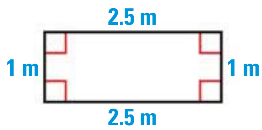
EXAMPLES 2 and 3 on pp. 43–44 for Exs. 32–35

Classify the polygon by the number of sides. Tell whether it is equilateral, equiangular, or regular. *Explain*.

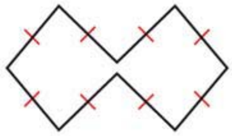
32.



33.



34.



35. Pentagon $ABCDE$ is a regular polygon. The length of $\overline{BC}$ is represented by the expression $5x - 4$. The length of $\overline{DE}$ is represented by the expression $2x + 11$. Find the length of $\overline{AB}$.

1.7 Find Perimeter, Circumference, and Area

pp. 49–56

EXAMPLE

The diameter of a circle is 10 feet. Find the circumference and area of the circle. Round to the nearest tenth.

The radius is half of the length of the diameter, so $r = \frac{1}{2}(10) = 5$ ft.

Circumference	**Area**
$C = 2\pi r \approx 2(3.14)(5) = 31.4$ ft	$A = \pi r^2 \approx 3.14(5^2) = 78.5\text{ ft}^2$

EXERCISES

EXAMPLES 1, 2, and 3 on pp. 49–50 for Exs. 36–40

In Exercises 36–38, find the perimeter (or circumference) and area of the figure described. If necessary, round to the nearest tenth.

36. Circle with diameter 15.6 meters

37. Rectangle with length $4\frac{1}{2}$ inches and width $2\frac{1}{2}$ inches

38. Triangle with vertices $U(1, 2)$, $V(-8, 2)$, and $W(-4, 6)$

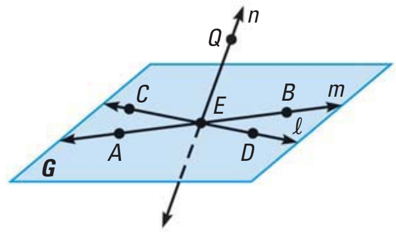
39. The height of a triangle is 18.6 meters. Its area is 46.5 square meters. Find the length of the triangle's base.

40. The area of a circle is 320 square meters. Find the radius of the circle. Then find the circumference. Round your answers to the nearest tenth.

1 CHAPTER TEST

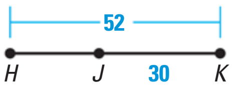
Use the diagram to decide whether the statement is *true* or *false*.

1. Point A lies on line m.
2. Point D lies on line n.
3. Points B, C, E, and Q are coplanar.
4. Points C, E, and B are collinear.
5. Another name for plane G is plane QEC.

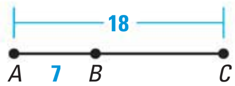


Find the indicated length.

6. Find HJ.



7. Find BC.



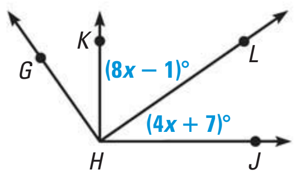
8. Find XZ.

In Exercises 9–11, find the distance between the two points.

9. $T(3, 4)$ and $W(2, 7)$
10. $C(5, 10)$ and $D(6, -1)$
11. $M(-8, 0)$ and $N(-1, 3)$

12. The midpoint of $\overline{AB}$ is $M(9, 7)$. One endpoint is $A(3, 9)$. Find the coordinates of endpoint B.
13. Line t bisects $\overline{CD}$ at point M, $CM = 3x$, and $MD = 27$. Find CD.

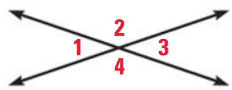
In Exercises 14 and 15, use the diagram.



14. Trace the diagram and extend the rays. Use a protractor to measure $\angle GHJ$. Classify it as *acute, obtuse, right,* or *straight*.
15. Given $m\angle KHJ = 90°$, find $m\angle LHJ$.

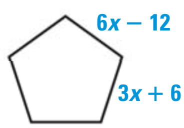
16. The measure of $\angle QRT$ is 154°, and $\overrightarrow{RS}$ bisects $\angle QRT$. What are the measures of $\angle QRS$ and $\angle SRT$?

In Exercises 17 and 18, use the diagram at the right.



17. Name four linear pairs.
18. Name two pairs of vertical angles.

19. The measure of an angle is 64°. What is the measure of its complement? What is the measure of its supplement?
20. A convex polygon has half as many sides as a concave 10-gon. Draw the concave polygon and the convex polygon. Classify the convex polygon by the number of sides it has.
21. Find the perimeter of the regular pentagon shown at the right.



22. **CARPET** You can afford to spend \$300 to carpet a room that is 5.5 yards long and 4.5 yards wide. The cost to purchase and install the carpet you like is \$1.50 per square foot. Can you afford to buy this carpet? *Explain.*

1 ALGEBRA REVIEW

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SOLVE LINEAR EQUATIONS AND WORD PROBLEMS

EXAMPLE 1 Solve linear equations

Solve the equation $-3(x + 5) + 4x = 25$.

$-3(x + 5) + 4x = 25$	Write original equation.
$-3x - 15 + 4x = 25$	Use the Distributive Property.
$x - 15 = 25$	Group and combine like terms.
$x = 40$	Add 15 to each side.

EXAMPLE 2 Solve a real-world problem

MEMBERSHIP COSTS A health club charges an initiation fee of \$50. Members then pay \$45 per month. You have \$400 to spend on a health club membership. For how many months can you afford to be a member?

Let n represent the number of months you can pay for a membership.

\$400 = Initiation fee + (Monthly Rate × Number of Months)

$400 = 50 + 45n$	Substitute.
$350 = 45n$	Subtract 50 from each side.
$7.8 = n$	Divide each side by 45.

▶ You can afford to be a member at the health club for 7 months.

EXERCISES

EXAMPLE 1 for Exs. 1–9

Solve the equation.

1. $9y + 1 - y = 49$
2. $5z + 7 + z = -8$
3. $-4(2 - t) = -16$
4. $7a - 2(a - 1) = 17$
5. $\frac{4x}{3} + 2(3 - x) = 5$
6. $\frac{2x - 5}{7} = 4$
7. $9c - 11 = -c + 29$
8. $2(0.3r + 1) = 23 - 0.1r$
9. $5(k + 2) = 3(k - 4)$

EXAMPLE 2 for Exs. 10–12

10. **GIFT CERTIFICATE** You have a \$50 gift certificate at a store. You want to buy a book that costs \$8.99 and boxes of stationery for your friends. Each box costs \$4.59. How many boxes can you buy with your gift certificate?

11. **CATERING** It costs \$350 to rent a room for a party. You also want to hire a caterer. The caterer charges \$8.75 per person. How many people can come to the party if you have \$500 to spend on the room and the caterer?

12. **JEWELRY** You are making a necklace out of glass beads. You use one bead that is $1\frac{1}{2}$ inches long and smaller beads that are each $\frac{3}{4}$ inch long. The necklace is 18 inches long. How many smaller beads do you need?

1 ★ Standardized TEST PREPARATION

SHORT RESPONSE QUESTIONS

Scoring Rubric

Full Credit
- solution is complete and correct

Partial Credit
- solution is complete but has errors,
or
- solution is without error but incomplete

No Credit
- no solution is given,
or
- solution makes no sense

PROBLEM

You want to rent portable flooring to set up a dance floor for a party. The table below shows the cost of renting portable flooring from a local company. You want to have a rectangular dance floor that is 5 yards long and 4 yards wide. How much will it cost to rent flooring? *Explain* your reasoning.

If the floor area is . . .	Then the cost is . . .
less than 100 square feet	\$6.50 per square foot
between 100 and 200 square feet	\$6.25 per square foot

Below are sample solutions to the problem. Read each solution and the comments in blue to see why the sample represents full credit, partial credit, or no credit.

SAMPLE 1: Full credit solution

Find the area of the dance floor. Area $= \ell w = 5(4) = 20 \text{ yd}^2$.

Then convert this area to square feet. There are $3^2 = 9 \text{ ft}^2$ in 1 yd^2.

The reasoning is correct, and the computations are accurate.

$$20 \cancel{\text{yd}^2} \cdot \frac{9 \text{ ft}^2}{1 \cancel{\text{yd}^2}} = 180 \text{ ft}^2$$

Because 180 ft^2 is between 100 ft^2 and 200 ft^2, the price of flooring is \$6.25 per square foot. Multiply the price per square foot by the area.

$$\text{Total cost} = \frac{\$6.25}{1 \cancel{\text{ft}^2}} \cdot 180 \cancel{\text{ft}^2} = \$1125$$

It will cost \$1125 to rent flooring.

The answer is correct.

SAMPLE 2: Partial credit solution

The area of the dance floor is $5(4) = 20$ square yards. Convert this area to square feet. There are 3 feet in 1 yard.

The reasoning is correct, but an incorrect conversion leads to an incorrect answer.

$$20 \cancel{\text{yd}^2} \cdot \frac{3 \text{ ft}^2}{1 \cancel{\text{yd}^2}} = 60 \text{ ft}^2$$

The flooring will cost \$6.50 per square foot because 60 ft^2 is less than 100 ft^2. To find the total cost, multiply the area by the cost per square foot.

$$60 \cancel{\text{ft}^2} \cdot \frac{\$6.50}{1 \cancel{\text{ft}^2}} = \$390$$

It will cost \$390 to rent flooring.

SAMPLE 3: Partial credit solution

The computations and the answer are correct, but the reasoning is incomplete.

The area of the room is 180 ft^2, so the flooring price is \$6.25. The total cost is $180 \cdot 6.25 = \$1125$.

It will cost \$1125 to rent flooring.

SAMPLE 4: No credit solution

The student's reasoning is incorrect, and the answer is incorrect.

Floor area $= 4 \times 5 = 20$.

Cost $= 20 \times \$650 = \$13{,}000$.

It will cost \$13,000 to rent flooring.

PRACTICE Apply the Scoring Rubric

Use the rubric on page 66 to score the solution to the problem below as *full credit*, *partial credit*, or *no credit*. *Explain* your reasoning.

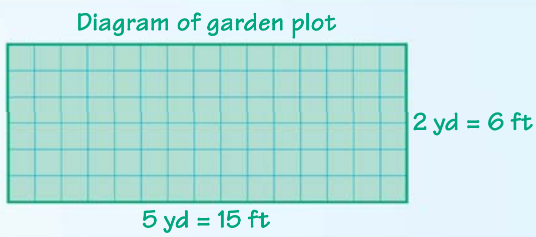
PROBLEM You have 450 daffodil bulbs. You divide a 5 yard by 2 yard rectangular garden into 1 foot by 1 foot squares. You want to plant the same number of bulbs in each square. How many bulbs should you plant in each square? *Explain* your reasoning.

1. First find the area of the plot in square feet. There are 3 feet in 1 yard, so the length is $5(3) = 15$ feet, and the width is $2(3) = 6$ feet. The area is $15(6) = 90$ square feet. The garden plot can be divided into 90 squares with side length 1 foot. Divide 450 by 90 to get 5 bulbs in each square.

2. The area of the garden plot is $5(2) = 10$ square yards. There are 3 feet in 1 yard, so you can multiply 10 square yards by 3 to get an area of 30 square feet. You can divide the garden plot into 30 squares. To find how many bulbs per square, divide 450 bulbs by 30 to get 15 bulbs.

3. Divide 450 by the area of the plot: 450 bulbs $\div$ 10 yards $= 45$ bulbs. You should plant 45 bulbs in each square.

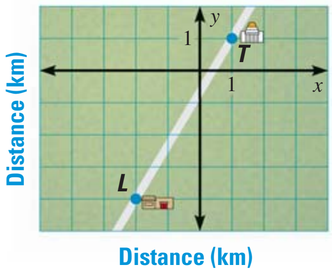
4. Multiply the length and width by 3 feet to convert yards to feet. The area is 15 ft $\times$ 6 ft $= 90$ ft^2. Divide the garden into 90 squares.



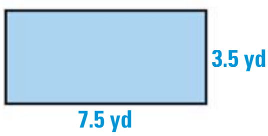
SHORT RESPONSE

1. It costs \$2 per square foot to refinish a hardwood floor if the area is less than 300 square feet, and \$1.75 per square foot if the area is greater than or equal to 300 square feet. How much does it cost to refinish a rectangular floor that is 6 yards long and 4.5 yards wide? *Explain* your reasoning.

2. As shown below, the library (point L) and the Town Hall (point T) are on the same straight road. Your house is on the same road, halfway between the library and the Town Hall. Let point H mark the location of your house. Find the coordinates of H and the approximate distance between the library and your house. *Explain* your reasoning.

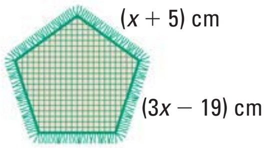


3. The water in a swimming pool evaporates over time if the pool is not covered. In one year, a swimming pool can lose about 17.6 gallons of water for every square foot of water that is exposed to air. About how much water would evaporate in one year from the surface of the water in the pool shown? *Explain* your reasoning.



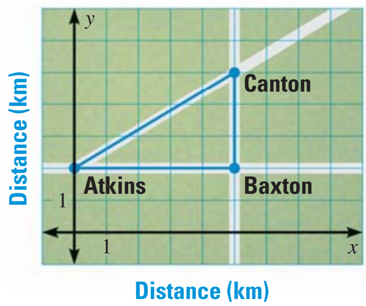
4. A company is designing a cover for a circular swimming pool. The diameter of the pool is 20 feet. The material for the cover costs \$4 per square yard. About how much will it cost the company to make the pool cover? *Explain* your reasoning.

5. You are making a mat with a fringed border. The mat is shaped like a regular pentagon, as shown below. Fringe costs \$1.50 per yard. How much will the fringe for the mat cost? *Explain* your reasoning.

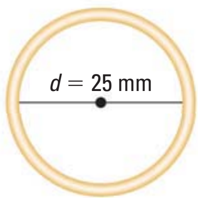


6. Angles A and B are complementary angles, $m\angle A = (2x - 4)°$, and $m\angle B = (4x - 8)°$. Find the measure of the supplement of $\angle B$. *Explain* your reasoning.

7. As shown on the map, you have two ways to drive from Atkins to Canton. You can either drive through Baxton, or you can drive directly from Atkins to Canton. About how much shorter is the trip from Atkins to Canton if you do not go through Baxton? *Explain* your reasoning.



8. A jeweler is making pairs of gold earrings. For each earring, the jeweler will make a circular hoop like the one shown below. The jeweler has 2 meters of gold wire. How many pairs of gold hoops can the jeweler make? *Justify* your reasoning.



MULTIPLE CHOICE

9. The midpoint of $\overline{AB}$ is $M(4, -2)$. One endpoint is $A(-2, 6)$. What is the length of $\overline{AB}$?

Ⓐ 5 units

Ⓑ 10 units

Ⓒ 20 units

Ⓓ 28 units

10. The perimeter of a rectangle is 85 feet. The length of the rectangle is 4 feet more than its width. Which equation can be used to find the width w of the rectangle?

Ⓐ $85 = 2(w + 4)$

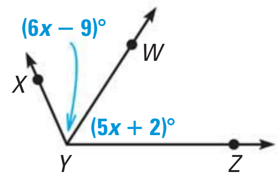
Ⓑ $85 = 2w + 2(w - 4)$

Ⓒ $85 = 2(2w + 4)$

Ⓓ $85 = w(w + 4)$

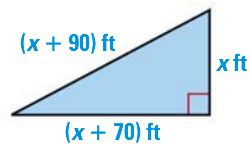
GRIDDED ANSWER

11. In the diagram, $\overrightarrow{YW}$ bisects $\angle XYZ$. Find $m\angle XYZ$ in degrees.



12. Angles A and B are complements, and the measure of $\angle A$ is 8 times the measure of $\angle B$. Find the measure (in degrees) of the supplement of $\angle A$.

13. The perimeter of the triangle shown is 400 feet. Find its area in square feet.



EXTENDED RESPONSE

14. The athletic director at a college wants to build an indoor playing field. The playing field will be twice as long as it is wide. Artificial turf costs \$4 per square foot. The director has \$50,000 to spend on artificial turf.

a. What is the largest area that the director can afford to cover with artificial turf? *Explain.*

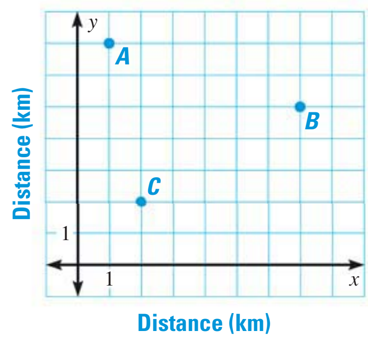
b. Find the approximate length and width of the field to the nearest foot.

15. An artist uses black ink to draw the outlines of 30 circles and 25 squares, and red ink to fill in the area of each circle and square. The diameter of each circle is 1 inch, and the side length of each square is 1 inch. Which group of drawings uses more black ink, the *circles* or the *squares*? Which group of drawings uses more red ink? *Explain.*

16. Points A and C represent the positions of two boats in a large lake. Point B represents the position of a fixed buoy.

a. Find the distance from each boat to the buoy.

b. The boat at point A travels toward the buoy in a straight line at a rate of 5 kilometers per hour. The boat at point C travels to the buoy at a rate of 5.2 kilometers per hour. Which boat reaches the buoy first? *Explain.*



2 Reasoning and Proof

Before

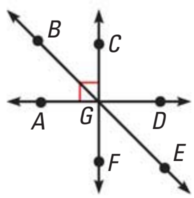
In previous courses and in Chapter 1, you learned the following skills, which you'll use in Chapter 2: naming figures, using notations, drawing diagrams, solving equations, and using postulates.

Prerequisite Skills

VOCABULARY CHECK

Use the diagram to name an example of the described figure.

1. A right angle
2. A pair of vertical angles
3. A pair of supplementary angles
4. A pair of complementary angles



SKILLS AND ALGEBRA CHECK

***Describe* what the notation means. Draw the figure.** *(Review p. 2 for 2.4.)*

5. $\overline{AB}$ **6.** $\overleftrightarrow{CD}$ **7.** EF **8.** $\overrightarrow{GH}$

Solve the equation. *(Review p. 875 for 2.5.)*

9. $3x + 5 = 20$ **10.** $4(x - 7) = -12$ **11.** $5(x + 8) = 4x$

Name the postulate used. Draw the figure. *(Review pp. 9, 24 for 2.5.)*

12. $m\angle ABD + m\angle DBC = m\angle ABC$ **13.** $ST + TU = SU$

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 2, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 133. You will also use the key vocabulary listed below.

Big Ideas

1. **Use inductive and deductive reasoning**
2. **Understanding geometric relationships in diagrams**
3. **Writing proofs of geometric relationships**

KEY VOCABULARY

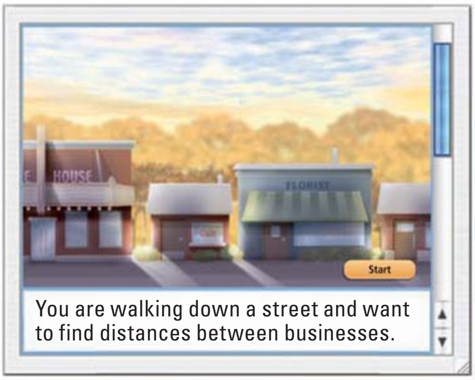
- conjecture, *p. 73*
- inductive reasoning, *p. 73*
- counterexample, *p. 74*
- conditional statement, *p. 79*
  converse, inverse, contrapositive
- if-then form, *p. 79*
  hypothesis, conclusion
- negation, *p. 79*
- equivalent statements, *p. 80*
- perpendicular lines, *p. 81*
- biconditional statement, *p. 82*
- deductive reasoning, *p. 87*
- proof, *p. 112*
- two-column proof, *p. 112*
- theorem, *p. 113*

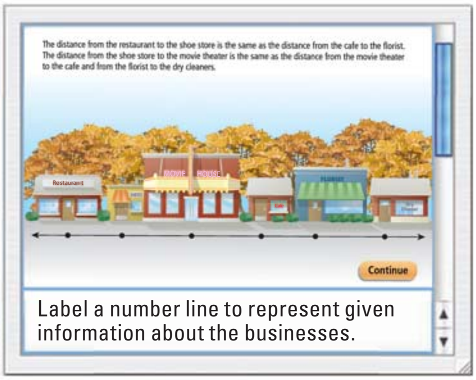
Why?

You can use reasoning to draw conclusions. For example, by making logical conclusions from organized information, you can make a layout of a city street.

Animated Geometry

The animation illustrated below for Exercise 29 on page 119 helps you answer this question: Is the distance from the restaurant to the movie theater the same as the distance from the cafe to the dry cleaners?

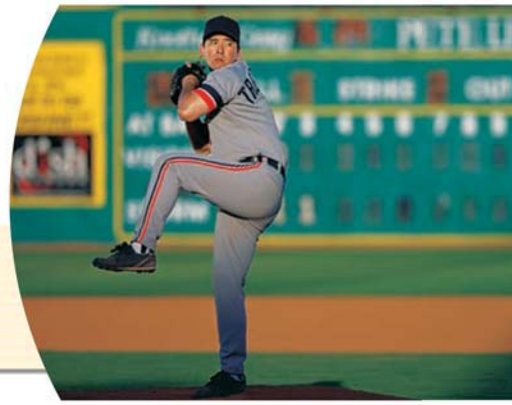




Animated Geometry at classzone.com

Other animations for Chapter 2: pages 72, 81, 88, 97, 106, and 125

2.1 Use Inductive Reasoning



Before You classified polygons by the number of sides.

Now You will describe patterns and use inductive reasoning.

Why? So you can make predictions about baseball, as in Ex. 32.

Key Vocabulary
- conjecture
- inductive reasoning
- counterexample

Geometry, like much of science and mathematics, was developed partly as a result of people recognizing and describing patterns. In this lesson, you will discover patterns yourself and use them to make predictions.

EXAMPLE 1 Describe a visual pattern

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.

Figure 1

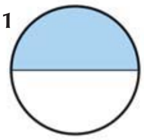



Figure 2

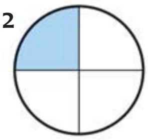
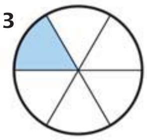



Figure 3


Solution

Each circle is divided into twice as many equal regions as the figure number. Sketch the fourth figure by dividing a circle into eighths. Shade the section just above the horizontal segment at the left.

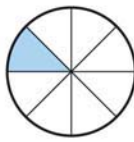


Figure 4

EXAMPLE 2 Describe a number pattern

READ SYMBOLS
The three dots (. . .) tell you that the pattern continues.

Describe the pattern in the numbers −7, −21, −63, −189, . . . and write the next three numbers in the pattern.

Notice that each number in the pattern is three times the previous number.

$$-7, \xrightarrow{\times 3} -21, \xrightarrow{\times 3} -63, \xrightarrow{\times 3} -189, \xrightarrow{\times 3} \ldots$$

▶ Continue the pattern. The next three numbers are −567, −1701, and −5103.

Animated Geometry at classzone.com

✓ GUIDED PRACTICE for Examples 1 and 2

1. Sketch the fifth figure in the pattern in Example 1.
2. *Describe* the pattern in the numbers 5.01, 5.03, 5.05, 5.07, ... Write the next three numbers in the pattern.

INDUCTIVE REASONING A **conjecture** is an unproven statement that is based on observations. You use **inductive reasoning** when you find a pattern in specific cases and then write a conjecture for the general case.

EXAMPLE 3 Make a conjecture

Given five collinear points, make a conjecture about the number of ways to connect different pairs of the points.

Solution

Make a table and look for a pattern. Notice the pattern in how the number of connections increases. You can use the pattern to make a conjecture.

Number of points	1	2	3	4	5
Picture					
Number of connections	0	1	3	6	?

$+1 \quad +2 \quad +3 \quad +?$

▶ **Conjecture** You can connect five collinear points 6 + **4**, or 10 different ways.

EXAMPLE 4 Make and test a conjecture

Numbers such as 3, 4, and 5 are called *consecutive numbers.* Make and test a conjecture about the sum of any three consecutive numbers.

Solution

STEP 1 **Find** a pattern using a few groups of small numbers.

$3 + 4 + 5 = 12 = 4 \cdot 3$ $\quad$ $7 + 8 + 9 = 24 = 8 \cdot 3$

$10 + 11 + 12 = 33 = 11 \cdot 3$ $\quad$ $16 + 17 + 18 = 51 = 17 \cdot 3$

▶ **Conjecture** The sum of any three consecutive integers is three times the second number.

STEP 2 **Test** your conjecture using other numbers. For example, test that it works with the groups −1, 0, 1 and 100, 101, 102.

$-1 + 0 + 1 = 0 = 0 \cdot 3$ ✓ $\quad$ $100 + 101 + 102 = 303 = 101 \cdot 3$ ✓

✓ GUIDED PRACTICE for Examples 3 and 4

3. Suppose you are given seven collinear points. Make a conjecture about the number of ways to connect different pairs of the points.

4. Make and test a conjecture about the sign of the product of any three negative integers.

DISPROVING CONJECTURES To show that a conjecture is true, you must show that it is true for all cases. You can show that a conjecture is false, however, by simply finding one *counterexample*. A **counterexample** is a specific case for which the conjecture is false.

EXAMPLE 5 Find a counterexample

A student makes the following conjecture about the sum of two numbers. Find a counterexample to disprove the student's conjecture.

Conjecture The sum of two numbers is always greater than the larger number.

Solution

To find a counterexample, you need to find a sum that is less than the larger number.

$-2 + -3 = -5$

$-5 \not> -3$

▶ Because a counterexample exists, the conjecture is false.

EXAMPLE 6 Standardized Test Practice

ELIMINATE CHOICES
Because the graph does not show data about boys or the World Cup games, you can eliminate choices A and C.

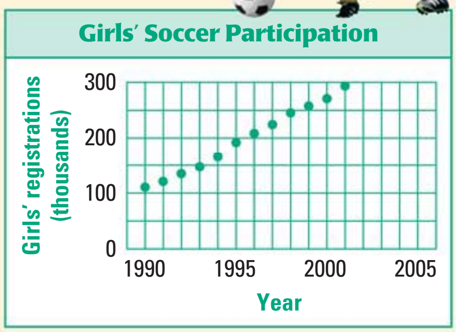
Which conjecture could a high school athletic director make based on the graph at the right?

Ⓐ More boys play soccer than girls.

Ⓑ More girls are playing soccer today than in 1995.

Ⓒ More people are playing soccer today than in the past because the 1994 World Cup games were held in the United States.

Ⓓ The number of girls playing soccer was more in 1995 than in 2001.



Solution

Choices A and C can be eliminated because they refer to facts not presented by the graph. Choice B is a reasonable conjecture because the graph shows an increase from 1990–2001, but does not give any reasons for that increase.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 5 and 6

5. Find a counterexample to show that the following conjecture is false.

   Conjecture The value of x^2 is always greater than the value of x.

6. Use the graph in Example 6 to make a conjecture that *could* be true. Give an explanation that supports your reasoning.

2.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 15, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 5, 19, 22, and 36

◆ = **MULTIPLE REPRESENTATIONS** Ex. 35

SKILL PRACTICE

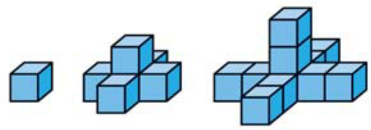
1. **VOCABULARY** Write a definition of *conjecture* in your own words.

2. ★ **WRITING** The word *counter* has several meanings. Look up the word in a dictionary. Identify which meaning helps you understand the definition of *counterexample*.

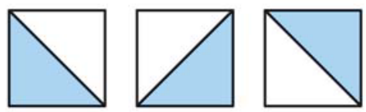
EXAMPLE 1 on p. 72 for Exs. 3–5

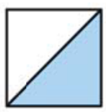
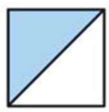
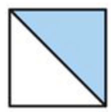
SKETCHING VISUAL PATTERNS **Sketch the next figure in the pattern.**

3. 

4. 

5. ★ **MULTIPLE CHOICE** What is the next figure in the pattern?



Ⓐ 
Ⓑ 
Ⓒ 
Ⓓ 

EXAMPLE 2 on p. 72 for Exs. 6–11

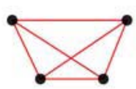
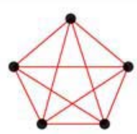
DESCRIBING NUMBER PATTERNS ***Describe* the pattern in the numbers. Write the next number in the pattern.**

6. 1, 5, 9, 13, . . .
7. 3, 12, 48, 192, . . .
8. 10, 5, 2.5, 1.25, . . .
9. 4, 3, 1, −2, . . .
10. $1, \frac{2}{3}, \frac{1}{3}, 0, \ldots$
11. −5, −2, 4, 13, . . .

MAKING CONJECTURES **In Exercises 12 and 13, copy and complete the conjecture based on the pattern you observe in the specific cases.**

EXAMPLE 3 on p. 73 for Ex. 12

12. Given seven noncollinear points, make a conjecture about the number of ways to connect different pairs of the points.

Number of points	3	4	5	6	7
Picture					?
Number of connections	3	6	10	15	?

Conjecture You can connect seven noncollinear points __?__ different ways.

EXAMPLE 4 on p. 73 for Ex. 13

13. Use these sums of odd integers: 3 + 7 = 10, 1 + 7 = 8, 17 + 21 = 38

Conjecture The sum of any two odd integers is __?__.

EXAMPLE 5
on p. 74
for Exs. 14–17

FINDING COUNTEREXAMPLES In Exercises 14–17, show the conjecture is false by finding a counterexample.

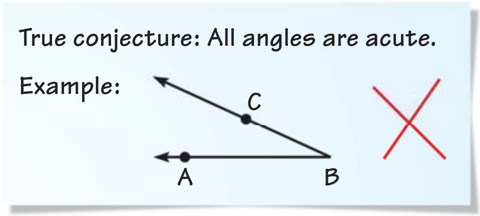
14. If the product of two numbers is positive, then the two numbers must both be positive.

15. The product $(a + b)^2$ is equal to $a^2 + b^2$, for $a \neq 0$ and $b \neq 0$.

16. All prime numbers are odd.

17. If the product of two numbers is even, then the two numbers must both be even.

18. **ERROR ANALYSIS** *Describe* and correct the error in the student's reasoning.



19. ★ **SHORT RESPONSE** *Explain* why only one counterexample is necessary to show that a conjecture is false.

ALGEBRA In Exercises 20 and 21, write a function rule relating x and y.

20.

x	1	2	3
y	−3	−2	−1

21.

x	1	2	3
y	2	4	6

22. ★ **MULTIPLE CHOICE** What is the first number in the pattern?

$\underline{\ ?\ }, \underline{\ ?\ }, \underline{\ ?\ }, 81, 243, 729$

Ⓐ 1 Ⓑ 3 Ⓒ 9 Ⓓ 27

MAKING PREDICTIONS *Describe* a pattern in the numbers. Write the next number in the pattern. Graph the pattern on a number line.

23. $2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \ldots$

24. 1, 8, 27, 64, 125, . . .

25. 0.45, 0.7, 0.95, 1.2, . . .

26. 1, 3, 6, 10, 15, . . .

27. 2, 20, 10, 100, 50, . . .

28. $0.4(6), 0.4(6)^2, 0.4(6)^3, \ldots$

29. **ALGEBRA** Consider the pattern $5, 5r, 5r^2, 5r^3, \ldots$. For what values of r will the values of the numbers in the pattern be increasing? For what values of r will the values of the numbers be decreasing? *Explain*.

30. **REASONING** A student claims that the next number in the pattern 1, 2, 4, . . . is 8, because each number shown is two times the previous number. Is there another description of the pattern that will give the same first three numbers but will lead to a different pattern? *Explain*.

31. **CHALLENGE** Consider the pattern $1, 1\frac{1}{2}, 1\frac{3}{4}, 1\frac{7}{8}, \ldots$.

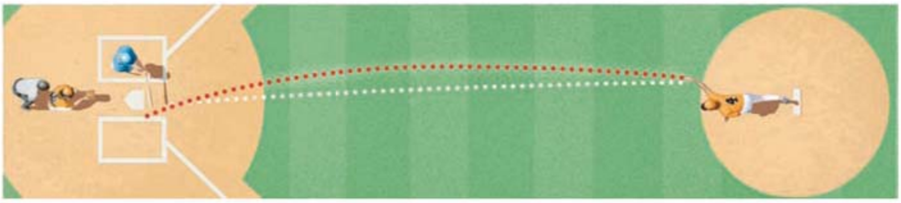
a. *Describe* the pattern. Write the next three numbers in the pattern.

b. What is happening to the values of the numbers?

c. Make a conjecture about later numbers. *Explain* your reasoning.

PROBLEM SOLVING

32. **BASEBALL** You are watching a pitcher who throws two types of pitches, a fastball (F, in white below) and a curveball (C, in red below). You notice that the order of pitches was F, C, F, F, C, C, F, F, F. Assuming that this pattern continues, predict the next five pitches.



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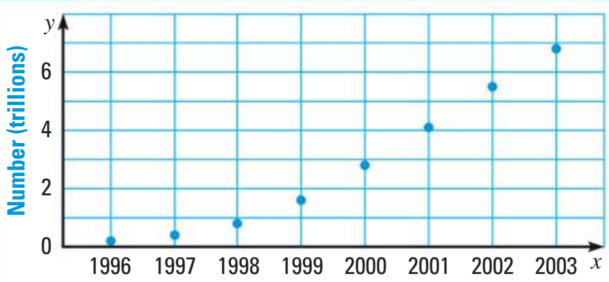
EXAMPLE 6 on p. 74 for Ex. 33

33. **STATISTICS** The scatter plot shows the number of person-to-person e-mail messages sent each year. Make a conjecture that *could* be true. Give an explanation that supports your reasoning.

Worldwide E-mail Messages Sent



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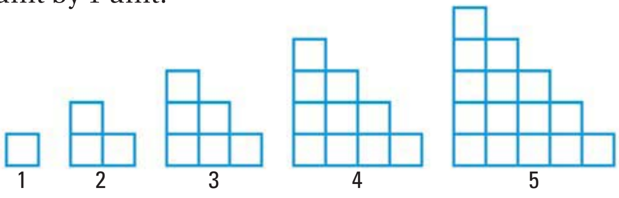
34. **VISUAL REASONING** Use the pattern below. Each figure is made of squares that are 1 unit by 1 unit.



a. Find the distance around each figure. Organize your results in a table.

b. Use your table to *describe* a pattern in the distances.

c. Predict the distance around the 20th figure in this pattern.

35. **MULTIPLE REPRESENTATIONS** Use the given function table relating x and y.

a. **Making a Table** Copy and complete the table.

b. **Drawing a Graph** Graph the table of values.

c. **Writing an Equation** *Describe* the pattern in words and then write an equation relating x and y.

x	y
−3	−5
?	1
5	11
?	15
12	?
15	31

36. ★ **EXTENDED RESPONSE** Your class is selling raffle tickets for $.25 each.

a. Make a table showing your income if you sold 0, 1, 2, 3, 4, 5, 10, or 20 raffle tickets.

b. Graph your results. *Describe* any pattern you see.

c. Write an equation for your income y if you sold x tickets.

d. If your class paid $14 for the raffle prize, at least how many tickets does your class need to sell to make a profit? *Explain*.

e. How many tickets does your class need to sell to make a profit of $50?

37. **FIBONACCI NUMBERS** The *Fibonacci numbers* are shown below. Use the Fibonacci numbers to answer the following questions.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, . . .

a. Copy and complete: After the first two numbers, each number is the __?__ of the __?__ previous numbers.

b. Write the next three numbers in the pattern.

c. **Research** This pattern has been used to describe the growth of the *nautilus shell*. Use an encyclopedia or the Internet to find another real-world example of this pattern.

38. **CHALLENGE** Set A consists of all multiples of 5 greater than 10 and less than 100. Set B consists of all multiples of 8 greater than 16 and less than 100. Show that each conjecture is false by finding a counterexample.

a. Any number in set A is also in set B.

b. Any number less than 100 is either in set A or in set B.

c. No number is in both set A and set B.

MIXED REVIEW

Use the Distributive Property to write the expression without parentheses. *(p. 872)*

39. $4(x - 5)$ 40. $-2(x - 7)$ 41. $(-2n + 5)4$ 42. $x(x + 8)$

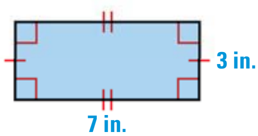
PREVIEW
Prepare for Lesson 2.2 in Exs. 43–46.

You ask your friends how many pets they have. The results are: 1, 5, 1, 0, 3, 6, 4, 2, 10, and 1. Use these data in Exercises 43–46. *(p. 887)*

43. Find the mean. 44. Find the median. 45. Find the mode(s).

46. Tell whether the *mean*, *median*, or *mode(s)* best represent(s) the data.

Find the perimeter and area of the figure. *(p. 49)*

47.


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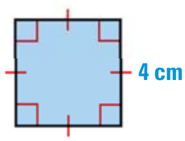

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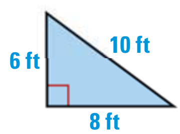


2.2 Analyze Conditional Statements

Before You used definitions.

Now You will write definitions as conditional statements.

Why? So you can verify statements, as in Example 2.

Key Vocabulary
- **conditional statement**
  converse, inverse, contrapositive
- **if-then form**
  hypothesis, conclusion
- **negation**
- **equivalent statements**
- **perpendicular lines**
- **biconditional statement**

A **conditional statement** is a logical statement that has two parts, a *hypothesis* and a *conclusion*. When a conditional statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**. Here is an example:

If $\underbrace{\text{it is raining}}_{\text{Hypothesis}}$, then $\underbrace{\text{there are clouds in the sky}}_{\text{Conclusion}}$.

EXAMPLE 1 Rewrite a statement in if-then form

Rewrite the conditional statement in if-then form.

a. All birds have feathers.

b. Two angles are supplementary if they are a linear pair.

Solution

First, identify the **hypothesis** and the **conclusion**. When you rewrite the statement in if-then form, you may need to reword the hypothesis or conclusion.

a. **All birds** have **feathers**.

If **an animal is a bird**, then **it has feathers**.

b. **Two angles are supplementary** if **they are a linear pair**.

If **two angles are a linear pair**, then **they are supplementary**.

✓ GUIDED PRACTICE for Example 1

Rewrite the conditional statement in if-then form.

1. All 90° angles are right angles.

2. $2x + 7 = 1$, because $x = -3$.

3. When $n = 9$, $n^2 = 81$.

4. Tourists at the Alamo are in Texas.

NEGATION The **negation** of a statement is the *opposite* of the original statement. Notice that Statement 2 is already negative, so its negation is positive.

Statement 1 The ball is red.

Negation 1 The ball is *not* red.

Statement 2 The cat is *not* black.

Negation 2 The cat is black.

VERIFYING STATEMENTS Conditional statements can be true or false. To show that a conditional statement is true, you must prove that the conclusion is true every time the hypothesis is true. To show that a conditional statement is false, you need to give *only one* counterexample.

RELATED CONDITIONALS To write the **converse** of a conditional statement, exchange the **hypothesis** and **conclusion**.

READ VOCABULARY
To *negate* part of a conditional statement, you write its negation.

To write the **inverse** of a conditional statement, negate both the hypothesis and the conclusion. To write the **contrapositive**, first write the converse and then negate both the hypothesis and the conclusion.

Conditional statement If $m\angle A = 99°$, then $\angle A$ is obtuse.	both true
Converse If $\angle A$ is obtuse, then $m\angle A = 99°$.	both false
Inverse If $m\angle A \neq 99°$, then $\angle A$ is not obtuse.	both false
Contrapositive If $\angle A$ is not obtuse, then $m\angle A \neq 99°$.	both true

EXAMPLE 2 Write four related conditional statements

Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement "Guitar players are musicians." Decide whether each statement is *true* or *false*.

Solution

If-then form If you are a guitar player, then you are a musician.
True, guitars players are musicians.

Converse If you are a musician, then you are a guitar player.
False, not all musicians play the guitar.

Inverse If you are not a guitar player, then you are not a musician.
False, even if you don't play a guitar, you can still be a musician.

Contrapositive If you are not a musician, then you are not a guitar player. *True*, a person who is not a musician cannot be a guitar player.

✓ GUIDED PRACTICE for Example 2

Write the converse, the inverse, and the contrapositive of the conditional statement. Tell whether each statement is *true* or *false*.

5. If a dog is a Great Dane, then it is large.
6. If a polygon is equilateral, then the polygon is regular.

EQUIVALENT STATEMENTS A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. Pairs of statements such as these are called *equivalent statements*. In general, when two statements are both true or both false, they are called **equivalent statements**.

DEFINITIONS You can write a definition as a conditional statement in if-then form or as its converse. Both the conditional statement and its converse are true. For example, consider the definition of *perpendicular lines*.

READ DIAGRAMS
In a diagram, a red square may be used to indicate a right angle or that two intersecting lines are perpendicular.

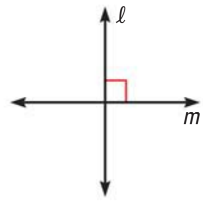
KEY CONCEPT — *For Your Notebook*

Perpendicular Lines

Definition If two lines intersect to form a right angle, then they are **perpendicular lines**.

The definition can also be written using the converse: If two lines are perpendicular lines, then they intersect to form a right angle.

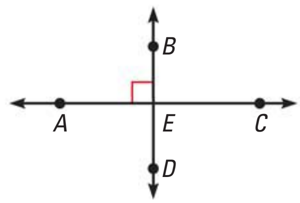
You can write "line ℓ is perpendicular to line m" as $\ell \perp m$.



$\ell \perp m$

EXAMPLE 3 Use definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.



a. $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

b. $\angle AEB$ and $\angle CEB$ are a linear pair.

c. $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are opposite rays.

Solution

a. This statement is *true*. The right angle symbol in the diagram indicates that the lines intersect to form a right angle. So you can say the lines are perpendicular.

b. This statement is *true*. By definition, if the noncommon sides of adjacent angles are opposite rays, then the angles are a linear pair. Because $\overrightarrow{EA}$ and $\overrightarrow{EC}$ are opposite rays, $\angle AEB$ and $\angle CEB$ are a linear pair.

c. This statement is *false*. Point E does not lie on the same line as A and B, so the rays are not opposite rays.

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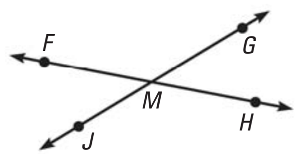
✓ GUIDED PRACTICE for Example 3

Use the diagram shown. Decide whether each statement is true. *Explain* your answer using the definitions you have learned.



7. $\angle JMF$ and $\angle FMG$ are supplementary.

8. Point M is the midpoint of $\overline{FH}$.

9. $\angle JMF$ and $\angle HMG$ are vertical angles.

10. $\overleftrightarrow{FH} \perp \overleftrightarrow{JG}$

READ DEFINITIONS
All definitions can be interpreted forward and backward in this way.

BICONDITIONAL STATEMENTS When a conditional statement and its converse are both true, you can write them as a single *biconditional statement*. A **biconditional statement** is a statement that contains the phrase "if and only if."

Any valid definition can be written as a biconditional statement.

EXAMPLE 4 Write a biconditional

Write the definition of perpendicular lines as a biconditional.

Solution

Definition	If two lines intersect to form a right angle, then they are perpendicular.
Converse	If two lines are perpendicular, then they intersect to form a right angle.
Biconditional	Two lines are perpendicular if and only if they intersect to form a right angle.

GUIDED PRACTICE for Example 4

11. Rewrite the definition of *right angle* as a biconditional statement.

12. Rewrite the statements as a biconditional.

If Mary is in theater class, she will be in the fall play. If Mary is in the fall play, she must be taking theater class.

2.2 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 17, and 33
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 25, 29, 33, 34, and 35

SKILL PRACTICE

1. VOCABULARY Copy and complete: The ___?___ of a conditional statement is found by switching the hypothesis and the conclusion.

2. ★ **WRITING** Write a definition for the term *collinear points*, and show how the definition can be interpreted as a biconditional.

EXAMPLE 1 on p. 79 for Exs. 3–6

REWRITING STATEMENTS **Rewrite the conditional statement in if-then form.**

3. When $x = 6$, $x^2 = 36$.

4. The measure of a straight angle is 180°.

5. Only people who are registered are allowed to vote.

6. ERROR ANALYSIS *Describe* and correct the error in writing the if-then statement.

> **Given statement:** All high school students take four English courses.
>
> **If-then statement:** If a high school student takes four courses, then all four are English courses.

EXAMPLE 2
on p. 80
for Exs. 7–15

WRITING RELATED STATEMENTS **For the given statement, write the if-then form, the converse, the inverse, and the contrapositive.**

7. The complementary angles add to 90°.

8. Ants are insects.

9. $3x + 10 = 16$, because $x = 2$.

10. A midpoint bisects a segment.

ANALYZING STATEMENTS **Decide whether the statement is *true* or *false*. If false, provide a counterexample.**

11. If a polygon has five sides, then it is a regular pentagon.

12. If $m\angle A$ is 85°, then the measure of the complement of $\angle A$ is 5°.

13. Supplementary angles are always linear pairs.

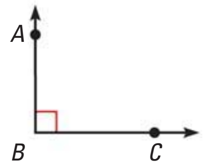
14. If a number is an integer, then it is rational.

15. If a number is a real number, then it is irrational.

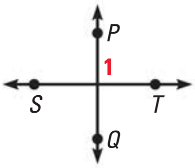
EXAMPLE 3
on p. 81
for Exs. 16–18

USING DEFINITIONS **Decide whether each statement about the diagram is true. *Explain* your answer using the definitions you have learned.**

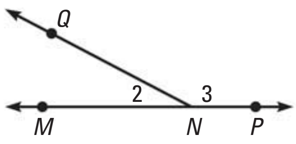
16. $m\angle ABC = 90°$



17. $\overleftrightarrow{PQ} \perp \overleftrightarrow{ST}$



18. $m\angle 2 + m\angle 3 = 180°$



EXAMPLE 4
on p. 82
for Exs. 19–21

REWRITING STATEMENTS **In Exercises 19–21, rewrite the definition as a biconditional statement.**

19. An angle with a measure between 90° and 180° is called *obtuse*.

20. Two angles are a *linear pair* if they are adjacent angles whose noncommon sides are opposite rays.

21. *Coplanar points* are points that lie in the same plane.

DEFINITIONS **Determine whether the statement is a valid definition.**

22. If two rays are *opposite rays*, then they have a common endpoint.

23. If the sides of a triangle are all the same length, then the triangle is *equilateral*.

24. If an angle is a *right angle*, then its measure is greater than that of an acute angle.

25. ★ **MULTIPLE CHOICE** Which statement has the same meaning as the given statement?

GIVEN ▶ You can go to the movie after you do your homework.

Ⓐ If you do your homework, then you can go to the movie afterwards.

Ⓑ If you do not do your homework, then you can go to the movie afterwards.

Ⓒ If you cannot go to the movie afterwards, then do your homework.

Ⓓ If you are going to the movie afterwards, then do not do your homework.

xy ALGEBRA Write the converse of each true statement. Tell whether the converse is true. If false, *explain* why.

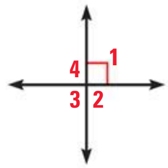
26. If $x > 4$, then $x > 0$.

27. If $x < 6$, then $-x > -6$.

28. If $x \leq -x$, then $x \leq 0$.

29. ★ **OPEN-ENDED MATH** Write a statement that is true but whose converse is false.

30. **CHALLENGE** Write a series of if-then statements that allow you to find the measure of each angle, given that $m\angle 1 = 90°$. Use the definition of linear pairs.



PROBLEM SOLVING

EXAMPLE 1
on p. 82
for Exs. 31–32

In Exercises 31 and 32, use the information about volcanoes to determine whether the biconditional statement is *true* or *false*. If false, provide a counterexample.

VOLCANOES Solid fragments are sometimes ejected from volcanoes during an eruption. The fragments are classified by size, as shown in the table.

31. A fragment is called a *block or bomb* if and only if its diameter is greater than 64 millimeters.

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32. A fragment is called a *lapilli* if and only if its diameter is less than 64 millimeters.

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Type of fragment	Diameter *d* (millimeters)
Ash	$d < 2$
Lapilli	$2 \leq d \leq 64$
Block or bomb	$d > 64$

33. ★ **SHORT RESPONSE** How can you show that the statement, "If you play a sport, then you wear a helmet." is false? *Explain*.

34. ★ **EXTENDED RESPONSE** You measure the heights of your classmates to get a data set.

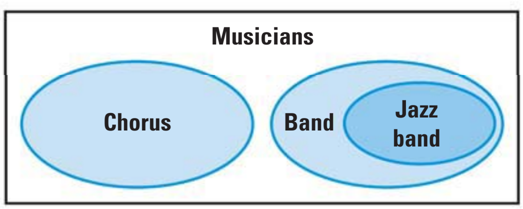
a. Tell whether this statement is true: If x and y are the least and greatest values in your data set, then the mean of the data is between x and y. *Explain* your reasoning.

b. Write the converse of the statement in part (a). Is the converse true? *Explain*.

c. Copy and complete the statement using *mean*, *median*, or *mode* to make a conditional that is true for any data set. *Explain* your reasoning.

Statement If a data set has a mean, a median, and a mode, then the __?__ of the data set will always be one of the measurements.

35. ★ **OPEN-ENDED MATH** The Venn diagram below represents all of the musicians at a high school. Write an if-then statement that describes a relationship between the various groups of musicians.



36. MULTI-STEP PROBLEM The statements below describe three ways that rocks are formed. Use these statements in parts (a)–(c).

Igneous rock is formed from the cooling of molten rock.

Sedimentary rock is formed from pieces of other rocks.

Metamorphic rock is formed by changing temperature, pressure, or chemistry.

a. Write each statement in if-then form.

b. Write the converse of each of the statements in part (a). Is the converse of each statement true? *Explain* your reasoning.

c. Write a true if-then statement about rocks. Is the converse of your statement *true* or *false*? *Explain* your reasoning.

37. ALGEBRA Can the statement, "If $x^2 - 10 = x + 2$, then $x = 4$," be combined with its converse to form a true biconditional?

38. REASONING You are given that the contrapositive of a statement is true. Will that help you determine whether the statement can be written as a true biconditional? *Explain*.

39. CHALLENGE Suppose each of the following statements is true. What can you conclude? *Explain* your answer.

If it is Tuesday, then I have art class.

It is Tuesday.

Each school day, I have either an art class or study hall.

If it is Friday, then I have gym class.

Today, I have either music class or study hall.

MIXED REVIEW

PREVIEW
Prepare for Lesson 2.3 in Exs. 40–45.

Find the product of the integers. *(p. 869)*

40. $(-2)(10)$ **41.** $(15)(-3)$ **42.** $(-12)(-4)$

43. $(-5)(-4)(10)$ **44.** $(-3)(6)(-2)$ **45.** $(-4)(-2)(-5)$

Sketch the figure described. *(p. 2)*

46. $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{CD}$ at point E. **47.** $\overleftrightarrow{XY}$ intersects plane P at point Z.

48. $\overleftrightarrow{GH}$ is parallel to $\overleftrightarrow{JK}$. **49.** Vertical planes X and Y intersect in $\overleftrightarrow{MN}$.

Find the coordinates of the midpoint of the segment with the given endpoints. *(p. 15)*

50. $A(10, 5)$ and $B(4, 5)$ **51.** $P(4, -1)$ and $Q(-2, 3)$ **52.** $L(2, 2)$ and $N(1, -2)$

Tell whether the figure is a polygon. If it is not, *explain* why. If it is a polygon, tell whether it is *convex* or *concave*. *(p. 42)*

53.  **54.**  **55.** 

Investigating Geometry ACTIVITY *Use before Lesson 2.3*

2.3 Logic Puzzles

MATERIALS • graph paper • pencils

QUESTION How can reasoning be used to solve a logic puzzle?

EXPLORE Solve a logic puzzle

Using the clues below, you can determine an important mathematical contribution and interesting fact about each of five mathematicians.

Copy the chart onto your graph paper. Use the chart to keep track of the information given in Clues 1–7. Place an X in a box to indicate a definite "no." Place an O in a box to indicate a definite "yes."

Clue 1 Pythagoras had his contribution named after him. He was known to avoid eating beans.

Clue 2 Albert Einstein considered Emmy Noether to be one of the greatest mathematicians and used her work to show the theory of relativity.

Clue 3 Anaxagoras was the first to theorize that the moon's light is actually the sun's light being reflected.

Clue 4 Julio Rey Pastor wrote a book at age 17.

Clue 5 The mathematician who is fluent in Latin contributed to the study of differential calculus.

Clue 6 The mathematician who did work with *n*-dimensional geometry was not the piano player.

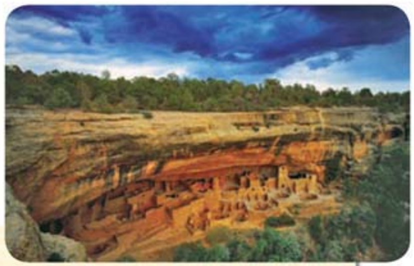
Clue 7 The person who first used perspective drawing to make scenery for plays was not Maria Agnesi or Julio Rey Pastor.

	n-dimensional geometry	Differential calculus	Math for theory of relativity	Perspective drawing	Pythagorean Theorem	Did not eat beans	Studied moonlight	Wrote a math book at 17	Fluent in Latin	Played piano
Maria Agnesi					X					
Anaxagoras					X					
Emmy Noether					X					
Julio Rey Pastor					X					
Pythagoras	X	X	X	X	O					
Did not eat beans										
Studied moonlight										
Wrote a math book at 17										
Fluent in Latin										
Played piano										

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Write Clue 4 as a conditional statement in if-then form. Then write the contrapositive of the statement. *Explain* why the contrapositive of this statement is a helpful clue.
2. *Explain* how you can use Clue 6 to figure out who played the piano.
3. *Explain* how you can use Clue 7 to figure out who worked with perspective drawing.

2.3 Apply Deductive Reasoning



Before You used inductive reasoning to form a conjecture.

Now You will use deductive reasoning to form a logical argument.

Why So you can reach logical conclusions about locations, as in Ex. 18.

Key Vocabulary
- **deductive reasoning**

Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument. This is different from *inductive reasoning*, which uses specific examples and patterns to form a conjecture.

READ VOCABULARY
The Law of Detachment is also called a *direct argument*. The Law of Syllogism is sometimes called the *chain rule*.

KEY CONCEPT *For Your Notebook*

Laws of Logic

Law of Detachment

If the hypothesis of a true conditional statement is true, then the conclusion is also true.

Law of Syllogism

If **hypothesis** p, then **conclusion** q.
If **hypothesis** q, then **conclusion** r. — If these statements are true,

If **hypothesis** p, then **conclusion** r. — then this statement is true.

EXAMPLE 1 Use the Law of Detachment

Use the Law of Detachment to make a valid conclusion in the true situation.

a. If two segments have the same length, then they are congruent. You know that $BC = XY$.

b. Mary goes to the movies every Friday and Saturday night. Today is Friday.

Solution

a. Because $BC = XY$ satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, $\overline{BC} \cong \overline{XY}$.

b. First, identify the hypothesis and the conclusion of the first statement. The hypothesis is "If it is Friday or Saturday night," and the conclusion is "then Mary goes to the movies."

"Today is Friday" satisfies the hypothesis of the conditional statement, so you can conclude that Mary will go to the movies tonight.

EXAMPLE 2 Use the Law of Syllogism

If possible, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

a. If Rick takes chemistry this year, then Jesse will be Rick's lab partner.

If Jesse is Rick's lab partner, then Rick will get an A in chemistry.

b. If $x^2 > 25$, then $x^2 > 20$.

If $x > 5$, then $x^2 > 25$.

c. If a polygon is regular, then all angles in the interior of the polygon are congruent.

If a polygon is regular, then all of its sides are congruent.

Solution

a. The conclusion of the first statement is the hypothesis of the second statement, so you can write the following new statement.

If Rick takes chemistry this year, then Rick will get an A in chemistry.

AVOID ERRORS
The order in which the statements are given does not affect whether you can use the Law of Syllogism.

b. Notice that the conclusion of the second statement is the hypothesis of the first statement, so you can write the following new statement.

If $x > 5$, then $x^2 > 20$.

c. Neither statement's conclusion is the same as the other statement's hypothesis. You cannot use the Law of Syllogism to write a new conditional statement.

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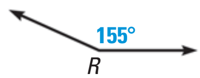
✓ GUIDED PRACTICE for Examples 1 and 2

1. If $90° < m\angle R < 180°$, then $\angle R$ is obtuse. The measure of $\angle R$ is 155°. Using the Law of Detachment, what statement can you make?



2. If Jenelle gets a job, then she can afford a car. If Jenelle can afford a car, then she will drive to school. Using the Law of Syllogism, what statement can you make?

State the law of logic that is illustrated.

3. If you get an A or better on your math test, then you can go to the movies. If you go to the movies, then you can watch your favorite actor.

   If you get an A or better on your math test, then you can watch your favorite actor.

4. If $x > 12$, then $x + 9 > 20$. The value of x is 14.

   Therefore, $x + 9 > 20$.

ANALYZING REASONING In Geometry, you will frequently use inductive reasoning to make conjectures. You will also be using deductive reasoning to show that conjectures are true or false. You will need to know which type of reasoning is being used.

EXAMPLE 3 Use inductive and deductive reasoning

ALGEBRA What conclusion can you make about the product of an even integer and any other integer?

Solution

STEP 1 **Look** for a pattern in several examples. Use inductive reasoning to make a conjecture.

$(-2)(2) = -4$, $(-1)(2) = -2$, $2(2) = 4$, $3(2) = 6$,

$(-2)(-4) = 8$, $(-1)(-4) = 4$, $2(-4) = -8$, $3(-4) = -12$

Conjecture Even integer • Any integer = Even integer

STEP 2 **Let** n and m each be any integer. Use deductive reasoning to show the conjecture is true.

$2n$ is an even integer because any integer multiplied by 2 is even.

$2nm$ represents the product of an even integer and any integer m.

$2nm$ is the product of 2 and an integer nm. So, $2nm$ is an even integer.

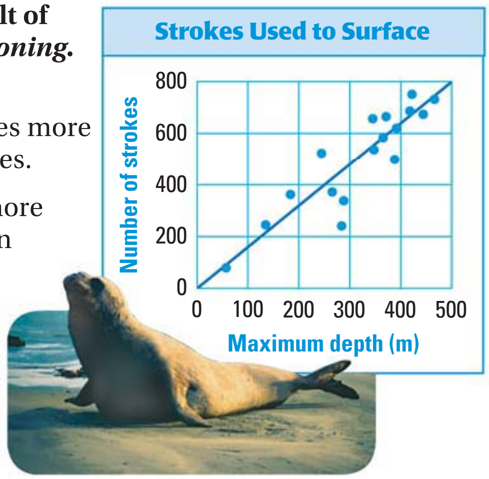
▶ The product of an even integer and any integer is an even integer.

EXAMPLE 4 Reasoning from a graph

Tell whether the statement is the result of *inductive reasoning* or *deductive reasoning*. Explain your choice.

a. The northern elephant seal requires more strokes to surface the deeper it dives.

b. The northern elephant seal uses more strokes to surface from 60 feet than from 250 feet.



Solution

a. Inductive reasoning, because it is based on a pattern in the data

b. Deductive reasoning, because you are comparing values that are given on the graph

✓ GUIDED PRACTICE for Examples 3 and 4

5. Use inductive reasoning to make a conjecture about the sum of a number and itself. Then use deductive reasoning to show the conjecture is true.

6. Use inductive reasoning to write another statement about the graph in Example 4. Then use deductive reasoning to write another statement.

2.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 17, and 21

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 3, 12, 20, and 23

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: If the hypothesis of a true if-then statement is true, then the conclusion is also true by the Law of __?__.

★ **WRITING** **Use deductive reasoning to make a statement about the picture.**

2. 

3. 

EXAMPLE 1 on p. 87 for Exs. 4–6

LAW OF DETACHMENT **Make a valid conclusion in the situation.**

4. If the measure of an angle is 90°, then it is a right angle. The measure of $\angle A$ is 90°.

5. If $x > 12$, then $-x < -12$. The value of x is 15.

6. If a book is a biography, then it is nonfiction. You are reading a biography.

EXAMPLE 2 on p. 88 for Exs. 7–10

LAW OF SYLLOGISM **In Exercises 7–10, write the statement that follows from the pair of statements that are given.**

7. If a rectangle has four equal side lengths, then it is a square. If a polygon is a square, then it is a regular polygon.

8. If $y > 0$, then $2y > 0$. If $2y > 0$, then $2y - 5 \neq -5$.

9. If you play the clarinet, then you play a woodwind instrument. If you play a woodwind instrument, then you are a musician.

10. If $a = 3$, then $5a = 15$. If $\frac{1}{2}a = 1\frac{1}{2}$, then $a = 3$.

EXAMPLE 3 on p. 89 for Ex. 11

11. **REASONING** What can you say about the sum of an even integer and an even integer? Use inductive reasoning to form a conjecture. Then use deductive reasoning to show that the conjecture is true.

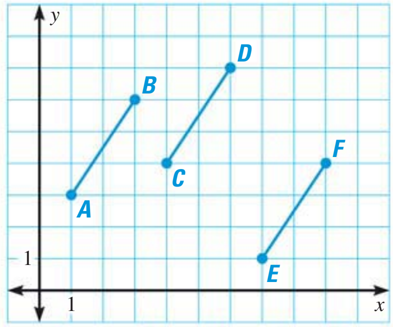
12. ★ **MULTIPLE CHOICE** If two angles are vertical angles, then they have the same measure. You know that $\angle A$ and $\angle B$ are vertical angles. Using the Law of Detachment, which conclusion could you make?

   (A) $m\angle A > m\angle B$ (B) $m\angle A = m\angle B$

   (C) $m\angle A + m\angle B = 90°$ (D) $m\angle A + m\angle B = 180°$

13. **ERROR ANALYSIS** *Describe* and correct the error in the argument: "If two angles are a linear pair, then they are supplementary. Angles C and D are supplementary, so the angles are a linear pair."

14. **ALGEBRA** Use the segments in the coordinate plane.



a. Use the distance formula to show that the segments are congruent.

b. Make a conjecture about some segments in the coordinate plane that are congruent to the given segments. Test your conjecture, and *explain* your reasoning.

c. Let one endpoint of a segment be (x, y). Use algebra to show that segments drawn using your conjecture will always be congruent.

d. A student states that the segments described below will each be congruent to the ones shown above. Determine whether the student is correct. *Explain* your reasoning.

$\overline{MN}$, with endpoints $M(3, 5)$ and $N(5, 2)$

$\overline{PQ}$, with endpoints $P(1, -1)$ and $Q(4, -3)$

$\overline{RS}$, with endpoints $R(-2, 2)$ and $S(1, 4)$

15. **CHALLENGE** Make a conjecture about whether the Law of Syllogism works when used with the contrapositives of a pair of statements. Use this pair of statements to *justify* your conjecture.

If a creature is a wombat, then it is a marsupial.

If a creature is a marsupial, then it has a pouch.

PROBLEM SOLVING

EXAMPLES 1 and 2 on pp. 87–88 for Exs. 16–17

USING THE LAWS OF LOGIC **In Exercises 16 and 17, what conclusions can you make using the true statement?**

16. **CAR COSTS** If you save \$2000, then you can buy a car. You have saved \$1200.

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17. **PROFIT** The bakery makes a profit if its revenue is greater than its costs. You will get a raise if the bakery makes a profit.

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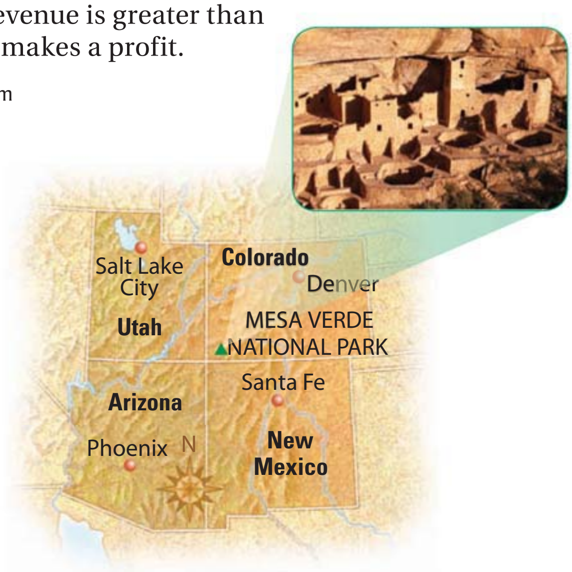
USING DEDUCTIVE REASONING **Select the word(s) that make(s) the conclusion true.**

18. Mesa Verde National Park is in Colorado. Simone vacationed in Colorado. So, Simone (*must have, may have,* or *never*) visited Mesa Verde National Park.



19. The cliff dwellings in Mesa Verde National Park are accessible to visitors only when accompanied by a park ranger. Billy is at a cliff dwelling in Mesa Verde National Park. So, Billy *(is, may be, is not)* with a park ranger.

EXAMPLE 4
on p. 89
for Ex. 20

20. ★ **EXTENDED RESPONSE** Geologists use the Mohs scale to determine a mineral's hardness. Using the scale, a mineral with a higher rating will leave a scratch on a mineral with a lower rating. Geologists use scratch tests to help identify an unknown mineral.

Mineral	 Talc	 Gypsum	 Calcite	 Fluorite
Mohs rating	1	2	3	4

a. Use the table to write three if-then statements such as "If talc is scratched against gypsum, then a scratch mark is left on the talc."

b. You must identify four minerals labeled *A*, *B*, *C*, and *D*. You know that the minerals are the ones shown in the table. The results of your scratch tests are shown below. What can you conclude? *Explain* your reasoning.

Mineral *A* is scratched by Mineral *B*.

Mineral *C* is scratched by all three of the other minerals.

c. What additional test(s) can you use to identify *all* the minerals in part (b)?

REASONING In Exercises 21 and 22, decide whether *inductive* or *deductive* reasoning is used to reach the conclusion. *Explain* your reasoning.

21. The rule at your school is that you must attend all of your classes in order to participate in sports after school. You played in a soccer game after school on Monday. Therefore, you went to all of your classes on Monday.

22. For the past 5 years, your neighbor goes on vacation every July 4th and asks you to feed her hamster. You conclude that you will be asked to feed her hamster on the next July 4th.

23. ★ **SHORT RESPONSE** Let an even integer be $2n$ and an odd integer be $2n + 1$. *Explain* why the sum of an even integer and an odd integer is an odd integer.

24. **LITERATURE** George Herbert wrote a poem, *Jacula Prudentum*, that includes the statements shown. Use the Law of Syllogism to write a new conditional statement. *Explain* your reasoning.

For want of a nail the shoe is lost,
for want of a shoe the horse is lost,
for want of a horse the rider is lost.

REASONING In Exercises 25–28, use the true statements below to determine whether you know the conclusion is *true* or *false*. *Explain* your reasoning.

If Arlo goes to the baseball game, then he will buy a hot dog.

If the baseball game is not sold out, then Arlo and Mia will go to the game.

If Mia goes to the baseball game, then she will buy popcorn.

The baseball game is not sold out.

25. Arlo bought a hot dog.

26. Arlo and Mia went to the game.

27. Mia bought a hot dog.

28. Arlo had some of Mia's popcorn.

◯ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

29. **CHALLENGE** Use these statements to answer parts (a)–(c).

Adam says Bob lies.

Bob says Charlie lies.

Charlie says Adam and Bob both lie.

a. If Adam is telling the truth, then Bob is lying. What can you conclude about Charlie's statement?

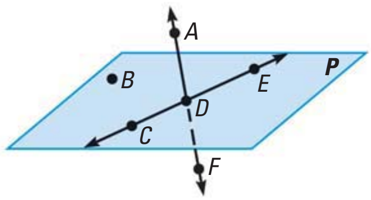
b. Assume Adam is telling the truth. *Explain* how this leads to a contradiction.

c. Who is telling the truth? Who is lying? How do you know?

MIXED REVIEW

PREVIEW
Prepare for Lesson 2.4 in Exs. 30–33.

In Exercises 30–33, use the diagram. *(p. 2)*



30. Name two lines.

31. Name four rays.

32. Name three collinear points.

33. Name four coplanar points.

Plot the given points in a coordinate plane. Then determine whether $\overline{AB}$ and $\overline{CD}$ are congruent. *(p. 9)*

34. $A(1, 4)$, $B(5, 4)$, $C(3, -4)$, $D(3, 0)$

35. $A(-1, 0)$, $B(-1, -5)$, $C(1, 2)$, $D(-5, 2)$

Rewrite the conditional statement in if-then form. *(p. 79)*

36. When $x = -2$, $x^2 = 4$.

37. The measure of an acute angle is less than 90°.

38. Only people who are members can access the website.

QUIZ *for Lessons 2.1–2.3*

Show the conjecture is false by finding a counterexample. *(p. 72)*

1. If the product of two numbers is positive, then the two numbers must be negative.

2. The sum of two numbers is always greater than the larger number.

In Exercises 3 and 4, write the if-then form and the contrapositive of the statement. *(p. 79)*

3. Points that lie on the same line are called collinear points.

4. $2x - 8 = 2$, because $x = 5$.

5. Make a valid conclusion about the following statements:
If it is above 90°F outside, then I will wear shorts. It is 98°F. *(p. 87)*

6. *Explain* why a number that is divisible by a multiple of 3 is also divisible by 3. *(p. 87)*

Extension
Use after Lesson 2.3

Symbolic Notation and Truth Tables

GOAL Use symbolic notation to represent logical statements.

Key Vocabulary
- truth value
- truth table

Conditional statements can be written using *symbolic notation*, where letters are used to represent statements. An arrow ($\rightarrow$), read "implies," connects the hypothesis and conclusion. To write the negation of a statement p you write the symbol for negation ($\sim$) before the letter. So, "not p" is written $\sim p$.

KEY CONCEPT *For Your Notebook*

Symbolic Notation

Let p be "the angle is a right angle" and let q be "the measure of the angle is 90°."

Conditional If p, then q. $p \rightarrow q$

Example: If an angle is a right angle, then its measure is 90°.

Converse If q, then p. $q \rightarrow p$

Example: If the measure of an angle is 90°, then the angle is a right angle.

Inverse If not p, then not q. $\sim p \rightarrow \sim q$

Example: If an angle is not a right angle, then its measure is not 90°.

Contrapositive If not q, then not p. $\sim q \rightarrow \sim p$

If the measure of an angle is not 90°, then the angle is not a right angle.

Biconditional p if and only if q $p \leftrightarrow q$

Example: An angle is a right angle if and only if its measure is 90°.

EXAMPLE 1 Use symbolic notation

Let p be "the car is running" and let q be "the key is in the ignition."

a. Write the conditional statement $p \rightarrow q$ in words.

b. Write the converse $q \rightarrow p$ in words.

c. Write the inverse $\sim p \rightarrow \sim q$ in words.

d. Write the contrapositive $\sim q \rightarrow \sim p$ in words.

Solution

a. Conditional: If the car is running, then the key is in the ignition.

b. Converse: If the key is in the ignition, then the car is running.

c. Inverse: If the car is not running, then the key is not in the ignition.

d. Contrapositive: If the key is not in the ignition, then the car is not running.

TRUTH TABLES The **truth value** of a statement is either true (T) or false (F). You can determine the conditions under which a conditional statement is true by using a **truth table**. The truth table at the right shows the truth values for hypothesis p and conclusion q. The conditional $p \to q$ is only false when a true hypothesis produces a false conclusion.

Conditional		
p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

EXAMPLE 2 Make a truth table

Use the truth table above to make truth tables for the converse, inverse, and contrapositive of a conditional statement $p \to q$.

Solution

READ TRUTH TABLES

A conditional statement and its contrapositive are *equivalent statements* because they have the same truth table. The same is true of the converse and the inverse.

Converse		
p	q	$q \to p$
T	T	T
T	F	T
F	T	F
F	F	T

Inverse				
p	q	$\sim p$	$\sim q$	$\sim p \to \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Contrapositive				
p	q	$\sim q$	$\sim p$	$\sim q \to \sim p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

PRACTICE

EXAMPLE 1
on p. 94
for Exs. 1–6

1. **WRITING** *Describe* how to use symbolic notation to represent the contrapositive of a conditional statement.

WRITING STATEMENTS **Use p and q to write the symbolic statement in words.**

p: Polygon *ABCDE* is equiangular and equilateral.

q: Polygon *ABCDE* is a regular polygon.

2. $p \to q$
3. $\sim p$
4. $\sim q \to \sim p$
5. $p \leftrightarrow q$

6. **LAW OF SYLLOGISM** Use the statements p, q, and r below to write a series of conditionals that would satisfy the Law of Syllogism. How could you write your reasoning using symbolic notation?

p: $x + 5 = 12$ $\quad$ q: $x = 7$ $\quad$ r: $3x = 21$

EXAMPLE 2
on p. 95
for Exs. 7–8

7. **WRITING** Is the truth value of a statement always true (T)? *Explain.*

8. **TRUTH TABLE** Use the statement "If an animal is a poodle, then it is a dog."
   a. Identify the hypothesis p and the conclusion q in the conditional.
   b. Make a truth table for the converse. *Explain* what each row in the table means in terms of the original statement.

2.4 Use Postulates and Diagrams

Before You used postulates involving angle and segment measures.

Now You will use postulates involving points, lines, and planes.

Why? So you can draw the layout of a neighborhood, as in Ex. 39.

Key Vocabulary
- **line perpendicular to a plane**
- **postulate,** *p. 8*

In geometry, rules that are accepted without proof are called *postulates* or *axioms*. Rules that are proved are called *theorems*. Postulates and theorems are often written in conditional form. Unlike the converse of a definition, the converse of a postulate or theorem cannot be assumed to be true.

You learned four postulates in Chapter 1.

POSTULATE 1	Ruler Postulate	page 9
POSTULATE 2	Segment Addition Postulate	page 10
POSTULATE 3	Protractor Postulate	page 24
POSTULATE 4	Angle Addition Postulate	page 25

Here are seven new postulates involving points, lines, and planes.

POSTULATES *For Your Notebook*

Point, Line, and Plane Postulates

POSTULATE 5 Through any two points there exists exactly one line.

POSTULATE 6 A line contains at least two points.

POSTULATE 7 If two lines intersect, then their intersection is exactly one point.

POSTULATE 8 Through any three noncollinear points there exists exactly one plane.

POSTULATE 9 A plane contains at least three noncollinear points.

POSTULATE 10 If two points lie in a plane, then the line containing them lies in the plane.

POSTULATE 11 If two planes intersect, then their intersection is a line.

ALGEBRA CONNECTION You have been using many of Postulates 5–11 in previous courses.

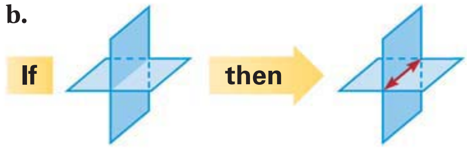
One way to graph a linear equation is to plot two points whose coordinates satisfy the equation and then connect them with a line. Postulate 5 guarantees that there is exactly one such line. A familiar way to find a common solution of two linear equations is to graph the lines and find the coordinates of their intersection. This process is guaranteed to work by Postulate 7.

EXAMPLE 1 Identify a postulate illustrated by a diagram

State the postulate illustrated by the diagram.

a.

b.



Solution

a. **Postulate 7** If two lines intersect, then their intersection is exactly one point.

b. **Postulate 11** If two planes intersect, then their intersection is a line.

EXAMPLE 2 Identify postulates from a diagram

Use the diagram to write examples of Postulates 9 and 10.

Postulate 9 Plane *P* contains at least three noncollinear points, *A*, *B*, and *C*.

Postulate 10 Point *A* and point *B* lie in plane *P*, so line *n* containing *A* and *B* also lies in plane *P*.

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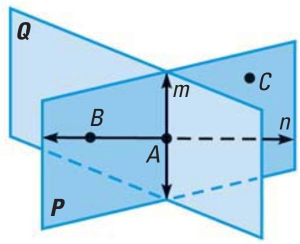


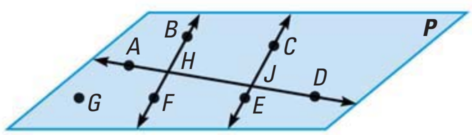
✓ GUIDED PRACTICE for Examples 1 and 2

1. Use the diagram in Example 2. Which postulate allows you to say that the intersection of plane *P* and plane *Q* is a line?
2. Use the diagram in Example 2 to write examples of Postulates 5, 6, and 7.

CONCEPT SUMMARY — For Your Notebook

Interpreting a Diagram

When you interpret a diagram, you can only assume information about size or measure if it is marked.



YOU CAN ASSUME	YOU CANNOT ASSUME
All points shown are coplanar.	*G*, *F*, and *E* are collinear.
$\angle AHB$ and $\angle BHD$ are a linear pair.	$\overleftrightarrow{BF}$ and $\overleftrightarrow{CE}$ intersect.
$\angle AHF$ and $\angle BHD$ are vertical angles.	$\overleftrightarrow{BF}$ and $\overleftrightarrow{CE}$ do not intersect.
A, *H*, *J*, and *D* are collinear.	$\angle BHA \cong \angle CJA$
$\overleftrightarrow{AD}$ and $\overleftrightarrow{BF}$ intersect at *H*.	$\overleftrightarrow{AD} \perp \overleftrightarrow{BF}$ or $m\angle AHB = 90°$

EXAMPLE 3 Use given information to sketch a diagram

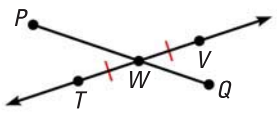
Sketch a diagram showing $\overleftrightarrow{TV}$ intersecting $\overline{PQ}$ at point W, so that $\overline{TW} \cong \overline{WV}$.

Solution

STEP 1 **Draw** $\overleftrightarrow{TV}$ and label points T and V.

STEP 2 **Draw** point W at the midpoint of $\overline{TV}$. Mark the congruent segments.

STEP 3 **Draw** $\overline{PQ}$ through W.

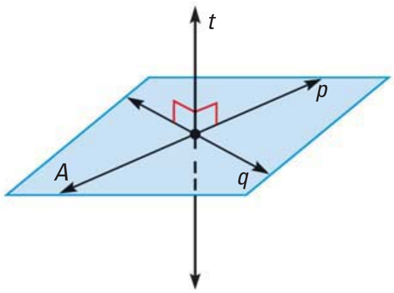


AVOID ERRORS

Notice that the picture was drawn so that W does not look like a midpoint of $\overline{PQ}$. Also, it was drawn so that $\overline{PQ}$ is not perpendicular to $\overline{TV}$.

PERPENDICULAR FIGURES A line is a **line perpendicular to a plane** if and only if the line intersects the plane in a point and is perpendicular to every line in the plane that intersects it at that point.

In a diagram, a line perpendicular to a plane must be marked with a right angle symbol.



EXAMPLE 4 Interpret a diagram in three dimensions

Which of the following statements *cannot* be assumed from the diagram?

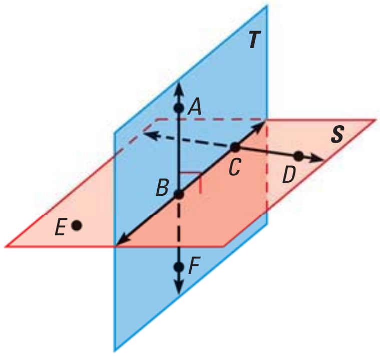
A, B, and F are collinear.

E, B, and D are collinear.

$\overline{AB} \perp$ plane S

$\overline{CD} \perp$ plane T

$\overleftrightarrow{AF}$ intersects $\overleftrightarrow{BC}$ at point B.



Solution

No drawn line connects E, B, and D, so you cannot assume they are collinear. With no right angle marked, you cannot assume $\overline{CD} \perp$ plane T.

✓ GUIDED PRACTICE for Examples 3 and 4

In Exercises 3 and 4, refer back to Example 3.

3. If the given information stated $\overline{PW}$ and $\overline{QW}$ are congruent, how would you indicate that in the diagram?
4. Name a pair of supplementary angles in the diagram. *Explain*.
5. In the diagram for Example 4, can you assume plane S intersects plane T at $\overleftrightarrow{BC}$?
6. *Explain* how you know that $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$ in Example 4.

2.4 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 13, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 10, 24, 25, 33, 39, and 41

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A __?__ is a line that intersects the plane in a point and is perpendicular to every line in the plane that intersects it.

2. ★ **WRITING** *Explain* why you cannot assume $\angle BHA \cong \angle CJA$ in the Concept Summary on page 97.

EXAMPLE 1 on p. 97 for Exs. 3–5

IDENTIFYING POSTULATES **State the postulate illustrated by the diagram.**

3. 

4. 

5. **CONDITIONAL STATEMENTS** Postulate 8 states that through any three noncollinear points there exists exactly one plane.

   a. Rewrite Postulate 8 in if-then form.

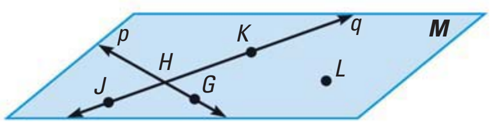
   b. Write the converse, inverse, and contrapositive of Postulate 8.

   c. Which statements in part (b) are true?

EXAMPLE 2 on p. 97 for Exs. 6–8

USING A DIAGRAM **Use the diagram to write an example of each postulate.**

6. Postulate 6
7. Postulate 7
8. Postulate 8



EXAMPLES 3 and 4 on p. 98 for Exs. 9–10

9. **SKETCHING** Sketch a diagram showing $\overleftrightarrow{XY}$ intersecting $\overleftrightarrow{WV}$ at point T, so $\overleftrightarrow{XY} \perp \overleftrightarrow{WV}$. In your diagram, does $\overline{WT}$ have to be congruent to $\overline{TV}$? *Explain* your reasoning.

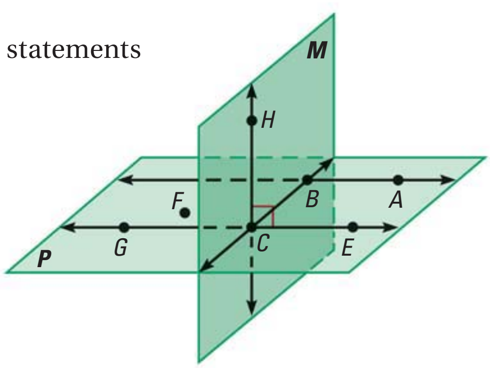
10. ★ **MULTIPLE CHOICE** Which of the following statements *cannot* be assumed from the diagram?

    (A) Points A, B, C, and E are coplanar.

    (B) Points F, B, and G are collinear.

    (C) $\overleftrightarrow{HC} \perp \overleftrightarrow{GE}$

    (D) $\overleftrightarrow{EC}$ intersects plane M at point C.



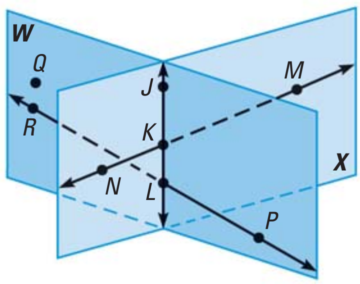
ANALYZING STATEMENTS **Decide whether the statement is true or false. If it is false, give a real-world counterexample.**

11. Through any three points, there exists exactly one line.

12. A point can be in more than one plane.

13. Any two planes intersect.

USING A DIAGRAM Use the diagram to determine if the statement is *true* or *false*.



14. Planes W and X intersect at $\overleftrightarrow{KL}$.
15. Points Q, J, and M are collinear.
16. Points K, L, M, and R are coplanar.
17. $\overleftrightarrow{MN}$ and $\overleftrightarrow{RP}$ intersect.
18. $\overleftrightarrow{RP} \perp$ plane W
19. $\overleftrightarrow{JK}$ lies in plane X.
20. $\angle PLK$ is a right angle.
21. $\angle NKL$ and $\angle JKM$ are vertical angles.
22. $\angle NKJ$ and $\angle JKM$ are supplementary angles.
23. $\angle JKM$ and $\angle KLP$ are congruent angles.

24. ★ **MULTIPLE CHOICE** Choose the diagram showing $\overleftrightarrow{LN}$, $\overleftrightarrow{AB}$, and $\overleftrightarrow{DC}$ intersecting at point M, $\overleftrightarrow{AB}$ bisecting $\overline{LN}$, and $\overleftrightarrow{DC} \perp \overleftrightarrow{LN}$.

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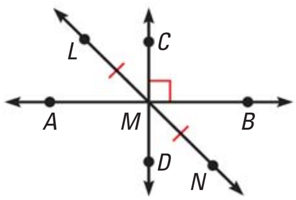

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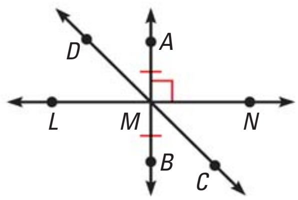

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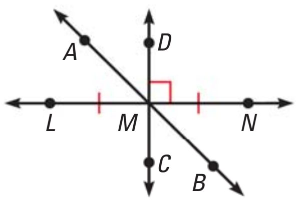
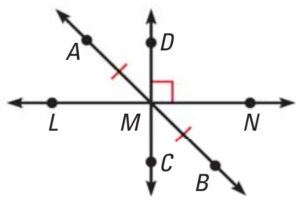


Ⓓ


25. ★ **OPEN-ENDED MATH** Sketch a diagram of a real-world object illustrating three of the postulates about points, lines, and planes. List the postulates used.

26. **ERROR ANALYSIS** A student made the false statement shown. Change the statement in two different ways to make it true.

> Three points are always contained in a line. ✗

27. **REASONING** Use Postulates 5 and 9 to *explain* why every plane contains at least one line.

28. **REASONING** Point X lies in plane M. Use Postulates 6 and 9 to *explain* why there are at least two lines in plane M that contain point X.

29. **CHALLENGE** Sketch a line m and a point C not on line m. Make a conjecture about how many planes can be drawn so that line m and point C lie in the plane. Use postulates to justify your conjecture.

PROBLEM SOLVING

REAL-WORLD SITUATIONS **Which postulate is suggested by the photo?**

30. 

31. 

32. 

33. ★ **SHORT RESPONSE** Give a real-world example of Postulate 6, which states that a line contains at least two points.

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34. **DRAW A DIAGRAM** Sketch two lines that intersect, and another line that does not intersect either one.

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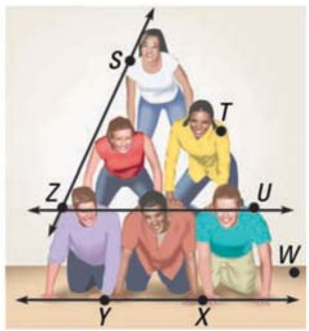
USING A DIAGRAM **Use the pyramid to write examples of the postulate indicated.**



35. Postulate 5

36. Postulate 7

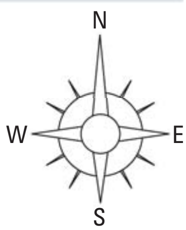
37. Postulate 9

38. Postulate 10

39. ★ **EXTENDED RESPONSE** A friend e-mailed you the following statements about a neighborhood. Use the statements to complete parts (a)–(e).

Subject Neighborhood

Building B is due west of Building A.
Buildings A and B are on Street 1.
Building D is due north of Building A.
Buildings A and D are on Street 2.
Building C is southwest of Building A.
Buildings A and C are on Street 3.
Building E is due east of Building B.
$\angle CAE$ formed by Streets 1 and 3 is obtuse.



a. Draw a diagram of the neighborhood.

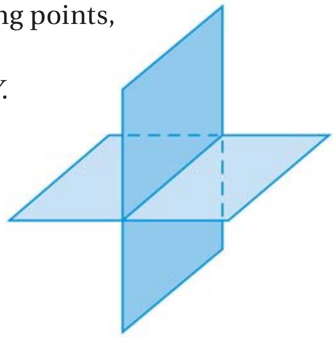
b. Where do Streets 1 and 2 intersect?

c. Classify the angle formed by Streets 1 and 2.

d. Is Building E between Buildings A and B? *Explain.*

e. What street is Building E on?

40. **MULTI-STEP PROBLEM** Copy the figure and label the following points, lines, and planes appropriately.



a. Label the horizontal plane as X and the vertical plane as Y.

b. Draw two points A and B on your diagram so they lie in plane Y, but not in plane X.

c. Illustrate Postulate 5 on your diagram.

d. If point C lies in both plane X and plane Y, where would it lie? Draw point C on your diagram.

e. Illustrate Postulate 9 for plane X on your diagram.

41. ★ **SHORT RESPONSE** Points E, F, and G all lie in plane P and in plane Q. What must be true about points E, F, and G if P and Q are different planes? What must be true about points E, F, and G to force P and Q to be the same plane? Make sketches to support your answers.

DRAWING DIAGRAMS $\overleftrightarrow{AC}$ **and** $\overleftrightarrow{DB}$ **intersect at point** E**. Draw one diagram that meets the additional condition(s) and another diagram that does not.**

42. $\angle AED$ and $\angle AEB$ are right angles.

43. Point E is the midpoint of $\overline{AC}$.

44. $\overrightarrow{EA}$ and $\overrightarrow{EC}$ are opposite rays. $\overrightarrow{EB}$ and $\overrightarrow{ED}$ are not opposite rays.

45. **CHALLENGE** Suppose none of the four legs of a chair are the same length. What is the maximum number of planes determined by the lower ends of the legs? Suppose exactly three of the legs of a second chair have the same length. What is the maximum number of planes determined by the lower ends of the legs of the second chair? *Explain* your reasoning.

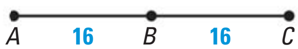
MIXED REVIEW

PREVIEW
Prepare for Lesson 2.5 in Exs. 46–48.

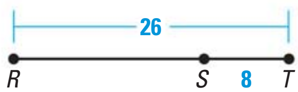
Draw an example of the type of angle described. *(p. 9)*

46. Find MP.

47. Find AC.

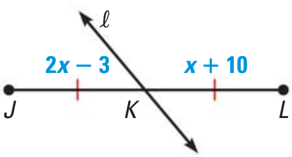


48. Find RS.

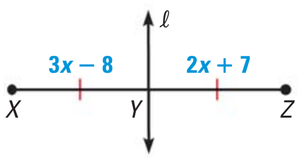


Line ℓ bisects the segment. Find the indicated length. *(p. 15)*

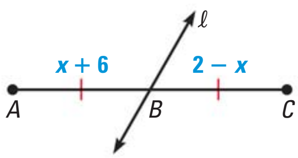
49. Find JK.



50. Find XZ.



51. Find BC.



Draw an example of the type of angle described. *(p. 24)*

52. Right angle
53. Acute angle
54. Obtuse angle
55. Straight angle

56. Two angles form a linear pair. The measure of one angle is 9 times the measure of the other angle. Find the measure of each angle. *(p. 35)*

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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Lessons 2.1–2.4

1. MULTI-STEP PROBLEM The table below shows the time of the sunrise on different days in Galveston, Texas.

Date in 2006	Time of sunrise (Central Standard Time)
Jan. 1	7:14 A.M.
Feb. 1	7:08 A.M.
Mar. 1	6:45 A.M.
Apr. 1	6:09 A.M.
May 1	5:37 A.M.
June 1	5:20 A.M.
July 1	5:23 A.M.
Aug. 1	5:40 A.M.

a. *Describe* the pattern, if any, in the times shown in the table.

b. Use the times in the table to make a reasonable prediction about the time of the sunrise on September 1, 2006.

2. SHORT RESPONSE As shown in the table below, hurricanes are categorized by the speed of the wind in the storm. Use the table to determine whether the statement is *true* or *false*. If false, provide a counterexample.

Hurricane category	Wind speed w (mi/h)
1	$74 \le w \le 95$
2	$96 \le w \le 110$
3	$111 \le w \le 130$
4	$131 \le w \le 155$
5	$w > 155$

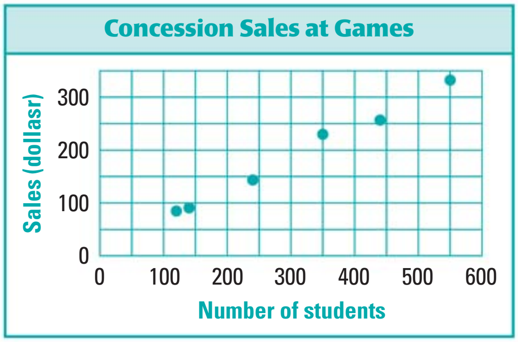
a. A hurricane is a category 5 hurricane if and only if its wind speed is greater than 155 miles per hour.

b. A hurricane is a category 3 hurricane if and only if its wind speed is less than 130 miles per hour.

3. GRIDDED ANSWER Write the next number in the pattern.

1, 2, 5, 10, 17, 26, . . .

4. EXTENDED RESPONSE The graph shows concession sales at six high school football games. Tell whether each statement is the result of *inductive reasoning* or *deductive reasoning*. *Explain* your thinking.



a. If 500 students attend a football game, the high school can expect concession sales to reach \$300.

b. Concession sales were highest at the game attended by 550 students.

c. The average number of students who come to a game is about 300.

5. SHORT RESPONSE Select the phrase that makes the conclusion true. *Explain* your reasoning.

a. A person needs a library card to check out books at the public library. You checked out a book at the public library. You (*must have, may have,* or *do not have*) a library card.

b. The islands of Hawaii are volcanoes. Bob has never been to the Hawaiian Islands. Bob (*has visited, may have visited,* or *has never visited*) volcanoes.

6. SHORT RESPONSE Sketch a diagram showing $\overleftrightarrow{PQ}$ intersecting $\overleftrightarrow{RS}$ at point N. In your diagram, $\angle PNS$ should be an obtuse angle. Identify two acute angles in your diagram. *Explain* how you know that these angles are acute.

2.5 Justify a Number Trick

MATERIALS • paper • pencil

QUESTION How can you use algebra to justify a number trick?

Number tricks can allow you to guess the result of a series of calculations.

EXPLORE Play the number trick

STEP 1 *Pick a number* Follow the directions below.

a. Pick any number between 11 and 98 that does not end in a zero.	23
b. Double the number.	$23 \cdot 2$
c. Add 4 to your answer.	$46 + 4$
d. Multiply your answer by 5.	$50 \cdot 5$
e. Add 12 to your answer.	$250 + 12$
f. Multiply your answer by 10.	$262 \cdot 10$
g. Subtract 320 from your answer.	$2620 - 320$
h. Cross out the zeros in your answer.	$23\cancel{00}$

STEP 2 *Repeat the trick* Repeat the trick three times using three different numbers. What do you notice?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Let x represent the number you chose in the Explore. Write algebraic expressions for each step. Remember to use the Order of Operations.
2. *Justify* each expression you wrote in Exercise 1.
3. Another number trick is as follows:

   Pick any number.
   Multiply your number by 2.
   Add 18 to your answer.
   Divide your answer by 2.
   Subtract your original number from your answer.

   What is your answer? Does your answer depend on the number you chose? How can you change the trick so your answer is always 15? *Explain*.
4. **REASONING** Write your own number trick.

2.5 Reason Using Properties from Algebra

Before	You used deductive reasoning to form logical arguments.
Now	You will use algebraic properties in logical arguments too.
Why	So you can apply a heart rate formula, as in Example 3.

Key Vocabulary
- **equation,** *p. 875*
- **solve an equation,** *p. 875*

When you *solve an equation*, you use properties of real numbers. Segment lengths and angle measures are real numbers, so you can also use these properties to write logical arguments about geometric figures.

KEY CONCEPT *For Your Notebook*

Algebraic Properties of Equality

Let a, b, and c be real numbers.

Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $ac = bc$.
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	If $a = b$, then a can be substituted for b in any equation or expression.

EXAMPLE 1 Write reasons for each step

Solve $2x + 5 = 20 - 3x$. Write a reason for each step.

Equation	Explanation	Reason
$2x + 5 = 20 - 3x$	Write original equation.	**Given**
$2x + 5 + 3x = 20 - 3x + 3x$	Add $3x$ to each side.	**Addition Property of Equality**
$5x + 5 = 20$	Combine like terms.	**Simplify.**
$5x = 15$	Subtract 5 from each side.	**Subtraction Property of Equality**
$x = 3$	Divide each side by 5.	**Division Property of Equality**

▶ The value of x is 3.

KEY CONCEPT — *For Your Notebook*

Distributive Property

$a(b + c) = ab + ac$, where a, b, and c are real numbers.

EXAMPLE 2 Use the Distributive Property

Solve $-4(11x + 2) = 80$. Write a reason for each step.

Solution

Equation	Explanation	Reason
$-4(11x + 2) = 80$	Write original equation.	Given
$-44x - 8 = 80$	Multiply.	Distributive Property
$-44x = 88$	Add 8 to each side.	Addition Property of Equality
$x = -2$	Divide each side by -44.	Division Property of Equality

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EXAMPLE 3 Use properties in the real world

HEART RATE When you exercise, your target heart rate should be between 50% to 70% of your maximum heart rate. Your target heart rate r at 70% can be determined by the formula $r = 0.70(220 - a)$ where a represents your age in years. Solve the formula for a.

Solution

Equation	Explanation	Reason
$r = 0.70(220 - a)$	Write original equation.	Given
$r = 154 - 0.70a$	Multiply.	Distributive Property
$r - 154 = -0.70a$	Subtract 154 from each side.	Subtraction Property of Equality
$\frac{r - 154}{-0.70} = a$	Divide each side by -0.70.	Division Property of Equality

✓ GUIDED PRACTICE for Examples 1, 2, and 3

In Exercises 1 and 2, solve the equation and write a reason for each step.

1. $4x + 9 = -3x + 2$
2. $14x + 3(7 - x) = -1$
3. Solve the formula $A = \frac{1}{2}bh$ for b.

PROPERTIES The following properties of equality are true for all real numbers. Segment lengths and angle measures are real numbers, so these properties of equality are true for segment lengths and angle measures.

KEY CONCEPT *For Your Notebook*

Reflexive Property of Equality

Real Numbers	For any real number a, $a = a$.
Segment Length	For any segment $\overline{AB}$, $AB = AB$.
Angle Measure	For any angle $\angle A$, $m\angle A = m\angle A$.

Symmetric Property of Equality

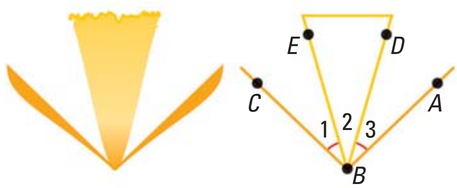
Real Numbers	For any real numbers a and b, if $a = b$, then $b = a$.
Segment Length	For any segments $\overline{AB}$ and $\overline{CD}$, if $AB = CD$, then $CD = AB$.
Angle Measure	For any angles $\angle A$ and $\angle B$, if $m\angle A = m\angle B$, then $m\angle B = m\angle A$.

Transitive Property of Equality

Real Numbers	For any real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.
Segment Length	For any segments $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$, if $AB = CD$ and $CD = EF$, then $AB = EF$.
Angle Measure	For any angles $\angle A$, $\angle B$, and $\angle C$, if $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

EXAMPLE 4 Use properties of equality

LOGO You are designing a logo to sell daffodils. Use the information given. Determine whether $m\angle EBA = m\angle DBC$.

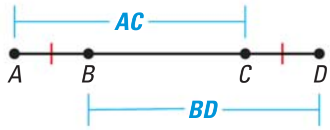


Solution

Equation	**Explanation**	**Reason**
$m\angle 1 = m\angle 3$	Marked in diagram.	**Given**
$m\angle EBA = m\angle 3 + m\angle 2$	Add measures of adjacent angles.	**Angle Addition Postulate**
$m\angle EBA = m\angle 1 + m\angle 2$	Substitute $m\angle 1$ for $m\angle 3$.	**Substitution Property of Equality**
$m\angle 1 + m\angle 2 = m\angle DBC$	Add measures of adjacent angles.	**Angle Addition Postulate**
$m\angle EBA = m\angle DBC$	Both measures are equal to the sum of $m\angle 1 + m\angle 2$.	**Transitive Property of Equality**

EXAMPLE 5 Use properties of equality

In the diagram, $AB = CD$. Show that $AC = BD$.



Solution

Equation	Explanation	Reason
$AB = CD$	Marked in diagram.	Given
$AC = AB + BC$	Add lengths of adjacent segments.	Segment Addition Postulate
$BD = BC + CD$	Add lengths of adjacent segments.	Segment Addition Postulate
$AB + BC = CD + BC$	Add BC to each side of $AB = CD$.	Addition Property of Equality
$AC = BD$	Substitute AC for $AB + BC$ and BD for $BC + CD$.	Substitution Property of Equality

✓ GUIDED PRACTICE for Examples 4 and 5

Name the property of equality the statement illustrates.

4. If $m\angle 6 = m\angle 7$, then $m\angle 7 = m\angle 6$.
5. If $JK = KL$ and $KL = 12$, then $JK = 12$.
6. $m\angle W = m\angle W$

2.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 21, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 5, 27, and 35

◆ = **MULTIPLE REPRESENTATIONS** Ex. 36

SKILL PRACTICE

1. **VOCABULARY** The following statement is true because of what property? The measure of an angle is equal to itself.

2. ★ **WRITING** *Explain* how to check the answer to Example 3 on page 106.

EXAMPLES 1 and 2 on pp. 105–106 for Exs. 3–14

WRITING REASONS **Copy the logical argument. Write a reason for each step.**

3. $3x - 12 = 7x + 8$ Given
   $-4x - 12 = 8$?
   $-4x = 20$?
   $x = -5$?

4. $5(x - 1) = 4x + 13$ Given
   $5x - 5 = 4x + 13$?
   $x - 5 = 13$?
   $x = 18$?

5. ★ **MULTIPLE CHOICE** Name the property of equality the statement illustrates: If $XY = AB$ and $AB = GH$, then $XY = GH$.

(A) Substitution (B) Reflexive (C) Symmetric (D) Transitive

WRITING REASONS **Solve the equation. Write a reason for each step.**

6. $5x - 10 = -40$
7. $4x + 9 = 16 - 3x$
8. $5(3x - 20) = -10$
9. $3(2x + 11) = 9$
10. $2(-x - 5) = 12$
11. $44 - 2(3x + 4) = -18x$
12. $4(5x - 9) = -2(x + 7)$
13. $2x - 15 - x = 21 + 10x$
14. $3(7x - 9) - 19x = -15$

EXAMPLE 3 on p. 106 for Exs. 15–20

ALGEBRA **Solve the equation for *y*. Write a reason for each step.**

15. $5x + y = 18$
16. $-4x + 2y = 8$
17. $12 - 3y = 30x$
18. $3x + 9y = -7$
19. $2y + 0.5x = 16$
20. $\frac{1}{2}x - \frac{3}{4}y = -2$

EXAMPLES 4 and 5 on pp. 107–108 for Exs. 21–25

COMPLETING STATEMENTS **In Exercises 21–25, use the property to copy and complete the statement.**

21. Substitution Property of Equality: If $AB = 20$, then $AB + CD = \underline{\ ?\ }$.
22. Symmetric Property of Equality: If $m\angle 1 = m\angle 2$, then $\underline{\ ?\ }$.
23. Addition Property of Equality: If $AB = CD$, then $\underline{\ ?\ } + EF = \underline{\ ?\ } + EF$.
24. Distributive Property: If $5(x + 8) = 2$, then $\underline{\ ?\ }x + \underline{\ ?\ } = 2$.
25. Transitive Property of Equality: If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $\underline{\ ?\ }$.

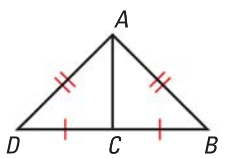
26. **ERROR ANALYSIS** *Describe* and correct the error in solving the equation for x.

$7x = x + 24$	Given
$8x = 24$	Addition Property of Equality
$x = 3$	Division Property of Equality

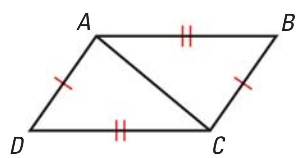
27. ★ **OPEN-ENDED MATH** Write examples from your everyday life that could help you remember the *Reflexive*, *Symmetric*, and *Transitive* Properties of Equality.

PERIMETER **In Exercises 28 and 29, show that the perimeter of triangle *ABC* is equal to the perimeter of triangle *ADC*.**

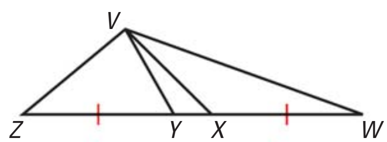
28.



29.



30. **CHALLENGE** In the figure at the right, $\overline{ZY} \cong \overline{XW}$, $ZX = 5x + 17$, $YW = 10 - 2x$, and $YX = 3$. Find ZY and XW.



PROBLEM SOLVING

EXAMPLE 3 on p. 106 for Exs. 31–32

31. **PERIMETER** The formula for the perimeter P of a rectangle is $P = 2\ell + 2w$ where ℓ is the length and w is the width. Solve the formula for ℓ and write a reason for each step. Then find the length of a rectangular lawn whose perimeter is 55 meters and whose width is 11 meters.

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32. **AREA** The formula for the area A of a triangle is $A = \frac{1}{2}bh$ where b is the base and h is the height. Solve the formula for h and write a reason for each step. Then find the height of a triangle whose area is 1768 square inches and whose base is 52 inches.

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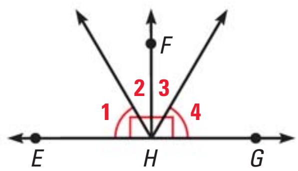
33. **PROPERTIES OF EQUALITY** Copy and complete the table to show $m\angle 2 = m\angle 3$.



Equation	Explanation	Reason
$m\angle 1 = m\angle 4$, $m\angle EHF = 90°$, $m\angle GHF = 90°$	?	Given
$m\angle EHF = m\angle GHF$	?	Substitution Property of Equality
$m\angle EHF = m\angle 1 + m\angle 2$ $m\angle GHF = m\angle 3 + m\angle 4$	Add measures of adjacent angles.	?
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	Write expressions equal to the angle measures.	?
?	Substitute $m\angle 1$ for $m\angle 4$.	?
$m\angle 2 = m\angle 3$	?	Subtraction Property of Equality

34. **MULTI-STEP PROBLEM** Points A, B, C, and D represent stops, in order, along a subway route. The distance between Stops A and C is the same as the distance between Stops B and D.

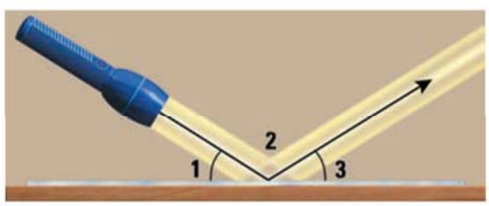
a. Draw a diagram to represent the situation.

b. Use the Segment Addition Postulate to show that the distance between Stops A and B is the same as the distance between Stops C and D.

c. *Justify* part (b) using the Properties of Equality.

EXAMPLE 4 on p. 107 for Ex. 35

35. ★ **SHORT RESPONSE** A flashlight beam is reflected off a mirror lying flat on the ground. Use the information given below to find $m\angle 2$.



$m\angle 1 + m\angle 2 + m\angle 3 = 180°$

$m\angle 1 + m\angle 2 = 148°$

$m\angle 1 = m\angle 3$

36. **MULTIPLE REPRESENTATIONS** The formula to convert a temperature in degrees Fahrenheit (°F) to degrees Celsius (°C) is $C = \frac{5}{9}(F - 32)$.

a. **Writing an Equation** Solve the formula for F. Write a reason for each step.

b. **Making a Table** Make a table that shows the conversion to Fahrenheit for each temperature: 0°C, 20°C, 32°C, and 41°C.

c. **Drawing a Graph** Use your table to graph the temperature in degrees Celsius (°C) as a function of the temperature in degrees Fahrenheit (°F). Is this a linear function?

CHALLENGE In Exercises 37 and 38, decide whether the relationship is *reflexive*, *symmetric*, or *transitive*.

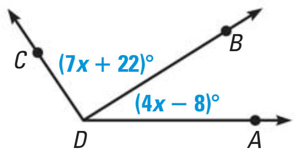
37. **Group:** two employees in a grocery store
Relationship: "worked the same hours as"
Example: Yen worked the same hours as Jim.

38. **Group:** negative numbers on a number line
Relationship: "is less than"
Example: -4 is less than -1.

MIXED REVIEW

PREVIEW
Prepare for Lesson 2.6 in Exs. 39–40.

In the diagram, $m\angle ADC = 124°$. *(p. 24)*



39. Find $m\angle ADB$.

40. Find $m\angle BDC$.

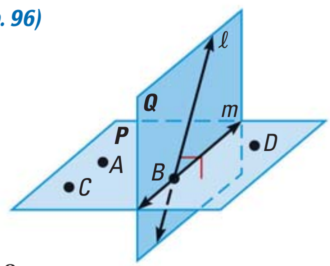
41. Find a counterexample to show the conjecture is false.

Conjecture All polygons have five sides. *(p. 72)*

42. Select the word(s) that make(s) the conclusion true. If $m\angle X = m\angle Y$ and $m\angle Y = m\angle Z$, then $m\angle X$ (*is*, *may be*, or *is not*) equal to $m\angle Z$. *(p. 87)*

QUIZ for Lessons 2.4–2.5

Use the diagram to determine if the statement is *true* or *false*. *(p. 96)*



1. Points B, C, and D are coplanar.
2. Point A is on line ℓ.
3. Plane P and plane Q are perpendicular.

Solve the equation. Write a reason for each step. *(p. 105)*

4. $x + 20 = 35$

5. $5x - 14 = 16 + 3x$

Use the property to copy and complete the statement. *(p. 105)*

6. Subtraction Property of Equality: If $AB = CD$, then $\underline{\ ?\ } - EF = \underline{\ ?\ } - EF$.
7. Transitive Property of Equality: If $a = b$ and $b = c$, then $\underline{\ ?\ } = \underline{\ ?\ }$.

2.6 Prove Statements about Segments and Angles

Before You used deductive reasoning.

Now You will write proofs using geometric theorems.

Why? So you can prove angles are congruent, as in Ex. 21.

Key Vocabulary
- **proof**
- **two-column proof**
- **theorem**

A **proof** is a logical argument that shows a statement is true. There are several formats for proofs. A **two-column proof** has numbered statements and corresponding reasons that show an argument in a logical order.

In a two-column proof, each statement in the left-hand column is either given information or the result of applying a known property or fact to statements already made. Each reason in the right-hand column is the explanation for the corresponding statement.

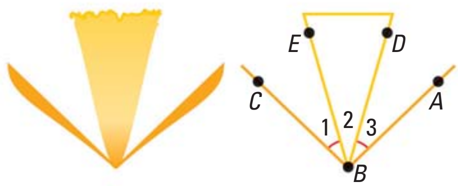
WRITE PROOFS
Writing a two-column proof is a formal way of organizing your reasons to show a statement is true.

EXAMPLE 1 Write a two-column proof

Write a two-column proof for the situation in Example 4 on page 107.

GIVEN ▶ $m\angle 1 = m\angle 3$

PROVE ▶ $m\angle EBA = m\angle DBC$



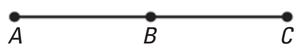
STATEMENTS	REASONS
1. $m\angle 1 = m\angle 3$	**1.** Given
2. $m\angle EBA = m\angle 3 + m\angle 2$	**2.** Angle Addition Postulate
3. $m\angle EBA = m\angle 1 + m\angle 2$	**3.** Substitution Property of Equality
4. $m\angle 1 + m\angle 2 = m\angle DBC$	**4.** Angle Addition Postulate
5. $m\angle EBA = m\angle DBC$	**5.** Transitive Property of Equality

✓ GUIDED PRACTICE for Example 1

1. Four steps of a proof are shown. Give the reasons for the last two steps.

GIVEN ▶ $AC = AB + AB$

PROVE ▶ $AB = BC$



STATEMENTS	REASONS
1. $AC = AB + AB$	**1.** Given
2. $AB + BC = AC$	**2.** Segment Addition Postulate
3. $AB + AB = AB + BC$	**3.** ?
4. $AB = BC$	**4.** ?

THEOREMS The reasons used in a proof can include definitions, properties, postulates, and *theorems*. A **theorem** is a statement that can be proven. Once you have proven a theorem, you can use the theorem as a reason in other proofs.

TAKE NOTES
Be sure to copy all new theorems in your notebook. Notice that the theorem box tells you where to find the proof(s).

THEOREMS *For Your Notebook*

THEOREM 2.1 Congruence of Segments

Segment congruence is reflexive, symmetric, and transitive.

Reflexive For any segment AB, $\overline{AB} \cong \overline{AB}$.

Symmetric If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Proofs: p. 137; Ex. 5, p. 121; Ex. 26, p. 118

THEOREM 2.2 Congruence of Angles

Angle congruence is reflexive, symmetric, and transitive.

Reflexive For any angle A, $\angle A \cong \angle A$.

Symmetric If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

Proofs: Ex. 25, p. 118; Concept Summary, p. 114; Ex. 21, p. 137

EXAMPLE 2 Name the property shown

Name the property illustrated by the statement.

a. If $\angle R \cong \angle T$ and $\angle T \cong \angle P$, then $\angle R \cong \angle P$.

b. If $\overline{NK} \cong \overline{BD}$, then $\overline{BD} \cong \overline{NK}$.

Solution

a. Transitive Property of Angle Congruence

b. Symmetric Property of Segment Congruence

✓ GUIDED PRACTICE for Example 2

Name the property illustrated by the statement.

2. $\overline{CD} \cong \overline{CD}$

3. If $\angle Q \cong \angle V$, then $\angle V \cong \angle Q$.

In this lesson, most of the proofs involve showing that congruence and equality are equivalent. You may find that what you are asked to prove seems to be obviously true. It is important to practice writing these proofs so that you will be prepared to write more complicated proofs in later chapters.

EXAMPLE 3 Use properties of equality

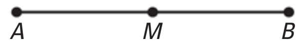
Prove this property of midpoints: If you know that M is the midpoint of $\overline{AB}$, prove that AB is two times AM and AM is one half of AB.

WRITE PROOFS
Before writing a proof, organize your reasoning by copying or drawing a diagram for the situation described. Then identify the GIVEN and PROVE statements.

GIVEN ▶ M is the midpoint of $\overline{AB}$.

PROVE ▶ **a.** $AB = 2 \cdot AM$

b. $AM = \frac{1}{2}AB$



STATEMENTS	REASONS
1. M is the midpoint of $\overline{AB}$.	**1.** Given
2. $\overline{AM} \cong \overline{MB}$	**2.** Definition of midpoint
3. $AM = MB$	**3.** Definition of congruent segments
4. $AM + MB = AB$	**4.** Segment Addition Postulate
5. $AM + AM = AB$	**5.** Substitution Property of Equality
a. 6. $2AM = AB$	**6.** Distributive Property
b. 7. $AM = \frac{1}{2}AB$	**7.** Division Property of Equality

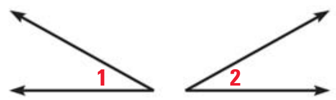
✓ GUIDED PRACTICE for Example 3

4. WHAT IF? Look back at Example 3. What would be different if you were proving that $AB = 2 \cdot MB$ and that $MB = \frac{1}{2}AB$ instead?

CONCEPT SUMMARY *For Your Notebook*

Writing a Two-Column Proof

In a proof, you make one statement at a time, until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.



Copy or draw diagrams and label given information to help develop proofs.

Proof of the Symmetric Property of Angle Congruence

GIVEN ▶ $\angle 1 \cong \angle 2$

PROVE ▶ $\angle 2 \cong \angle 1$

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	**1.** Given
2. $m\angle 1 = m\angle 2$	**2.** Definition of congruent angles
3. $m\angle 2 = m\angle 1$	**3.** Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	**4.** Definition of congruent angles

Statements based on facts that you know or on conclusions from deductive reasoning

Definitions, postulates, or proven theorems that allow you to state the corresponding statement

The number of statements will vary.

Remember to give a reason for the last statement.

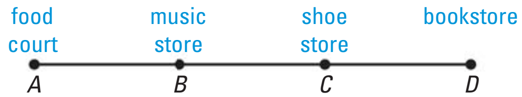
EXAMPLE 4 Solve a multi-step problem

SHOPPING MALL Walking down a hallway at the mall, you notice the music store is halfway between the food court and the shoe store. The shoe store is halfway between the music store and the bookstore. Prove that the distance between the entrances of the food court and music store is the same as the distance between the entrances of the shoe store and bookstore.

ANOTHER WAY
For an alternative method for solving the problem in Example 4, turn to page 120 for the **Problem Solving Workshop**.

Solution

STEP 1 **Draw** and label a diagram.



STEP 2 **Draw** separate diagrams to show mathematical relationships.

STEP 3 **State** what is given and what is to be proved for the situation. Then write a proof.

GIVEN ▶ B is the midpoint of $\overline{AC}$.
C is the midpoint of $\overline{BD}$.

PROVE ▶ $AB = CD$

STATEMENTS	REASONS
1. B is the midpoint of $\overline{AC}$. C is the midpoint of $\overline{BD}$.	**1.** Given
2. $\overline{AB} \cong \overline{BC}$	**2.** Definition of midpoint
3. $\overline{BC} \cong \overline{CD}$	**3.** Definition of midpoint
4. $\overline{AB} \cong \overline{CD}$	**4.** Transitive Property of Congruence
5. $AB = CD$	**5.** Definition of congruent segments

✓ GUIDED PRACTICE for Example 4

5. In Example 4, does it matter what the actual distances are in order to prove the relationship between AB and CD? *Explain.*

6. In Example 4, there is a clothing store halfway between the music store and the shoe store. What other two store entrances are the same distance from the entrance of the clothing store?

2.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 15, and 21

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 4, 12, 19, 27, and 28

SKILL PRACTICE

1. **VOCABULARY** What is a *theorem*? How is it different from a *postulate*?

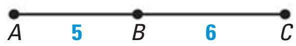
2. ★ **WRITING** You can use theorems as reasons in a two-column proof. What other types of statements can you use as reasons in a two-column proof? Give examples.

EXAMPLE 1 on p. 112 for Exs. 3–4

3. **DEVELOPING PROOF** Copy and complete the proof.

GIVEN ▶ $AB = 5$, $BC = 6$

PROVE ▶ $AC = 11$



STATEMENTS	REASONS
1. $AB = 5$, $BC = 6$	1. Given
2. $AC = AB + BC$	2. Segment Addition Postulate
3. $AC = 5 + 6$	3. ___?___
4. ___?___	4. Simplify.

4. ★ **MULTIPLE CHOICE** Which property listed is the reason for the last step in the proof?

GIVEN ▶ $m\angle 1 = 59°$, $m\angle 2 = 59°$

PROVE ▶ $m\angle 1 = m\angle 2$

STATEMENTS	REASONS
1. $m\angle 1 = 59°$, $m\angle 2 = 59°$	1. Given
2. $59° = m\angle 2$	2. Symmetric Property of Equality
3. $m\angle 1 = m\angle 2$	3. ___?___

(A) Transitive Property of Equality (B) Reflexive Property of Equality

(C) Symmetric Property of Equality (D) Distributive Property

EXAMPLES 2 and 3 on pp. 113–114 for Exs. 5–13

USING PROPERTIES **Use the property to copy and complete the statement.**

5. Reflexive Property of Congruence: ___?___ $\cong \overline{SE}$

6. Symmetric Property of Congruence: If ___?___ $\cong$ ___?___, then $\angle RST \cong \angle JKL$.

7. Transitive Property of Congruence: If $\angle F \cong \angle J$ and ___?___ $\cong$ ___?___, then $\angle F \cong \angle L$.

NAMING PROPERTIES **Name the property illustrated by the statement.**

8. If $\overline{DG} \cong \overline{CT}$, then $\overline{CT} \cong \overline{DG}$.

9. $\angle VWX \cong \angle VWX$

10. If $\overline{JK} \cong \overline{MN}$ and $\overline{MN} \cong \overline{XY}$, then $\overline{JK} \cong \overline{XY}$.

11. $YZ = ZY$

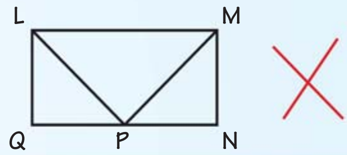
12. ★ **MULTIPLE CHOICE** Name the property illustrated by the statement "If $\overline{CD} \cong \overline{MN}$, then $\overline{MN} \cong \overline{CD}$."

(A) Reflexive Property of Equality (B) Symmetric Property of Equality

(C) Symmetric Property of Congruence (D) Transitive Property of Congruence

13. **ERROR ANALYSIS** In the diagram below, $\overline{MN} \cong \overline{LQ}$ and $\overline{LQ} \cong \overline{PN}$. *Describe* and correct the error in the reasoning.

> Because $\overline{MN} \cong \overline{LQ}$ and $\overline{LQ} \cong \overline{PN}$, then $\overline{MN} \cong \overline{PN}$ by the Reflexive Property of Segment Congruence.



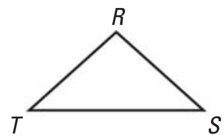
EXAMPLE 4 on p. 115 for Exs. 14–15

MAKING A SKETCH **In Exercises 14 and 15, sketch a diagram that represents the given information.**

14. **CRYSTALS** The shape of a crystal can be represented by intersecting lines and planes. Suppose a crystal is *cubic*, which means it can be represented by six planes that intersect at right angles.

15. **BEACH VACATION** You are on vacation at the beach. Along the boardwalk, the bike rentals are halfway between your cottage and the kite shop. The snack shop is halfway between your cottage and the bike rentals. The arcade is halfway between the bike rentals and the kite shop.

16. **DEVELOPING PROOF** Copy and complete the proof.



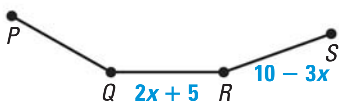
GIVEN ▶ $RT = 5$, $RS = 5$, $\overline{RT} \cong \overline{TS}$

PROVE ▶ $\overline{RS} \cong \overline{TS}$

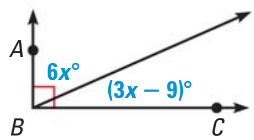
STATEMENTS	REASONS
1. $RT = 5$, $RS = 5$, $\overline{RT} \cong \overline{TS}$	1. ?
2. $RS = RT$	2. Transitive Property of Equality
3. $RT = TS$	3. Definition of congruent segments
4. $RS = TS$	4. Transitive Property of Equality
5. $\overline{RS} \cong \overline{TS}$	5. ?

ALGEBRA **Solve for *x* using the given information. *Explain* your steps.**

17. **GIVEN** ▶ $\overline{QR} \cong \overline{PQ}$, $\overline{RS} \cong \overline{PQ}$



18. **GIVEN** ▶ $m\angle ABC = 90°$



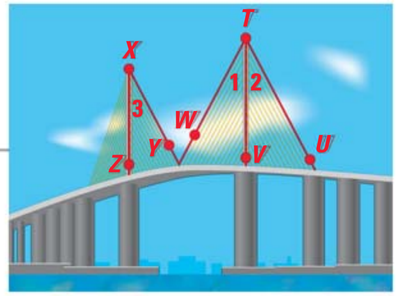
19. ★ **SHORT RESPONSE** *Explain* why writing a proof is an example of deductive reasoning, not inductive reasoning.

20. **CHALLENGE** Point P is the midpoint of $\overline{MN}$ and point Q is the midpoint of $\overline{MP}$. Suppose $\overline{AB}$ is congruent to $\overline{MP}$, and $\overline{PN}$ has length x. Write the length of the segments in terms of x. *Explain.*

a. $\overline{AB}$ b. $\overline{MN}$ c. $\overline{MQ}$ d. $\overline{NQ}$

PROBLEM SOLVING

21. **BRIDGE** In the bridge in the illustration, it is known that $\angle 2 \cong \angle 3$ and $\overrightarrow{TV}$ bisects $\angle UTW$. Copy and complete the proof to show that $\angle 1 \cong \angle 3$.



STATEMENTS	REASONS
1. $\overrightarrow{TV}$ bisects $\angle UTW$.	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** ___?___
3. $\angle 2 \cong \angle 3$	**3.** Given
4. $\angle 1 \cong \angle 3$	**4.** ___?___

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EXAMPLE 3 on p. 114 for Ex. 22

22. **DEVELOPING PROOF** Write a complete proof by matching each statement with its corresponding reason.

GIVEN ▶ $\overrightarrow{QS}$ is an angle bisector of $\angle PQR$.

PROVE ▶ $m\angle PQS = \frac{1}{2}m\angle PQR$

STATEMENTS	REASONS
1. $\overrightarrow{QS}$ is an angle bisector of $\angle PQR$.	**A.** Definition of angle bisector
2. $\angle PQS \cong \angle SQR$	**B.** Distributive Property
3. $m\angle PQS = m\angle SQR$	**C.** Angle Addition Postulate
4. $m\angle PQS + m\angle SQR = m\angle PQR$	**D.** Given
5. $m\angle PQS + m\angle PQS = m\angle PQR$	**E.** Division Property of Equality
6. $2 \cdot m\angle PQS = m\angle PQR$	**F.** Definition of congruent angles
7. $m\angle PQS = \frac{1}{2}m\angle PQR$	**G.** Substitution Property of Equality

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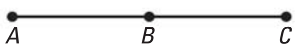
PROOF **Use the given information and the diagram to prove the statement.**

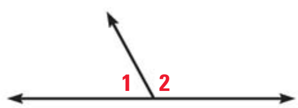
23. **GIVEN** ▶ $2AB = AC$

PROVE ▶ $AB = BC$



24. **GIVEN** ▶ $m\angle 1 + m\angle 2 = 180°$
$m\angle 1 = 62°$

PROVE ▶ $m\angle 2 = 118°$

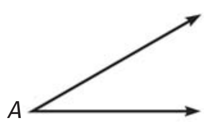


PROVING PROPERTIES **Prove the indicated property of congruence.**

25. Reflexive Property of Angle Congruence

GIVEN ▶ A is an angle.

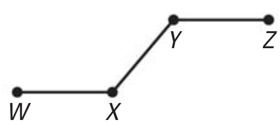
PROVE ▶ $\angle A \cong \angle A$



26. Transitive Property of Segment Congruence

GIVEN ▶ $\overline{WX} \cong \overline{XY}$ and $\overline{XY} \cong \overline{YZ}$

PROVE ▶ $\overline{WX} \cong \overline{YZ}$



○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

27. ★ **SHORT RESPONSE** In the sculpture shown, $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$. Classify the triangle and *justify* your reasoning.

28. ★ **SHORT RESPONSE** You use a computer drawing program to create a line segment. You copy the segment and paste it. You copy the pasted segment and then paste it, and so on. How do you know all the line segments are congruent?

EXAMPLE 4
on p. 115
for Ex. 29

29. **MULTI-STEP PROBLEM** The distance from the restaurant to the shoe store is the same as the distance from the cafe to the florist. The distance from the shoe store to the movie theater is the same as the distance from the movie theater to the cafe, and from the florist to the dry cleaners.

Use the steps below to prove that the distance from the restaurant to the movie theater is the same as the distance from the cafe to the dry cleaners.

a. Draw and label a diagram to show the mathematical relationships.

b. State what is given and what is to be proved for the situation.

c. Write a two-column proof.

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30. **CHALLENGE** The distance from Springfield to Lakewood City is equal to the distance from Springfield to Bettsville. Janisburg is 50 miles farther from Springfield than Bettsville is. Moon Valley is 50 miles farther from Springfield than Lakewood City is.

a. Assume all five cities lie in a straight line. Draw a diagram that represents this situation.

b. Suppose you do not know that all five cities lie in a straight line. Draw a diagram that is different from the one in part (a) to represent the situation.

c. *Explain* the differences in the two diagrams.

MIXED REVIEW

PREVIEW
Prepare for Lesson 2.7 in Exs. 31–33.

Given $m\angle 1$, find the measure of an angle that is complementary to $\angle 1$ and the measure of an angle that is supplementary to $\angle 1$. *(p. 35)*

31. $m\angle 1 = 47°$ 32. $m\angle 1 = 29°$ 33. $m\angle 1 = 89°$

Solve the equation. Write a reason for each step. *(p. 105)*

34. $5x + 14 = -16$ 35. $2x - 9 = 15 - 4x$ 36. $x + 28 = -11 - 3x - 17$

PROBLEM SOLVING WORKSHOP
LESSON 2.6

Using ALTERNATIVE METHODS

Another Way to Solve Example 4, page 115

MULTIPLE REPRESENTATIONS The first step in writing any proof is to make a plan. A diagram or *visual organizer* can help you plan your proof. The steps of a proof must be in a logical order, but there may be more than one correct order.

PROBLEM

SHOPPING MALL Walking down a hallway at the mall, you notice the music store is halfway between the food court and the shoe store. The shoe store is halfway between the music store and the bookstore. Prove that the distance between the entrances of the food court and music store is the same as the distance between the entrances of the shoe store and bookstore.

METHOD

Using a Visual Organizer

STEP 1 **Use** a visual organizer to map out your proof.

The music store is halfway between the food court and the shoe store. The shoe store is halfway between the music store and the bookstore.

Given information	M is halfway between F and S.	S is halfway between M and B.
Deductions from given information	M is the midpoint of $\overline{FS}$. So, $FM = MS$.	S is the midpoint of $\overline{MB}$. So, $MS = SB$.
Statement to prove	$FM = SB$	

STEP 2 **Write** a proof using the lengths of the segments.

GIVEN ▶ M is halfway between F and S.
S is halfway between M and B.

PROVE ▶ $FM = SB$

STATEMENTS	REASONS
1. M is halfway between F and S.	**1.** Given
2. S is halfway between M and B.	**2.** Given
3. M is the midpoint of $\overline{FS}$.	**3.** Definition of midpoint
4. S is the midpoint of $\overline{MB}$.	**4.** Definition of midpoint
5. $FM = MS$ and $MS = SB$	**5.** Definition of midpoint
6. $MS = MS$	**6.** Reflexive Property of Equality
7. $FM = SB$	**7.** Substitution Property of Equality

PRACTICE

1. **COMPARE PROOFS** *Compare* the proof on the previous page and the proof in Example 4 on page 115.

   a. How are the proofs the same? How are they different?

   b. Which proof is easier for you to understand? *Explain*.

2. **REASONING** Below is a proof of the Transitive Property of Angle Congruence. What is another reason you could give for Statement 3? *Explain*.

   GIVEN ▶ $\angle A \cong \angle B$ and $\angle B \cong \angle C$

   PROVE ▶ $\angle A \cong \angle C$

STATEMENTS	REASONS
1. $\angle A \cong \angle B$, $\angle B \cong \angle C$	1. Given
2. $m\angle A = m\angle B$, $m\angle B = m\angle C$	2. Definition of congruent angles
3. $m\angle A = m\angle C$	3. Transitive Property of Equality
4. $\angle A \cong \angle C$	4. Definition of congruent angles

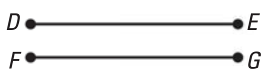
3. **SHOPPING MALL** You are at the same mall as on page 120 and you notice that the bookstore is halfway between the shoe store and the toy store. Draw a diagram or make a visual organizer, then write a proof to show that the distance from the entrances of the food court and music store is the same as the distance from the entrances of the book store and toy store.

4. **WINDOW DESIGN** The entrance to the mall has a decorative window above the main doors as shown. The colored dividers form congruent angles. Draw a diagram or make a visual organizer, then write a proof to show that the angle measure between the red dividers is half the measure of the angle between the blue dividers.

5. **COMPARE PROOFS** Below is a proof of the Symmetric Property of Segment Congruence.

   GIVEN ▶ $\overline{DE} \cong \overline{FG}$

   PROVE ▶ $\overline{FG} \cong \overline{DE}$



STATEMENTS	REASONS
1. $\overline{DE} \cong \overline{FG}$	1. Given
2. $DE = FG$	2. Definition of congruent segments
3. $FG = DE$	3. Symmetric Property of Equality
4. $\overline{FG} \cong \overline{DE}$	4. Definition of congruent segments

   a. *Compare* this proof to the proof of the Symmetric Property of Angle Congruence in the Concept Summary on page 114. What makes the proofs different? *Explain*.

   b. *Explain* why Statement 2 above cannot be $\overline{FG} \cong \overline{DE}$.

Investigating Geometry ACTIVITY Use before Lesson 2.7

2.7 Angles and Intersecting Lines

MATERIALS • graphing calculator or computer

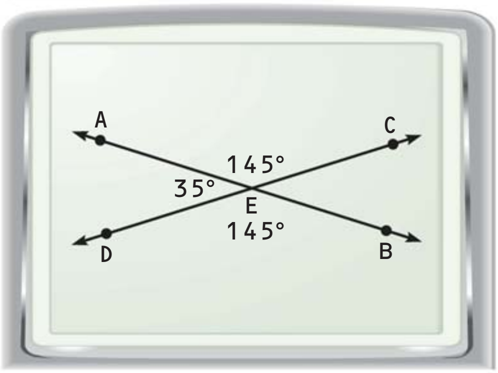
QUESTION **What is the relationship between the measures of the angles formed by intersecting lines?**

You can use geometry drawing software to investigate the measures of angles formed when lines intersect.

EXPLORE 1 Measure linear pairs formed by intersecting lines

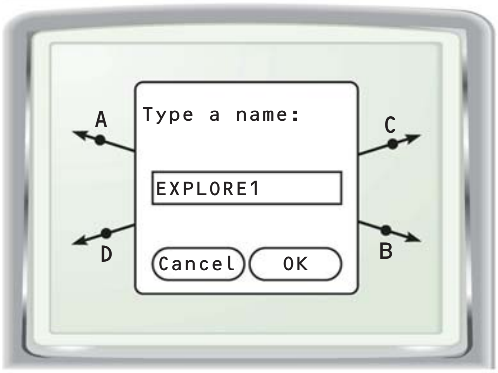
STEP 1 ***Draw two intersecting lines*** Draw and label $\overleftrightarrow{AB}$. Draw and label $\overleftrightarrow{CD}$ so that it intersects $\overleftrightarrow{AB}$. Draw and label the point of intersection E.

STEP 2



Measure angles Measure $\angle AEC$, $\angle AED$, and $\angle DEB$. Move point C to change the angles.

STEP 3



Save Save as "EXPLORE1" by choosing Save from the F1 menu and typing the name.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. *Describe* the relationship between $\angle AEC$ and $\angle AED$.
2. *Describe* the relationship between $\angle AED$ and $\angle DEB$.
3. What do you notice about $\angle AEC$ and $\angle DEB$?
4. In Explore 1, what happens when you move C to a different position? Do the angle relationships stay the same? Make a conjecture about two angles supplementary to the same angle.
5. Do you think your conjecture will be true for supplementary angles that are not adjacent? *Explain*.

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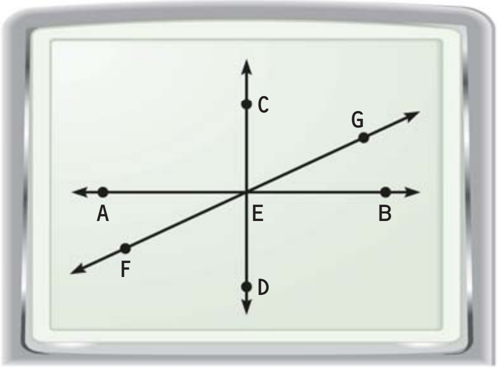
Keystrokes

EXPLORE 2 Measure complementary angles

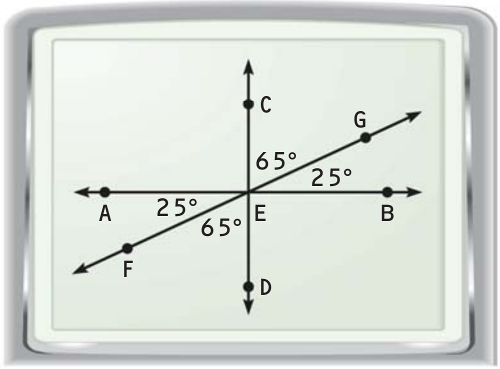
STEP 1 ***Draw two perpendicular lines*** Draw and label $\overleftrightarrow{AB}$. Draw point E on $\overleftrightarrow{AB}$. Draw and label $\overleftrightarrow{EC} \perp \overleftrightarrow{AB}$. Draw and label point D on $\overleftrightarrow{EC}$ so that E is between C and D as shown in Step 2.

STEP 2



Draw another line Draw and label $\overleftrightarrow{EG}$ so that G is in the interior of $\angle CEB$. Draw point F on $\overleftrightarrow{EG}$ as shown. Save as "EXPLORE2".

STEP 3



Measure angles Measure $\angle AEF$, $\angle FED$, $\angle CEG$, and $\angle GEB$. Move point G to change the angles.

EXPLORE 3 Measure vertical angles formed by intersecting lines

STEP 1 ***Draw two intersecting lines*** Draw and label $\overleftrightarrow{AB}$. Draw and label $\overleftrightarrow{CD}$ so that it intersects $\overleftrightarrow{AB}$. Draw and label the point of intersection E.

STEP 2 ***Measure angles*** Measure $\angle AEC$, $\angle AED$, $\angle BEC$, and $\angle DEB$. Move point C to change the angles. Save as "EXPLORE3".

DRAW CONCLUSIONS Use your observations to complete these exercises

6. In Explore 2, does the angle relationship stay the same as you move G?

7. In Explore 2, make a conjecture about the relationship between $\angle CEG$ and $\angle GEB$. Write your conjecture in if-then form.

8. In Explore 3, the intersecting lines form two pairs of vertical angles. Make a conjecture about the relationship between any two vertical angles. Write your conjecture in if-then form.

9. Name the pairs of vertical angles in Explore 2. Use this drawing to test your conjecture from Exercise 8.

2.7 Prove Angle Pair Relationships

Before You identified relationships between pairs of angles.

Now You will use properties of special pairs of angles.

Why? So you can describe angles found in a home, as in Ex. 44.

Key Vocabulary
- **complementary angles,** *p. 35*
- **supplementary angles,** *p. 35*
- **linear pair,** *p. 37*
- **vertical angles,** *p. 37*

Sometimes, a new theorem describes a relationship that is useful in writing proofs. For example, using the *Right Angles Congruence Theorem* will reduce the number of steps you need to include in a proof involving right angles.

THEOREM *For Your Notebook*

THEOREM 2.3 Right Angles Congruence Theorem

All right angles are congruent.

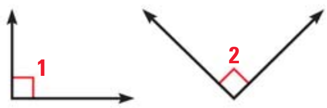
Proof: below

PROOF Right Angles Congruence Theorem

WRITE PROOFS
When you prove a theorem, write the hypothesis of the theorem as the GIVEN statement. The conclusion is what you must PROVE.

GIVEN ▶ $\angle 1$ and $\angle 2$ are right angles.

PROVE ▶ $\angle 1 \cong \angle 2$



STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are right angles.	**1.** Given
2. $m\angle 1 = 90°$, $m\angle 2 = 90°$	**2.** Definition of right angle
3. $m\angle 1 = m\angle 2$	**3.** Transitive Property of Equality
4. $\angle 1 \cong \angle 2$	**4.** Definition of congruent angles

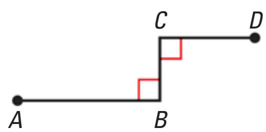
EXAMPLE 1 Use right angle congruence

AVOID ERRORS
The given information in Example 1 is about perpendicular lines. You must then use deductive reasoning to show the angles are right angles.

Write a proof.

GIVEN ▶ $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$

PROVE ▶ $\angle B \cong \angle C$

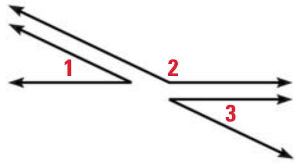


STATEMENTS	REASONS
1. $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$	**1.** Given
2. $\angle B$ and $\angle C$ are right angles.	**2.** Definition of perpendicular lines
3. $\angle B \cong \angle C$	**3.** Right Angles Congruence Theorem

THEOREMS *For Your Notebook*

THEOREM 2.4 Congruent Supplements Theorem

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

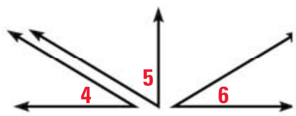


If $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 2$ are supplementary, then $\angle 1 \cong \angle 3$.

Proof: Example 2, below; Ex. 36, p. 129

THEOREM 2.5 Congruent Complements Theorem

If two angles are complementary to the same angle (or to congruent angles), then they are congruent.



If $\angle 4$ and $\angle 5$ are complementary and $\angle 6$ and $\angle 5$ are complementary, then $\angle 4 \cong \angle 6$.

Proof: Ex. 37, p. 129; Ex. 41, p. 130

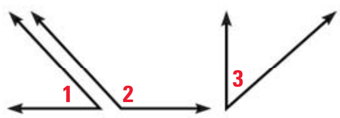
To prove Theorem 2.4, you must prove two cases: one with angles supplementary to the same angle and one with angles supplementary to congruent angles. The proof of Theorem 2.5 also requires two cases.

EXAMPLE 2 Prove a case of Congruent Supplements Theorem

Prove that two angles supplementary to the same angle are congruent.

GIVEN ▶ $\angle 1$ and $\angle 2$ are supplements.
$\angle 3$ and $\angle 2$ are supplements.

PROVE ▶ $\angle 1 \cong \angle 3$



STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are supplements. $\angle 3$ and $\angle 2$ are supplements.	**1.** Given
2. $m\angle 1 + m\angle 2 = 180°$ $m\angle 3 + m\angle 2 = 180°$	**2.** Definition of supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	**3.** Transitive Property of Equality
4. $m\angle 1 = m\angle 3$	**4.** Subtraction Property of Equality
5. $\angle 1 \cong \angle 3$	**5.** Definition of congruent angles

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✓ GUIDED PRACTICE for Examples 1 and 2

1. How many steps do you save in the proof in Example 1 by using the *Right Angles Congruence Theorem*?

2. Draw a diagram and write GIVEN and PROVE statements for a proof of each case of the *Congruent Complements Theorem*.

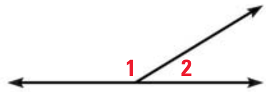
INTERSECTING LINES When two lines intersect, pairs of vertical angles and linear pairs are formed. The relationship that you used in Lesson 1.5 for linear pairs is formally stated below as the *Linear Pair Postulate*. This postulate is used in the proof of the *Vertical Angles Congruence Theorem*.

POSTULATE — For Your Notebook

POSTULATE 12 Linear Pair Postulate

If two angles form a linear pair, then they are supplementary.

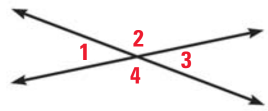
$\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 = 180°$.



THEOREM — For Your Notebook

THEOREM 2.6 Vertical Angles Congruence Theorem

Vertical angles are congruent.



Proof: Example 3, below

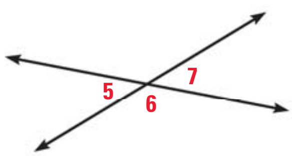
$\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$

EXAMPLE 3 Prove the Vertical Angles Congruence Theorem

Prove vertical angles are congruent.

GIVEN ▶ $\angle 5$ and $\angle 7$ are vertical angles.

PROVE ▶ $\angle 5 \cong \angle 7$

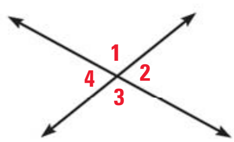


STATEMENTS	REASONS
1. $\angle 5$ and $\angle 7$ are vertical angles.	**1.** Given
2. $\angle 5$ and $\angle 6$ are a linear pair. $\angle 6$ and $\angle 7$ are a linear pair.	**2.** Definition of linear pair, as shown in the diagram
3. $\angle 5$ and $\angle 6$ are supplementary. $\angle 6$ and $\angle 7$ are supplementary.	**3.** Linear Pair Postulate
4. $\angle 5 \cong \angle 7$	**4.** Congruent Supplements Theorem

USE A DIAGRAM
You can use information labeled in a diagram in your proof.

✓ GUIDED PRACTICE for Example 3

In Exercises 3–5, use the diagram.



3. If $m\angle 1 = 112°$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

4. If $m\angle 2 = 67°$, find $m\angle 1$, $m\angle 3$, and $m\angle 4$.

5. If $m\angle 4 = 71°$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$.

6. Which previously proven theorem is used in Example 3 as a reason?

★ EXAMPLE 4 Standardized Test Practice

ELIMINATE CHOICES
Look for angle pair relationships in the diagram. The angles in the diagram are supplementary, not complementary or congruent, so eliminate choices A and C.

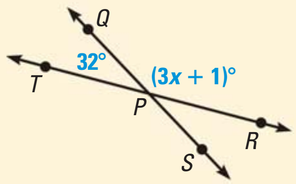
Which equation can be used to find x?

Ⓐ $32 + (3x + 1) = 90$

Ⓑ $32 + (3x + 1) = 180$

Ⓒ $32 = 3x + 1$

Ⓓ $3x + 1 = 212$



Solution

Because $\angle TPQ$ and $\angle QPR$ form a linear pair, the sum of their measures is 180°.

▸ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Example 4

Use the diagram in Example 4.

7. Solve for x.

8. Find $m\angle TPS$.

2.7 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 13, and 39
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 16, 30, and 45

SKILL PRACTICE

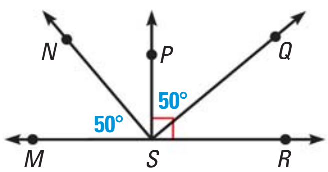
1. VOCABULARY Copy and complete: If two lines intersect at a point, then the __?__ angles formed by the intersecting lines are congruent.

2. ★ WRITING *Describe* the relationship between the angle measures of complementary angles, supplementary angles, vertical angles, and linear pairs.

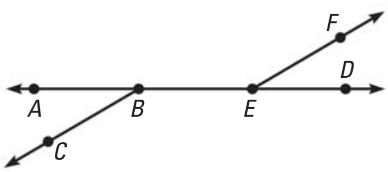
EXAMPLES 1 and 2 on pp. 124–125 for Exs. 3–7

IDENTIFY ANGLES **Identify the pair(s) of congruent angles in the figures below. *Explain* how you know they are congruent.**

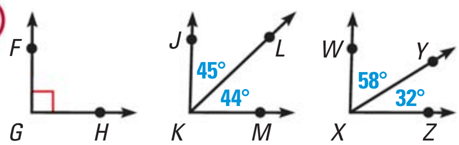
3.



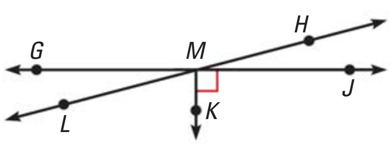
4. $\angle ABC$ is supplementary to $\angle CBD$. $\angle CBD$ is supplementary to $\angle DEF$.



5.



6.



7. ★ **SHORT RESPONSE** The x-axis and y-axis in a coordinate plane are perpendicular to each other. The axes form four angles. Are the four angles congruent right angles? *Explain.*

EXAMPLE 3
on p. 126
for Exs. 8–11

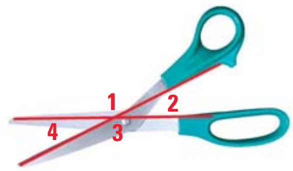
FINDING ANGLE MEASURES **In Exercises 8–11, use the diagram at the right.**

8. If $m\angle 1 = 145°$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

9. If $m\angle 3 = 168°$, find $m\angle 1$, $m\angle 2$, and $m\angle 4$.

10. If $m\angle 4 = 37°$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$.

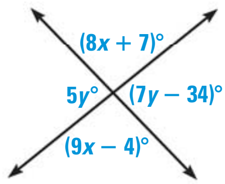
11. If $m\angle 2 = 62°$, find $m\angle 1$, $m\angle 3$, and $m\angle 4$.



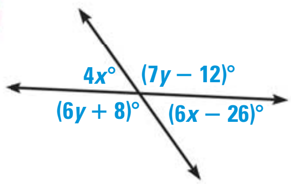
EXAMPLE 4
on p. 127
for Exs. 12–14

xy ALGEBRA **Find the values of x and y.**

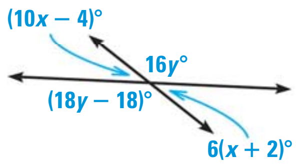
12.



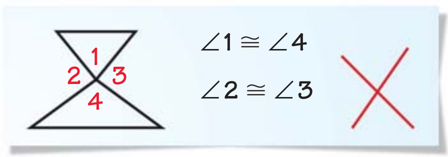
13.



14.



15. **ERROR ANALYSIS** *Describe* the error in stating that $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$.



16. ★ **MULTIPLE CHOICE** In a figure, $\angle A$ and $\angle D$ are complementary angles and $m\angle A = 4x°$. Which expression can be used to find $m\angle D$?

Ⓐ $(4x + 90)°$ Ⓑ $(180 - 4x)°$ Ⓒ $(180 + 4x)°$ Ⓓ $(90 - 4x)°$

FINDING ANGLE MEASURES **In Exercises 17–21, copy and complete the statement given that $m\angle FHE = m\angle BHG = m\angle AHF = 90°$.**

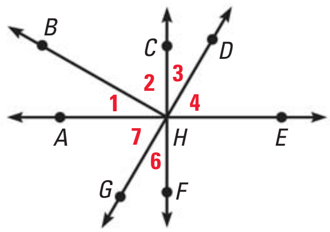
17. If $m\angle 3 = 30°$, then $m\angle 6 =$ _?_.

18. If $m\angle BHF = 115°$, then $m\angle 3 =$ _?_.

19. If $m\angle 6 = 27°$, then $m\angle 1 =$ _?_.

20. If $m\angle DHF = 133°$, then $m\angle CHG =$ _?_.

21. If $m\angle 3 = 32°$, then $m\angle 2 =$ _?_.



ANALYZING STATEMENTS **Two lines that are not perpendicular intersect such that $\angle 1$ and $\angle 2$ are a linear pair, $\angle 1$ and $\angle 4$ are a linear pair, and $\angle 1$ and $\angle 3$ are vertical angles. Tell whether the statement is true.**

22. $\angle 1 \cong \angle 2$

23. $\angle 1 \cong \angle 3$

24. $\angle 1 \cong \angle 4$

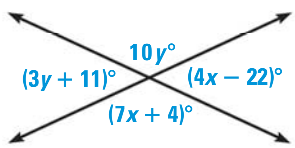
25. $\angle 3 \cong \angle 2$

26. $\angle 2 \cong \angle 4$

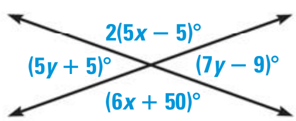
27. $m\angle 3 + m\angle 4 = 180°$

xy ALGEBRA **Find the measure of each angle in the diagram.**

28.



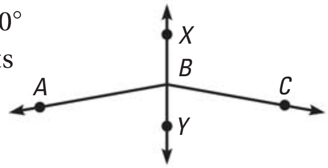
29.



○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

30. ★ **OPEN-ENDED MATH** In the diagram, $m\angle CBY = 80°$ and $\overleftrightarrow{XY}$ bisects $\angle ABC$. Give two more true statements about the diagram.



DRAWING CONCLUSIONS In Exercises 31–34, use the given statement to name two congruent angles. Then give a reason that justifies your conclusion.

31. In triangle GFE, $\overrightarrow{GH}$ bisects $\angle EGF$.

32. $\angle 1$ is a supplement of $\angle 6$, and $\angle 9$ is a supplement of $\angle 6$.

33. $\overline{AB}$ is perpendicular to $\overline{CD}$, and $\overline{AB}$ and $\overline{CD}$ intersect at E.

34. $\angle 5$ is complementary to $\angle 12$, and $\angle 1$ is complementary to $\angle 12$.

35. **CHALLENGE** Sketch two intersecting lines j and k. Sketch another pair of lines ℓ and m that intersect at the same point as j and k and that bisect the angles formed by j and k. Line ℓ is perpendicular to line m. *Explain* why this is true.

PROBLEM SOLVING

EXAMPLE 2
on p. 125
for Ex. 36

36. **PROVING THEOREM 2.4** Prove the second case of the Congruent Supplements Theorem where two angles are supplementary to congruent angles.

GIVEN ▶ $\angle 1$ and $\angle 2$ are supplements.
$\angle 3$ and $\angle 4$ are supplements.
$\angle 1 \cong \angle 4$

PROVE ▶ $\angle 2 \cong \angle 3$

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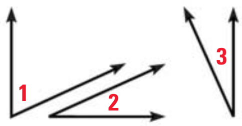
37. **PROVING THEOREM 2.5** Copy and complete the proof of the first case of the Congruent Complements Theorem where two angles are complementary to the same angles.

GIVEN ▶ $\angle 1$ and $\angle 2$ are complements.
$\angle 1$ and $\angle 3$ are complements.

PROVE ▶ $\angle 2 \cong \angle 3$



STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are complements. $\angle 1$ and $\angle 3$ are complements.	**1.** ?
2. $m\angle 1 + m\angle 2 = 90°$ $m\angle 1 + m\angle 3 = 90°$	**2.** ?
3. ?	**3.** Transitive Property of Equality
4. ?	**4.** Subtraction Property of Equality
5. $\angle 2 \cong \angle 3$	**5.** ?

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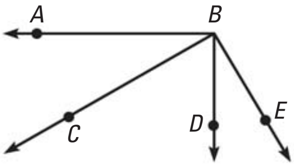
PROOF **Use the given information and the diagram to prove the statement.**

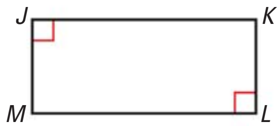
38. **GIVEN ▶** $\angle ABD$ is a right angle.
$\angle CBE$ is a right angle.

PROVE ▶ $\angle ABC \cong \angle DBE$



39. **GIVEN ▶** $\overline{JK} \perp \overline{JM}$, $\overline{KL} \perp \overline{ML}$,
$\angle J \cong \angle M$, $\angle K \cong \angle L$

PROVE ▶ $\overline{JM} \perp \overline{ML}$ and $\overline{JK} \perp \overline{KL}$



40. **MULTI-STEP PROBLEM** Use the photo of the folding table.

a. If $m\angle 1 = x°$, write expressions for the other three angle measures.

b. Estimate the value of x. What are the measures of the other angles?

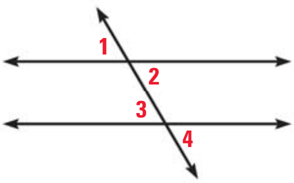
c. As the table is folded up, $\angle 4$ gets smaller. What happens to the other three angles? *Explain* your reasoning.

41. **PROVING THEOREM 2.5** Write a two-column proof for the second case of Theorem 2.5 where two angles are complementary to congruent angles.

WRITING PROOFS **Write a two-column proof.**

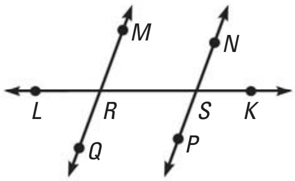
42. **GIVEN ▶** $\angle 1 \cong \angle 3$

PROVE ▶ $\angle 2 \cong \angle 4$



43. **GIVEN ▶** $\angle QRS$ and $\angle PSR$ are supplementary.

PROVE ▶ $\angle QRL \cong \angle PSR$



44. **STAIRCASE** Use the photo and the given information to prove the statement.

GIVEN ▶ $\angle 1$ is complementary to $\angle 3$.
$\angle 2$ is complementary to $\angle 4$.

PROVE ▶ $\angle 1 \cong \angle 4$

45. ★ **EXTENDED RESPONSE** $\angle STV$ is bisected by $\overrightarrow{TW}$, and $\overrightarrow{TX}$ and $\overrightarrow{TW}$ are opposite rays. You want to show $\angle STX \cong \angle VTX$.

a. Draw a diagram.

b. Identify the GIVEN and PROVE statements for the situation.

c. Write a two-column proof.

◯ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

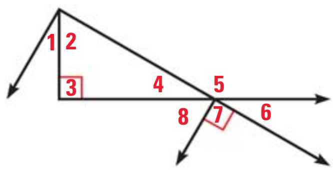
46. **USING DIAGRAMS** Copy and complete the statement with <, >, or =.

a. $m\angle 3$ _?_ $m\angle 7$

b. $m\angle 4$ _?_ $m\angle 6$

c. $m\angle 8 + m\angle 6$ _?_ $150°$

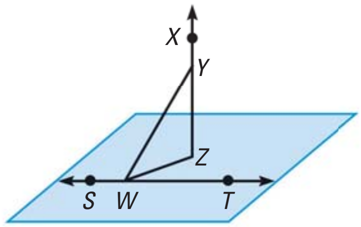
d. If $m\angle 4 = 30°$, then $m\angle 5$ _?_ $m\angle 4$



CHALLENGE **In Exercises 47 and 48, write a two-column proof.**

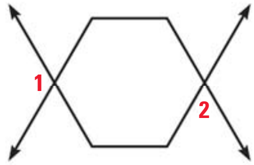
47. **GIVEN** ▶ $m\angle WYZ = m\angle TWZ = 45°$

PROVE ▶ $\angle SWZ \cong \angle XYW$



48. **GIVEN** ▶ The hexagon is regular.

PROVE ▶ $m\angle 1 + m\angle 2 = 180°$



MIXED REVIEW

PREVIEW
Prepare for Lesson 3.1 in Exs. 49–52.

In Exercises 49–52, sketch a plane. Then sketch the described situation. *(p. 2)*

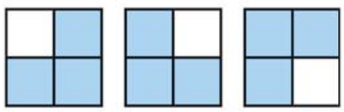
49. Three noncollinear points that lie in the plane

50. A line that intersects the plane at one point

51. Two perpendicular lines that lie in the plane

52. A plane perpendicular to the given plane

53. Sketch the next figure in the pattern. *(p. 72)*



QUIZ *for Lessons 2.6–2.7*

Match the statement with the property that it illustrates. *(p. 112)*

1. If $\overline{HJ} \cong \overline{LM}$, then $\overline{LM} \cong \overline{HJ}$.

2. If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 4$, then $\angle 1 \cong \angle 4$.

3. $\angle XYZ \cong \angle XYZ$

A. Reflexive Property of Congruence

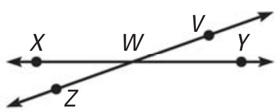
B. Symmetric Property of Congruence

C. Transitive Property of Congruence

4. Write a two-column proof. *(p. 124)*

GIVEN ▶ $\angle XWY$ is a straight angle.
$\angle ZWV$ is a straight angle.

PROVE ▶ $\angle XWV \cong \angle ZWY$



MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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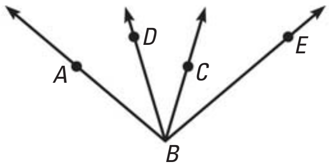
Lessons 2.5–2.7

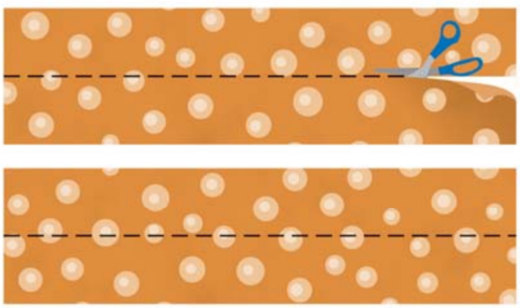
1. **MULTI-STEP PROBLEM** In the diagram below, $\overrightarrow{BD}$ bisects $\angle ABC$ and $\overrightarrow{BC}$ bisects $\angle DBE$.



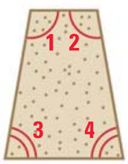
   a. Prove $m\angle ABD = m\angle CBE$.

   b. If $m\angle ABE = 99°$, what is $m\angle DBC$? *Explain*.

2. **SHORT RESPONSE** You are cutting a rectangular piece of fabric into strips that you will weave together to make a placemat. As shown, you cut the fabric in half lengthwise to create two congruent pieces. You then cut each of these pieces in half lengthwise. Do all of the strips have the same width? *Explain* your reasoning.



3. **GRIDDED ANSWER** The cross section of a concrete retaining wall is shown below. Use the given information to find the measure of $\angle 1$ in degrees.



   $m\angle 1 = m\angle 2$

   $m\angle 3 = m\angle 4$

   $m\angle 3 = 80°$

   $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$

4. **EXTENDED RESPONSE** *Explain* how the Congruent Supplements Theorem and the Transitive Property of Angle Congruence can both be used to show how angles that are supplementary to the same angle are congruent.

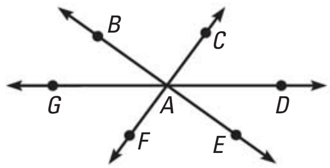
5. **EXTENDED RESPONSE** A formula you can use to calculate the total cost of an item including sales tax is $T = c(1 + s)$, where T is the total cost including sales tax, c is the cost not including sales tax, and s is the sales tax rate written as a decimal.

   a. Solve the formula for s. Give a reason for each step.

   b. Use your formula to find the sales tax rate on a purchase that was \$26.75 with tax and \$25 without tax.

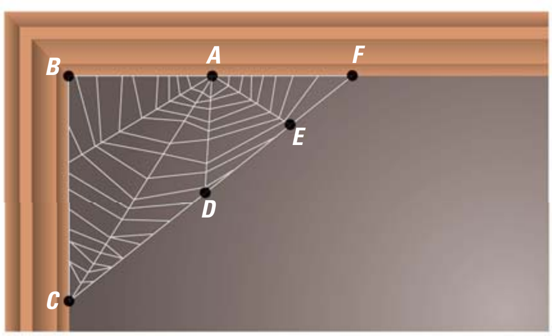
   c. Look back at the steps you used to solve the formula for s. Could you have solved for s in a different way? *Explain*.

6. **OPEN-ENDED** In the diagram below, $m\angle GAB = 36°$. What additional information do you need to find $m\angle BAC$ and $m\angle CAD$? *Explain* your reasoning.



7. **SHORT RESPONSE** Two lines intersect to form $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$. The measure of $\angle 3$ is three times the measure of $\angle 1$ and $m\angle 1 = m\angle 2$. Find all four angle measures. *Explain* your reasoning.

8. **SHORT RESPONSE** Part of a spider web is shown below. If you know that $\angle CAD$ and $\angle DAE$ are complements and that $\overrightarrow{AB}$ and $\overrightarrow{AF}$ are opposite rays, what can you conclude about $\angle BAC$ and $\angle EAF$? *Explain* your reasoning.



2 CHAPTER SUMMARY

BIG IDEAS — *For Your Notebook*

Big Idea 1

Using Inductive and Deductive Reasoning

When you make a conjecture based on a pattern, you use inductive reasoning. You use deductive reasoning to show whether the conjecture is true or false by using facts, definitions, postulates, or proven theorems. If you can find one counterexample to the conjecture, then you know the conjecture is false.

Big Idea 2

Understanding Geometric Relationships in Diagrams

The following can be assumed from the diagram:

A, B, and C are coplanar.

$\angle ABH$ and $\angle HBF$ are a linear pair.

Plane T and plane S intersect in $\overleftrightarrow{BC}$.

$\overleftrightarrow{CD}$ lies in plane S.

$\angle ABC$ and $\angle HBF$ are vertical angles.

$\overleftrightarrow{AB} \perp$ plane S.

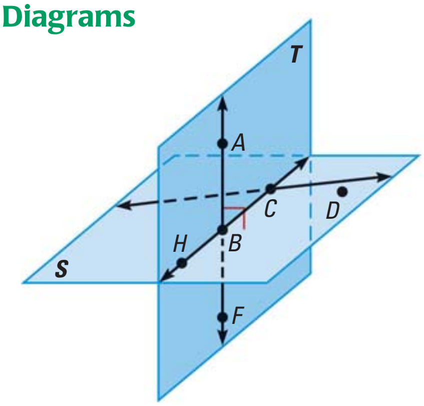


Diagram assumptions are reviewed on page 97.

Big Idea 3

Writing Proofs of Geometric Relationships

You can write a logical argument to show a geometric relationship is true. In a two-column proof, you use deductive reasoning to work from GIVEN information to reach a conjecture you want to PROVE.

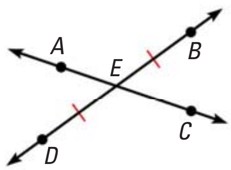


Diagram of geometric relationship with given information labeled to help you write the proof

GIVEN ▶ The hypothesis of an if-then statement

PROVE ▶ The conclusion of an if-then statement

STATEMENTS	REASONS
1. Hypothesis	1. Given
______	______
______	______
______	______
n. Conclusion	*n.* ______

Statements based on facts that you know or conclusions from deductive reasoning

Use postulates, proven theorems, definitions, and properties of numbers and congruence as reasons.

Proof summary is on page 114.

2 CHAPTER REVIEW

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• Vocabulary practice

REVIEW KEY VOCABULARY

See pp. 926–931 for a list of postulates and theorems.

- conjecture, *p. 73*
- inductive reasoning, *p. 73*
- counterexample, *p. 74*
- conditional statement, *p. 79*
  converse, inverse, contrapositive
- if-then form, *p. 79*
  hypothesis, conclusion
- negation, *p. 79*
- equivalent statements, *p. 80*
- perpendicular lines, *p. 81*
- biconditional statement, *p. 82*
- deductive reasoning, *p. 87*
- line perpendicular to a plane, *p. 98*
- proof, *p. 112*
- two-column proof, *p. 112*
- theorem, *p. 113*

VOCABULARY EXERCISES

1. Copy and complete: A statement that can be proven is called a(n) __?__.
2. **WRITING** *Compare* the inverse of a conditional statement to the converse of the conditional statement.
3. You know $m\angle A = m\angle B$ and $m\angle B = m\angle C$. What does the Transitive Property of Equality tell you about the measures of the angles?

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 2.

2.1 Use Inductive Reasoning *pp. 72–78*

EXAMPLE

Describe the pattern in the numbers 3, 21, 147, 1029, ..., and write the next three numbers in the pattern.

Each number is seven times the previous number.

$$3 \xrightarrow{\times 7} 21, \xrightarrow{\times 7} 147, \xrightarrow{\times 7} 1029, \ldots \xrightarrow{\times 7}$$

So, the next three numbers are 7203, 50,421, and 352,947.

EXERCISES

EXAMPLES 2 and 5 on pp. 72–74 for Exs. 4–5

4. *Describe* the pattern in the numbers −20,480, −5120, −1280, −320, ... Write the next three numbers.
5. Find a counterexample to disprove the conjecture:

   If the quotient of two numbers is positive, then the two numbers must both be positive.

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Chapter Review Practice

2.2 Analyze Conditional Statements

pp. 79–85

EXAMPLE

Write the if-then form, the converse, the inverse, and the contrapositive of the statement "Black bears live in North America."

a. If-then form: If a bear is a black bear, then it lives in North America.

b. Converse: If a bear lives in North America, then it is a black bear.

c. Inverse: If a bear is not a black bear, then it does not live in North America.

d. Contrapositive: If a bear does not live in North America, then it is not a black bear.

EXERCISES

EXAMPLES 2, 3, and 4 on pp. 80–82 for Exs. 6–8

6. Write the if-then form, the converse, the inverse, and the contrapositive of the statement "An angle whose measure is 34° is an acute angle."

7. Is this a valid definition? *Explain* why or why not.

"If the sum of the measures of two angles is 90°, then the angles are complementary."

8. Write the definition of *equiangular* as a biconditional statement.

2.3 Apply Deductive Reasoning

pp. 87–93

EXAMPLE

Use the Law of Detachment to make a valid conclusion in the true situation.

If two angles have the same measure, then they are congruent. You know that $m\angle A = m\angle B$.

▶ Because $m\angle A = m\angle B$ satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, $\angle A \cong \angle B$.

EXERCISES

EXAMPLES 1, 2, and 4 on pp. 87–89 for Exs. 9–11

9. Use the Law of Detachment to make a valid conclusion.

If an angle is a right angle, then the angle measures 90°. $\angle B$ is a right angle.

10. Use the Law of Syllogism to write the statement that follows from the pair of true statements.

If $x = 3$, then $2x = 6$.

If $4x = 12$, then $x = 3$.

11. What can you say about the sum of any two odd integers? Use inductive reasoning to form a conjecture. Then use deductive reasoning to show that the conjecture is true.

2 CHAPTER REVIEW

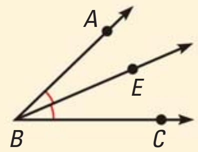
2.4 Use Postulates and Diagrams *pp. 96–102*

EXAMPLE

$\angle ABC$, an acute angle, is bisected by $\overrightarrow{BE}$. Sketch a diagram that represents the given information.

1. Draw $\angle ABC$, an acute angle, and label points A, B, and C.

2. Draw angle bisector $\overrightarrow{BE}$. Mark congruent angles.



EXERCISES

EXAMPLES 3 and 4 on p. 98 for Exs. 12–13

12. Straight angle CDE is bisected by $\overrightarrow{DK}$. Sketch a diagram that represents the given information.

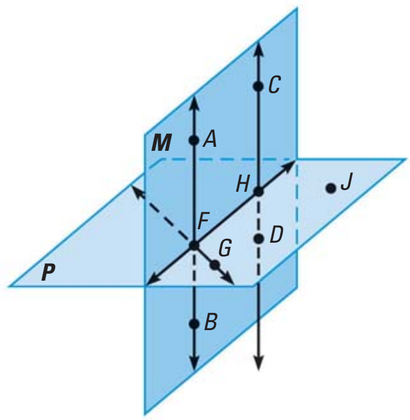
13. Which of the following statements *cannot* be assumed from the diagram?

(A) A, B, and C are coplanar.

(B) $\overleftrightarrow{CD} \perp$ plane P

(C) A, F, and B are collinear.

(D) Plane M intersects plane P in $\overleftrightarrow{FH}$.



2.5 Reason Using Properties from Algebra *pp. 105–111*

EXAMPLE

Solve $3x + 2(2x + 9) = -10$. Write a reason for each step.

$3x + 2(2x + 9) = -10$	**Write original equation.**
$3x + 4x + 18 = -10$	**Distributive Property**
$7x + 18 = -10$	**Simplify.**
$7x = -28$	**Subtraction Property of Equality**
$x = -4$	**Division Property of Equality**

EXERCISES

EXAMPLES 1 and 2 on pp. 105–106 for Exs. 14–17

Solve the equation. Write a reason for each step.

14. $-9x - 21 = -20x - 87$

15. $15x + 22 = 7x + 62$

16. $3(2x + 9) = 30$

17. $5x + 2(2x - 23) = -154$

2.6 Prove Statements about Segments and Angles *pp. 112–119*

EXAMPLE

Prove the Reflexive Property of Segment Congruence.

GIVEN ▶ $\overline{AB}$ is a line segment.

PROVE ▶ $\overline{AB} \cong \overline{AB}$

STATEMENTS	REASONS
1. $\overline{AB}$ is a line segment.	**1.** Given
2. AB is the length of $\overline{AB}$.	**2.** Ruler Postulate
3. $AB = AB$	**3.** Reflexive Property of Equality
4. $\overline{AB} \cong \overline{AB}$	**4.** Definition of congruent segments

EXERCISES

EXAMPLES 2 and 3 on pp. 113–114 for Exs. 18–21

Name the property illustrated by the statement.

18. If $\angle DEF \cong \angle JKL$, then $\angle JKL \cong \angle DEF$.

19. $\angle C \cong \angle C$

20. If $MN = PQ$ and $PQ = RS$, then $MN = RS$.

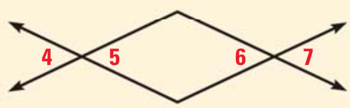
21. Prove the Transitive Property of Angle Congruence.

2.7 Prove Angle Pair Relationships *pp. 124–131*

EXAMPLE

GIVEN ▶ $\angle 5 \cong \angle 6$

PROVE ▶ $\angle 4 \cong \angle 7$

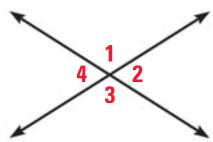


STATEMENTS	REASONS
1. $\angle 5 \cong \angle 6$	**1.** Given
2. $\angle 4 \cong \angle 5$	**2.** Vertical Angles Congruence Theorem
3. $\angle 4 \cong \angle 6$	**3.** Transitive Property of Congruence
4. $\angle 6 \cong \angle 7$	**4.** Vertical Angles Congruence Theorem
5. $\angle 4 \cong \angle 7$	**5.** Transitive Property of Congruence

EXERCISES

EXAMPLES 2 and 3 on pp. 125–126 for Exs. 22–24

In Exercises 22 and 23, use the diagram at the right.



22. If $m\angle 1 = 114°$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

23. If $m\angle 4 = 57°$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$.

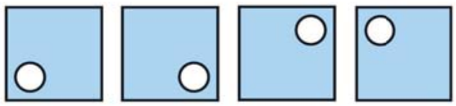
24. Write a two-column proof.

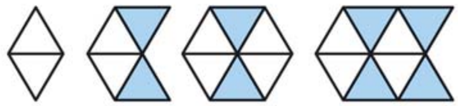
GIVEN ▶ $\angle 3$ and $\angle 2$ are complementary.
$m\angle 1 + m\angle 2 = 90°$

PROVE ▶ $\angle 3 \cong \angle 1$

2 CHAPTER TEST

Sketch the next figure in the pattern.

1. 

2. 

Describe **the pattern in the numbers. Write the next number.**

3. $-6, -1, 4, 9, \ldots$

4. $100, -50, 25, -12.5, \ldots$

In Exercises 5–8, write the if-then form, the converse, the inverse, and the contrapositive for the given statement.

5. All right angles are congruent.

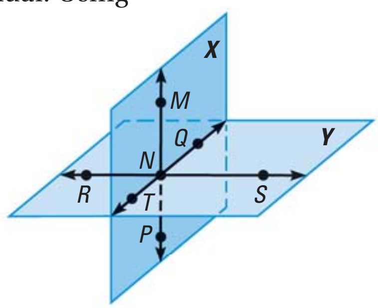
6. Frogs are amphibians.

7. $5x + 4 = -6$, because $x = -2$.

8. A regular polygon is equilateral.

9. If you decide to go to the football game, then you will miss band practice. Tonight, you are going the football game. Using the Law of Detachment, what statement can you make?

10. If Margot goes to college, then she will major in Chemistry. If Margot majors in Chemistry, then she will need to buy a lab manual. Using the Law of Syllogism, what statement can you make?



Use the diagram to write examples of the stated postulate.

11. A line contains at least two points.

12. A plane contains at least three noncollinear points.

13. If two planes intersect, then their intersection is a line.

Solve the equation. Write a reason for each step.

14. $9x + 31 = -23$

15. $-7(-x + 2) = 42$

16. $26 + 2(3x + 11) = -18x$

In Exercises 17–19, match the statement with the property that it illustrates.

17. If $\angle RST \cong \angle XYZ$, then $\angle XYZ \cong \angle RST$.

18. $\overline{PQ} \cong \overline{PQ}$

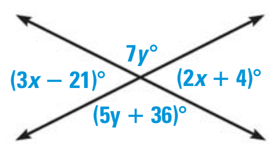
19. If $\overline{FG} \cong \overline{JK}$ and $\overline{JK} \cong \overline{LM}$, then $\overline{FG} \cong \overline{LM}$.

A. Reflexive Property of Congruence

B. Symmetric Property of Congruence

C. Transitive Property of Congruence

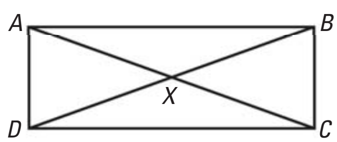
20. Use the Vertical Angles Congruence Theorem to find the measure of each angle in the diagram at the right.



21. Write a two-column proof.

GIVEN ▶ $\overline{AX} \cong \overline{DX}$, $\overline{XB} \cong \overline{XC}$

PROVE ▶ $\overline{AC} \cong \overline{BD}$



2 ALGEBRA REVIEW

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SIMPLIFY RATIONAL AND RADICAL EXPRESSIONS

EXAMPLE 1 Simplify rational expressions

a. $\frac{2x^2}{4xy}$ **b.** $\frac{3x^2 + 2x}{9x + 6}$

Solution

To simplify a rational expression, factor the numerator and denominator. Then divide out any common factors.

a. $\frac{2x^2}{4xy} = \frac{\cancel{2} \cdot \cancel{x} \cdot x}{\cancel{2} \cdot 2 \cdot \cancel{x} \cdot y} = \frac{x}{2y}$ **b.** $\frac{3x^2 + 2x}{9x + 6} = \frac{x\cancel{(3x + 2)}}{3\cancel{(3x + 2)}} = \frac{x}{3}$

EXAMPLE 2 Simplify radical expressions

a. $\sqrt{54}$ **b.** $2\sqrt{5} - 5\sqrt{2} - 3\sqrt{5}$ **c.** $(3\sqrt{2})(-6\sqrt{6})$

Solution

a. $\sqrt{54} = \sqrt{9} \cdot \sqrt{6}$ **Use product property of radicals.**

$= 3\sqrt{6}$ **Simplify.**

b. $2\sqrt{5} - 5\sqrt{2} - 3\sqrt{5} = -\sqrt{5} - 5\sqrt{2}$ **Combine like terms.**

c. $(3\sqrt{2})(-6\sqrt{6}) = -18\sqrt{12}$ **Use product property and associative property.**

$= -18 \cdot 2\sqrt{3}$ **Simplify $\sqrt{12}$.**

$= -36\sqrt{3}$ **Simplify.**

EXERCISES

EXAMPLE 1 for Exs. 1–9

Simplify the expression, if possible.

1. $\frac{5x^4}{20x^2}$ **2.** $\frac{-12ab^3}{9a^2b}$ **3.** $\frac{5m + 35}{5}$

4. $\frac{36m - 48m}{6m}$ **5.** $\frac{k + 3}{-2k + 3}$ **6.** $\frac{m + 4}{m^2 + 4m}$

7. $\frac{12x + 16}{8 + 6x}$ **8.** $\frac{3x^3}{5x + 8x^2}$ **9.** $\frac{3x^2 - 6x}{6x^2 - 3x}$

EXAMPLE 2 for Exs. 10–24

Simplify the expression, if possible. All variables are positive.

10. $\sqrt{75}$ **11.** $-\sqrt{180}$ **12.** $\pm\sqrt{128}$

13. $\sqrt{2} - \sqrt{18} + \sqrt{6}$ **14.** $\sqrt{28} - \sqrt{63} - \sqrt{35}$ **15.** $4\sqrt{8} + 3\sqrt{32}$

16. $(6\sqrt{5})(2\sqrt{2})$ **17.** $(-4\sqrt{10})(-5\sqrt{5})$ **18.** $(2\sqrt{6})^2$

19. $\sqrt{(25)^2}$ **20.** $\sqrt{x^2}$ **21.** $\sqrt{-(a)^2}$

22. $\sqrt{(3y)^2}$ **23.** $\sqrt{3^2 + 2^2}$ **24.** $\sqrt{h^2 + k^2}$

Scoring Rubric

Full Credit
- solution is complete and correct

Partial Credit
- solution is complete but has errors,
or
- solution is without error but incomplete

No Credit
- no solution is given,
or
- solution makes no sense

EXTENDED RESPONSE QUESTIONS

PROBLEM

Seven members of the student government (Frank, Gina, Henry, Isabelle, Jack, Katie, and Leah) are posing for a picture for the school yearbook. For the picture, the photographer will arrange the students in a row according to the following restrictions:

Henry must stand in the middle spot.

Katie must stand in the right-most spot.

There must be exactly two spots between Gina and Frank.

Isabelle cannot stand next to Henry.

Frank must stand next to Katie.

a. *Describe* one possible ordering of the students.

b. Which student(s) can stand in the second spot from the left?

c. If the condition that Leah must stand in the left-most spot is added, will there be exactly one ordering of the students? *Justify* your answer.

Below are sample solutions to the problem. Read each solution and the comments in blue to see why the sample represents full credit, partial credit, or no credit.

SAMPLE 1: Full credit solution

The method of representation is clearly explained.

a. Using the first letters of the students' names, here is one possible ordering of the students:

I L G H J F K

The conclusion is correct and shows understanding of the problem.

b. The only students without fixed positions are Isabelle, Leah, and Jack. There are no restrictions on placement in the second spot from the left, so any of these three students can occupy that location.

c. Henry, Frank, Katie, and Gina have fixed positions according to the restrictions. If Leah must stand in the left-most spot, the ordering looks like:

L _ G H _ F K

The reasoning behind the answer is explained clearly.

Because Isabelle cannot stand next to Henry, she must occupy the spot next to Leah. Therefore, Jack stands next to Henry and the only possible order would have to be:

L I G H J F K.

Yes, there would be exactly one ordering of the students.

SAMPLE 2: Partial credit solution

The answer to part (a) is correct.

Part (b) is correct but not explained.

The student did not recall that Isabelle cannot stand next to Henry; therefore, the conclusion is incorrect.

a. One possible ordering of the students is:
Jack, Isabelle, Gina, Henry, Leah, Frank, and Katie.

b. There are three students who could stand in the second spot from the left. They are Isabelle, Leah, and Jack.

c. No, there would be two possible orderings of the students. With Leah in the left-most spot, the ordering looks like:

Leah, ____, Gina, Henry, ____, Frank, and Katie

Therefore, the two possible orderings are
Leah, Isabelle, Gina, Henry, Jack, Frank, and Katie
or
Leah, Jack, Gina, Henry, Isabelle, Frank, and Katie.

SAMPLE 3: No credit solution

The answer to part (a) is incorrect because Isabelle is next to Henry.

Parts (b) and (c) are based on the incorrect conclusion in part (a).

a. One possible ordering of the students is **L G J H I F K**.

b. There are four students who can stand in the second spot from the left. Those students are Leah, Gina, Isabelle, and Jack.

c. The two possible orderings are **L G J H I F K** and **L J G H I F K**.

PRACTICE Apply the Scoring Rubric

1. A student's solution to the problem on the previous page is given below. Score the solution as *full credit*, *partial credit*, or *no credit*. *Explain* your reasoning. If you choose *partial credit* or *no credit*, explain how you would change the solution so that it earns a score of full credit.

a. A possible ordering of the students is I - J - G - H - L - F - K.

b. There are no restrictions on the second spot from the left. Leah, Isabelle, and Jack could all potentially stand in this location.

c. The positions of Gina, Henry, Frank, and Katie are fixed.

_ - _ - G - H - _ - F - K.

Because Isabelle cannot stand next to Henry, she must occupy the left-most spot or the second spot from the left. There are no restrictions on Leah or Jack. That leaves four possible orderings:

I - J - G - H - L - F - K I - L - G - H - J - F - K
L - I - G - H - J - F - K J - I - G - H - L - F - K.

If the restriction is added that Leah must occupy the left-most spot, there is exactly one ordering that would satisfy all conditions:

L - I - G - H - J - F - K.

2 ★ Standardized TEST PRACTICE

EXTENDED RESPONSE

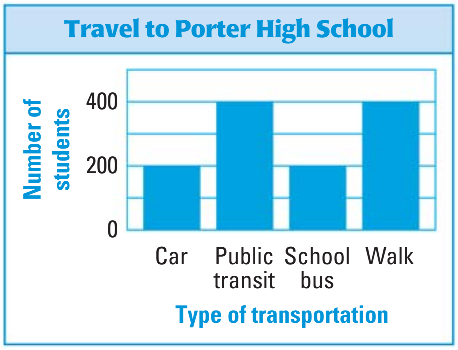
1. In some bowling leagues, the handicap H of a bowler with an average score A is found using the formula $H = \frac{4}{5}(200 - A)$. The handicap is then added to the bowler's score.

a. Solve the formula for A. Write a reason for each step.

b. Use your formula to find a bowler's average score with a handicap of 12.

c. Using this formula, is it possible to calculate a handicap for a bowler with an average score above 200? *Explain* your reasoning.

2. A survey was conducted at Porter High School asking students what form of transportation they use to go to school. All students in the high school were surveyed. The results are shown in the bar graph.



a. Does the statement "About 1500 students attend Porter High School" follow from the data? *Explain*.

b. Does the statement "About one third of all students at Porter take public transit to school" follow from the data? *Explain*.

c. John makes the conclusion that Porter High School is located in a city or a city suburb. *Explain* his reasoning and tell if his conclusion is the result of *inductive reasoning* or *deductive reasoning*.

d. Betty makes the conclusion that there are twice as many students who walk as take a car to school. *Explain* her reasoning and tell if her conclusion is the result of *inductive reasoning* or *deductive reasoning*.

3. The senior class officers are planning a meeting with the principal and some class officers from the other grades. The senior class president, vice president, treasurer, and secretary will all be present. The junior class president and treasurer will attend. The sophomore class president and vice president, and freshmen treasurer will attend. The secretary makes a seating chart for the meeting using the following conditions.

The principal will sit in chair 10. The senior class treasurer will sit at the other end.

The senior class president will sit to the left of the principal, next to the junior class president, and across from the sophomore class president.

All three treasurers will sit together. The two sophomores will sit next to each other.

The two vice presidents and the freshman treasurer will sit on the same side of the table.

a. Draw a diagram to show where everyone will sit.

b. *Explain* why the senior class secretary must sit between the junior class president and junior class treasurer.

c. Can the senior class vice-president sit across from the junior class president? *Justify* your answer.

MULTIPLE CHOICE

4. If d represents an odd integer, which of the expressions represents an even integer?

Ⓐ $d + 2$

Ⓑ $2d - 1$

Ⓒ $3d + 1$

Ⓓ $3d + 2$

5. In the repeating decimal 0.23142314. . . , where the digits 2314 repeat, which digit is in the 300th place to the right of the decimal point?

Ⓐ 1

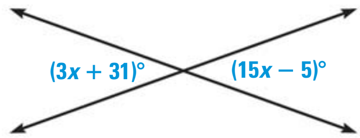
Ⓑ 2

Ⓒ 3

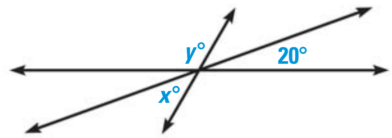
Ⓓ 4

GRIDDED ANSWER

6. Use the diagram to find the value of x.



7. Three lines intersect in the figure shown. What is the value of $x + y$?



8. R is the midpoint of $\overline{PQ}$, and S and T are the midpoints of $\overline{PR}$ and $\overline{RQ}$, respectively. If $ST = 20$, what is PT?

SHORT RESPONSE

9. Is this a correct conclusion from the given information? If so, *explain* why. If not, *explain* the error in the reasoning.

If you are a soccer player, then you wear shin guards. Your friend is wearing shin guards. Therefore, she is a soccer player.

10. *Describe* the pattern in the numbers. Write the next number in the pattern.

192, −48, 12, −3, . . .

11. Points A, B, C, D, E, and F are coplanar. Points A, B, and F are collinear. The line through A and B is perpendicular to the line through C and D, and the line through C and D is perpendicular to the line through E and F. Which four points must lie on the same line? *Justify* your answer.

12. Westville High School offers after-school tutoring with five student volunteer tutors for this program: Jen, Kim, Lou, Mike, and Nina. On any given weekday, three tutors are scheduled to work. Due to the students' other commitments after school, the tutoring work schedule must meet the following conditions.

Jen can work any day except every other Monday and Wednesday.

Kim can only work on Thursdays and Fridays.

Lou can work on Tuesdays and Wednesdays.

Mike cannot work on Fridays.

Nina cannot work on Tuesdays.

Name three tutors who can work on *any* Wednesday. *Justify* your answer.