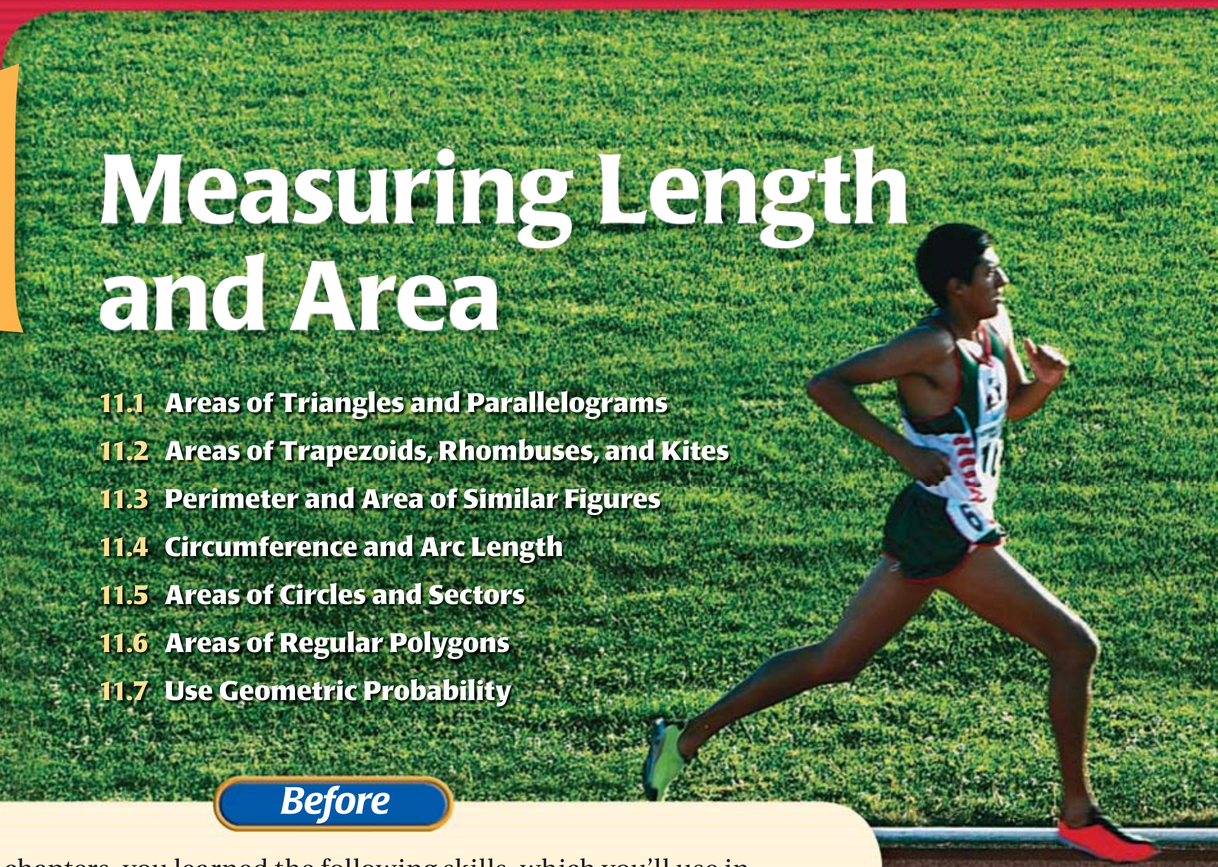


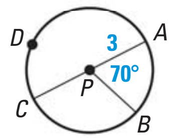
11 Measuring Length and Area

Before

In previous chapters, you learned the following skills, which you'll use in Chapter 11: applying properties of circles and polygons, using formulas, solving for lengths in right triangles, and using ratios and proportions.

Prerequisite Skills

VOCABULARY CHECK



Give the indicated measure for $\odot P$.

1. The radius 2. The diameter 3. $m\widehat{ADB}$

SKILLS AND ALGEBRA CHECK

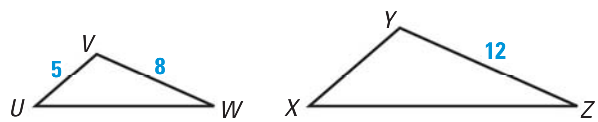
4. Use a formula to find the width w of the rectangle that has a perimeter of 24 centimeters and a length of 9 centimeters. *(Review p. 49 for 11.1.)*

In $\triangle ABC$, angle C is a right angle. Use the given information to find AC. *(Review pp. 433, 457, 473 for 11.1, 11.6.)*

5. $AB = 14, BC = 6$ 6. $m\angle A = 35°, AB = 25$ 7. $m\angle B = 60°, BC = 5$

8. Which special quadrilaterals have diagonals that bisect each other? *(Review pp. 533, 542 for 11.2.)*

9. Use a proportion to find XY if $\triangle UVW \sim \triangle XYZ$. *(Review p. 372 for 11.3.)*



@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 11, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 779. You will also use the key vocabulary listed below.

Big Ideas

1. **Using area formulas for polygons**
2. **Relating length, perimeter, and area ratios in similar polygons**
3. **Comparing measures for parts of circles and the whole circle**

KEY VOCABULARY

- bases of a parallelogram, *p. 720*
- height of a parallelogram, *p. 720*
- height of a trapezoid, *p. 730*
- circumference, *p. 746*
- arc length, *p. 747*
- sector of a circle, *p. 756*
- center of a polygon, *p. 762*
- radius of a polygon, *p. 762*
- apothem of a polygon, *p. 762*
- central angle of a regular polygon, *p. 762*
- probability, *p. 771*
- geometric probability, *p. 771*

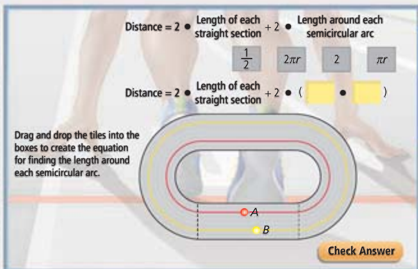
Why?

You can apply formulas for perimeter, circumference, and area to find and compare measures. To find lengths along a running track, you can break the track into straight sides and semicircles.

Animated Geometry

The animation illustrated below for Example 5 on page 749 helps you answer this question: How far does a runner travel to go around a track?

Your goal is to find the distances traveled by two runners in different track lanes.



Choose the correct expressions to complete the equation.

Animated Geometry at classzone.com

Other animations for Chapter 11: pages 720, 739, 759, 765, and 771

11.1 Areas of Triangles and Parallelograms

Before You learned properties of triangles and parallelograms.

Now You will find areas of triangles and parallelograms.

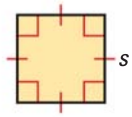
Why? So you can plan a jewelry making project, as in Ex. 44.

Key Vocabulary
- **bases of a parallelogram**
- **height of a parallelogram**
- **area,** *p. 49*
- **perimeter,** *p. 49*

POSTULATES — *For Your Notebook*

POSTULATE 24 Area of a Square Postulate

The area of a square is the square of the length of its side.



$A = s^2$

POSTULATE 25 Area Congruence Postulate

If two polygons are congruent, then they have the same area.

POSTULATE 26 Area Addition Postulate

The area of a region is the sum of the areas of its nonoverlapping parts.

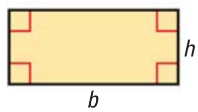
RECTANGLES A rectangle that is b units by h units can be split into $b \cdot h$ unit squares, so the area formula for a rectangle follows from Postulates 24 and 26.

THEOREM — *For Your Notebook*

THEOREM 11.1 Area of a Rectangle

The area of a rectangle is the product of its base and height.

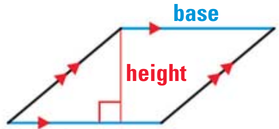
Justification: Ex. 46, p. 726



$A = bh$

READ DIAGRAMS
The word *base* can refer to a segment or to its length. The segment used for the height must be perpendicular to the bases used.

PARALLELOGRAMS Either pair of parallel sides can be used as the **bases** of a parallelogram. The **height** is the perpendicular distance between these bases.



If you transform a rectangle to form other parallelograms with the same base and height, the area stays the same.

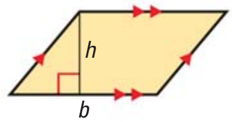
Animated Geometry at classzone.com

THEOREMS *For Your Notebook*

THEOREM 11.2 Area of a Parallelogram

The area of a parallelogram is the product of a base and its corresponding height.

Justification: Ex. 42, p. 725

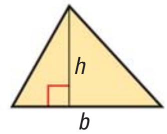


$A = bh$

THEOREM 11.3 Area of a Triangle

The area of a triangle is one half the product of a base and its corresponding height.

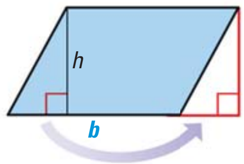
Justification: Ex. 43, p. 726



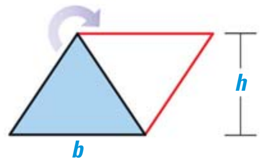
$A = \frac{1}{2}bh$

READ VOCABULARY
The *height* of a triangle is the length of the altitude drawn to the given *base*.

RELATING AREA FORMULAS As illustrated below, the area formula for a parallelogram is related to the formula for a rectangle, and the area formula for a triangle is related to the formula for a parallelogram. You will write a justification of these relationships in Exercises 42 and 43 on pages 725–726.



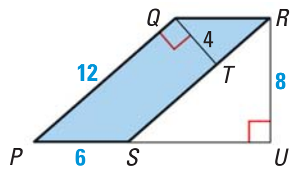
Area of ▱ = Area of Rectangle



Area of △ = $\frac{1}{2}$ • Area of ▱

EXAMPLE 1 Use a formula to find area

Find the area of ▱*PQRS*.



Solution

Method 1 Use $\overline{PS}$ as the base.
The base is extended to measure the height *RU*. So, $b = 6$ and $h = 8$.

Area $= bh = 6(8) = 48$ square units

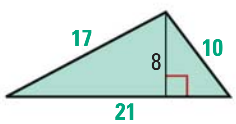
Method 2 Use $\overline{PQ}$ as the base.
Then the height is *QT*. So, $b = 12$ and $h = 4$.

Area $= bh = 12(4) = 48$ square units

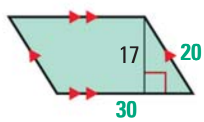
✓ GUIDED PRACTICE for Example 1

Find the perimeter and area of the polygon.

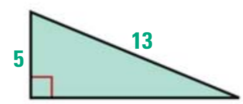
1.



2.

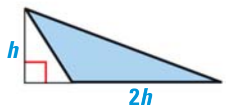


3.



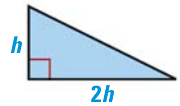
EXAMPLE 2 Solve for unknown measures

DRAW DIAGRAMS
Note that there are other ways you can draw the triangle described in Example 2.



ALGEBRA **The base of a triangle is twice its height. The area of the triangle is 36 square inches. Find the base and height.**

Let h represent the height of the triangle. Then the base is $2h$.



$A = \frac{1}{2}bh$ **Write formula.**

$36 = \frac{1}{2}(2h)(h)$ **Substitute 36 for A and $2h$ for b.**

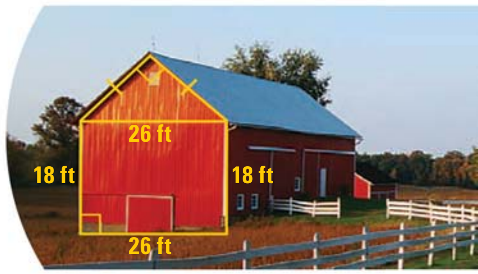
$36 = h^2$ **Simplify.**

$6 = h$ **Find positive square root of each side.**

▶ The height of the triangle is 6 inches, and the base is $6 \cdot 2 = 12$ inches.

EXAMPLE 3 Solve a multi-step problem

PAINTING You need to buy paint so that you can paint the side of a barn. A gallon of paint covers 350 square feet. How many gallons should you buy?



Solution

You can use a right triangle and a rectangle to approximate the area of the side of the barn.

ANOTHER WAY
In Example 3, you have a 45°-45°-90° triangle, so you can also find x by using trigonometry or special right angles.

STEP 1 **Find** the length x of each leg of the triangle.

$26^2 = x^2 + x^2$ **Use Pythagorean Theorem.**

$676 = 2x^2$ **Simplify.**

$\sqrt{338} = x$ **Solve for the positive value of x.**

STEP 2 **Find** the approximate area of the side of the barn.

Area = **Area of rectangle** + **Area of triangle**

$= 26(18) + \frac{1}{2} \cdot \left[(\sqrt{338})(\sqrt{338})\right] = 637 \text{ ft}^2$

STEP 3 **Determine** how many gallons of paint you need.

$637 \cancel{\text{ft}^2} \cdot \frac{1 \text{ gal}}{350 \cancel{\text{ft}^2}} \approx 1.82 \text{ gal}$ **Use unit analysis.**

▶ Round up so you will have enough paint. You need to buy 2 gallons of paint.

✓ GUIDED PRACTICE for Examples 2 and 3

4. A parallelogram has an area of 153 square inches and a height of 17 inches. What is the length of the base?

5. **WHAT IF?** In Example 3, suppose there is a 5 foot by 10 foot rectangular window on the side of the barn. What is the approximate area you need to paint?

11.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 23, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 21, 30, 39, and 45

SKILL PRACTICE

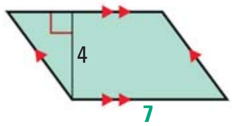
1. **VOCABULARY** Copy and complete: Either pair of parallel sides of a parallelogram can be called its __?__, and the perpendicular distance between these sides is called the __?__.

2. ★ **WRITING** What are the two formulas you have learned for the area of a rectangle? *Explain* why these formulas give the same results.

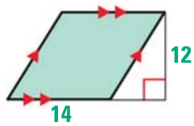
EXAMPLE 1 on p. 721 for Exs. 3–15

FINDING AREA **Find the area of the polygon.**

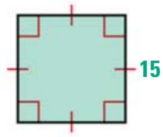
3.



4.



5.



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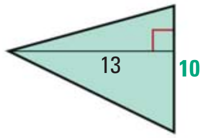

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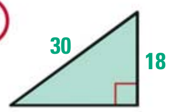
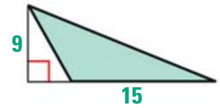
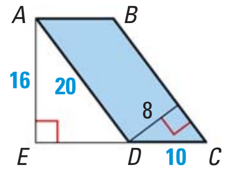


8.

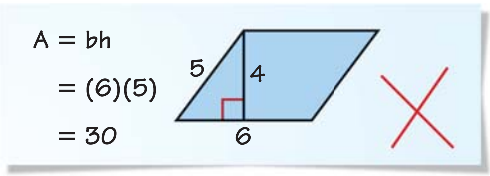


9. **COMPARING METHODS** Show two different ways to calculate the area of parallelogram *ABCD*. *Compare* your results.

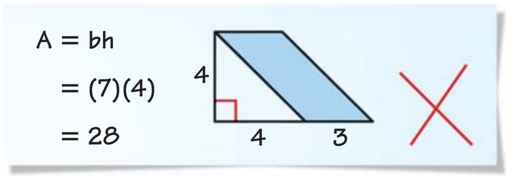


ERROR ANALYSIS ***Describe*** **and correct the error in finding the area of the parallelogram.**

10.



11.



PYTHAGOREAN THEOREM **The lengths of the hypotenuse and one leg of a right triangle are given. Find the perimeter and area of the triangle.**

12. Hypotenuse: 15 in.; leg: 12 in.

13. Hypotenuse: 34 ft; leg: 16 ft

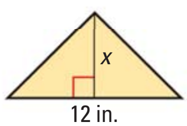
14. Hypotenuse: 85 m; leg: 84 m

15. Hypotenuse: 29 cm; leg: 20 cm

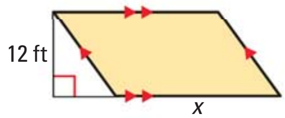
EXAMPLE 2 on p. 722 for Exs. 16–21

xy **ALGEBRA** **Find the value of *x*.**

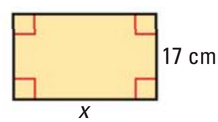
16. $A = 36 \text{ in.}^2$



17. $A = 276 \text{ ft}^2$



18. $A = 476 \text{ cm}^2$



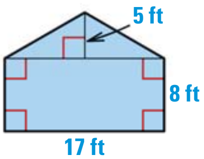
19. **ALGEBRA** The area of a triangle is 4 square feet. The height of the triangle is half its base. Find the base and the height.

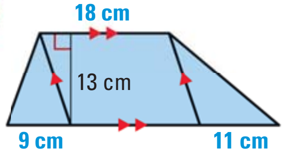
20. **ALGEBRA** The area of a parallelogram is 507 square centimeters, and its height is three times its base. Find the base and the height.

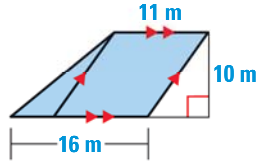
21. ★ **OPEN-ENDED MATH** A polygon has an area of 80 square meters and a height of 10 meters. Make scale drawings of three different triangles and three different parallelograms that match this description. Label the base and the height.

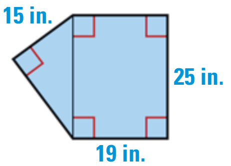
EXAMPLE 3 on p. 722 for Exs. 22–27

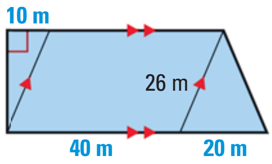
FINDING AREA **Find the area of the shaded polygon.**

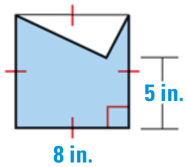
22. 

23. 

24. 

25. 

26. 

27. 

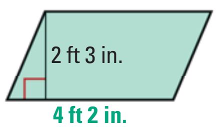
COORDINATE GRAPHING **Graph the points and connect them to form a polygon. Find the area of the polygon.**

28. $A(3, 3)$, $B(10, 3)$, $C(8, -3)$, $D(1, -3)$

29. $E(-2, -2)$, $F(5, 1)$, $G(3, -2)$

30. ★ **MULTIPLE CHOICE** What is the area of the parallelogram shown at the right?

(A) 8 ft^2 6 in.^{2}
(B) 1350 in.
(C) 675 in.^{2}
(D) 9.375 ft^2



31. **TECHNOLOGY** Use geometry drawing software to draw a line ℓ and a line m parallel to ℓ. Then draw $\triangle ABC$ so that C is on line ℓ and $\overline{AB}$ is on line m. Find the base AB, the height CD, and the area of $\triangle ABC$. Move point C to change the shape of $\triangle ABC$. What do you notice about the base, height, and area of $\triangle ABC$?

32. **USING TRIGONOMETRY** In $\square ABCD$, base AD is 15 and AB is 8. What are the height and area of $\square ABCD$ if $m\angle DAB$ is 20°? if $m\angle DAB$ is 50°?

33. **ALGEBRA** Find the area of a right triangle with side lengths 12 centimeters, 35 centimeters, and 37 centimeters. Then find the length of the altitude drawn to the hypotenuse.

34. **ALGEBRA** Find the area of a triangle with side lengths 5 feet, 5 feet, and 8 feet. Then find the lengths of all three altitudes of the triangle.

35. **CHALLENGE** The vertices of quadrilateral $ABCD$ are $A(2, -2)$, $B(6, 4)$, $C(-1, 5)$, and $D(-5, 2)$. Without using the Distance Formula, find the area of $ABCD$. Show your steps.

Problem Solving

36. **SAILING** Sails A and B are right triangles. The lengths of the legs of Sail A are 65 feet and 35 feet. The lengths of the legs of Sail B are 29.5 feet and 10.5 feet. Find the area of each sail to the nearest square foot. About how many times as great is the area of Sail A as the area of Sail B?

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EXAMPLE 3 on p. 722 for Ex. 37

37. **MOWING** You can mow 10 square yards of grass in one minute. How long does it take you to mow a triangular plot with height 25 yards and base 24 yards? How long does it take you to mow a rectangular plot with base 24 yards and height 36 yards?

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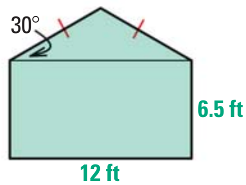
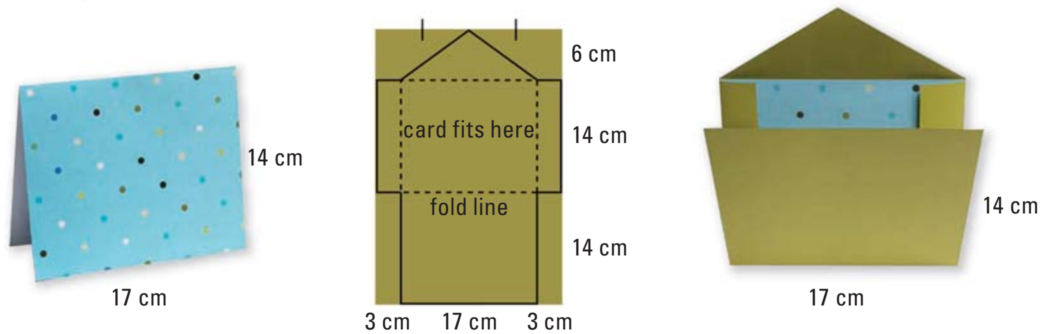
38. **CARPENTRY** You are making a table in the shape of a parallelogram to replace an old 24 inch by 15 inch rectangular table. You want the areas of two tables to be equal. The base of the parallelogram is 20 inches. What should the height be?

39. ★ **SHORT RESPONSE** A *4 inch square* is a square that has a side length of 4 inches. Does a 4 inch square have an area of 4 square inches? If not, what size square does have an area of 4 square inches? *Explain*.

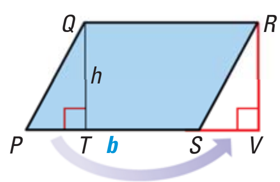
40. **PAINTING** You are earning money by painting a shed. You plan to paint two sides of the shed today. Each of the two sides has the dimensions shown at the right. You can paint 200 square feet per hour, and you charge $20 per hour. How much will you get paid for painting those two sides of the shed?



41. **ENVELOPES** The pattern below shows how to make an envelope to fit a card that is 17 centimeters by 14 centimeters. What are the dimensions of the rectangle you need to start with? What is the area of the paper that is actually used in the envelope? of the paper that is cut off?



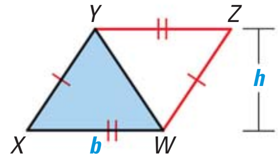
42. **JUSTIFYING THEOREM 11.2** You can use the area formula for a rectangle to justify the area formula for a parallelogram. First draw $\square PQRS$ with base b and height h, as shown. Then draw a segment perpendicular to $\overleftrightarrow{PS}$ through point R. Label point V.



   a. In the diagram, *explain* how you know that $\triangle PQT \cong \triangle SRV$.

   b. *Explain* how you know that the area of $PQRS$ is equal to the area of $QRVT$. How do you know that Area of $PQRS = bh$?

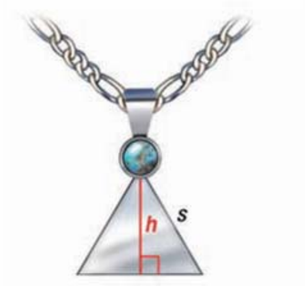
43. **JUSTIFYING THEOREM 11.3** You can use the area formula for a parallelogram to justify the area formula for a triangle. Start with two congruent triangles with base b and height h. Place and label them as shown. *Explain* how you know that $XYZW$ is a parallelogram and that Area of $\triangle XYW = \frac{1}{2}bh$.



44. **MULTI-STEP PROBLEM** You have enough silver to make a pendant with an area of 4 square centimeters. The pendant will be an equilateral triangle. Let s be the side length of the triangle.

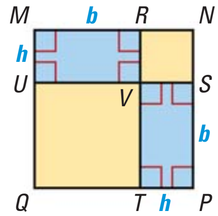
a. Find the height h of the triangle in terms of s. Then write a formula for the area of the triangle in terms of s.

b. Find the side length of the triangle. Round to the nearest centimeter.



45. ★ **EXTENDED RESPONSE** The base of a parallelogram is 7 feet and the height is 3 feet. *Explain* why the perimeter cannot be determined from the given information. Is there a least possible perimeter for the parallelogram? Is there a greatest possible perimeter? *Explain*.

46. **JUSTIFYING THEOREM 11.1** You can use the diagram to show that the area of a rectangle is the product of its base b and height h.

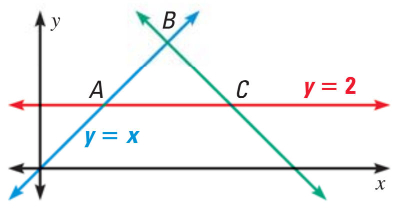


a. Figures $MRVU$ and $VSPT$ are congruent rectangles with base b and height h. *Explain* why $RNSV$, $UVTQ$, and $MNPQ$ are squares. Write expressions in terms of b and h for the areas of the squares.

b. Let A be the area of $MRVU$. Substitute A and the expressions from part (a) into the equation below. Solve to find an expression for A.

Area of $MNPQ$ = Area of $MRVU$ + Area of $UVTQ$ + Area of $RNSV$ + Area of $VSPT$

47. **CHALLENGE** An equation of $\overleftrightarrow{AB}$ is $y = x$. An equation of $\overleftrightarrow{AC}$ is $y = 2$. Suppose $\overleftrightarrow{BC}$ is placed so that $\triangle ABC$ is isosceles with an area of 4 square units. Find two different lines that fit these conditions. Give an equation for each line. Is there another line that could fit this requirement for $\overleftrightarrow{BC}$? *Explain*.

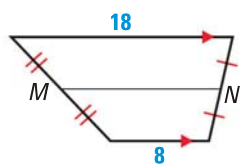


MIXED REVIEW

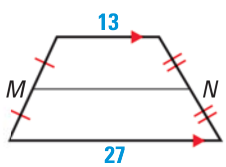
PREVIEW
Prepare for Lesson 11.2 in Exs. 48–50.

Find the length of the midsegment $\overline{MN}$ of the trapezoid. *(p. 542)*

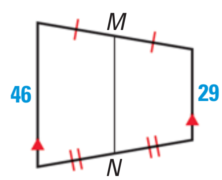
48.



49.



50.



The coordinates of $\triangle PQR$ are $P(-4, 1)$, $Q(2, 5)$, and $R(1, -4)$. Graph the image of the triangle after the translation. Use prime notation. *(p. 572)*

51. $(x, y) \rightarrow (x + 1, y + 4)$

52. $(x, y) \rightarrow (x + 3, y - 5)$

53. $(x, y) \rightarrow (x - 3, y - 2)$

54. $(x, y) \rightarrow (x - 2, y + 3)$

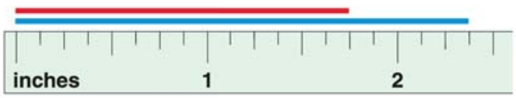
Extension
Use after Lesson 11.1

Determine Precision and Accuracy

GOAL Determine the precision and accuracy of measurements.

Key Vocabulary
- unit of measure
- greatest possible error
- relative error

All measurements are approximations. The length of each segment below, *to the nearest inch*, is 2 inches. The measurement is to the nearest inch, so the **unit of measure** is 1 inch.



If you are told that an object is 2 inches long, you know that its exact length is between $1\frac{1}{2}$ inches and $2\frac{1}{2}$ inches, or within $\frac{1}{2}$ inch of 2 inches. The **greatest possible error** of a measurement is equal to one half of the unit of measure.

When the unit of measure is smaller, the greatest possible error is smaller and the measurement is *more precise*. Using one-eighth inch as the unit of measure for the segments above gives lengths of $1\frac{6}{8}$ inches and $2\frac{3}{8}$ inches and a greatest possible error of $\frac{1}{16}$ inch.

EXAMPLE 1 Find greatest possible error

AMUSEMENT PARK The final drop of a log flume ride is listed in the park guide as 52.3 feet. Find the unit of measure and the greatest possible error.

Solution

The measurement 52.3 feet is given to the nearest tenth of a foot. So, the unit of measure is $\frac{1}{10}$ foot. The greatest possible error is half the unit of measure. Because $\frac{1}{2}\left(\frac{1}{10}\right) = \frac{1}{20} = 0.05$, the greatest possible error is 0.05 foot.

READ VOCABULARY
The *precision* of a measurement depends only on the unit of measure. The *accuracy* of a measurement depends on both the unit of measure and on the size of the object being measured.

RELATIVE ERROR The diameter of a bicycle tire is 26 inches. The diameter of a key ring is 1 inch. In each case, the greatest possible error is $\frac{1}{2}$ inch, but a half-inch error has a much greater effect on the diameter of a smaller object. The **relative error** of a measurement is the ratio $\frac{\text{greatest possible error}}{\text{measured length}}$.

Bicycle tire diameter	Key ring diameter
Rel. error $= \frac{0.5 \text{ in.}}{26 \text{ in.}} \approx 0.01923 \approx 1.9\%$	Rel. error $= \frac{0.5 \text{ in.}}{1 \text{ in.}} = 0.5 = 50\%$

The measurement with the smaller relative error is said to be *more accurate*.

EXAMPLE 2 Find relative error

PLAYING AREAS An air hockey table is 3.7 feet wide. An ice rink is 85 feet wide. Find the relative error of each measurement. Which measurement is more accurate?

	Air hockey table (3.7 feet)	Ice rink (85 feet)
Unit of measure	0.1 ft	1 ft
Greatest possible error $\frac{1}{2}\cdot$ (unit of measure)	$\frac{1}{2}(0.1\text{ ft}) = 0.05\text{ ft}$	$\frac{1}{2}(1\text{ ft}) = 0.5\text{ ft}$
Relative error $\frac{\text{greatest possible error}}{\text{measured length}}$	$\frac{0.05\text{ ft}}{3.7\text{ ft}} \approx 0.0135 \approx 1.4\%$	$\frac{0.5\text{ ft}}{85\text{ ft}} \approx 0.00588 \approx 0.6\%$

▶ The ice rink width has the smaller relative error, so it is more accurate.

PRACTICE

1. **VOCABULARY** *Describe* the difference between the *precision* of a measurement and the *accuracy* of a measurement. Give an example that illustrates the difference.

EXAMPLE 1 on p. 727 for Exs. 2–5

GREATEST POSSIBLE ERROR Find the unit of measure. Then find the greatest possible error.

2. 14.6 in.
3. 6 m
4. 8.217 km
5. $4\frac{5}{16}$ yd

EXAMPLE 2 on p. 728 for Exs. 6–9

RELATIVE ERROR Find the relative error of the measurement.

6. 4.0 cm
7. 28 in.
8. 4.6 m
9. 12.16 mm

10. **CHOOSING A UNIT** You are estimating the amount of paper needed to make book covers for your textbooks. Which unit of measure, 1 foot, 1 inch, or $\frac{1}{16}$ inch, should you use to measure your textbooks? *Explain*.

11. **REASONING** The greatest possible error of a measurement is $\frac{1}{16}$ inch. *Explain* how such a measurement could be more accurate in one situation than in another situation.

PRECISION AND ACCURACY Tell which measurement is more precise. Then tell which of the two measurements is more accurate.

12. 17 cm; 12 cm
13. 18.65 ft; 25.6 ft
14. 6.8 in.; 13.4 ft
15. 3.5 ft; 35 in.

16. **PERIMETER** A side of the eraser shown is a parallelogram. What is the greatest possible error for the length of each side of the parallelogram? for the perimeter of the parallelogram? Find the greatest and least possible perimeter of the parallelogram.

11.2 Areas of Trapezoids and Kites

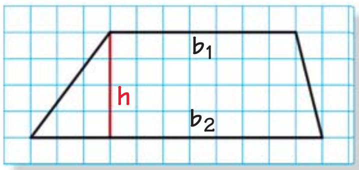
MATERIALS • graph paper • straightedge • scissors • tape

QUESTION How can you use a parallelogram to find other areas?

A trapezoid or a kite can be cut out and rearranged to form a parallelogram.

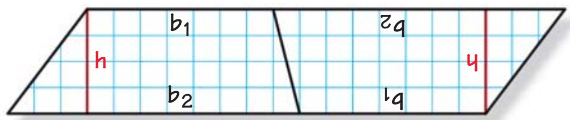
EXPLORE 1 Use two congruent trapezoids to form a parallelogram

STEP 1



Draw a trapezoid Fold graph paper in half and draw a trapezoid. Cut out two congruent trapezoids. Label as shown.

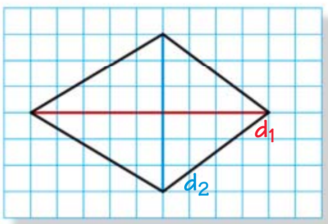
STEP 2



Create a parallelogram Arrange the two trapezoids from Step 1 to form a parallelogram. Then tape them together.

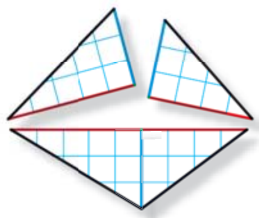
EXPLORE 2 Use one kite to form a rectangle

STEP 1



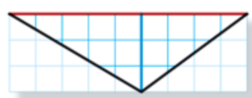
Draw a kite Draw a kite and its perpendicular diagonals. Label the diagonal that is a line of symmetry d_1. Label the other diagonal d_2.

STEP 2



Cut triangles Cut out the kite. Cut along d_1 to form two congruent triangles. Then cut one triangle along part of d_2 to form two right triangles.

STEP 3



Create a rectangle Turn over the right triangles. Place each with its hypotenuse along a side of the larger triangle to form a rectangle. Then tape the pieces together.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. In Explore 1, how does the area of one trapezoid compare to the area of the parallelogram formed from two trapezoids? Write expressions in terms of b_1, b_2, and h for the base, height, and area of the parallelogram. Then write a formula for the area of a trapezoid.

2. In Explore 2, how do the base and height of the rectangle compare to d_1 and d_2? Write an expression for the area of the rectangle in terms of d_1 and d_2. Then use that expression to write a formula for the area of a kite.

11.2 Areas of Trapezoids, Rhombuses, and Kites

Before You found areas of triangles and parallelograms.

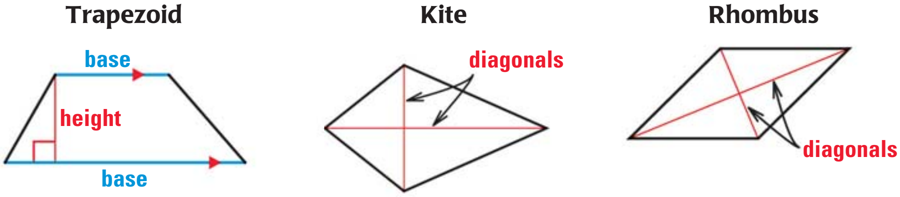
Now You will find areas of other types of quadrilaterals.

Why? So you can find the area of a free-throw lane, as in Example 1.

Key Vocabulary
- **height of a trapezoid**
- **diagonal,** *p. 507*
- **bases of a trapezoid,** *p. 542*

As you saw in the Activity on page 729, you can use the area formula for a parallelogram to develop area formulas for other special quadrilaterals. The areas of the figures below are related to the lengths of the marked segments.

The **height of a trapezoid** is the perpendicular distance between its bases.

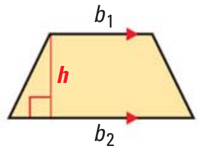


THEOREM *For Your Notebook*

THEOREM 11.4 Area of a Trapezoid

The area of a trapezoid is one half the product of the height and the sum of the lengths of the bases.

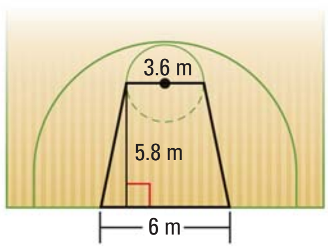
Proof: Ex. 40, p. 736



$A = \frac{1}{2}h(b_1 + b_2)$

EXAMPLE 1 Find the area of a trapezoid

BASKETBALL The free-throw lane on an international basketball court is shaped like a trapezoid. Find the area of the free-throw lane.



Solution

The height of the trapezoid is 5.8 meters. The lengths of the bases are 3.6 meters and 6 meters.

$A = \frac{1}{2}h(b_1 + b_2)$	**Formula for area of a trapezoid**
$= \frac{1}{2}(5.8)(3.6 + 6)$	**Substitute 5.8 for h, 3.6 for b_1, and 6 for b_2.**
$= 27.84$	**Simplify.**

▶ The area of the free-throw lane is about 27.8 square meters.

ANOTHER WAY

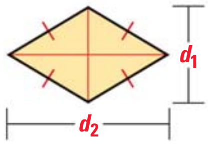
In a trapezoid, the average of the lengths of the bases is also the length of the midsegment. So, you can also find the area by multiplying the midsegment by the height.

ANOTHER WAY Remember that a rhombus is also a parallelogram, so you can also use the formula $A = bh$.

THEOREMS — *For Your Notebook*

THEOREM 11.5 Area of a Rhombus

The area of a rhombus is one half the product of the lengths of its diagonals.

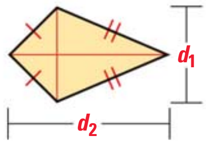


$A = \frac{1}{2}d_1d_2$

Justification: Ex. 39, p. 735

THEOREM 11.6 Area of a Kite

The area of a kite is one half the product of the lengths of its diagonals.

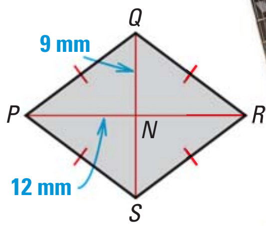


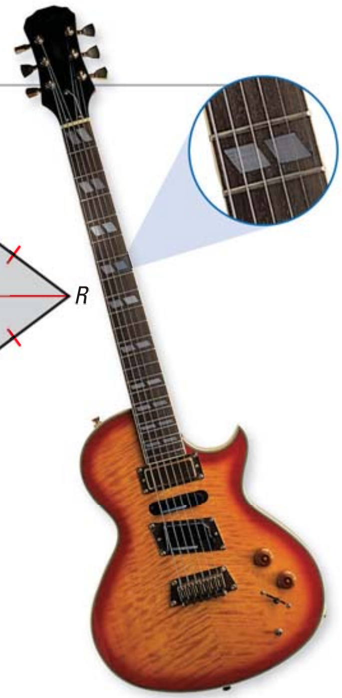
$A = \frac{1}{2}d_1d_2$

Proof: Ex. 41, p. 736

EXAMPLE 2 Find the area of a rhombus

MUSIC Rhombus $PQRS$ represents one of the inlays on the guitar in the photo. Find the area of the inlay.





READ DIAGRAMS When you read a diagram, look for information you need to find. The diagram gives the lengths of $\overline{QN}$ and $\overline{PN}$, but not the lengths of $\overline{QS}$ and $\overline{PR}$.

Solution

STEP 1 **Find** the length of each diagonal. The diagonals of a rhombus bisect each other, so $QN = NS$ and $PN = NR$.

$QS = QN + NS = 9 + 9 = 18$ mm

$PR = PN + NR = 12 + 12 = 24$ mm

STEP 2 **Find** the area of the rhombus. Let d_1 represent QS and d_2 represent PR.

$A = \frac{1}{2}d_1d_2$ — **Formula for area of a rhombus**

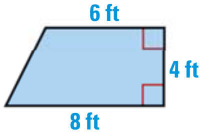
$= \frac{1}{2}(18)(24)$ — **Substitute.**

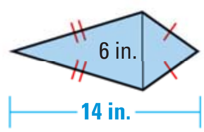
$= 216$ — **Simplify.**

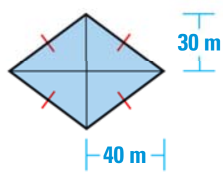
▶ The area of the inlay is 216 square millimeters.

✓ GUIDED PRACTICE for Examples 1 and 2

Find the area of the figure.

1. 

2. 

3. 

EXAMPLE 3 Standardized Test Practice

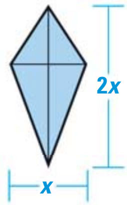
One diagonal of a kite is twice as long as the other diagonal. The area of the kite is 72.25 square inches. What are the lengths of the diagonals?

Ⓐ 6 in., 6 in. Ⓑ 8.5 in., 8.5 in. Ⓒ 8.5 in., 17 in. Ⓓ 6 in., 12 in.

ELIMINATE CHOICES
In Example 3, you can eliminate choices A and B because in each case, one diagonal is not twice as long as the other diagonal.

Solution

Draw and label a diagram. Let x be the length of one diagonal. The other diagonal is twice as long, so label it $2x$. Use the formula for the area of a kite to find the value of x.



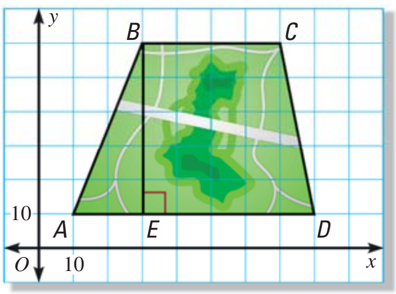
$A = \frac{1}{2}d_1d_2$	**Formula for area of a kite**
$72.25 = \frac{1}{2}(x)(2x)$	**Substitute 72.25 for A, x for d_1, and $2x$ for d_2.**
$72.25 = x^2$	**Simplify.**
$8.5 = x$	**Find the positive square root of each side.**

The lengths of the diagonals are 8.5 inches and $2(8.5) = 17$ inches.

▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

EXAMPLE 4 Find an area in the coordinate plane

CITY PLANNING You have a map of a city park. Each grid square represents a 10 meter by 10 meter square. Find the area of the park.



Solution

STEP 1 **Find** the lengths of the bases and the height of trapezoid $ABCD$.

$b_1 = BC = |70 - 30| = 40$ m

$b_2 = AD = |80 - 10| = 70$ m

$h = BE = |60 - 10| = 50$ m

STEP 2 **Find** the area of $ABCD$.

$$A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(50)(40 + 70) = 2750$$

▶ The area of the park is 2750 square meters.

✓ GUIDED PRACTICE for Examples 3 and 4

4. The area of a kite is 80 square feet. One diagonal is 4 times as long as the other. Find the diagonal lengths.

5. Find the area of a rhombus with vertices $M(1, 3)$, $N(5, 5)$, $P(9, 3)$, and $Q(5, 1)$.

11.2 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 17, and 35
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 30, 39, and 42

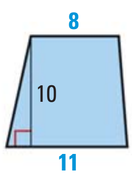
SKILL PRACTICE

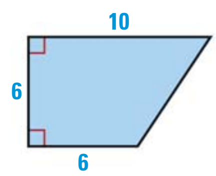
1. **VOCABULARY** Copy and complete: The perpendicular distance between the bases of a trapezoid is called the _?_ of the trapezoid.

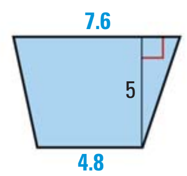
2. ★ **WRITING** Sketch a kite and its diagonals. *Describe* what you know about the segments and angles formed by the intersecting diagonals.

EXAMPLE 1 on p. 730 for Exs. 3–6

FINDING AREA **Find the area of the trapezoid.**

3. 

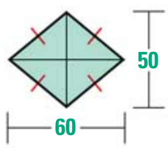
4. 

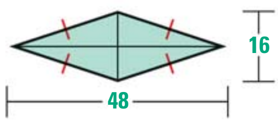
5. 

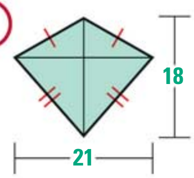
6. **DRAWING DIAGRAMS** The lengths of the bases of a trapezoid are 5.4 centimeters and 10.2 centimeters. The height is 8 centimeters. Draw and label a trapezoid that matches this description. Then find its area.

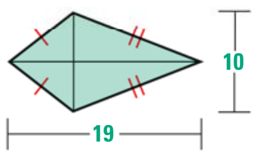
EXAMPLE 2 on p. 731 for Exs. 7–14

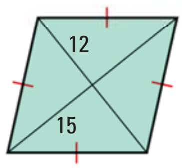
FINDING AREA **Find the area of the rhombus or kite.**

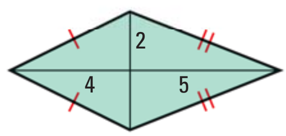
7. 

8. 

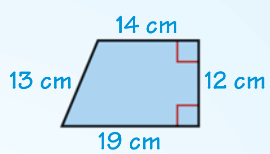
9. 

10. 

11. 

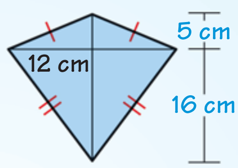
12. 

ERROR ANALYSIS ***Describe*** **and correct the error in finding the area.**

13. 

$A = \frac{1}{2}(13)(14 + 19)$

$= 214.5\ cm^2$

14. 

$A = \frac{1}{2}(12)(21)$

$= 126\ cm^2$

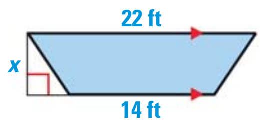
EXAMPLE 3 on p. 732 for Exs. 15–18

15. ★ **MULTIPLE CHOICE** One diagonal of a rhombus is three times as long as the other diagonal. The area of the rhombus is 24 square feet. What are the lengths of the diagonals?

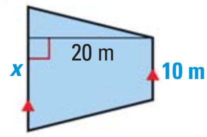
Ⓐ 8 ft, 11 ft Ⓑ 4 ft, 12 ft Ⓒ 2 ft, 6 ft Ⓓ 6 ft, 24 ft

ALGEBRA **Use the given information to find the value of x.**

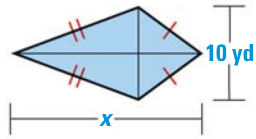
16. Area $= 108\text{ ft}^2$



17. Area $= 300\text{ m}^2$



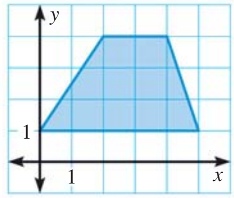
18. Area $= 100\text{ yd}^2$



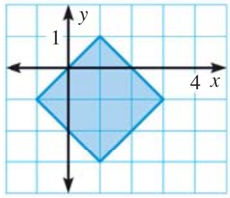
EXAMPLE 4
on p. 732
for Exs. 19–21

COORDINATE GEOMETRY **Find the area of the figure.**

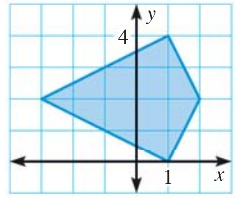
19.



20.



21.



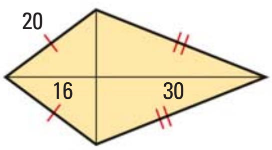
ALGEBRA **Find the lengths of the bases of the trapezoid described.**

22. The height is 3 feet. One base is twice as long as the other base. The area is 13.5 square feet.

23. One base is 8 centimeters longer than the other base. The height is 6 centimeters and the area is 54 square centimeters.

FINDING AREA **Find the area of the shaded region.**

24.



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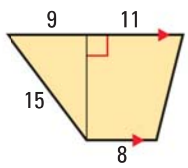

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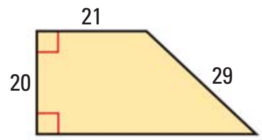
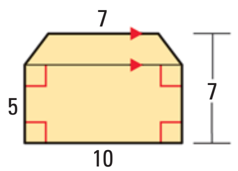


27.



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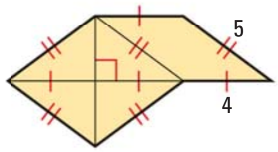

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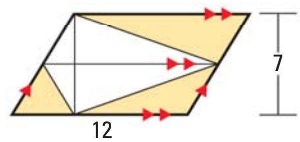


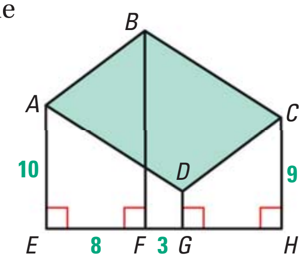
30. ★ **OPEN-ENDED MATH** Draw three examples of trapezoids that match this description: The height of the trapezoid is 3 units and its area is the same as the area of a parallelogram with height 3 units and base 8 units.

VISUALIZING **Sketch the figure. Then determine its perimeter and area.**

31. The figure is a trapezoid. It has two right angles. The lengths of its bases are 7 and 15. Its height is 6.

32. The figure is a rhombus. Its side length is 13. The length of one of its diagonals is 24.

33. **CHALLENGE** In the diagram shown at the right, $ABCD$ is a parallelogram and $BF = 16$. Find the area of $\square ABCD$. *Explain* your reasoning. (*Hint:* Draw auxiliary lines through point A and through point D that are parallel to $\overline{EH}$.)



○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

Problem Solving

EXAMPLE 1 on p. 730 for Ex. 34

34. TRUCKS The windshield in a truck is in the shape of a trapezoid. The lengths of the bases of the trapezoid are 70 inches and 79 inches. The height is 35 inches. Find the area of the glass in the windshield.

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EXAMPLE 2 on p. 731 for Ex. 35

35. INTERNET You are creating a kite-shaped logo for your school's website. The diagonals of the logo are 8 millimeters and 5 millimeters long. Find the area of the logo. Draw two different possible shapes for the logo.

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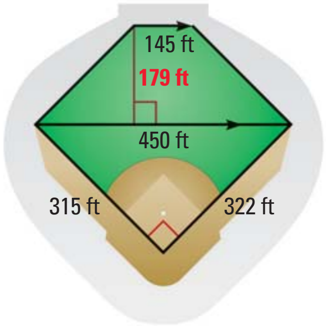
36. DESIGN You are designing a wall hanging that is in the shape of a rhombus. The area of the wall hanging is 432 square inches and the length of one diagonal is 36 inches. Find the length of the other diagonal.

37. MULTI-STEP PROBLEM As shown, a baseball stadium's playing field is shaped like a pentagon. To find the area of the playing field shown at the right, you can divide the field into two smaller polygons.

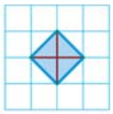
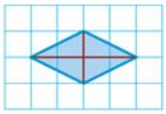
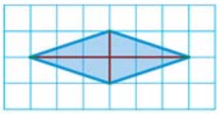
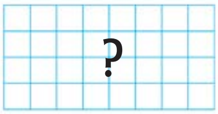
a. Classify the two polygons.

b. Find the area of the playing field in square feet. Then express your answer in square yards. Round to the nearest square foot.



38. VISUAL REASONING Follow the steps in parts (a)–(c).

a. **Analyze** Copy the table and extend it to include a column for $n = 5$. Complete the table for $n = 4$ and $n = 5$.

Rhombus number, *n*	1	2	3	4
Diagram				 ?
Area, *A*	2	4	6	?

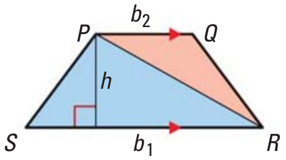
b. **Use Algebra** *Describe* the relationship between the rhombus number n and the area of the rhombus. Then write an algebraic rule for finding the area of the nth rhombus.

c. **Compare** In each rhombus, the length of one diagonal (d_1) is 2. What is the length of the other diagonal (d_2) for the nth rhombus? Use the formula for the area of a rhombus to write a rule for finding the area of the nth rhombus. *Compare* this rule with the one you wrote in part (b).

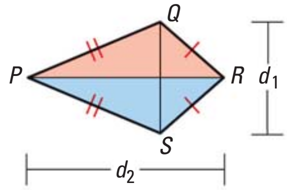
39. ★ SHORT RESPONSE Look back at the Activity on page 729. *Explain* how the results for kites in Explore 2 can be used to justify Theorem 11.5, the formula for the area of a rhombus.

PROVING THEOREMS 11.4 AND 11.6 **Use the triangle area formula and the triangles in the diagram to write a plan for the proof.**

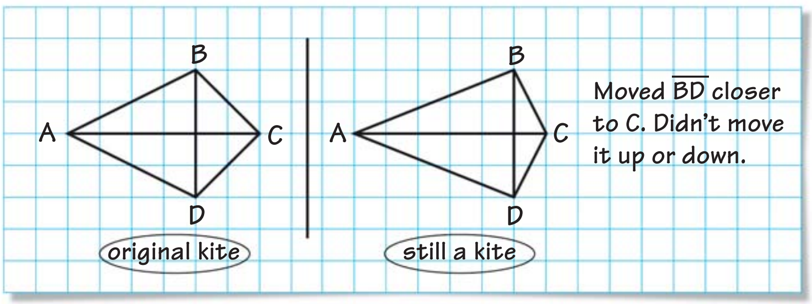
40. Show that the area A of the trapezoid shown is $\frac{1}{2}h(b_1 + b_2)$.



41. Show that the area A of the kite shown is $\frac{1}{2}d_1d_2$.



42. ★ **EXTENDED RESPONSE** You will explore the effect of moving a diagonal.

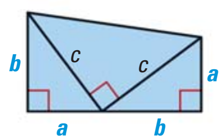


a. Investigate Draw a kite in which the longer diagonal is horizontal. Suppose this diagonal is fixed and you can slide the vertical diagonal left or right and up or down. You can keep sliding as long as the diagonals continue to intersect. Draw and identify each type of figure you can form.

b. Justify Is it possible to form any shapes that are not quadrilaterals? *Explain.*

c. Compare Compare the areas of the different shapes you found in part (b). What do you notice about the areas? *Explain.*

43. **CHALLENGE** James A. Garfield, the twentieth president of the United States, discovered a proof of the Pythagorean Theorem in 1876. His proof involved the fact that a trapezoid can be formed from two congruent right triangles and an isosceles right triangle. Use the diagram to show that $a^2 + b^2 = c^2$.



MIXED REVIEW

Solve for the indicated variable. Write a reason for each step. *(p. 105)*

44. $d = rt$; solve for t

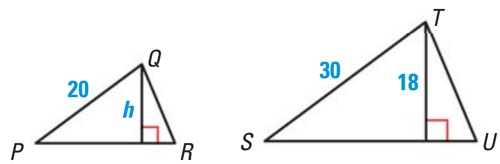
45. $A = \frac{1}{2}bh$; solve for h

46. $P = 2\ell + 2w$; solve for w

47. Find the angle measures of an isosceles triangle if the measure of a base angle is 4 times the measure of the vertex angle. *(p. 264)*

PREVIEW
Prepare for Lesson 11.3 in Ex. 48.

48. In the diagram at the right, $\triangle PQR \sim \triangle STU$. The perimeter of $\triangle STU$ is 81 inches. Find the height h and the perimeter of $\triangle PQR$. *(p. 372)*



11.3 Perimeter and Area of Similar Figures

Before You used ratios to find perimeters of similar figures.

Now You will use ratios to find areas of similar figures.

Why So you can apply similarity in cooking, as in Example 3.

Key Vocabulary
- **regular polygon,** *p. 43*
- **corresponding sides,** *p. 225*
- **similar polygons,** *p. 372*

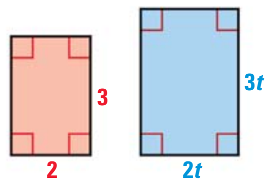
In Chapter 6 you learned that if two polygons are similar, then the ratio of their perimeters, or of any two corresponding lengths, is equal to the ratio of their corresponding side lengths. As shown below, the areas have a different ratio.

Ratio of perimeters

$$\frac{\text{Blue}}{\text{Red}} = \frac{10t}{10} = t$$

Ratio of areas

$$\frac{\text{Blue}}{\text{Red}} = \frac{6t^2}{6} = t^2$$



THEOREM *For Your Notebook*

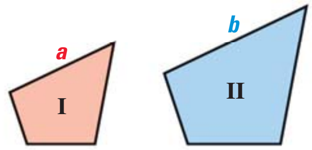
THEOREM 11.7 Areas of Similar Polygons

If two polygons are similar with the lengths of corresponding sides in the ratio of $a:b$, then the ratio of their areas is $a^2:b^2$.

$$\frac{\text{Side length of Polygon I}}{\text{Side length of Polygon II}} = \frac{a}{b}$$

$$\frac{\text{Area of Polygon I}}{\text{Area of Polygon II}} = \frac{a^2}{b^2}$$

Justification: Ex. 30, p. 742



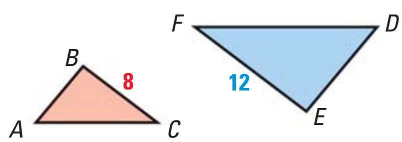
Polygon I ~ Polygon II

EXAMPLE 1 Find ratios of similar polygons

In the diagram, $\triangle ABC \sim \triangle DEF$. Find the indicated ratio.

a. Ratio (red to blue) of the perimeters

b. Ratio (red to blue) of the areas



Solution

The ratio of the lengths of corresponding sides is $\frac{8}{12} = \frac{2}{3}$, or $2:3$.

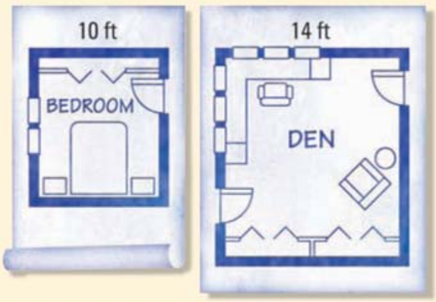
a. By Theorem 6.1 on page 374, the ratio of the perimeters is $2:3$.

b. By Theorem 11.7 above, the ratio of the areas is $2^2:3^2$, or $4:9$.

INTERPRET RATIOS
You can also compare the measures with fractions. The perimeter of $\triangle ABC$ is two thirds of the perimeter of $\triangle DEF$. The area of $\triangle ABC$ is four ninths of the area of $\triangle DEF$.

EXAMPLE 2 Standardized Test Practice

You are installing the same carpet in a bedroom and den. The floors of the rooms are similar. The carpet for the bedroom costs \$225. Carpet is sold by the square foot. How much does it cost to carpet the den?



Ⓐ \$115 Ⓑ \$161

Ⓒ \$315 Ⓓ \$441

USE ESTIMATION

The cost for the den is $\frac{49}{25}$ times the cost for the bedroom. Because $\frac{49}{25}$ is a little less than 2, the cost for the den is a little less than twice \$225. The only possible choice is D.

Solution

The ratio of a side length of the den to the corresponding side length of the bedroom is 14 : 10, or 7 : 5. So, the ratio of the areas is $7^2 : 5^2$, or 49 : 25. This ratio is also the ratio of the carpeting costs. Let x be the cost for the den.

$\frac{49}{25} = \frac{x}{225}$ ← **cost of carpet for den** / ← **cost of carpet for bedroom**

$x = 441$ **Solve for x.**

▶ It costs \$441 to carpet the den. The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 1 and 2

1. The perimeter of $\triangle ABC$ is 16 feet, and its area is 64 feet. The perimeter of $\triangle DEF$ is 12 feet. Given $\triangle ABC \sim \triangle DEF$, find the ratio of the area of $\triangle ABC$ to the area of $\triangle DEF$. Then find the area of $\triangle DEF$.

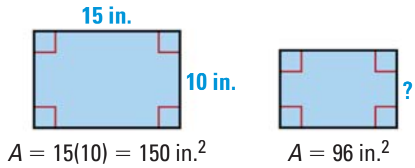
EXAMPLE 3 Use a ratio of areas

COOKING A large rectangular baking pan is 15 inches long and 10 inches wide. A smaller pan is similar to the large pan. The area of the smaller pan is 96 square inches. Find the width of the smaller pan.

ANOTHER WAY

For an alternative method for solving the problem in Example 3, turn to page 744 for the **Problem Solving Workshop**.

Solution



First draw a diagram to represent the problem. Label dimensions and areas.

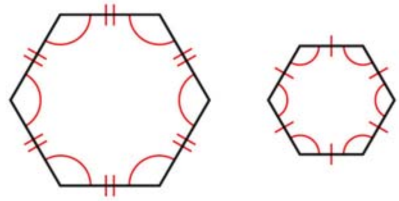
Then use Theorem 11.7. If the area ratio is $a^2 : b^2$, then the length ratio is $a : b$.

$$\frac{\text{Area of smaller pan}}{\text{Area of large pan}} = \frac{96}{150} = \frac{16}{25}$$ **Write ratio of known areas. Then simplify.**

$$\frac{\text{Length in smaller pan}}{\text{Length in large pan}} = \frac{4}{5}$$ **Find square root of area ratio.**

▶ Any length in the smaller pan is $\frac{4}{5}$, or 0.8, of the corresponding length in the large pan. So, the width of the smaller pan is $0.8(10 \text{ inches}) = 8$ inches.

REGULAR POLYGONS Consider two regular polygons with the same number of sides. All of the angles are congruent. The lengths of all pairs of corresponding sides are in the same ratio. So, any two such polygons are similar. Also, any two circles are similar.



EXAMPLE 4 Solve a multi-step problem

GAZEBO The floor of the gazebo shown is a regular octagon. Each side of the floor is 8 feet, and the area is about 309 square feet. You build a small model gazebo in the shape of a regular octagon. The perimeter of the floor of the model gazebo is 24 inches. Find the area of the floor of the model gazebo to the nearest tenth of a square inch.

Solution

All regular octagons are similar, so the floor of the model is similar to the floor of the full-sized gazebo.

ANOTHER WAY
In Step 1, instead of finding the perimeter of the full-sized and comparing perimeters, you can find the side length of the model and compare side lengths. $24 \div 8 = 3$, so the ratio of side lengths is $\frac{8\text{ ft.}}{3\text{ in.}} = \frac{96\text{ in.}}{3\text{ in.}} = \frac{32}{1}$.

STEP 1 **Find** the ratio of the lengths of the two floors by finding the ratio of the perimeters. Use the same units for both lengths in the ratio.

$$\frac{\text{Perimeter of full-sized}}{\text{Perimeter of model}} = \frac{8(8\text{ ft})}{24\text{ in.}} = \frac{64\text{ ft}}{24\text{ in.}} = \frac{64\text{ ft}}{2\text{ ft}} = \frac{32}{1}$$

So, the ratio of corresponding lengths (full-sized to model) is 32 : 1.

STEP 2 **Calculate** the area of the model gazebo's floor. Let x be this area.

$$\frac{(\text{Length in full-sized})^2}{(\text{Length in model})^2} = \frac{\text{Area of full-sized}}{\text{Area of model}} \quad \textbf{Theorem 11.7}$$

$$\frac{32^2}{1^2} = \frac{309\text{ ft}^2}{x\text{ ft}^2} \quad \textbf{Substitute.}$$

$$1024x = 309 \quad \textbf{Cross Products Property}$$

$$x \approx 0.302\text{ ft}^2 \quad \textbf{Solve for } \boldsymbol{x}\textbf{.}$$

STEP 3 **Convert** the area to square inches.

$$0.302\ \cancel{\text{ft}^2} \cdot \frac{144\text{ in.}^2}{1\ \cancel{\text{ft}^2}} \approx 43.5\text{ in.}^2$$

▶ The area of the floor of the model gazebo is about 43.5 square inches.

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✓ GUIDED PRACTICE for Examples 3 and 4

2. The ratio of the areas of two regular decagons is 20 : 36. What is the ratio of their corresponding side lengths in simplest radical form?

3. Rectangles I and II are similar. The perimeter of Rectangle I is 66 inches. Rectangle II is 35 feet long and 20 feet wide. Show the steps you would use to find the ratio of the areas and then find the area of Rectangle I.

11.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 17, and 27

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 12, 18, 28, 32, and 33

SKILL PRACTICE

1. **VOCABULARY** Sketch two similar triangles. Use your sketch to explain what is meant by *corresponding side lengths*.

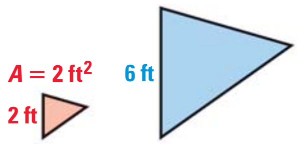
2. ★ **WRITING** Two regular n-gons are similar. The ratio of their side lengths is is $3:4$. Do you need to know the value of n to find the ratio of the perimeters or the ratio of the areas of the polygons? *Explain*.

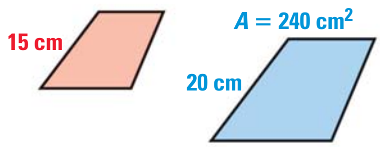
EXAMPLES 1 and 2 on pp. 737–738 for Exs. 3–8

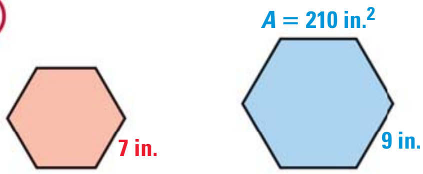
FINDING RATIOS **Copy and complete the table of ratios for similar polygons.**

	Ratio of corresponding side lengths	Ratio of perimeters	Ratio of areas
3.	$6:11$	?	?
4.	?	$20:36 = ?$	?

RATIOS AND AREAS **Corresponding lengths in similar figures are given. Find the ratios (red to blue) of the perimeters and areas. Find the unknown area.**

5.


6.


7.


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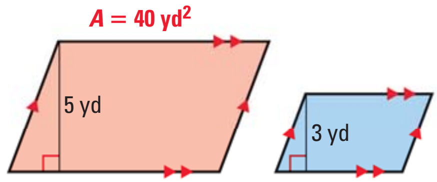


EXAMPLE 3 on p. 738 for Exs. 9–15

FINDING LENGTH RATIOS **The ratio of the areas of two similar figures is given. Write the ratio of the lengths of corresponding sides.**

9. Ratio of areas $= 49:16$

10. Ratio of areas $= 16:121$

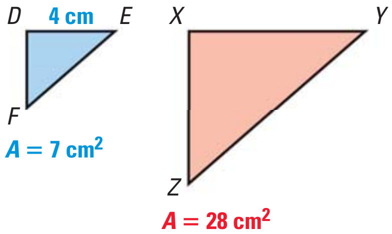
11. Ratio of areas $= 121:144$

12. ★ **MULTIPLE CHOICE** The area of $\triangle LMN$ is 18 ft^2 and the area of $\triangle FGH$ is 24 ft^2. If $\triangle LMN \sim \triangle FGH$, what is the ratio of LM to FG?

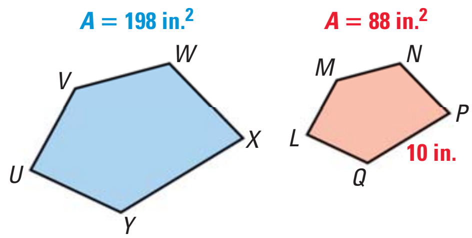
(A) $3:4$ (B) $9:16$ (C) $\sqrt{3}:2$ (D) $4:3$

FINDING SIDE LENGTHS **Use the given area to find *XY*.**

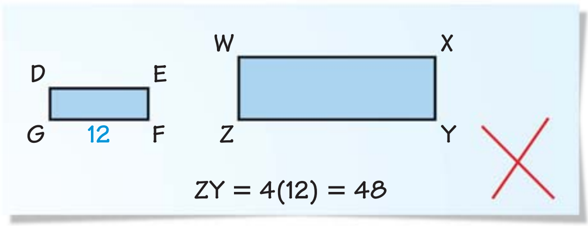
13. $\triangle DEF \sim \triangle XYZ$



14. $UVWXY \sim LMNPQ$



15. ERROR ANALYSIS In the diagram, Rectangles *DEFG* and *WXYZ* are similar. The ratio of the area of *DEFG* to the area of *WXYZ* is 1 : 4. *Describe* and correct the error in finding *ZY*.



EXAMPLE 4
on p. 739
for Exs. 16–17

16. REGULAR PENTAGONS Regular pentagon *QRSTU* has a side length of 12 centimeters and an area of about 248 square centimeters. Regular pentagon *VWXYZ* has a perimeter of 140 centimeters. Find its area.

17. RHOMBUSES Rhombuses *MNPQ* and *RSTU* are similar. The area of *RSTU* is 28 square feet. The diagonals of *MNPQ* are 25 feet long and 14 feet long. Find the area of *MNPQ*. Then use the ratio of the areas to find the lengths of the diagonals of *RSTU*.

18. ★ SHORT RESPONSE You enlarge the same figure three different ways. In each case, the enlarged figure is similar to the original. List the enlargements in order from smallest to largest. *Explain*.

Case 1 The side lengths of the original figure are multiplied by 3.
Case 2 The perimeter of the original figure is multiplied by 4.
Case 3 The area of the original figure is multiplied by 5.

REASONING In Exercises 19 and 20, copy and complete the statement using *always*, *sometimes*, or *never*. *Explain* your reasoning.

19. Doubling the side length of a square __?__ doubles the area.

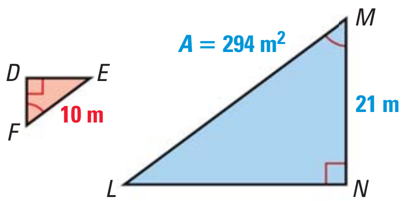
20. Two similar octagons __?__ have the same perimeter.

21. FINDING AREA The sides of $\triangle ABC$ are 4.5 feet, 7.5 feet, and 9 feet long. The area is about 17 square feet. *Explain* how to use the area of $\triangle ABC$ to find the area of a $\triangle DEF$ with side lengths 6 feet, 10 feet, and 12 feet.

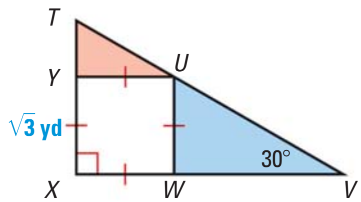
22. RECTANGLES Rectangles *ABCD* and *DEFG* are similar. The length of *ABCD* is 24 feet and the perimeter is 84 square feet. The width of *DEFG* is 3 yards. Find the ratio of the area of *ABCD* to the area of *DEFG*.

SIMILAR TRIANGLES *Explain* why the red and blue triangles are similar. Find the ratio (red to blue) of the areas of the triangles. Show your steps.

23.



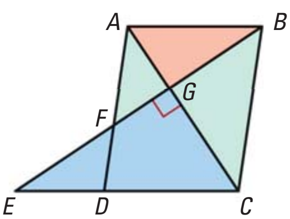
24.



25. CHALLENGE In the diagram shown, *ABCD* is a parallelogram. The ratio of the area of $\triangle AGB$ to the area of $\triangle CGE$ is 9 : 25, $CG = 10$, and $GE = 15$.

a. Find *AG*, *GB*, *GF*, and *FE*. Show your methods.

b. Give two area ratios other than 9 : 25 or 25 : 9 for pairs of similar triangles in the figure. *Explain*.



PROBLEM SOLVING

26. **BANNER** Two rectangular banners from this year's music festival are shown. Organizers of next year's festival want to design a new banner that will be similar to the banner whose dimensions are given in the photograph. The length of the longest side of the new banner will be 5 feet. Find the area of the new banner.

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EXAMPLE 3 on p. 738 for Ex. 27

27. **PATIO** A new patio will be an irregular hexagon. The patio will have two long parallel sides and an area of 360 square feet. The area of a similar shaped patio is 250 square feet, and its long parallel sides are 12.5 feet apart. What will be the corresponding distance on the new patio?

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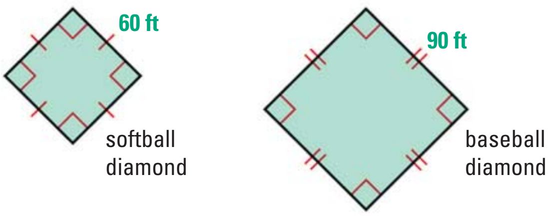
28. ★ **MULTIPLE CHOICE** You need 20 pounds of grass seed to plant grass inside the baseball diamond shown. About how many pounds do you need to plant grass inside the softball diamond?

Ⓐ 6 Ⓑ 9

Ⓒ 13 Ⓓ 20



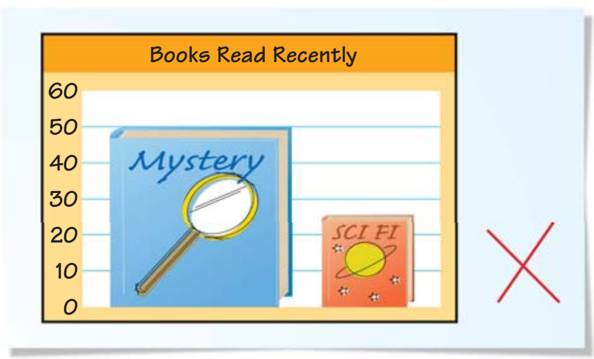
29. **MULTI-STEP PROBLEM** Use graph paper for parts (a) and (b).

a. Draw a triangle and label its vertices. Find the area of the triangle.

b. Mark and label the midpoints of each side of the triangle. Connect the midpoints to form a smaller triangle. Show that the larger and smaller triangles are similar. Then use the fact that the triangles are similar to find the area of the smaller triangle.

30. **JUSTIFYING THEOREM 11.7** Choose a type of polygon for which you know the area formula. Use algebra and the area formula to prove Theorem 11.7 for that polygon. (*Hint:* Use the ratio for the corresponding side lengths in two similar polygons to express each dimension in one polygon as $\frac{a}{b}$ times the corresponding dimension in the other polygon.)

31. **MISLEADING GRAPHS** A student wants to show that the students in a science class prefer mysteries to science fiction books. Over a two month period, the students in the class read 50 mysteries, but only 25 science fiction books. The student makes a bar graph of these data. *Explain* why the graph is visually misleading. Show how the student could redraw the bar graph.

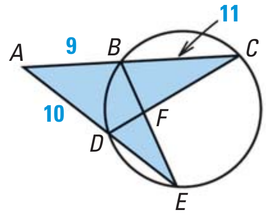


○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

32. ★ **OPEN-ENDED MATH** The ratio of the areas of two similar polygons is 9 : 6. Draw two polygons that fit this description. Find the ratio of their perimeters. Then write the ratio in simplest radical form.

33. ★ **EXTENDED RESPONSE** Use the diagram shown at the right.

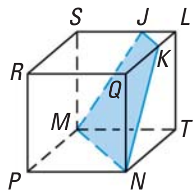


a. Name as many pairs of similar triangles as you can. *Explain* your reasoning.

b. Find the ratio of the areas for one pair of similar triangles.

c. Show two ways to find the length of $\overline{DE}$.

34. **CHALLENGE** In the diagram, the solid figure is a cube. Quadrilateral *JKNM* is on a plane that cuts through the cube, with $JL = KL$.



a. *Explain* how you know that $\triangle JKL \sim \triangle MNP$.

b. Suppose $\frac{JK}{MN} = \frac{1}{3}$. Find the ratio of the area of $\triangle JKL$ to the area of one face of the cube.

c. Find the ratio of the area of $\triangle JKL$ to the area of pentagon *JKQRS*.

MIXED REVIEW

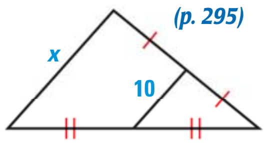
PREVIEW
Prepare for Lesson 11.4 in Exs. 35–38.

Find the circumference of the circle with the given radius *r* or diameter *d*. Use $\pi \approx 3.14$. Round your answers to the nearest hundredth. *(p. 49)*

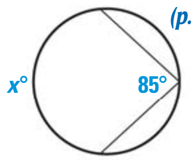
35. $d = 4$ cm
36. $d = 10$ ft
37. $r = 2.5$ yd
38. $r = 3.1$ m

Find the value of *x*.

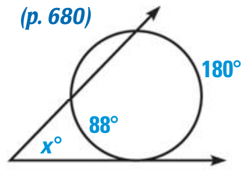
39. *(p. 295)*



40. *(p. 672)*



41. *(p. 680)*

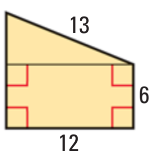


QUIZ for Lessons 11.1–11.3

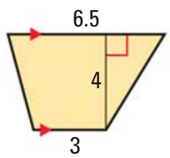
1. The height of ▱*ABCD* is 3 times its base. Its area is 108 square feet. Find the base and the height. *(p. 720)*

Find the area of the figure.

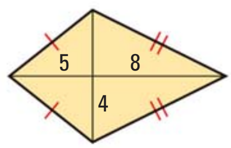
2. *(p. 720)*



3. *(p. 730)*



4. *(p. 730)*



5. The ratio of the lengths of corresponding sides of two similar heptagons is 7 : 20. Find the ratio of their perimeters and their areas. *(p. 737)*

6. Triangles *PQR* and *XYZ* are similar. The area of $\triangle PQR$ is 1200 ft^2 and the area of $\triangle XYZ$ is 48 ft^2. Given $PQ = 50$ ft, find *XY*. *(p. 737)*

PROBLEM SOLVING WORKSHOP
LESSON 11.3

Using ALTERNATIVE METHODS

Another Way to Solve Example 3, page 738

MULTIPLE REPRESENTATIONS In Example 3 on page 738, you used proportional reasoning to solve a problem about cooking. You can also solve the problem by using an area formula.

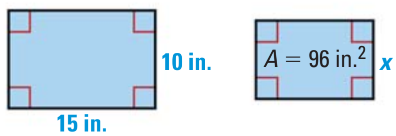
PROBLEM

COOKING A large rectangular baking pan is 15 inches long and 10 inches wide. A smaller pan is similar to the large pan. The area of the smaller pan is 96 square inches. Find the width of the smaller pan.

METHOD

Using a Formula You can use what you know about side lengths of similar figures to find the width of the pan.

STEP 1 **Use** the given dimensions of the large pan to write expressions for the dimensions of the smaller pan. Let x represent the width of the smaller pan.



The length of the larger pan is 1.5 times its width. So, the length of the smaller pan is also 1.5 times its width, or $1.5x$.

STEP 2 **Use** the formula for the area of a rectangle to write an equation.

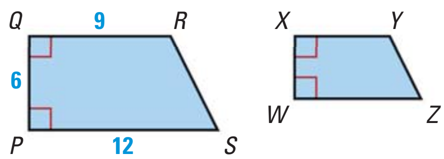
$A = \ell w$	**Formula for area of a rectangle**
$96 = 1.5x \cdot x$	**Substitute $1.5x$ for ℓ and x for w.**
$8 = x$	**Solve for a positive value of x.**

▶ The width of the smaller pan is 8 inches.

PRACTICE

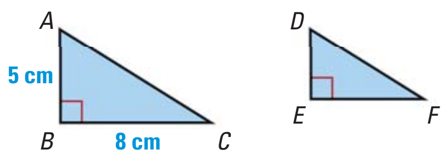
1. **COOKING** A third pan is similar to the large pan shown above and has 1.44 times its area. Find the length of the third pan.

2. **TRAPEZOIDS** Trapezoid $PQRS$ is similar to trapezoid $WXYZ$. The area of $WXYZ$ is 28 square units. Find WZ.



3. **SQUARES** One square has sides of length s. If another square has twice the area of the first square, what is its side length?

4. **REASONING** $\triangle ABC \sim \triangle DEF$ and the area of $\triangle DEF$ is 11.25 square centimeters. Find DE and DF. *Explain* your reasoning.



MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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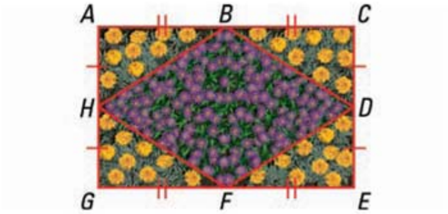
Lessons 11.1–11.3

1. MULTI-STEP PROBLEM The diagram below represents a rectangular flower bed. In the diagram, $AG = 9.5$ feet and $GE = 15$ feet.



a. *Explain* how you know that $BDFH$ is a rhombus.

b. Find the area of rectangle $ACEG$ and the area of rhombus $BDFH$.

c. You want to plant asters inside rhombus $BDFH$ and marigolds in the other parts of the flower bed. It costs about \$.30 per square foot to plant marigolds and about \$.40 per square foot to plant asters. How much will you spend on flowers?

2. OPEN-ENDED A polygon has an area of 48 square meters and a height of 8 meters. Draw three different triangles that fit this description and three different parallelograms. *Explain* your thinking.

3. EXTENDED RESPONSE You are tiling a 12 foot by 21 foot rectangular floor. Prices are shown below for two sizes of square tiles.

a. How many small tiles would you need for the floor? How many large tiles?

b. Find the cost of buying large tiles for the floor and the cost of buying small tiles for the floor. Which tile should you use if you want to spend as little as possible?

c. *Compare* the side lengths, the areas, and the costs of the two tiles. Is the cost per tile based on side length or on area? *Explain*.

4. SHORT RESPONSE What happens to the area of a rhombus if you double the length of each diagonal? if you triple the length of each diagonal? *Explain* what happens to the area of a rhombus if each diagonal is multiplied by the same number n.

5. MULTI-STEP PROBLEM The pool shown is a right triangle with legs of length 40 feet and 41 feet. The path around the pool is 40 inches wide.

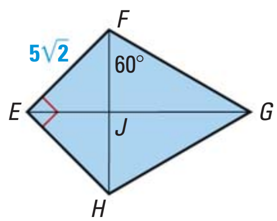
a. Find the area of $\triangle STU$.

b. In the diagram, $\triangle PQR \sim \triangle STU$, and the scale factor of the two triangles is $1.3:1$. Find the perimeter of $\triangle PQR$.

c. Find the area of $\triangle PQR$. Then find the area of the path around the pool.

6. GRIDDED ANSWER In trapezoid $ABCD$, $\overline{AB} \parallel \overline{CD}$, $m\angle D = 90°$, $AD = 5$ inches, and $CD = 3 \cdot AB$. The area of trapezoid $ABCD$ is 1250 square inches. Find the length (in inches) of $\overline{CD}$.

7. EXTENDED RESPONSE In the diagram below, $\triangle EFH$ is an isosceles right triangle, and $\triangle FGH$ is an equilateral triangle.



a. Find FH. *Explain* your reasoning.

b. Find EG. *Explain* your reasoning.

c. Find the area of $EFGH$.

11.4 Circumference and Arc Length

Before You found the circumference of a circle.

Now You will find arc lengths and other measures.

Why? So you can find a running distance, as in Example 5.

Key Vocabulary
- **circumference**
- **arc length**
- **radius,** *p. 651*
- **diameter,** *p. 651*
- **measure of an arc,** *p. 659*

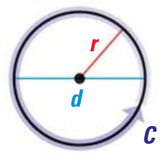
The **circumference** of a circle is the distance around the circle. For all circles, the ratio of the circumference to the diameter is the same. This ratio is known as π, or *pi*. In Chapter 1, you used 3.14 to approximate the value of π. Throughout this chapter, you should use the π key on a calculator, then round to the hundredths place unless instructed otherwise.

THEOREM *For Your Notebook*

THEOREM 11.8 Circumference of a Circle

The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$, where d is the diameter of the circle and r is the radius of the circle.

Justification: Ex. 2, p. 769



$C = \pi d = 2\pi r$

EXAMPLE 1 Use the formula for circumference

Find the indicated measure.

a. Circumference of a circle with radius 9 centimeters

b. Radius of a circle with circumference 26 meters

Solution

a.

$C = 2\pi r$	Write circumference formula.
$= 2 \cdot \pi \cdot 9$	Substitute 9 for r.
$= 18\pi$	Simplify.
≈ 56.55	Use a calculator.

▶ The circumference is about 56.55 centimeters.

b.

$C = 2\pi r$	Write circumference formula.
$26 = 2\pi r$	Substitute 26 for C.
$\frac{26}{2\pi} = r$	Divide each side by 2π.
$4.14 \approx r$	Use a calculator.

▶ The radius is about 4.14 meters.

ANOTHER WAY
You can give an exact measure in terms of π. In Example 1, part (a), the exact circumference is 18π. The exact radius in Example 1, part (b) is $\frac{26}{2\pi}$, or $\frac{13}{\pi}$.

EXAMPLE 2 Use circumference to find distance traveled

TIRE REVOLUTIONS The dimensions of a car tire are shown at the right. To the nearest foot, how far does the tire travel when it makes 15 revolutions?

Solution

STEP 1 **Find** the diameter of the tire.

$d = 15 + 2(5.5) = 26$ in.

STEP 2 **Find** the circumference of the tire.

$C = \pi d = \pi(26) \approx 81.68$ in.

STEP 3 **Find** the distance the tire travels in 15 revolutions. In one revolution, the tire travels a distance equal to its circumference. In 15 revolutions, the tire travels a distance equal to 15 times its circumference.

Distance traveled = Number of revolutions • Circumference

$\approx 15 \cdot 81.68$ in.

$= 1225.2$ in.

AVOID ERRORS
Always pay attention to units. In Example 2, you need to convert units to get a correct answer.

STEP 4 **Use** unit analysis. Change 1225.2 inches to feet.

$$1225.2 \text{ } \cancel{\text{in.}} \cdot \frac{1 \text{ ft}}{12 \text{ } \cancel{\text{in.}}} = 102.1 \text{ ft}$$

▶ The tire travels approximately 102 feet.

✓ GUIDED PRACTICE for Examples 1 and 2

1. Find the circumference of a circle with diameter 5 inches. Find the diameter of a circle with circumference 17 feet.

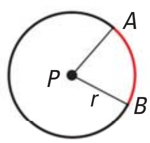
2. A car tire has a diameter of 28 inches. How many revolutions does the tire make while traveling 500 feet?

ARC LENGTH An **arc length** is a portion of the circumference of a circle. You can use the measure of the arc (in degrees) to find its length (in linear units).

COROLLARY *For Your Notebook*

ARC LENGTH COROLLARY

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.

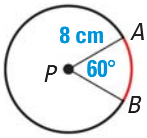


$$\frac{\text{Arc length of } \widehat{AB}}{2\pi r} = \frac{m\widehat{AB}}{360^\circ}, \text{ or Arc length of } \widehat{AB} = \frac{m\widehat{AB}}{360^\circ} \cdot 2\pi r$$

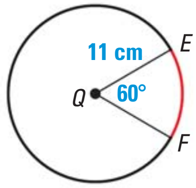
EXAMPLE 3 Find arc lengths

Find the length of each red arc.

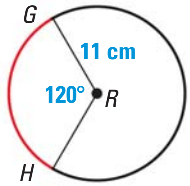
a.



b.



c.



INTERPRET DIAGRAMS

In Example 3, $\overset{\frown}{AB}$ and $\overset{\frown}{EF}$ have the same measure. However, they have different lengths because they are in circles with different circumferences.

Solution

a. Arc length of $\overset{\frown}{AB} = \frac{60°}{360°} \cdot 2\pi(8) \approx 8.38$ centimeters

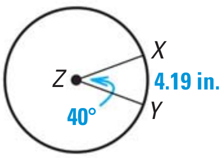
b. Arc length of $\overset{\frown}{EF} = \frac{60°}{360°} \cdot 2\pi(11) \approx 11.52$ centimeters

c. Arc length of $\overset{\frown}{GH} = \frac{120°}{360°} \cdot 2\pi(11) \approx 23.04$ centimeters

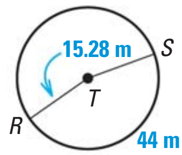
EXAMPLE 4 Use arc lengths to find measures

Find the indicated measure.

a. Circumference C of $\odot Z$



b. $m\overset{\frown}{RS}$



Solution

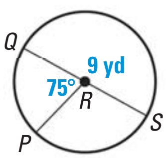
a.
$$\frac{\text{Arc length of } \overset{\frown}{XY}}{C} = \frac{m\overset{\frown}{XY}}{360°}$$
$$\frac{4.19}{C} = \frac{40°}{360°}$$
$$\frac{4.19}{C} = \frac{1}{9}$$
▶ $37.71 = C$

b.
$$\frac{\text{Arc length of } \overset{\frown}{RS}}{2\pi r} = \frac{m\overset{\frown}{RS}}{360°}$$
$$\frac{44}{2\pi(15.28)} = \frac{m\overset{\frown}{RS}}{360°}$$
$$360° \cdot \frac{44}{2\pi(15.28)} = m\overset{\frown}{RS}$$
▶ $165° \approx m\overset{\frown}{RS}$

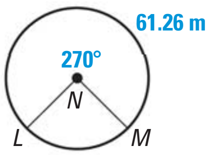
✓ GUIDED PRACTICE for Examples 3 and 4

Find the indicated measure.

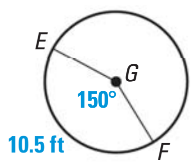
3. Length of $\overset{\frown}{PQ}$



4. Circumference of $\odot N$

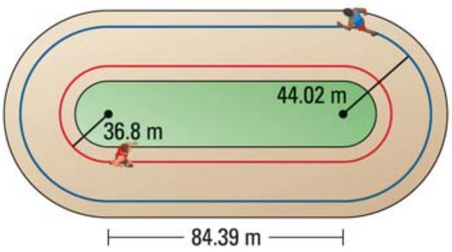


5. Radius of $\odot G$



EXAMPLE 5 Use arc length to find distances

TRACK The curves at the ends of the track shown are 180° arcs of circles. The radius of the arc for a runner on the red path shown is 36.8 meters. About how far does this runner travel to go once around the track? Round to the nearest tenth of a meter.



Solution

USE FORMULAS
The arc length of a semicircle is half the circumference of the circle with the same radius. So, the arc length of a semicircle is $\frac{1}{2} \cdot 2\pi r$, or πr.

The path of a runner is made of two straight sections and two semicircles. To find the total distance, find the sum of the lengths of each part.

$$\text{Distance} = \text{2 • Length of each straight section} + \text{2 • Length of each semicircle}$$

$$= 2(84.39) + 2 \cdot \left(\frac{1}{2} \cdot 2\pi \cdot 36.8\right)$$

$$\approx 400.0 \text{ meters}$$

▶ The runner on the red path travels about 400 meters.

Animated **Geometry** at classzone.com

✓ GUIDED PRACTICE for Example 5

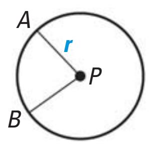
6. In Example 5, the radius of the arc for a runner on the blue path is 44.02 meters, as shown in the diagram. About how far does this runner travel to go once around the track? Round to the nearest tenth of a meter.

11.4 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 23, 25, and 35
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 31, 32, and 38

SKILL PRACTICE

In Exercises 1 and 2, refer to the diagram of $\odot P$ shown.



1. VOCABULARY Copy and complete the equation: $\frac{?}{2\pi r} = \frac{m\overset{\frown}{AB}}{?}$.

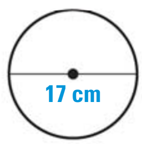
2. ★ WRITING *Describe* the difference between the *arc measure* and the *arc length* of $\overset{\frown}{AB}$.

EXAMPLE 1
on p. 746 for Exs. 3–7

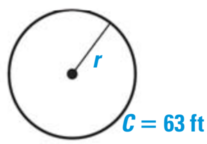
USING CIRCUMFERENCE **Use the diagram to find the indicated measure.**

3. Find the circumference.

4. Find the circumference.



5. Find the radius.



FINDING EXACT MEASURES **Find the indicated measure.**

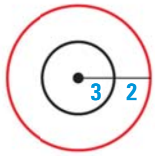
6. The exact circumference of a circle with diameter 5 inches

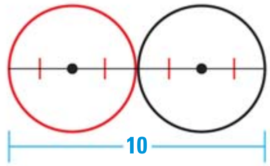
7. The exact radius of a circle with circumference 28π meters

EXAMPLE 2
on p. 747
for Exs. 8–10

FINDING CIRCUMFERENCE **Find the circumference of the red circle.**

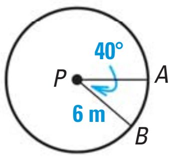
8. 

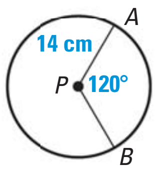
9. 

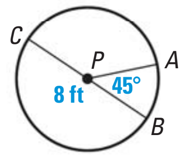
10. 

EXAMPLE 3
on p. 748
for Exs. 11–20

FINDING ARC LENGTHS **Find the length of $\widehat{AB}$.**

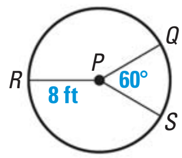
11. 

12. 

13. 

14. **ERROR ANALYSIS** A student says that two arcs from different circles have the same arc length if their central angles have the same measure. *Explain* the error in the student's reasoning.

FINDING MEASURES **In $\odot P$ shown at the right, $\angle QPR \cong \angle RPS$. Find the indicated measure.**



15. $m\widehat{QRS}$

16. Length of $\widehat{QRS}$

17. $m\widehat{QR}$

18. $m\widehat{RSQ}$

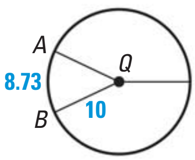
19. Length of $\widehat{QR}$

20. Length of $\widehat{RSQ}$

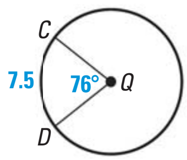
EXAMPLE 4
on p. 748
for Exs. 21–23

USING ARC LENGTH **Find the indicated measure.**

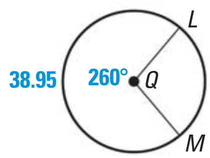
21. $m\widehat{AB}$



22. Circumference of $\odot Q$

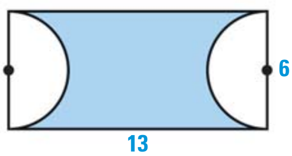


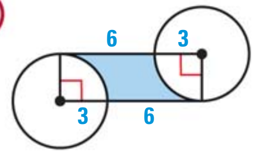
23. Radius of $\odot Q$



EXAMPLE 5
on p. 749
for Exs. 24–25

FINDING PERIMETERS **Find the perimeter of the shaded region.**

24. 

25. 

COORDINATE GEOMETRY **The equation of a circle is given. Find the circumference of the circle. Write the circumference in terms of π.**

26. $x^2 + y^2 = 16$

27. $(x + 2)^2 + (y - 3)^2 = 9$

28. $x^2 + y^2 = 18$

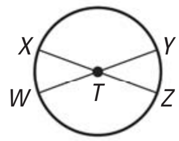
29. **ALGEBRA** Solve the formula $C = 2\pi r$ for r. Solve the formula $C = \pi d$ for d. Use the rewritten formulas to find r and d when $C = 26\pi$.

30. **FINDING VALUES** In the table below, $\overset{\frown}{AB}$ refers to the arc of a circle. Copy and complete the table.

Radius	?	2	0.8	4.2	?	$4\sqrt{2}$
$m\overset{\frown}{AB}$	45°	60°	?	183°	90°	?
Length of $\overset{\frown}{AB}$	4	?	0.3	?	3.22	2.86

31. ★ **SHORT RESPONSE** Suppose $\overset{\frown}{EF}$ is an arc on a circle with radius r. Let $x°$ be the measure of $\overset{\frown}{EF}$. *Describe* the effect on the length of $\overset{\frown}{EF}$ if you (a) double the radius of the circle, and (b) double the measure of $\overset{\frown}{EF}$.

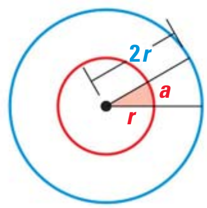
32. ★ **MULTIPLE CHOICE** In the diagram, $\overline{WY}$ and $\overline{XZ}$ are diameters of $\odot T$, and $WY = XZ = 6$. If $m\overset{\frown}{XY} = 140°$, what is the length of $\overset{\frown}{YZ}$?



Ⓐ $\frac{2}{3}\pi$ Ⓑ $\frac{4}{3}\pi$ Ⓒ 6π Ⓓ 4π

33. **CHALLENGE** Find the circumference of a circle inscribed in a rhombus with diagonals that are 12 centimeters and 16 centimeters long. *Explain*.

34. **FINDING CIRCUMFERENCE** In the diagram, the measure of the shaded red angle is 30°. The arc length a is 2. *Explain* how to find the circumference of the blue circle without finding the radius of either the red or the blue circles.



PROBLEM SOLVING

35. **TREES** A group of students wants to find the diameter of the trunk of a young sequoia tree. The students wrap a rope around the tree trunk, then measure the length of rope needed to wrap one time around the trunk. This length is 21 feet 8 inches. *Explain* how they can use this length to estimate the diameter of the tree trunk to the nearest half foot.

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36. **INSCRIBED SQUARE** A square with side length 6 units is inscribed in a circle so that all four vertices are on the circle. Draw a sketch to represent this problem. Find the circumference of the circle.

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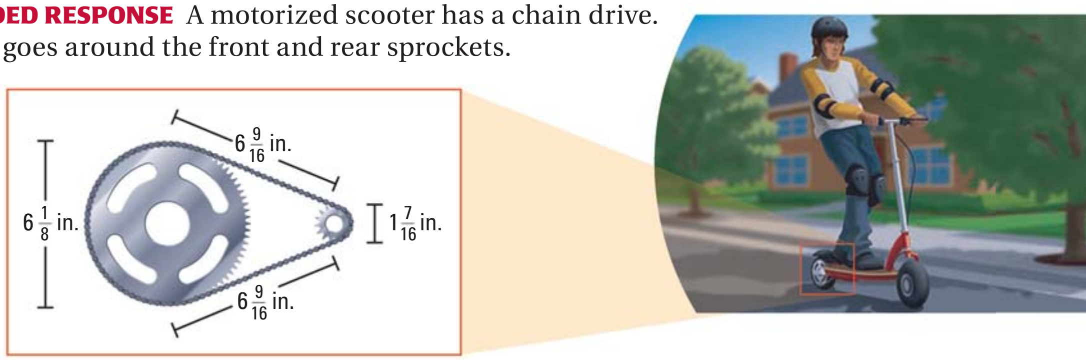
EXAMPLE 2
on p. 747
for Ex. 37

37. **MEASURING WHEEL** As shown, a measuring wheel is used to calculate the length of a path. The diameter of the wheel is 8 inches. The wheel rotates 87 times along the length of the path. About how long is the path?

38. ★ **EXTENDED RESPONSE** A motorized scooter has a chain drive. The chain goes around the front and rear sprockets.

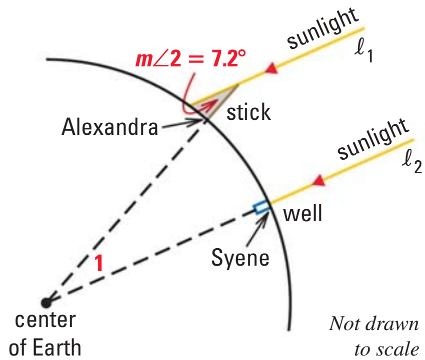


a. About how long is the chain? *Explain.*

b. Each sprocket has teeth that grip the chain. There are 76 teeth on the larger sprocket, and 15 teeth on the smaller sprocket. About how many teeth are gripping the chain at any given time? *Explain.*

39. **SCIENCE** Over 2000 years ago, the Greek scholar Eratosthenes estimated Earth's circumference by assuming that the Sun's rays are parallel. He chose a day when the Sun shone straight down into a well in the city of Syene. At noon, he measured the angle the Sun's rays made with a vertical stick in the city of Alexandria. Eratosthenes assumed that the distance from Syene to Alexandria was equal to about 575 miles.

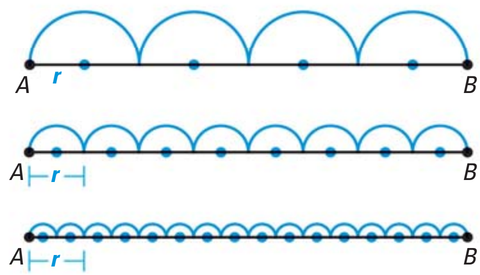
Find $m\angle 1$. Then estimate Earth's circumference.



CHALLENGE **Suppose $\overline{AB}$ is divided into four congruent segments, and semicircles with radius *r* are drawn.**

40. What is the sum of the four arc lengths if the radius of each arc is r?

41. Suppose that $\overline{AB}$ is divided into n congruent segments and that semicircles are drawn, as shown. What will the sum of the arc lengths be for 8 segments? for 16 segments? for n segments? *Explain* your thinking.



MIXED REVIEW

PREVIEW
Prepare for Lesson 11.5 in Exs. 42–45.

Find the area of a circle with radius *r*. Round to the nearest hundredth. *(p. 49)*

42. $r = 6$ cm

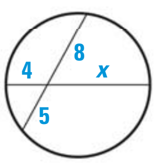
43. $r = 4.2$ in.

44. $r = 8\frac{3}{4}$ mi

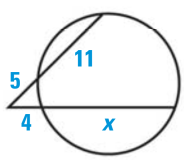
45. $r = 1\frac{3}{8}$ in.

Find the value of *x*. *(p. 689)*

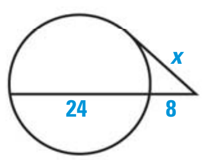
46.



47.



48.



Extension
Use after Lesson 11.4

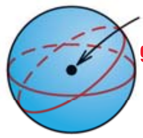
Geometry on a Sphere

GOAL Compare Euclidean and spherical geometries.

Key Vocabulary
- great circle

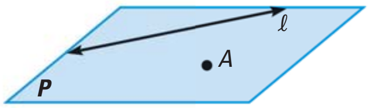
In Euclidean geometry, a plane is a flat surface that extends without end in all directions, and a line in the plane is a set of points that extends without end in two directions. Geometry on a sphere is different.

In *spherical geometry*, a plane is the surface of a sphere. A line is defined as a **great circle**, which is a circle on the sphere whose center is the center of the sphere.



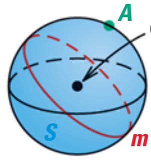
KEY CONCEPT — *For Your Notebook*

Euclidean Geometry



Plane P contains line ℓ and point A not on the line ℓ.

Spherical Geometry



Sphere S contains great circle m and point A not on m. Great circle m is a line.

HISTORY NOTE
Spherical geometry is sometimes called *Riemann geometry* after Bernhard Riemann, who wrote the first description of it in 1854.

Some properties and postulates in Euclidean geometry are true in spherical geometry. Others are not, or are true only under certain circumstances. For example, in Euclidean geometry, Postulate 5 states that through any two points there exists exactly one line. On a sphere, this postulate is true only for points that are not the endpoints of a diameter of the sphere.

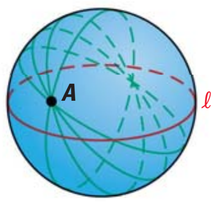
EXAMPLE 1 Compare Euclidean and spherical geometry

Tell whether the following postulate in Euclidean geometry is also true in spherical geometry. Draw a diagram to support your answer.

Parallel Postulate: If there is a line ℓ and a point A not on the line, then there is exactly one line through the point A parallel to the given line ℓ.

Solution

Parallel lines do not intersect. The sphere shows a line ℓ (a great circle) and a point A not on ℓ. Several lines are drawn through A. Each great circle containing A intersects ℓ. So, there can be no line parallel to ℓ. The parallel postulate is not true in spherical geometry.

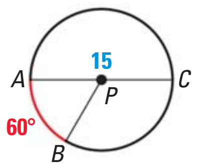


DISTANCES In Euclidean geometry, there is exactly one distance that can be measured between any two points. On a sphere, there are two distances that can be measured between two points. These distances are the lengths of the major and minor arcs of the great circle drawn through the points.

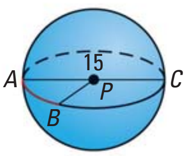
EXAMPLE 2 Find distances on a sphere

READ DIAGRAMS

The diagram below is a cross section of the sphere in Example 2. It shows $\overset{\frown}{AB}$ and $\overset{\frown}{ACB}$ on a great circle.



The diameter of the sphere shown is 15, and $m\overset{\frown}{AB} = 60°$. Find the distances between A and B.



Solution

Find the lengths of the minor arc $\overset{\frown}{AB}$ and the major arc $\overset{\frown}{ACB}$ of the great circle shown. In each case, let x be the arc length.

$$\frac{\text{Arc length of } \overset{\frown}{AB}}{2\pi r} = \frac{m\overset{\frown}{AB}}{360°} \qquad \frac{\text{Arc length of } \overset{\frown}{ACB}}{2\pi r} = \frac{m\overset{\frown}{ACB}}{360°}$$

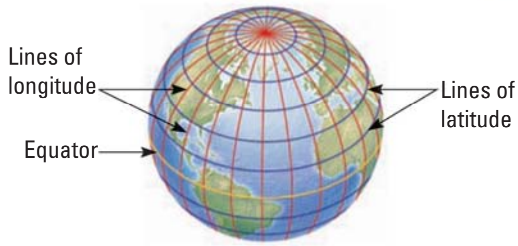
$$\frac{x}{15\pi} = \frac{60°}{360°} \qquad \frac{x}{15\pi} = \frac{360° - 60°}{360°}$$

$$x = 2.5\pi \qquad x = 12.5\pi$$

▶ The distances are 2.5π and 12.5π.

PRACTICE

1. **WRITING** Lines of latitude and longitude are used to identify positions on Earth. Which of the lines shown in the figure are great circles. Which are not? *Explain* your reasoning.



EXAMPLE 1
on p. 753 for Exs. 2–3

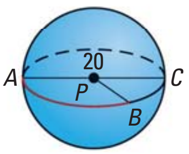
2. **COMPARING GEOMETRIES** Draw sketches to show that there is more than one line through the endpoints of a diameter of a sphere, but only one line through two points that are *not* endpoints of a diameter.

3. **COMPARING GEOMETRIES** The following statement is true in Euclidean geometry: If two lines intersect, then their intersection is exactly one point. Rewrite this statement to be true for lines on a sphere. *Explain*.

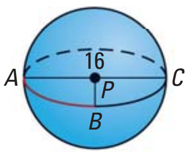
EXAMPLE 2
on p. 754 for Exs. 4–6

FINDING DISTANCES Use the diagram and the given arc measure to find the distances between points A and B. Leave your answers in terms of π.

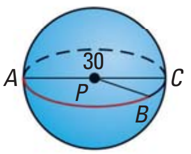
4. $m\overset{\frown}{AB} = 120°$



5. $m\overset{\frown}{AB} = 90°$



6. $m\overset{\frown}{AB} = 140°$



11.5 Areas of Circles and Sectors

Before You found circumferences of circles.

Now You will find the areas of circles and sectors.

Why So you can estimate walking distances, as in Ex. 38.

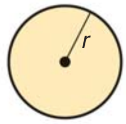
Key Vocabulary
- **sector of a circle**

In Chapter 1, you used the formula for the area of a circle. This formula is presented below as Theorem 11.9.

THEOREM *For Your Notebook*

THEOREM 11.9 Area of a Circle

The area of a circle is π times the square of the radius.



$A = \pi r^2$

Justification: Ex. 43, p. 761; Ex. 3, p. 769

EXAMPLE 1 Use the formula for area of a circle

Find the indicated measure.

a. Area

$r = 2.5$ cm

b. Diameter

$A = 113.1\text{ cm}^2$

Solution

a. $A = \pi r^2$ — Write formula for the area of a circle.

$= \pi \cdot (2.5)^2$ — Substitute 2.5 for r.

$= 6.25\pi$ — Simplify.

≈ 19.63 — Use a calculator.

▶ The area of $\odot A$ is about 19.63 square centimeters.

b. $A = \pi r^2$ — Write formula for the area of a circle.

$113.1 = \pi r^2$ — Substitute 113.1 for A.

$\frac{113.1}{\pi} = r^2$ — Divide each side by π.

$6 \approx r$ — Find the positive square root of each side.

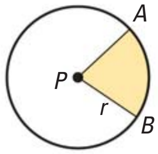
▶ The radius is about 6 inches, so the diameter is about 12 centimeters.

SECTORS A **sector of a circle** is the region bounded by two radii of the circle and their intercepted arc. In the diagram below, sector APB is bounded by $\overline{AP}$, $\overline{BP}$, and $\overset{\frown}{AB}$. Theorem 11.10 gives a method for finding the area of a sector.

THEOREM — *For Your Notebook*

THEOREM 11.10 Area of a Sector

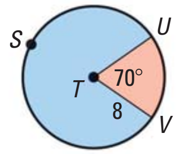
The ratio of the area of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc to 360°.



$$\frac{\text{Area of sector } APB}{\pi r^2} = \frac{m\overset{\frown}{AB}}{360^\circ}, \text{ or Area of sector } APB = \frac{m\overset{\frown}{AB}}{360^\circ} \cdot \pi r^2$$

EXAMPLE 2 Find areas of sectors

Find the areas of the sectors formed by $\angle UTV$.



Solution

STEP 1 **Find** the measures of the minor and major arcs.

Because $m\angle UTV = 70^\circ$, $m\overset{\frown}{UV} = 70^\circ$ and $m\overset{\frown}{USV} = 360^\circ - 70^\circ = 290^\circ$.

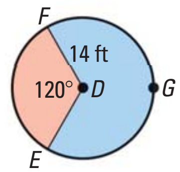
STEP 2 **Find** the areas of the small and large sectors.

Area of small sector $= \frac{m\overset{\frown}{UV}}{360^\circ} \cdot \pi r^2$	Write formula for area of a sector.
$= \frac{70^\circ}{360^\circ} \cdot \pi \cdot 8^2$	Substitute.
≈ 39.10	Use a calculator.
Area of large sector $= \frac{m\overset{\frown}{USV}}{360^\circ} \cdot \pi r^2$	Write formula for area of a sector.
$= \frac{290^\circ}{360^\circ} \cdot \pi \cdot 8^2$	Substitute.
≈ 161.97	Use a calculator.

▶ The areas of the small and large sectors are about 39.10 square units and 161.97 square units, respectively.

✓ GUIDED PRACTICE for Examples 1 and 2

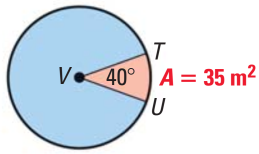
Use the diagram to find the indicated measure.



1. Area of $\odot D$
2. Area of red sector
3. Area of blue sector

EXAMPLE 3 Use the Area of a Sector Theorem

Use the diagram to find the area of $\odot V$.



Solution

Area of sector $TVU = \frac{m\overparen{TU}}{360^\circ} \cdot \text{Area of } \odot V$	**Write formula for area of a sector.**
$35 = \frac{40^\circ}{360^\circ} \cdot \text{Area of } \odot V$	**Substitute.**
$315 = \text{Area of } \odot V$	**Solve for Area of $\odot V$.**

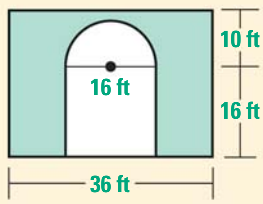
▶ The area of $\odot V$ is 315 square meters.

EXAMPLE 4 Standardized Test Practice

A rectangular wall has an entrance cut into it. You want to paint the wall. To the nearest square foot, what is the area of the region you need to paint?

Ⓐ 357 ft² Ⓑ 479 ft²

Ⓒ 579 ft² Ⓓ 936 ft²



Solution

AVOID ERRORS
Use the radius (8 ft), not the diameter (16 ft) when you calculate the area of the semicircle.

The area you need to paint is the area of the rectangle minus the area of the entrance. The entrance can be divided into a **semicircle** and a **square**.

$$\text{Area of wall} = \text{Area of rectangle} - (\text{Area of semicircle} + \text{Area of square})$$

$$= 36(26) - \left[\frac{180^\circ}{360^\circ} \cdot (\pi \cdot 8^2) + 16^2\right]$$

$$= 936 - [32\pi + 256]$$

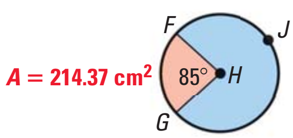
$$\approx 579.47$$

The area is about 579 square feet.

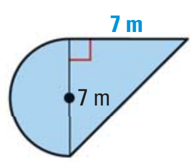
▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 3 and 4

4. Find the area of $\odot H$.



5. Find the area of the figure.



6. If you know the area and radius of a sector of a circle, can you find the measure of the intercepted arc? *Explain.*

11.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 17, and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 19, 40, and 42

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A _?_ of a circle is the region bounded by two radii of the circle and their intercepted arc.

2. ★ **WRITING** Suppose you double the arc measure of a sector in a given circle. Will the area of the sector also be doubled? *Explain.*

EXAMPLE 1 on p. 755 for Exs. 3–9

FINDING AREA **Find the exact area of a circle with the given radius *r* or diameter *d*. Then find the area to the nearest hundredth.**

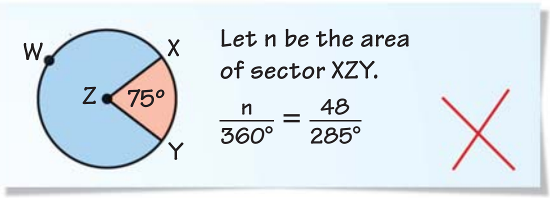
3. $r = 5$ in.
4. $d = 16$ ft
5. $d = 23$ cm
6. $r = 1.5$ km

USING AREA **In Exercises 7–9, find the indicated measure.**

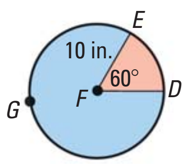
7. The area of a circle is 154 square meters. Find the radius.
8. The area of a circle is 380 square inches. Find the radius.
9. The area of a circle is 676π square centimeters. Find the diameter.

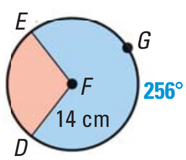
EXAMPLE 2 on p. 756 for Exs. 10–13

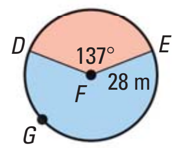
10. **ERROR ANALYSIS** In the diagram at the right, the area of $\odot Z$ is 48 square feet. A student writes a proportion to find the area of sector *XZY*. *Describe* and correct the error in writing the proportion. Then find the area of sector *XZY*.



FINDING AREA OF SECTORS **Find the areas of the sectors formed by $\angle DFE$.**

11. 

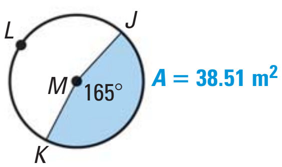
12. 

13. 

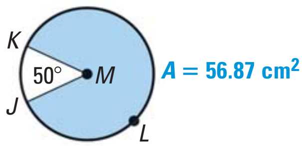
EXAMPLE 3 on p. 757 for Exs. 14–16

USING AREA OF A SECTOR **Use the diagram to find the indicated measure.**

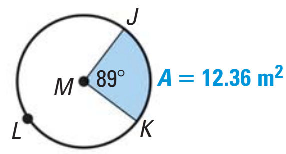
14. Find the area of $\odot M$.



15. Find the area of $\odot M$.

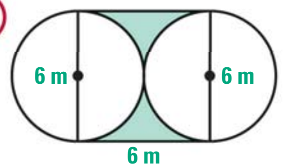


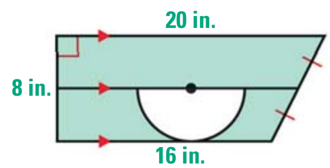
16. Find the radius of $\odot M$.



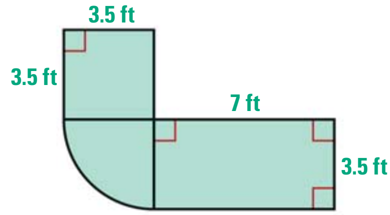
EXAMPLE 4 on p. 757 for Exs. 17–19

FINDING AREA **Find the area of the shaded region.**

17. 

18. 

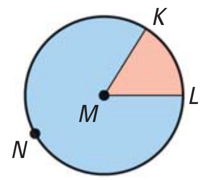
19. ★ **MULTIPLE CHOICE** The diagram shows the shape of a putting green at a miniature golf course. One part of the green is a sector of a circle. To the nearest square foot, what is the area of the putting green?



Ⓐ 46 ft^2 Ⓑ 49 ft^2

Ⓒ 56 ft^2 Ⓓ 75 ft^2

FINDING MEASURES **The area of $\odot M$ is 260.67 square inches. The area of sector KML is 42 square inches. Find the indicated measure.**



20. Radius of $\odot M$

21. Circumference of $\odot M$

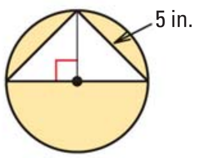
22. $m\overparen{KL}$

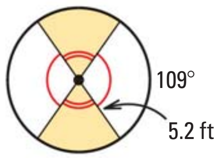
23. Perimeter of blue region

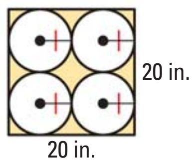
24. Length of $\overparen{KL}$

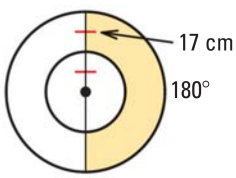
25. Perimeter of red region

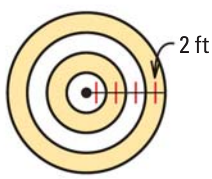
FINDING AREA **Find the area of the shaded region.**

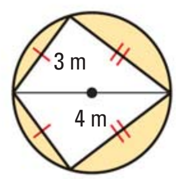
26. 

27. 

28. 

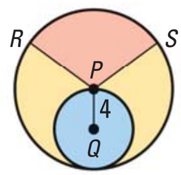
29. 

30. 

31. 

Animated Geometry at classzone.com

32. **TANGENT CIRCLES** In the diagram at the right, $\odot Q$ and $\odot P$ are tangent, and P lies on $\odot Q$. The measure of $\overparen{RS}$ is 108°. Find the area of the red region, the area of the blue region, and the area of the yellow region. Leave your answers in terms of π.

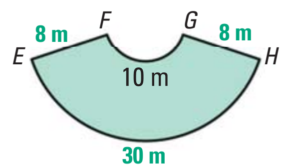


33. **SIMILARITY** Look back at the Perimeters of Similar Polygons Theorem on page 374 and the Areas of Similar Polygons Theorem on page 737. How would you rewrite these theorems to apply to circles? *Explain*.

34. **ERROR ANALYSIS** The ratio of the lengths of two arcs in a circle is 2 : 1. A student claims that the ratio of the areas of the sectors bounded by these arcs is 4 : 1, because $\left(\frac{2}{1}\right)^2 = \frac{4}{1}$. *Describe* and correct the error.

35. **DRAWING A DIAGRAM** A square is inscribed in a circle. The same square is also circumscribed about a smaller circle. Draw a diagram. Find the ratio of the area of the large circle to the area of the small circle.

36. **CHALLENGE** In the diagram at the right, $\overparen{FG}$ and $\overparen{EH}$ are arcs of concentric circles, and $\overline{EF}$ and $\overline{GH}$ lie on radii of the larger circle. Find the area of the shaded region.



PROBLEM SOLVING

EXAMPLE 1
on p. 755
for Ex. 37

37. METEOROLOGY The *eye of a hurricane* is a relatively calm circular region in the center of the storm. The diameter of the eye is typically about 20 miles. If the eye of a hurricane is 20 miles in diameter, what is the area of the land that is underneath the eye?

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38. WALKING The area of a circular pond is about 138,656 square feet. You are going to walk around the entire edge of the pond. About how far will you walk? Give your answer to the nearest foot.

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39. CIRCLE GRAPH The table shows how students get to school.

Method	% of Students
Bus	65%
Walk	25%
Other	10%

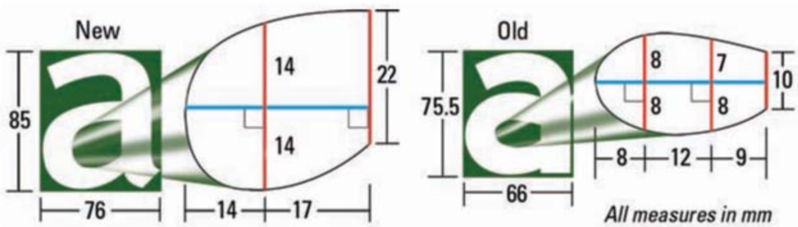
a. *Explain* why a circle graph is appropriate for the data.

b. You will represent each method by a sector of a circle graph. Find the central angle to use for each sector. Then use a protractor and a compass to construct the graph. Use a radius of 2 inches.

c. Find the area of each sector in your graph.

40. ★ SHORT RESPONSE It takes about $\frac{1}{4}$ cup of dough to make a tortilla with a 6 inch diameter. How much dough does it take to make a tortilla with a 12 inch diameter? *Explain* your reasoning.

41. HIGHWAY SIGNS A new typeface has been designed to make highway signs more readable. One change was to redesign the form of the letters to increase the space inside letters.



a. Estimate the interior area for the old and the new "a." Then find the percent increase in interior area.

b. Do you think the change in interior area is just a result of a change in height and width of the letter *a*? *Explain*.

42. ★ EXTENDED RESPONSE A circular pizza with a 12 inch diameter is enough for you and 2 friends. You want to buy pizza for yourself and 7 friends. A 10 inch diameter pizza with one topping costs $6.99 and a 14 inch diameter pizza with one topping costs $12.99. How many 10 inch and 14 inch pizzas should you buy in each situation below? *Explain*.

a. You want to spend as little money as possible.

b. You want to have three pizzas, each with a different topping.

c. You want to have as much of the thick outer crust as possible.

○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

43. **JUSTIFYING THEOREM 11.9** You can follow the steps below to justify the formula for the area of a circle with radius r.

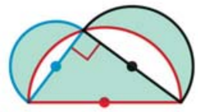
Divide a circle into 16 congruent sectors. Cut out the sectors.

Rearrange the 16 sectors to form a shape resembling a parallelogram.

a. Write expressions in terms of r for the approximate height and base of the parallelogram. Then write an expression for its area.

b. *Explain* how your answers to part (a) justify Theorem 11.9.

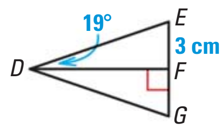
44. **CHALLENGE** Semicircles with diameters equal to the three sides of a right triangle are drawn, as shown. Prove that the sum of the area of the two shaded crescents equals the area of the triangle.



MIXED REVIEW

PREVIEW
Prepare for Lesson 11.6 In Exs. 45–47.

Triangle DEG is isosceles with altitude $\overline{DF}$. Find the given measurement. *Explain* your reasoning. *(p. 319)*



45. $m\angle DFG$ 46. $m\angle FDG$ 47. FG

Sketch the indicated figure. Draw all of its lines of symmetry. *(p. 619)*

48. Isosceles trapezoid 49. Regular hexagon

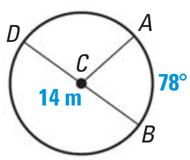
Graph $\triangle ABC$. Then find its area. *(p. 720)*

50. $A(2, 2)$, $B(9, 2)$, $C(4, 16)$ 51. $A(-8, 3)$, $B(-3, 3)$, $C(-1, -10)$

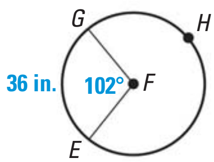
QUIZ for Lessons 11.4–11.5

Find the indicated measure. *(p. 746)*

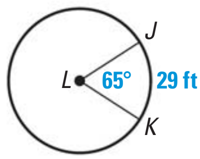
1. Length of $\widehat{AB}$



2. Circumference of $\odot F$

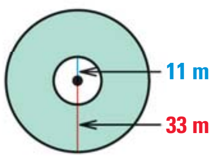


3. Radius of $\odot L$

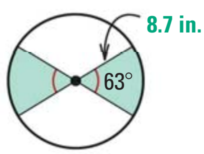


Find the area of the shaded region. *(p. 755)*

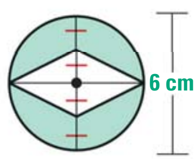
4.



5.



6.



11.6 Areas of Regular Polygons

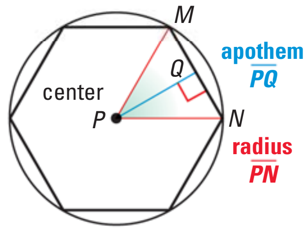
Before You found areas of circles.

Now You will find areas of regular polygons inscribed in circles.

Why? So you can understand the structure of a honeycomb, as in Ex. 44.

Key Vocabulary
- center of a polygon
- radius of a polygon
- apothem of a polygon
- central angle of a regular polygon

The diagram shows a regular polygon inscribed in a circle. The **center of the polygon** and the **radius of the polygon** are the center and the radius of its circumscribed circle.



$\angle MPN$ is a central angle.

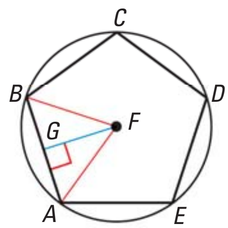
The distance from the center to any side of the polygon is called the **apothem of the polygon**. The apothem is the height to the base of an isosceles triangle that has two radii as legs.

A **central angle of a regular polygon** is an angle formed by two radii drawn to consecutive vertices of the polygon. To find the measure of each central angle, divide 360° by the number of sides.

EXAMPLE 1 Find angle measures in a regular polygon

In the diagram, *ABCDE* is a regular pentagon inscribed in $\odot F$. Find each angle measure.

a. $m\angle AFB$ **b.** $m\angle AFG$ **c.** $m\angle GAF$



Solution

a. $\angle AFB$ is a central angle, so $m\angle AFB = \frac{360°}{5}$, or 72°.

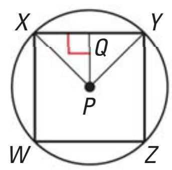
b. $\overline{FG}$ is an apothem, which makes it an altitude of isosceles $\triangle AFB$.
So, $\overline{FG}$ bisects $\angle AFB$ and $m\angle AFG = \frac{1}{2} m\angle AFB = 36°$.

c. The sum of the measures of right $\triangle GAF$ is 180°.
So, $90° + 36° + m\angle GAF = 180°$, and $m\angle GAF = 54°$.

READ DIAGRAMS
A segment whose length is *the apothem* is sometimes called *an apothem*. The segment is an altitude of an isosceles triangle, so it is also a median and angle bisector of the isosceles triangle.

✓ GUIDED PRACTICE for Example 1

In the diagram, *WXYZ* is a square inscribed in $\odot P$.



1. Identify the center, a radius, an apothem, and a central angle of the polygon.
2. Find $m\angle XPY$, $m\angle XPQ$, and $m\angle PXQ$.

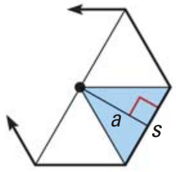
AREA OF AN *n*-GON You can find the area of any regular n-gon by dividing it into congruent triangles.

$A =$ Area of one triangle • Number of triangles

$= \left(\frac{1}{2} \cdot s \cdot a\right) \cdot n$ — **Base of triangle is *s* and height of triangle is *a*. Number of triangles is *n*.**

$= \frac{1}{2} \cdot a \cdot (n \cdot s)$ — **Commutative and Associative Properties of Equality**

$= \frac{1}{2}a \cdot P$ — **There are *n* congruent sides of length *s*, so perimeter *P* is *n* • *s*.**

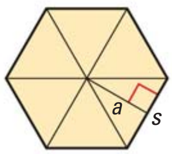


READ DIAGRAMS
In this book, a point shown inside a regular polygon marks the center of the circle that can be circumscribed about the polygon.

THEOREM — *For Your Notebook*

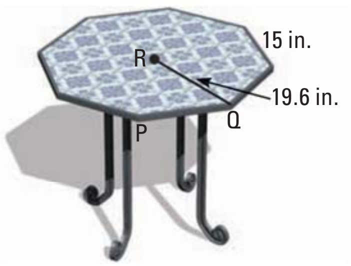
THEOREM 11.11 Area of a Regular Polygon

The area of a regular n-gon with side length s is half the product of the apothem a and the perimeter P, so $A = \frac{1}{2}aP$, or $A = \frac{1}{2}a \cdot ns$.



EXAMPLE 2 Find the area of a regular polygon

DECORATING You are decorating the top of a table by covering it with small ceramic tiles. The table top is a regular octagon with 15 inch sides and a radius of about 19.6 inches. What is the area you are covering?



Solution

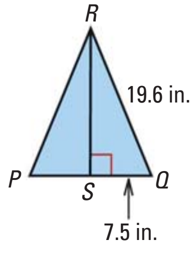
STEP 1 **Find** the perimeter P of the table top.
An octagon has 8 sides, so $P = 8(15) = 120$ inches.

STEP 2 **Find** the apothem a. The apothem is height RS of $\triangle PQR$.
Because $\triangle PQR$ is isosceles, altitude $\overline{RS}$ bisects $\overline{QP}$.

So, $QS = \frac{1}{2}(QP) = \frac{1}{2}(15) = 7.5$ inches.

To find RS, use the Pythagorean Theorem for $\triangle RQS$.

$a = RS \approx \sqrt{19.6^2 - 7.5^2} = \sqrt{327.91} \approx 18.108$



STEP 3 **Find** the area A of the table top.

$A = \frac{1}{2}aP$ — **Formula for area of regular polygon**

$\approx \frac{1}{2}(18.108)(120)$ — **Substitute.**

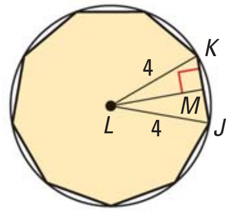
≈ 1086.5 — **Simplify.**

▸ So, the area you are covering with tiles is about 1086.5 square inches.

ROUNDING
In general, your answer will be more accurate if you avoid rounding until the last step. Round your final answers to the nearest tenth unless you are told otherwise.

EXAMPLE 3 Find the perimeter and area of a regular polygon

A regular nonagon is inscribed in a circle with radius 4 units. Find the perimeter and area of the nonagon.



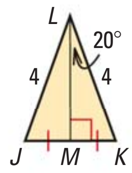
Solution

The measure of central $\angle JLK$ is $\frac{360°}{9}$, or 40°. Apothem $\overline{LM}$ bisects the central angle, so $m\angle KLM$ is 20°. To find the lengths of the legs, use trigonometric ratios for right $\triangle KLM$.

$$\sin 20° = \frac{MK}{LK} \qquad \cos 20° = \frac{LM}{LK}$$

$$\sin 20° = \frac{MK}{4} \qquad \cos 20° = \frac{LM}{4}$$

$$4 \cdot \sin 20° = MK \qquad 4 \cdot \cos 20° = LM$$



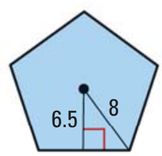
The regular nonagon has side length $s = 2MK = 2(4 \cdot \sin 20°) = 8 \cdot \sin 20°$ and apothem $a = LM = 4 \cdot \cos 20°$.

▶ So, the perimeter is $P = 9s = 9(8 \cdot \sin 20°) = 72 \cdot \sin 20° \approx 24.6$ units, and the area is $A = \frac{1}{2}aP = \frac{1}{2}(4 \cdot \cos 20°)(72 \cdot \sin 20°) \approx 46.3$ square units.

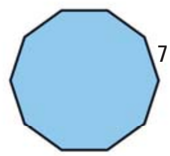
✓ GUIDED PRACTICE for Examples 2 and 3

Find the perimeter and the area of the regular polygon.

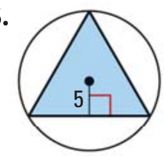
3.



4.



5.



6. Which of Exercises 3–5 above can be solved using special right triangles?

CONCEPT SUMMARY *For Your Notebook*

Finding Lengths in a Regular *n*-gon

To find the area of a regular *n*-gon with radius *r*, you may need to first find the apothem ***a*** or the side length ***s***.

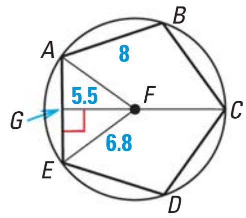
You can use . . .	. . . when you know *n* and . . .	. . . as in . . .
Pythagorean Theorem: $\left(\frac{1}{2}s\right)^2 + a^2 = r^2$	Two measures: *r* and ***a***, or *r* and ***s***	Example 2 and Guided Practice Ex. 3.
Special Right Triangles	Any one measure: *r* or ***a*** or ***s*** **And** the value of *n* is 3, 4, or 6	Guided Practice Ex. 5.
Trigonometry	Any one measure: *r* or ***a*** or ***s***	Example 3 and Guided Practice Exs. 4 and 5.

11.6 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 21, and 37
★ = **STANDARDIZED TEST PRACTICE** Exs. 5, 18, 22, and 44

SKILL PRACTICE

VOCABULARY **In Exercises 1–4, use the diagram shown.**



1. Identify the *center* of regular polygon *ABCDE*.
2. Identify a *central angle* of the polygon.
3. What is the *radius* of the polygon?
4. What is the *apothem*?

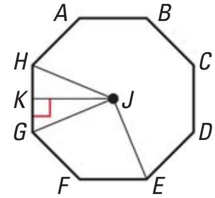
5. ★ **WRITING** *Explain* how to find the measure of a *central angle* of a regular polygon with n sides.

EXAMPLE 1 on p. 762 for Exs. 6–13

MEASURES OF CENTRAL ANGLES **Find the measure of a central angle of a regular polygon with the given number of sides. Round answers to the nearest tenth of a degree, if necessary.**

6. 10 sides
7. 18 sides
8. 24 sides
9. 7 sides

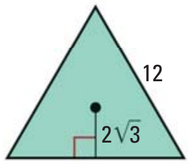
FINDING ANGLE MEASURES **Find the given angle measure for the regular octagon shown.**

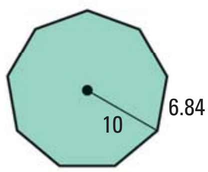


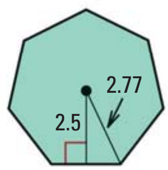
10. $m\angle GJH$
11. $m\angle GJK$
12. $m\angle KGJ$
13. $m\angle EJH$

EXAMPLE 2 on p. 763 for Exs. 14–17

FINDING AREA **Find the area of the regular polygon.**

14.


15.


16.


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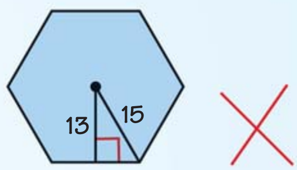
17. **ERROR ANALYSIS** *Describe* and correct the error in finding the area of the regular hexagon.

$$\sqrt{15^2 - 13^2} \approx 7.5$$

$$A = \frac{1}{2}a \cdot ns$$

$$A = \frac{1}{2}(13)(6)(7.5) = 292.5$$

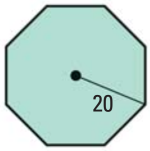


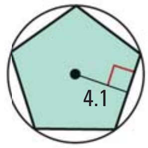
EXAMPLE 3 on p. 764 for Exs. 18–25

18. ★ **MULTIPLE CHOICE** Which expression gives the apothem for a regular dodecagon with side length 8?

Ⓐ $a = \dfrac{4}{\tan 30°}$ Ⓑ $a = \dfrac{4}{\tan 15°}$ Ⓒ $a = \dfrac{8}{\tan 15°}$ Ⓓ $a = 8 \cdot \cos 15°$

PERIMETER AND AREA **Find the perimeter and area of the regular polygon.**

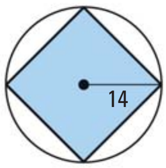
19.


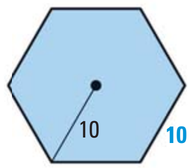
20.


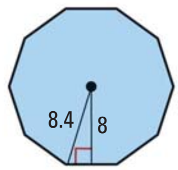
21.


22. ★ **SHORT RESPONSE** The perimeter of a regular nonagon is 18 inches. Is that enough information to find the area? If so, find the area and *explain* your steps. If not, *explain* why not.

CHOOSE A METHOD **Identify any unknown length(s) you need to know to find the area of the regular polygon. Which methods in the table on page 764 can you use to find those lengths? Choose a method and find the area.**

23.


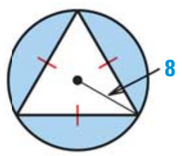
24.


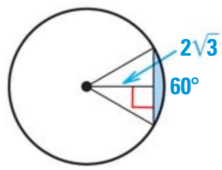
25.


26. **INSCRIBED SQUARE** Find the area of the *unshaded* region in Exercise 23.

POLYGONS IN CIRCLES **Find the area of the shaded region.**

27.


28.


29.


30. **COORDINATE GEOMETRY** Find the area of a regular pentagon inscribed in a circle whose equation is given by $(x - 4)^2 + (y + 2)^2 = 25$.

REASONING **Decide whether the statement is *true* or *false*. *Explain*.**

31. The area of a regular n-gon of fixed radius r increases as n increases.

32. The apothem of a regular polygon is always less than the radius.

33. The radius of a regular polygon is always less than the side length.

34. **FORMULAS** In Exercise 44 on page 726, the formula $A = \frac{\sqrt{3}s^2}{4}$ for the area A of an equilateral triangle with side length s was developed. Show that the formulas for the area of a triangle and for the area of a regular polygon, $A = \frac{1}{2}bh$ and $A = \frac{1}{2}a \cdot ns$, also result in this formula when they are applied to an equilateral triangle with side length s.

35. **CHALLENGE** An equilateral triangle is shown inside a square inside a regular pentagon inside a regular hexagon. Write an expression for the exact area of the shaded regions in the figure. Then find the approximate area of the entire shaded region, rounded to the nearest whole unit.

◯ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

PROBLEM SOLVING

EXAMPLE 3
on p. 764
for Ex. 36

36. BASALTIC COLUMNS Basaltic columns are geological formations that result from rapidly cooling lava. The Giant's Causeway in Ireland, pictured here, contains many hexagonal columns. Suppose that one of the columns is in the shape of a regular hexagon with radius 8 inches.

a. What is the apothem of the column?

b. Find the perimeter and area of the column. Round the area to the nearest square inch.

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37. WATCH A watch has a circular face on a background that is a regular octagon. Find the apothem and the area of the octagon. Then find the area of the silver border around the circular face.

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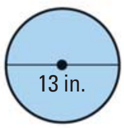
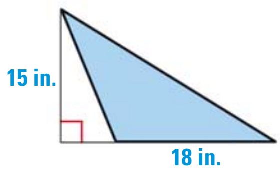
38. COMPARING AREAS *Predict* which figure has the greatest area and which has the smallest area. Check by finding the area of each figure.

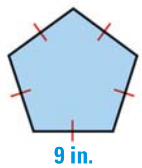
a.



b.



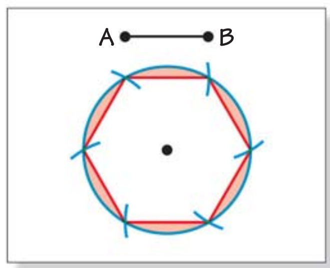
c.



39. CRAFTS You want to make two wooden trivets, a large one and a small one. Both trivets will be shaped like regular pentagons. The perimeter of the small trivet is 15 inches, and the perimeter of the large trivet is 25 inches. Find the area of the small trivet. Then use the Areas of Similar Polygons Theorem to find the area of the large trivet. Round your answers to the nearest tenth.

40. CONSTRUCTION Use a ruler and compass.

a. Draw $\overline{AB}$ with a length of 1 inch. Open the compass to 1 inch and draw a circle with that radius. Using the same compass setting, mark off equal parts along the circle. Then connect the six points where the compass marks and circle intersect to draw a regular hexagon as shown.



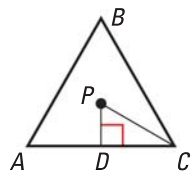
b. What is the area of the hexagon? of the shaded region?

c. *Explain* how to construct an equilateral triangle.

41. HEXAGONS AND TRIANGLES Show that a regular hexagon can be divided into six equilateral triangles with the same side length.

42. ALTERNATIVE METHODS Find the area of a regular hexagon with side length 2 and apothem $\sqrt{3}$ in at least four different ways.

43. **APPLYING TRIANGLE PROPERTIES** In Chapter 5, you learned properties of special segments in triangles. Use what you know about special segments in triangles to show that radius CP in equilateral $\triangle ABC$ is twice the apothem DP.



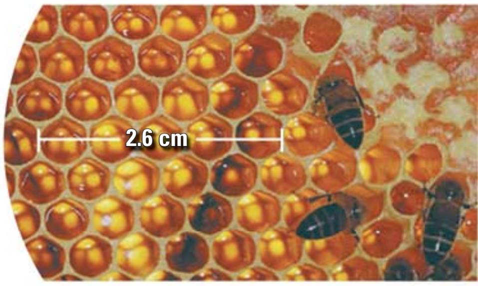
44. ★ **EXTENDED RESPONSE** Assume that each honeycomb cell is a regular hexagon. The distance is measured through the center of each cell.

a. Find the average distance across a cell in centimeters.

b. Find the area of a "typical" cell in square centimeters. Show your steps.

c. What is the area of 100 cells in square centimeters? in square decimeters? (1 decimeter = 10 centimeters.)

d. Scientists are often interested in the number of cells per square decimeter. *Explain* how to rewrite your results in this form.



45. **CONSTANT PERIMETER** Use a piece of string that is 60 centimeters long.

a. Arrange the string to form an equilateral triangle and find the area. Next form a square and find the area. Then do the same for a regular pentagon, a regular hexagon, and a regular decagon. What is happening to the area?

b. Predict and then find the areas of a regular 60-gon and a regular 120-gon.

c. Graph the area A as a function of the number of sides n. The graph approaches a limiting value. What shape do you think will have the greatest area? What will that area be?

46. **CHALLENGE** Two regular polygons both have n sides. One of the polygons is inscribed in, and the other is circumscribed about, a circle of radius r. Find the area between the two polygons in terms of n and r.

MIXED REVIEW

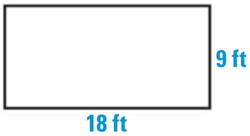
PREVIEW
Prepare for Lesson 11.7 in Exs. 47–51.

A jar contains 10 red marbles, 6 blue marbles, and 2 white marbles. Find the probability of the event described. *(p. 893)*

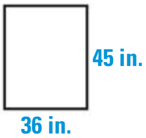
47. You randomly choose one red marble from the jar, put it back in the jar, and then randomly choose a red marble.

48. You randomly choose one blue marble from the jar, keep it, and then randomly choose one white marble.

Find the ratio of the width to the length of the rectangle. Then simplify the ratio. *(p. 356)*

49.


50.


51.


52. The vertices of quadrilateral $ABCD$ are $A(-3, 3)$, $B(1, 1)$, $C(1, -3)$, and $D(-3, -1)$. Draw $ABCD$ and determine whether it is a parallelogram. *(p. 522)*

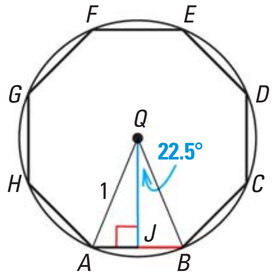
@HomeTutor classzone.com Keystrokes

11.6 Perimeter and Area of Polygons

MATERIALS • computer

QUESTION How can you use a spreadsheet to find perimeters and areas of regular *n*-gons?

First consider a regular octagon with radius 1.



Because there are 8 central angles, $m\angle JQB$ is $\frac{1}{2}\left(\frac{360°}{8}\right) = \frac{180°}{8}$, or 22.5°.

You can express the side length and apothem using trigonometric functions.

$\sin 22.5° = \frac{JB}{QB} = \frac{JB}{1} = JB$ $\cos 22.5° = \frac{QJ}{QB} = \frac{QJ}{1} = QJ$

So, **side length *s*** $= 2(JB) = 2 \cdot \sin 22.5°$ So, **apothem *a*** is $QJ = \cos 22.5°$

Perimeter *P* $= 8s = 8(2 \cdot \sin 22.5°) = 16 \cdot \sin 22.5°$

Area *A* $= \frac{1}{2}aP = \frac{1}{2}(\cos 22.5°)(16 \cdot \sin 22.5°) = 8(\cos 22.5°)(\sin 22.5°)$

Using these steps for any regular *n*-gon inscribed in a circle of radius 1 gives

$$P = 2n \cdot \sin\left(\frac{180°}{n}\right) \quad \text{and} \quad A = n \cdot \sin\left(\frac{180°}{n}\right) \cdot \cos\left(\frac{180°}{n}\right).$$

EXAMPLE Use a spreadsheet to find measures of regular *n*-gons

STEP 1 ***Make a table*** Use a spreadsheet to make a table with three columns.

	A	B	C
1	Number of sides	Perimeter	Area
2	n	2*n*sin(180/n)	n*sin(180/n)*cos(180/n)
3	3	=2*A3*sin(180/A3)	=A3*sin(180/A3)*cos(180/A3)
4	=A3+1	=2*A4*sin(180/A4)	=A4*sin(180/A4)*cos(180/A4)

If your spreadsheet uses radian measure, use "pi()" instead of "180."

STEP 2 ***Enter formulas*** Enter the formulas shown in cells A4, B3, and C3. Then use the Fill Down feature to create more rows.

PRACTICE

1. What shape do the regular *n*-gons approach as the value of *n* gets very large? *Explain* your reasoning.
2. What value do the perimeters approach as the value of *n* gets very large? *Explain* how this result justifies the formula for the circumference of a circle.
3. What value do the areas approach as the value of *n* gets very large? *Explain* how this result justifies the formula for the area of a circle.

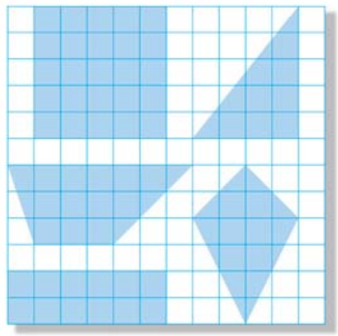
11.7 Investigate Geometric Probability

MATERIALS • graph paper • small dried bean

QUESTION How do theoretical and experimental probabilities compare?

EXPLORE Find geometric probabilities

STEP 1 ***Draw a target*** On a piece of graph paper, make a target by drawing some polygons. Choose polygons whose area you can calculate and make them as large as possible. Shade in the polygons. An example is shown.



Sample target

STEP 2 ***Calculate theoretical probability*** Calculate the *theoretical* probability that a randomly tossed bean that lands on the target will land in a shaded region.

$$\text{Theoretical probability} = \frac{\text{Sum of areas of polygons}}{\text{Area of paper}}$$

STEP 3 ***Perform an experiment*** Place the target on the floor against a wall. Toss a dried bean so that it hits the wall and then bounces onto the target. Determine whether the bean lands on a shaded or unshaded region of the target. If the bean lands so that it lies in both a shaded and unshaded region, use the region in which most of the bean lies. If the bean does not land completely on the target, repeat the toss.

STEP 4 ***Make a table*** Record the results of the toss in a table. Repeat until you have recorded the results of 50 tosses.

Toss	Shaded area	Unshaded area
1	X	
2		X
. . .	. . .	. . .
50	X	

STEP 5 ***Calculate experimental probability*** Use the results from your table to calculate the *experimental* probability that a randomly tossed bean that lands on the target will land in a shaded region.

$$\text{Experimental probability} = \frac{\text{Number of times a bean landed on a shaded region}}{\text{Total number of tosses}}$$

DRAW CONCLUSIONS Use your observations to complete these exercises

1. *Compare* the theoretical probability from Step 2 with the experimental probability from Step 5. What do you notice?
2. Repeat Steps 3–5, this time using only 10 tosses. Calculate the experimental probability for those 10 tosses. *Compare* the experimental probability and the theoretical probability.
3. **REASONING** How does the number of tosses affect the relationship between the experimental and theoretical probabilities? *Explain.*

11.7 Use Geometric Probability

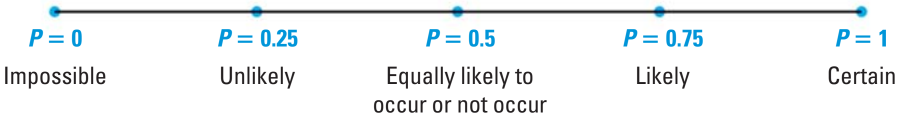
Before You found lengths and areas.

Now You will use lengths and areas to find geometric probabilities.

Why? So you can calculate real-world probabilities, as in Example 2.

Key Vocabulary
- **probability**
- **geometric probability**

The **probability** of an event is a measure of the likelihood that the event will occur. It is a number between 0 and 1, inclusive, and can be expressed as a fraction, decimal, or percent. The probability of event A is written as $P(A)$.



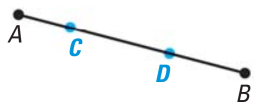
In a previous course, you may have found probability by calculating the ratio of the number of favorable outcomes to the total number of possible outcomes. In this lesson, you will find *geometric probabilities*.

A **geometric probability** is a ratio that involves a geometric measure such as length or area.

KEY CONCEPT — For Your Notebook

Probability and Length

Let $\overline{AB}$ be a segment that contains the segment $\overline{CD}$. If a point K on $\overline{AB}$ is chosen at random, then the probability that it is on $\overline{CD}$ is the ratio of the length of $\overline{CD}$ to the length of $\overline{AB}$.



$$P(K \text{ is on } \overline{CD}) = \frac{\text{Length of } \overline{CD}}{\text{Length of } \overline{AB}}$$

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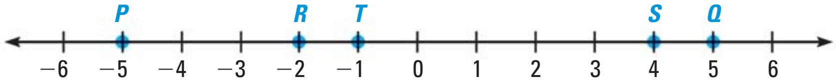
EXAMPLE 1 Use lengths to find a geometric probability

USE A FORMULA
To apply the geometric probability formulas on this page and on page 772, you need to know that every point on the segment or in the region is *equally likely* to be chosen.

Find the probability that a point chosen at random on $\overline{PQ}$ is on $\overline{RS}$.



Solution

$$P(\text{Point is on } \overline{RS}) = \frac{\text{Length of } \overline{RS}}{\text{Length of } \overline{PQ}} = \frac{|4-(-2)|}{|5-(-5)|} = \frac{6}{10} = \frac{3}{5}, \text{ 0.6, or 60\%.}$$

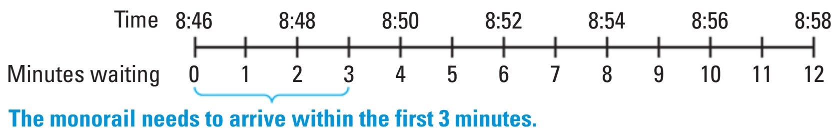
EXAMPLE 2 Use a segment to model a real-world probability

MONORAIL A monorail runs every 12 minutes. The ride from the station near your home to the station near your work takes 9 minutes. One morning, you arrive at the station near your home at 8:46. You want to get to the station near your work by 8:58. What is the probability you will get there by 8:58?

Solution

STEP 1 **Find** the longest you can wait for the monorail and still get to the station near your work by 8:58. The ride takes 9 minutes, so you need to catch the monorail no later than 9 minutes before 8:58, or by 8:49. The longest you can wait is 3 minutes (8:49 − 8:46 = 3 min).

STEP 2 **Model** the situation. The monorail runs every 12 minutes, so it will arrive in 12 minutes or less. You need it to arrive within 3 minutes.



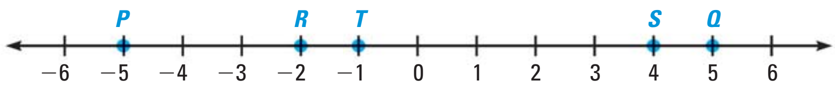
STEP 3 **Find** the probability.

$$P(\text{You get to the station by 8:58}) = \frac{\text{Favorable waiting time}}{\text{Maximum waiting time}} = \frac{3}{12} = \frac{1}{4}$$

▶ The probability that you will get to the station by 8:58. is $\frac{1}{4}$, or 25%.

✓ GUIDED PRACTICE for Examples 1 and 2

Find the probability that a point chosen at random on $\overline{PQ}$ is on the given segment. Express your answer as a fraction, a decimal, and a percent.



1. $\overline{RT}$ **2.** $\overline{TS}$ **3.** $\overline{PT}$ **4.** $\overline{RQ}$

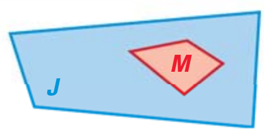
5. **WHAT IF?** In Example 2, suppose you arrive at the station near your home at 8:43. What is the probability that you will get to the station near your work by 8:58?

PROBABILITY AND AREA Another formula for geometric probability involves the ratio of the areas of two regions.

KEY CONCEPT *For Your Notebook*

Probability and Area

Let J be a region that contains region M. If a point K in J is chosen at random, then the probability that it is in region M is the ratio of the area of M to the area of J.



$$P(K \text{ is in region } M) = \frac{\text{Area of } M}{\text{Area of } J}$$

EXAMPLE 3 Use areas to find a geometric probability

ARCHERY The diameter of the target shown at the right is 80 centimeters. The diameter of the red circle on the target is 16 centimeters. An arrow is shot and hits the target. If the arrow is equally likely to land on any point on the target, what is the probability that it lands in the red circle?

Solution

ANOTHER WAY
All circles are similar and the Area of Similar Polygons Theorem also applies to circles. The ratio of radii is 8 : 40, or 1 : 5, so the ratio of areas is $1^2 : 5^2$, or 1 : 25.

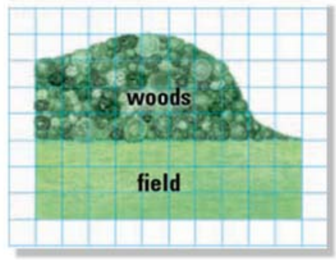
Find the ratio of the area of the red circle to the area of the target.

$$P(\text{arrow lands in red region}) = \frac{\text{Area of red circle}}{\text{Area of target}} = \frac{\pi(8^2)}{\pi(40^2)} = \frac{64\pi}{1600\pi} = \frac{1}{25}$$

▶ The probability that the arrow lands in the red region is $\frac{1}{25}$, or 4%.

EXAMPLE 4 Estimate area on a grid to find a probability

SCALE DRAWING Your dog dropped a ball in a park. A scale drawing of the park is shown. If the ball is equally likely to be anywhere in the park, estimate the probability that it is in the field.

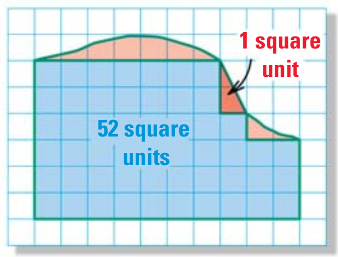


Solution

STEP 1 **Find** the area of the field. The shape is a rectangle, so the area is $bh = 10 \cdot 3 = 30$ square units.

STEP 2 **Find** the total area of the park.

Count the squares that are fully covered. There are 30 squares in the field and 22 in the woods. So, there are 52 full squares.



Make groups of partially covered squares so the combined area of each group is about 1 square unit. The total area of the partial squares is about 6 or 7 square units. So, use $52 + 6.5 = 58.5$ square units for the total area.

STEP 3 **Write** a ratio of the areas to find the probability.

$$P(\text{ball in field}) = \frac{\text{Area of field}}{\text{Total area of park}} \approx \frac{30}{58.5} = \frac{300}{585} = \frac{20}{39}$$

▶ The probability that the ball is in the field is about $\frac{20}{39}$, or 51.3%.

CHECK RESULTS
The ball must be either in the field or in the woods, so check that the probabilities in Example 4 and Guided Practice Exercise 7 add up to 100%.

✓ GUIDED PRACTICE for Examples 3 and 4

6. In the target in Example 3, each ring is 8 centimeters wide. Find the probability that an arrow lands in the black region.

7. In Example 4, estimate the probability that the ball is in the woods.

11.7 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 9, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 23, 34, and 35

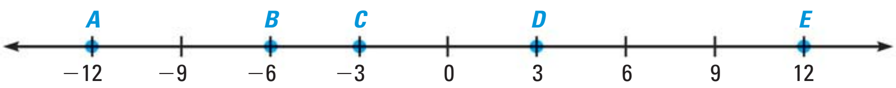
SKILL PRACTICE

1. VOCABULARY Copy and complete: If an event cannot occur, its probability is _?_. If an event is certain to occur, its probability is _?_.

2. ★ WRITING *Compare* a geometric probability and a probability found by dividing the number of favorable outcomes by the total number of possible outcomes.

EXAMPLE 1 on p. 771 for Exs. 3–7

PROBABILITY ON A SEGMENT **In Exercises 3–6, find the probability that a point K, selected randomly on $\overline{AE}$, is on the given segment. Express your answer as a fraction, decimal, and percent.**



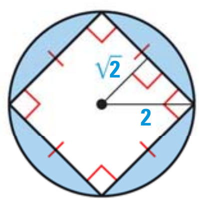
3. $\overline{AD}$ **4.** $\overline{BC}$ **5.** $\overline{DE}$ **6.** $\overline{AE}$

7. ★ WRITING Look at your answers to Exercises 3 and 5. *Describe* how the two probabilities are related.

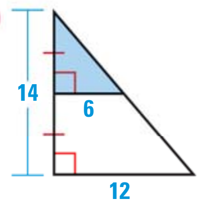
EXAMPLE 3 on p. 773 for Exs. 8–11

FIND A GEOMETRIC PROBABILITY **Find the probability that a randomly chosen point in the figure lies in the shaded region.**

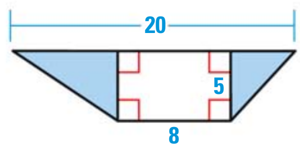
8.



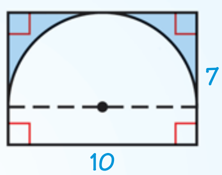
9.



10.



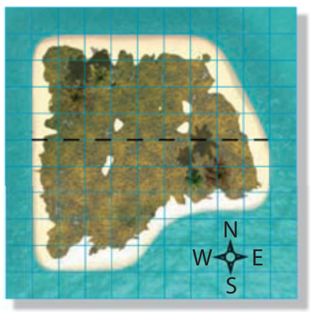
11. ERROR ANALYSIS Three sides of the rectangle are tangent to the semicircle. *Describe* and correct the error in finding the probability that a randomly chosen point in the figure lies in the shaded region.



$$\frac{10(7) - \frac{1}{2}\pi(5)^2}{10(7)} = \frac{70 - 12.5\pi}{70} \approx 43.9\%$$

EXAMPLE 4 on p. 773 for Exs. 12–14

ESTIMATING AREA **Use the scale drawing.**



12. What is the approximate area of the north side of the island? the south side of the island? the whole island?

13. Find the probability that a randomly chosen location on the island lies on the north side.

14. Find the probability that a randomly chosen location on the island lies on the south side.

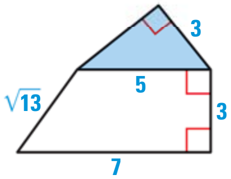
15. **SIMILAR TRIANGLES** In Exercise 9, how do you know that the shaded triangle is similar to the whole triangle? *Explain* how you can use the Area of Similar Polygons Theorem to find the desired probability.

xy ALGEBRA In Exercises 16–19, find the probability that a point chosen at random on the segment satisfies the inequality.

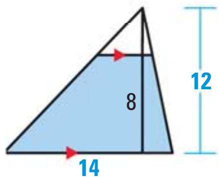
16. $x - 6 \leq 1$ **17.** $1 \leq 2x - 3 \leq 5$ **18.** $\frac{x}{2} \geq 7$ **19.** $3x \leq 27$

FIND A GEOMETRIC PROBABILITY Find the probability that a randomly chosen point in the figure lies in the shaded region. *Explain* your steps.

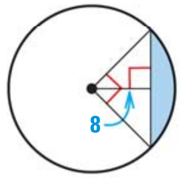
20.



21.



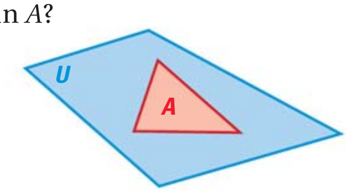
22.



23. ★ **MULTIPLE CHOICE** A point X is chosen at random in region U, and U includes region A. What is the probability that X is not in A?

Ⓐ $\frac{\text{Area of } A}{\text{Area of } U}$ Ⓑ $\frac{\text{Area of } A}{\text{Area of } U - \text{Area of } A}$

Ⓒ $\frac{1}{\text{Area of } A}$ Ⓓ $\frac{\text{Area of } U - \text{Area of } A}{\text{Area of } U}$



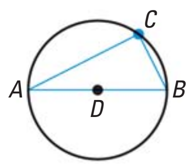
24. **ARCS AND SECTORS** A sector of a circle intercepts an arc of 80°. Find the probability that a randomly chosen point on the circle lies on the arc. Find the probability that a randomly chosen point in the circle lies in the sector. *Explain* why the probabilities do not depend on the radius.

INSCRIBED POLYGONS Find the probability that a randomly chosen point in the circle described lies in the inscribed polygon.

25. Regular hexagon inscribed in circle with circumference $C \approx 188.5$

26. Regular octagon inscribed in circle with radius r

27. **INSCRIBED ANGLES** Points A and B are the endpoints of a diameter of $\odot D$. Point C is chosen at random from the other points on the circle. What is the probability that $\triangle ABC$ is a right triangle? What is the probability that $m\angle CAB \leq 45°$?

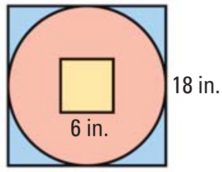


28. **COORDINATE GRAPHS** Graph the system of inequalities $0 \leq x \leq 2$, $0 \leq y \leq 3$, and $y \geq x$. If a point (x, y) is chosen at random in the solution region, what is the probability that $x^2 + y^2 \geq 4$?

29. **CHALLENGE** You carry out a series of steps to paint a walking stick. In the first step, you paint half the length of the stick. For each following step, you paint half of the remaining unpainted portion of the stick. After n steps, you choose a point at random on the stick. Find a value of n so that the probability of choosing a point on the painted portion of the stick after the nth step is greater than 99.95%.

PROBLEM SOLVING

30. **DARTBOARD** A dart is thrown and hits the target shown. If the dart is equally likely to hit any point on the target, what is the probability that it hits inside the inner square? that it hits outside the inner square but inside the circle?



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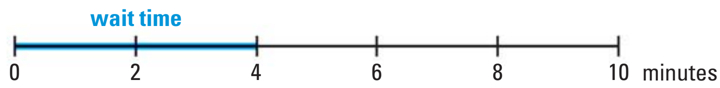
EXAMPLE 2 on p. 772 for Exs. 31–33

31. **TRANSPORTATION** A fair provides a shuttle bus from a parking lot to the fair entrance. Buses arrive at the parking lot every 10 minutes. They wait for 4 minutes while passengers get on and get off. Then the buses depart.



a. What is the probability that there is a bus waiting when a passenger arrives at a random time?

b. What is the probability that there is not a bus waiting when a passenger arrives at a random time?

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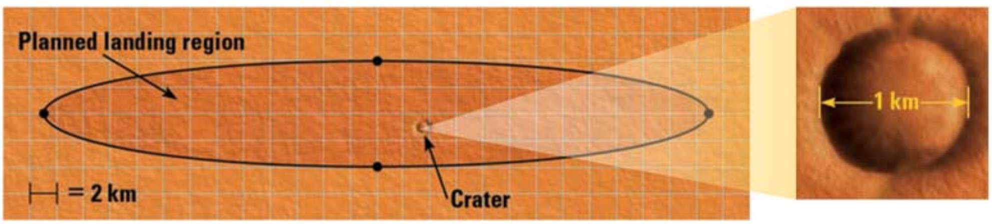
32. **FIRE ALARM** Suppose that your school day is from 8:00 A.M. until 3:00 P.M. You eat lunch at 12:00 P.M. If there is a fire drill at a random time during the day, what is the probability that it begins before lunch?

33. **PHONE CALL** You are expecting a call from a friend anytime between 7:00 P.M. and 8:00 P.M. You are practicing the drums and cannot hear the phone from 6:55 P.M. to 7:10 P.M. What is the probability that you missed your friend's call?

34. ★ **EXTENDED RESPONSE** Scientists lost contact with the space probe Beagle 2 when it was landing on Mars in 2003. They have been unable to locate it since. Early in the search, some scientists thought that it was possible, though unlikely, that Beagle had landed in a circular crater inside the planned landing region. The diameter of the crater is 1 km.



a. In the scale drawing, each square has side length 2 kilometers. Estimate the area of the planned landing region. *Explain* your steps.

b. Estimate the probability of Beagle 2 landing in the crater if it was equally likely to land anywhere in the planned landing region.

35. ★ **SHORT RESPONSE** If the central angle of a sector of a circle stays the same and the radius of the circle doubles, what can you conclude about the probability of a randomly selected point being in the sector? *Explain*. Include an example with your explanation.

36. **PROBABILITY AND LENGTH** A 6 inch long rope is cut into two pieces at a random point. Find the probability both pieces are at least 1 inch long.

37. **COMPOUND EVENTS** You throw two darts at the dartboard in Exercise 30 on page 776. Each dart hits the dartboard. The throws are independent of each other. Find the probability of the compound event described.

a. Both darts hit the yellow square.

b. The first dart hits the yellow square and the second hits outside the circle.

c. Both darts hit inside the circle but outside the yellow square.

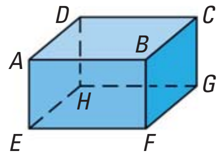
38. **CHALLENGE** A researcher used a 1 hour tape to record birdcalls. Eight minutes after the recorder was turned on, a 5 minute birdcall began. Later, the researcher accidentally erased 10 continuous minutes of the tape. What is the probability that part of the birdcall was erased? What is the probability that all of the birdcall was erased?

MIXED REVIEW

PREVIEW
Prepare for Lesson 12.1 in Exs. 39–41.

39. Draw a concave hexagon and a concave pentagon. *(p. 42)*

Think of each segment shown as part of a line.

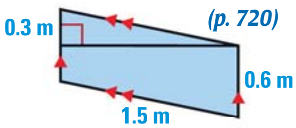


40. Name the intersection of plane *DCH* and plane *ADE*. *(p. 96)*

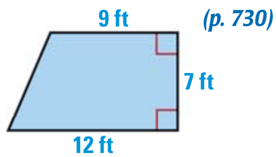
41. Name a plane that appears to be parallel to plane *ADH*. *(p. 147)*

Find the area of the polygon.

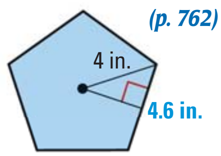
42. *(p. 720)*



43. *(p. 730)*



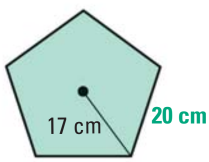
44. *(p. 762)*



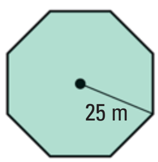
QUIZ for Lessons 11.6–11.7

Find the area of the regular polygon. *(p. 762)*

1.

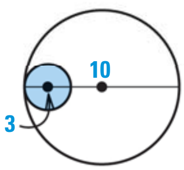


2.

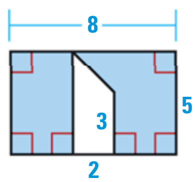


Find the probability that a randomly chosen point in the figure lies in the shaded region. *(p. 771)*

3.



4.



MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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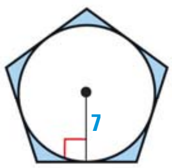
Lessons 11.4–11.7

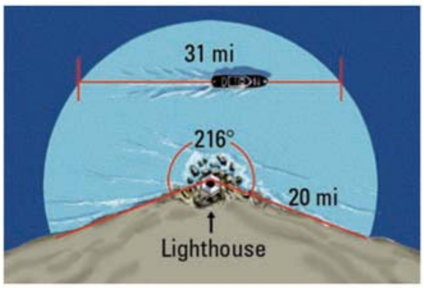
1. **MULTI-STEP PROBLEM** The Hobby-Eberly optical telescope is located in Fort Davis, Texas. The telescope's primary mirror is made of 91 small mirrors that form a hexagon. Each small mirror is a regular hexagon with side length 0.5 meter.

   a. Find the apothem of a small mirror.
   b. Find the area of one of the small mirrors.
   c. Find the area of the primary mirror.

2. **GRIDDED ANSWER** As shown, a circle is inscribed in a regular pentagon. The circle and the pentagon have the same center. Find the area of the shaded region. Round to the nearest tenth.

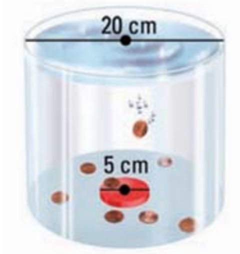


3. **EXTENDED RESPONSE** The diagram shows a projected beam of light from a lighthouse.



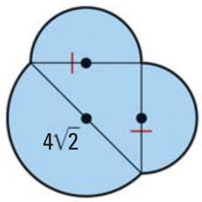
   a. Find the area of the water's surface that is illuminated by the lighthouse.
   b. A boat traveling along a straight line is illuminated by the lighthouse for about 31 miles. Find the closest distance between the lighthouse and the boat. *Explain* your steps.

4. **SHORT RESPONSE** At a school fundraiser, a glass jar with a circular base is filled with water. A circular red dish is placed at the bottom of the jar. A person donates a coin by dropping it into the jar. If the coin lands in the dish, the person wins a small prize.



   a. Suppose a coin tossed into the jar has an equally likely chance of landing anywhere on the bottom of the jar, including in the dish. What is the probability that it will land in the dish?
   b. Suppose 400 coins are dropped into the jar. About how many prizes would you expect people to win? *Explain*.

5. **SHORT RESPONSE** The figure is made of a right triangle and three semicircles. Write expressions for the perimeter and area of the figure in terms of π. *Explain* your reasoning.



6. **OPEN-ENDED** In general, a fan with a greater area does a better job of moving air and cooling you. The fan below is a sector of a cardboard circle. Give an example of a cardboard fan with a smaller radius that will do a better job of cooling you. The intercepted arc should be less than 180°.

11 CHAPTER SUMMARY

BIG IDEAS

For Your Notebook

Big Idea 1

Using Area Formulas for Polygons

Polygon	Formula	
Triangle	$A = \frac{1}{2}bh$,	with base b and height h
Parallelogram	$A = bh$,	with base b and height h
Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$,	with bases b_1 and b_2 and height h
Rhombus	$A = \frac{1}{2}d_1d_2$,	with diagonals d_1 and d_2
Kite	$A = \frac{1}{2}d_1d_2$,	with diagonals d_1 and d_2
Regular polygon	$A = \frac{1}{2}a \cdot ns$,	with apothem a, n sides, and side length s

Sometimes you need to use the Pythagorean Theorem, special right triangles, or trigonometry to find a length in a polygon before you can find its area.

Big Idea 2

Relating Length, Perimeter, and Area Ratios in Similar Polygons

You can use ratios of corresponding measures to find other ratios of measures. You can solve proportions to find unknown lengths or areas.

If two figures are similar and . . .	then . . .
the ratio of side lengths is $a:b$	• the ratio of perimeters is also $a:b$. • the ratio of areas is $a^2:b^2$.
the ratio of perimeters is $c:d$	• the ratio of side lengths is also $c:d$. • the ratio of areas is $c^2:d^2$.
the ratio of areas is $e:f$	• the ratio of side lengths is $\sqrt{e}:\sqrt{f}$. • the ratio of perimeters is $\sqrt{e}:\sqrt{f}$.

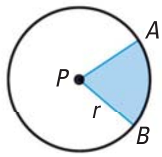
Big Idea 3

Comparing Measures for Parts of Circles and the Whole Circle

Given $\odot P$ with radius r, you can use proportional reasoning to find measures of parts of the circle.

Arc length $\quad \dfrac{\text{Arc length of } \overset{\frown}{AB}}{2\pi r} = \dfrac{m\overset{\frown}{AB}}{360^\circ}$ ← Part ← Whole

Area of sector $\quad \dfrac{\text{Area of sector } APB}{\pi r^2} = \dfrac{m\overset{\frown}{AB}}{360^\circ}$ ← Part ← Whole



11 CHAPTER REVIEW

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• Multi-Language Glossary
• Vocabulary practice

REVIEW KEY VOCABULARY

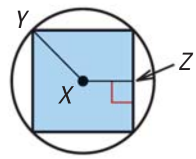
For a list of postulates and theorems, see pp. 926–931.

- bases of a parallelogram, *p. 720*
- height of a parallelogram, *p. 720*
- height of a trapezoid, *p. 730*
- circumference, *p. 746*
- arc length, *p. 747*
- sector of a circle, *p. 756*
- center of a polygon, *p. 762*
- radius of a polygon, *p. 762*
- apothem of a polygon, *p. 762*
- central angle of a regular polygon, *p. 762*
- probability, *p. 771*
- geometric probability, *p. 771*

VOCABULARY EXERCISES

1. Copy and complete: A *sector of a circle* is the region bounded by __?__.
2. **WRITING** *Explain* the relationship between the height of a parallelogram and the bases of a parallelogram.

The diagram shows a square inscribed in a circle. Name an example of the given segment.



3. An apothem of the square
4. A radius of the square

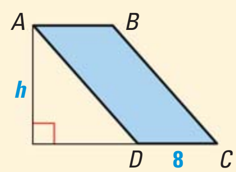
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 11.

11.1 Areas of Triangles and Parallelograms *pp. 720–726*

EXAMPLE

The area of ▱*ABCD* is 96 square units. Find its height *h*.

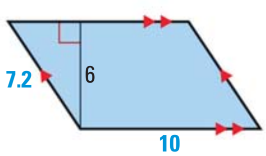


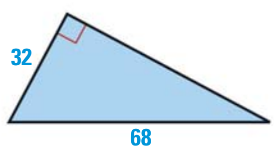
$A = bh$	**Formula for area of a parallelogram**
$96 = 8h$	**Substitute 96 for *A* and 8 for *b*.**
$h = 12$	**Solve.**

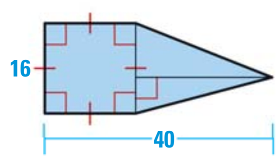
EXERCISES

EXAMPLES 1, 2, and 3 on pp. 721–722 for Exs. 5–8

Find the area of the polygon.

5. 

6. 

7. 

8. The area of a triangle is 147 square inches and its height is 1.5 times its base. Find the base and the height of the triangle.

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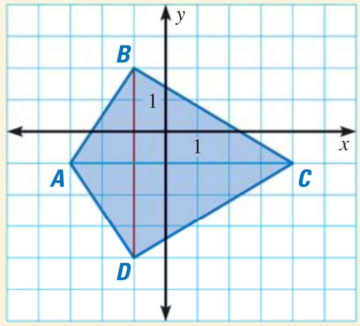
Chapter Review Practice

11.2 Areas of Trapezoids, Rhombuses, and Kites *pp. 730–736*

EXAMPLE

Find the area of the kite.



Find the lengths of the diagonals of the kite.

$d_1 = BD = |2 - (-4)| = 6$

$d_2 = AC = |4 - (-3)| = 7$

Find the area of $ABCD$.

$A = \frac{1}{2}d_1d_2$ **Formula for area of a kite**

$= \frac{1}{2}(6)(7) = 21$ **Substitute and simplify.**

▶ The area of the kite is 21 square units.

EXERCISES

EXAMPLE 4 on p. 732 for Exs. 9–11

Graph the polygon with the given vertices and find its area.

9. $L(2, 2)$, $M(6, 2)$, $N(8, 4)$, $P(4, 4)$

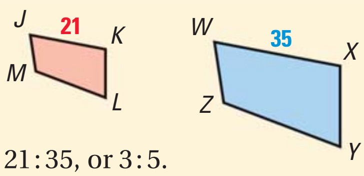
10. $Q(-3, 0)$, $R(-2, 3)$, $S(-1, 0)$, $T(-2, -2)$

11. $D(-1, 4)$, $E(5, 4)$, $F(3, -2)$, $G(1, -2)$

11.3 Perimeter and Area of Similar Figures *pp. 737–743*

EXAMPLE

Quadrilaterals *JKLM* and *WXYZ* are similar. Find the ratios (red to blue) of the perimeters and of the areas.



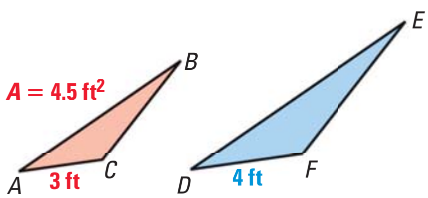
The ratio of the lengths of the corresponding sides is 21 : 35, or 3 : 5. Using Theorem 6.1, the ratio of the perimeters is 3 : 5. Using Theorem 11.7, the ratio of the areas is $3^2 : 5^2$, or 9 : 25.

EXERCISES

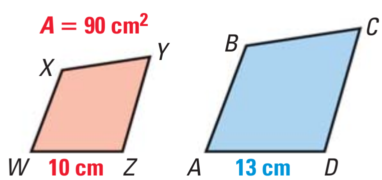
EXAMPLES 1, 2, and 3 on pp. 737–738 for Exs. 12–14

The polygons are similar. Find the ratio (red to blue) of the perimeters and of the areas. Then find the unknown area.

12. $\triangle ABC \sim \triangle DEF$



13. $WXYZ \sim ABCD$



14. The ratio of the areas of two similar figures is 144 : 49. Write the ratio of the lengths of corresponding sides.

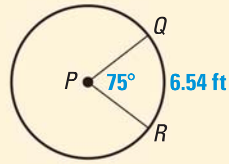
11 CHAPTER REVIEW

11.4 Circumference and Arc Length

pp. 746–752

EXAMPLE

The arc length of $\widehat{QR}$ is 6.54 feet. Find the radius of $\odot P$.



$$\frac{\text{Arc length of } \widehat{QR}}{2\pi r} = \frac{m\widehat{QR}}{360^\circ}$$ **Arc Length Corollary**

$$\frac{6.54}{2\pi r} = \frac{75^\circ}{360^\circ}$$ **Substitute.**

$$6.54(360^\circ) = 75^\circ(2\pi r)$$ **Cross Products Property**

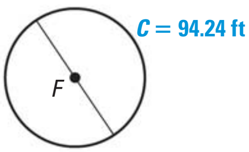
$$r \approx 5.00 \text{ ft}$$ **Solve.**

EXERCISES

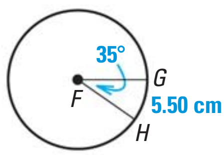
EXAMPLES 1, 3, and 4 on pp. 746, 748 for Exs. 15–17

Find the indicated measure.

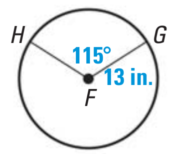
15. Diameter of $\odot F$



16. Circumference of $\odot F$



17. Length of $\widehat{GH}$

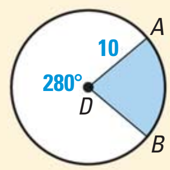


11.5 Areas of Circles and Sectors

pp. 755–761

EXAMPLE

Find the area of sector *ADB*.



First find the measure of the minor arc.

$m\angle ADB = 360^\circ - 280^\circ = 80^\circ$, so $m\widehat{AB} = 80^\circ$.

$$\text{Area of sector } ADB = \frac{m\widehat{AB}}{360^\circ} \cdot \pi r^2$$ **Formula for area of a sector**

$$= \frac{80^\circ}{360^\circ} \cdot \pi \cdot 10^2$$ **Substitute.**

$$\approx 69.81 \text{ units}^2$$ **Use a calculator.**

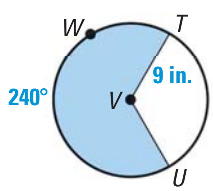
▶ The area of the small sector is about 69.81 square units.

EXERCISES

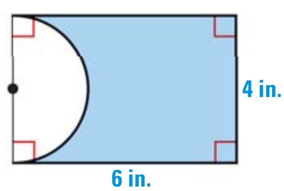
EXAMPLES 2, 3, and 4 on pp. 756–757 for Exs. 18–20

Find the area of the blue shaded region.

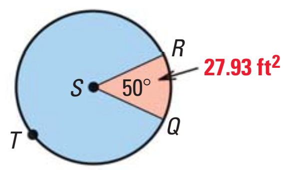
18.



19.



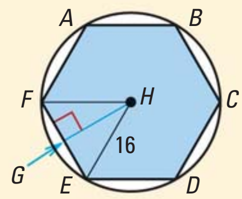
20.



11.6 Areas of Regular Polygons

pp. 762–768

EXAMPLE



A regular hexagon is inscribed in $\odot H$. Find (a) $m\angle EHG$, and (b) the area of the hexagon.

a. $\angle FHE$ is a central angle, so $m\angle FHE = \frac{360°}{6} = 60°$.
Apothem $\overline{GH}$ bisects $\angle FHE$. So, $m\angle EHG = 30°$.

b. Because $\triangle EHG$ is a 30°-60°-90° triangle, $GE = \frac{1}{2} \cdot HE = 8$ and $GH = \sqrt{3} \cdot GE = 8\sqrt{3}$. So, $s = 16$ and $a = 8\sqrt{3}$. Then use the area formula.

$$A = \frac{1}{2}a \cdot ns = \frac{1}{2}(8\sqrt{3})(6)(16) \approx 665.1 \text{ square units}$$

EXERCISES

EXAMPLES 2 and 3 on pp. 763–764 for Exs. 21–22

21. PLATTER A platter is in the shape of a regular octagon. Find the perimeter and area of the platter if its apothem is 6 inches.

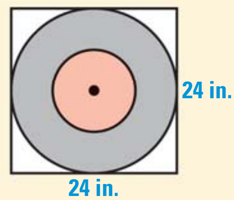
22. PUZZLE A jigsaw puzzle is in the shape of a regular pentagon. Find its area if its radius is 17 centimeters and its side length is 20 centimeters.

11.7 Use Geometric Probability

pp. 771–777

EXAMPLE

A dart is thrown and hits the square dartboard shown. The dart is equally likely to land on any point on the board. Find the probability that the dart lands in the white region outside the concentric circles.



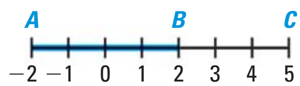
$$P(\text{dart lands in white region}) = \frac{\text{Area of white region}}{\text{Area of dart board}} = \frac{24^2 - \pi(12^2)}{24^2} \approx 0.215$$

▶ The probability that the dart lands in the white region is about 21.5%.

EXERCISES

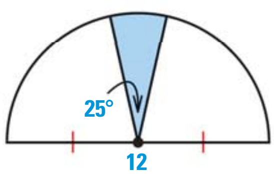
EXAMPLES 1 and 3 on pp. 771, 773 for Exs. 23–26

23. A point K is selected randomly on $\overline{AC}$ at the right. What is the probability that K is on $\overline{AB}$?



Find the probability that a randomly chosen point in the figure lies in the shaded region.

24.



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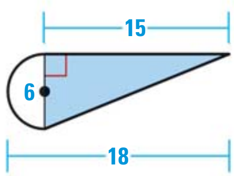

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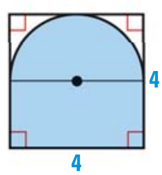
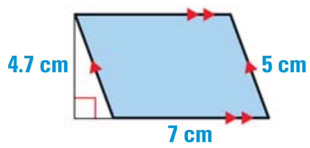


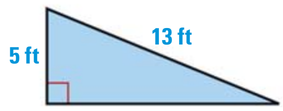
11 CHAPTER TEST

In Exercises 1–6, find the area of the shaded polygon.

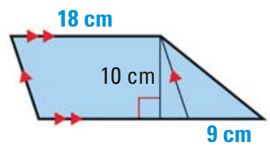
1.



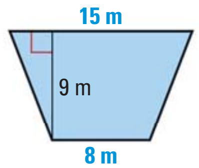
2.



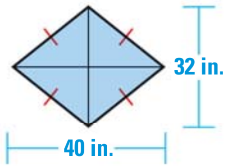
3.



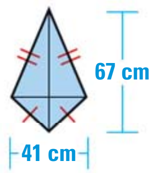
4.



5.



6.



7. The base of a parallelogram is 3 times its height. The area of the parallelogram is 108 square inches. Find the base and the height.

Quadrilaterals *ABCD* and *EFGH* are similar. The perimeter of *ABCD* is 40 inches and the perimeter of *EFGH* is 16 inches.

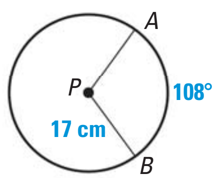
8. Find the ratio of the perimeters of *ABCD* to *EFGH*.

9. Find the ratio of the corresponding side lengths of *ABCD* to *EFGH*.

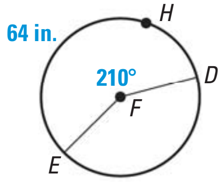
10. Find the ratio of the areas of *ABCD* to *EFGH*.

Find the indicated measure for the circle shown.

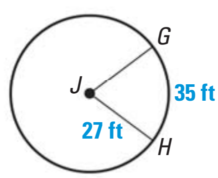
11. Length of $\overset{\frown}{AB}$



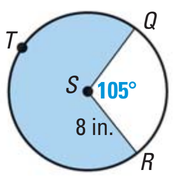
12. Circumference of $\odot F$



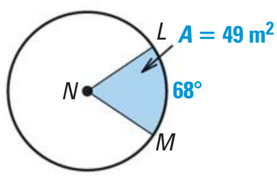
13. $m\overset{\frown}{GH}$



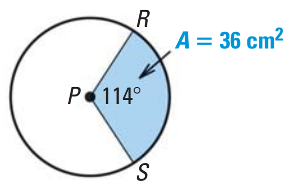
14. Area of shaded sector



15. Area of $\odot N$

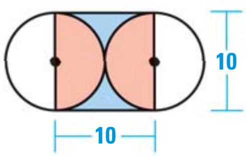


16. Radius of $\odot P$



17. **TILING** A floor tile is in the shape of a regular hexagon and has a perimeter of 18 inches. Find the side length, apothem, and area of the tile.

Find the probability that a randomly chosen point in the figure lies in the region described.



18. In the red region

19. In the blue region

11 ALGEBRA REVIEW

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USE ALGEBRAIC MODELS TO SOLVE PROBLEMS

EXAMPLE 1 *Write and solve an algebraic model for a problem*

FUNDRAISER You are baking cakes to sell at a fundraiser. It costs \$3 to make each cake, and you plan to sell the cakes for \$8 each. You spent \$20 on pans and utensils. How many cakes do you need to sell to make a profit of \$50?

Solution

Let x represent the number of cakes sold.

Income − Expenses = Profit	**Write verbal model.**
$8x - (3x + 20) = 50$	**Substitute $8x$ for income, $3x + 20$ for expenses, and 50 for profit.**
$8x - 3x - 20 = 50$	**Distributive Property**
$5x - 20 = 50$	**Combine like terms.**
$x = 14$	**Solve for x.**

▶ You need to sell 14 cakes to make a profit of \$50.

EXERCISES

EXAMPLE 1 for Exs. 1–7

Write an algebraic model to represent the situation. Then solve the problem.

1. **BICYCLES** You ride your bike 14.25 miles in 90 minutes. At this rate, how far can you bike in 2 hours?

2. **SHOPPING** Alma spent \$39 on a shirt and a jacket. The shirt cost \$12. Find the original cost of a jacket if Alma bought it on sale for 25% off.

3. **CELL PHONES** Your cell phone provider charges \$29.50 per month for 200 minutes. You pay \$.25 per minute for each minute over 200 minutes. In May, your bill was \$32.75. How many additional minutes did you use?

4. **EXERCISE** Jaime burns 12.1 calories per minute running and 7.6 calories per minute swimming. He wants to burn at least 400 calories and plans to swim for 20 minutes. How long does he need to run to meet his goal?

5. **CARS** You buy a car for \$18,000. The value of the car decreases 10% each year. What will the value of the car be after 5 years?

6. **TICKETS** Student tickets for a show cost \$5 and adult tickets cost \$8. At one show, \$2065 was collected in ticket sales. If 62 more student tickets were sold than adult tickets, how many of each type of ticket was sold?

7. **TENNIS** The height h in feet of a tennis ball is $h = -16t^2 + 47t + 6$, where t is the time in seconds after being hit. If the ball is not first hit by another player, how long does it take to reach the ground?

Scoring Rubric

Full Credit
- solution is complete and correct

Partial Credit
- solution is complete but has errors,

or
- solution is without error but incomplete

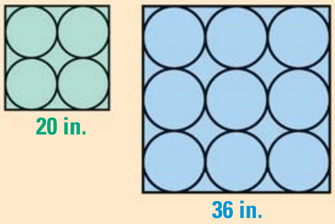
No Credit
- no solution is given,

or
- solution makes no sense

EXTENDED RESPONSE QUESTIONS

PROBLEM

You are making circular signs for a pep rally at your school. You can cut 4 circles with diameter 10 inches from a cardboard square that is 20 inches long on each side, or 9 circles with diameter 12 inches from a cardboard square that is 36 inches long on each side.



a. For each cardboard square, find the area of the cardboard that is used for the signs. Round to the nearest square inch. Show your work.

b. You want to waste as little of a cardboard square as possible. Does it matter which size of cardboard you use? If so, which size of cardboard should you choose if you want to use a greater percent of the cardboard's area for the signs? *Explain*.

Below are sample solutions to the problem. Read each solution and the comments in blue to see why the sample represents *full credit*, *partial credit*, or *no credit*.

SAMPLE 1: Full credit solution

In part (a), the student's work is shown and the calculations are correct.

a. For each cardboard square, multiply the number of circles by the area of one circle.

For the 20 inch square, the radius of each of the 4 circles is 5 inches.

$$\text{Area of 4 circles} = 4 \cdot \pi r^2 = 4 \cdot \pi(5)^2 \approx 314 \text{ in.}^2$$

For the 36 inch square, the radius of each of the 9 circles is 6 inches.

$$\text{Area of 9 circles} = 9 \cdot \pi r^2 = 9 \cdot \pi(6)^2 \approx 1018 \text{ in.}^2$$

The reasoning in part (b) is correct and the answer is correct.

b. For each cardboard square, find the percent of the cardboard square's area that is used for the circles.

$$\text{Percent for 20 inch square: } \frac{\text{Area of 4 circles}}{\text{Area of cardboard}} = \frac{314}{20^2} = 0.785 = 78.5\%$$

$$\text{Percent for 36 inch square: } \frac{\text{Area of 9 circles}}{\text{Area of cardboard}} = \frac{1018}{36^2} \approx 0.785 = 78.5\%$$

It doesn't matter which size of cardboard you use. In each case, you will use about 78.5% of the cardboard's area.

SAMPLE 2: Partial credit solution

In part (a), the answer is incomplete because the student does not find the area of all the circles.

The reasoning in part (b) is correct, but the answer is wrong because the student did not consider the area of all the circles.

a. Use the formula $A = \pi r^2$ to find the area of each circle. Divide each diameter in half to get the radius of the circle.

Area of 10 inch diameter circle $= \pi(5)^2 \approx 79$ in.^{2}
Area of 12 inch diameter circle $= \pi(6)^2 \approx 113$ in.^{2}

b. Find and compare the percents.

$$\frac{\text{Area of circles}}{\text{Area of 20 in. square}} = \frac{79}{20^2} = 0.1975 = 19.75\%$$

$$\frac{\text{Area of circles}}{\text{Area of 36 in. square}} = \frac{113}{36^2} \approx 0.0872 = 8.72\%$$

You use 19.75% of the 20 inch cardboard's area, but only 8.72% of the 36 inch cardboard's area. So, you should use the 20 inch cardboard.

SAMPLE 3: No credit solution

In part (a), the wrong formula is used.

In part (b), the reasoning and the answer are incorrect.

a. Area $= \pi d = \pi(10) \approx 31$ in.^{2} Multiply by 4 to get 124 in.^{2}
Area $= \pi d = \pi(12) \approx 38$ in.^{2} Multiply by 9 to get 342 in.^{2}

b. You use 342 in.^{2} of cardboard for 9 signs, and only 124 in.^{2} for 4 signs. You should use the 36 inch cardboard because you will use more of it.

PRACTICE Apply the Scoring Rubric

1. A student's solution to the problem on the previous page is given below. Score the solution as *full credit, partial credit,* or *no credit*. *Explain* your reasoning. If you choose *partial credit* or *no credit*, *explain* how you would change the solution so that it earns a score of full credit.

a. There are two sizes of circles you can make. Find the area of each.

Area of a circle made from the 20 inch square $= \pi(5)^2 \approx 78.5$ in.^{2}
Area of a circle made from the 36 inch square $= \pi(6)^2 \approx 113.1$ in.^{2}

Then multiply each area by the number of circles that have that area.

Area of circles in 20 inch square $\approx 4 \cdot 78.5 = 314$ in.^{2}
Area of circles in 36 inch square $\approx 9 \cdot 113.1 \approx 1018$ in.^{2}

b. Find the percent of each square's area that is used for the signs.

$$\frac{\text{Area of 4 circles}}{\text{Area of 20 in. square}} = \frac{314}{20} = 15.7\%$$

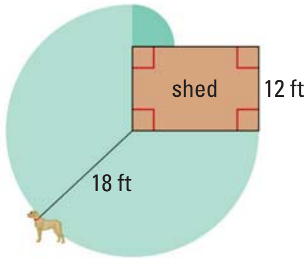
$$\frac{\text{Area of 9 circles}}{\text{Area of 36 in. square}} = \frac{1018}{36} \approx 28.3\%$$

Because 28.3% > 15.7%, you use a greater percent of the cardboard's area when you use the 36 inch square.

★ Standardized TEST PRACTICE

EXTENDED RESPONSE

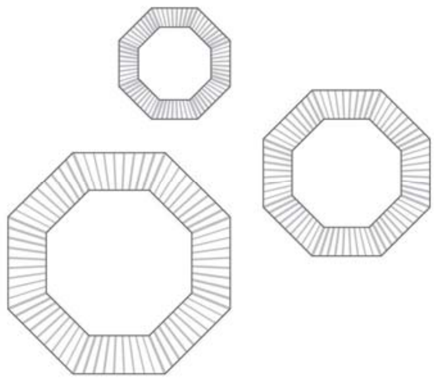
1. A dog is tied to the corner of a shed with a leash. The leash prevents the dog from moving more than 18 feet from the corner. In the diagram, the shaded sectors show the region over which the dog can roam.



   a. Find the area of the sector with radius 18 feet.
   b. What is the radius of the smaller sector? Find its area. *Explain*.
   c. Find the area over which the dog can move. *Explain*.

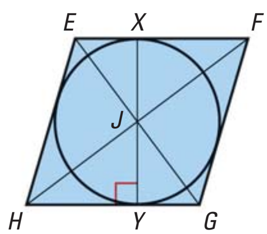
2. A circle passes through the points (3, 0), (9, 0), (6, 3), and (6, −3).
   a. Graph the circle in a coordinate plane. Give the coordinates of its center.
   b. Sketch the image of the circle after a dilation centered at the origin with a scale factor of 2. How are the coordinates of the center of the dilated circle related to the coordinates of the center of the original circle? *Explain*.
   c. How are the circumferences of the circle and its image after the dilation related? How are the areas related? *Explain*.

3. A caterer uses a set of three different-sized trays. Each tray is a regular octagon. The areas of the trays are in the ratio 2 : 3 : 4.



   a. The area of the smallest tray is about 483 square centimeters. Find the areas of the other trays to the nearest square centimeter. *Explain* your reasoning.
   b. The perimeter of the smallest tray is 80 centimeters. Find the approximate perimeters of the other trays. Round to the nearest tenth of a centimeter. *Explain* your reasoning.

4. In the diagram, the diagonals of rhombus $EFGH$ intersect at point J, $EG = 6$, and $FH = 8$. A circle with center J is inscribed in $EFGH$, and $\overline{XY}$ is a diameter of $\odot J$.



   a. Find EF. *Explain* your reasoning.
   b. Use the formula for the area of a rhombus to find the area of $EFGH$.
   c. Use the formula for the area of a parallelogram to write an equation relating the area of $EFGH$ from part (b) to EF and XY.
   d. Find XY. Then find the area of the inscribed circle. *Explain* your reasoning.

MULTIPLE CHOICE

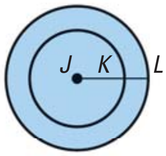
5. In the diagram, J is the center of two circles, and K lies on $\overline{JL}$. Given $JL = 6$ and $KL = 2$, what is the ratio of the area of the smaller circle to the area of the larger circle?

Ⓐ $\sqrt{2} : \sqrt{3}$

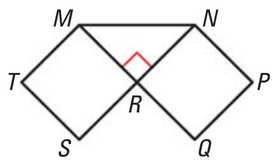
Ⓑ $1 : 3$

Ⓒ $2 : 3$

Ⓓ $4 : 9$



6. In the diagram, $TMRS$ and $RNPQ$ are congruent squares, and $\triangle MNR$ is a right triangle. What is the probability that a randomly chosen point on the diagram lies inside $\triangle MNR$?



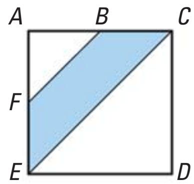
Ⓐ 0.2 Ⓑ 0.25

Ⓒ 0.5 Ⓓ 0.75

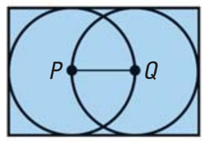
GRIDDED ANSWER

7. You are buying fertilizer for a lawn that is shaped like a parallelogram. Two sides of the parallelogram are each 300 feet long, and the perpendicular distance between these sides is 150 feet. One bag of fertilizer covers 5000 square feet and costs \$14. How much (in dollars) will you spend?

8. In square $ACDE$, $ED = 2$, $AB = BC$, and $AF = FE$. What is the area (in square units) of the shaded region?

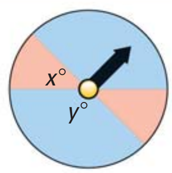


9. In the diagram, a rectangle's sides are tangent to two circles with centers at points P and Q. The circumference of each circle is 8π square units. What is the area (in square units) of the rectangle?



SHORT RESPONSE

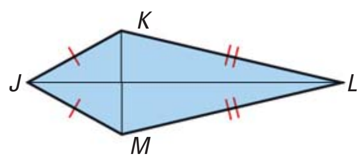
10. You are designing a spinner for a board game. An arrow is attached to the center of a circle with diameter 7 inches. The arrow is spun until it stops. The arrow has an equally likely chance of stopping anywhere.



a. If $x° = 45°$, what is the probability that the arrow points to a red sector? *Explain*.

b. You want to change the spinner so the probability that the arrow points to a blue sector is half the probability that it points to a red sector. What values should you use for x and y? *Explain*.

11. In quadrilateral $JKLM$, $JL = 3 \cdot KM$. The area of $JKLM$ is 54 square centimeters.



a. Find JL and KM.

b. Quadrilateral $NPQR$ is similar to $JKLM$, and its area is 486 square centimeters. Sketch $NPQR$ and its diagonals. Then find the length of $\overline{NQ}$. *Explain* your reasoning.

12 Surface Area and Volume of Solids

Before

In previous chapters, you learned the following skills, which you'll use in Chapter 12: properties of similar polygons, areas and perimeters of two-dimensional figures, and right triangle trigonometry.

Prerequisite Skills

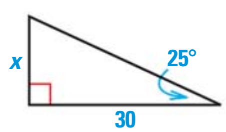
VOCABULARY CHECK

1. Copy and complete: The area of a regular polygon is given by the formula $A =$ _?_.
2. *Explain* what it means for two polygons to be similar.

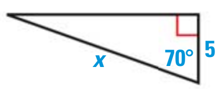
SKILLS AND ALGEBRA CHECK

Use trigonometry to find the value of *x*. *(Review pp. 466, 473 for 12.2–12.5.)*

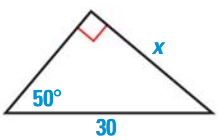
3.



4.



5.



Find the circumference and area of the circle with the given dimension. *(Review pp. 746, 755 for 12.2–12.5.)*

6. $r = 2$ m

7. $d = 3$ in.

8. $r = 2\sqrt{5}$ cm

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 12, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 856. You will also use the key vocabulary listed below.

Big Ideas

1. **Exploring solids and their properties**
2. **Solving problems using surface area and volume**
3. **Connecting similarity to solids**

KEY VOCABULARY

- polyhedron, *p. 794* face, edge, vertex
- Platonic solids, *p. 796*
- cross section, *p. 797*
- prism, *p. 803*
- surface area, *p. 803*
- lateral area, *p. 803*
- net, *p. 803*
- right prism, *p. 804*
- oblique prism, *p. 804*
- cylinder, *p. 805*
- right cylinder, *p. 805*
- pyramid, *p. 810*
- regular pyramid, *p. 810*
- cone, *p. 812*
- right cone, *p. 812*
- volume, *p. 819*
- sphere, *p. 838*
- great circle, *p. 839*
- hemisphere, *p. 839*
- similar solids, *p. 847*

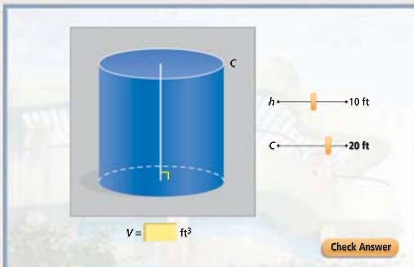
Why?

Knowing how to use surface area and volume formulas can help you solve problems in three dimensions. For example, you can use a formula to find the volume of a column in a building.

Animated Geometry

The animation illustrated below for Exercise 31 on page 825 helps you answer this question: What is the volume of the column?

You can use the height and circumference of a column to find its volume.



Drag the sliders to change the height and circumference of the cylinder.

Animated Geometry at classzone.com

Other animations for Chapter 12: pages 795, 805, 821, 833, 841, and 852

12.1 Investigate Solids

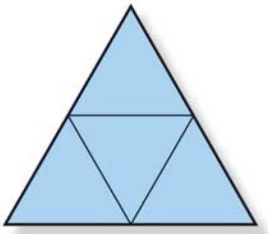
MATERIALS • poster board • scissors • tape • straightedge

QUESTION **What solids can be made using congruent regular polygons?**

Platonic solids, named after the Greek philosopher Plato (427 B.C.–347 B.C.), are solids that have the same congruent regular polygon as each *face*, or side of the solid.

EXPLORE 1 Make a solid using four equilateral triangles

STEP 1



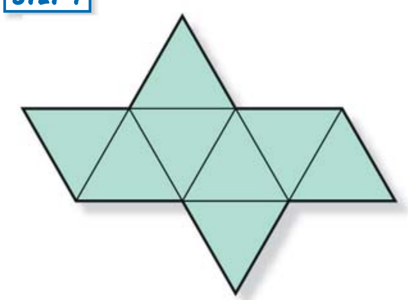
Make a net Copy the full-sized triangle from page 793 on poster board to make a template. Trace the triangle four times to make a *net* like the one shown.

STEP 2

Make a solid Cut out your net. Fold along the lines. Tape the edges together to form a solid. How many faces meet at each *vertex*?

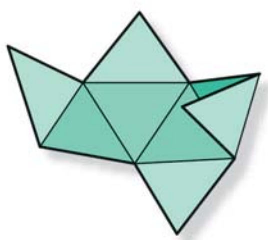
EXPLORE 2 Make a solid using eight equilateral triangles

STEP 1



Make a net Trace your triangle template from Explore 1 eight times to make a net like the one shown.

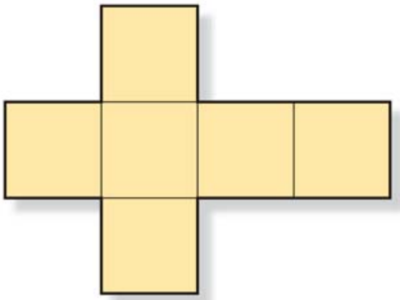
STEP 2



Make a solid Cut out your net. Fold along the lines. Tape the edges together to form a solid. How many faces meet at each vertex?

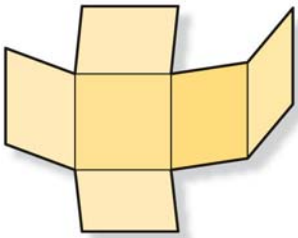
EXPLORE 3 Make a solid using six squares

STEP 1



Make a net Copy the full-sized square from the bottom of the page on poster board to make a template. Trace the square six times to make a net like the one shown.

STEP 2

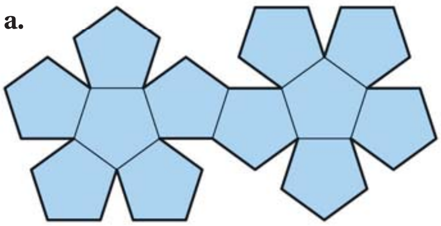


Make a solid Cut out your net. Fold along the lines. Tape the edges together to form a solid. How many faces meet at each vertex?

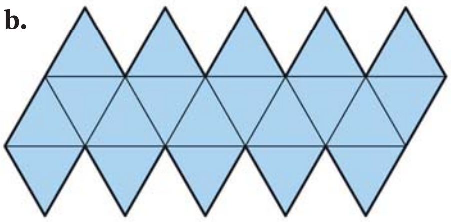
DRAW CONCLUSIONS Use your observations to complete these exercises

1. The two other convex solids that you can make using congruent, regular faces are shown below. For each of these solids, how many faces meet at each vertex?

a.

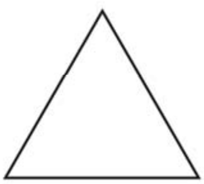


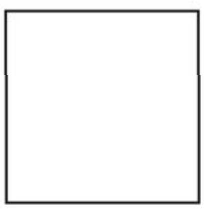
b.



2. *Explain* why it is not possible to make a solid that has six congruent equilateral triangles meeting at each vertex.
3. *Explain* why it is not possible to make a solid that has three congruent regular hexagons meeting at each vertex.
4. Count the number of vertices V, edges E, and faces F for each solid you made. Make a conjecture about the relationship between the sum $F + V$ and the value of E.

Templates:



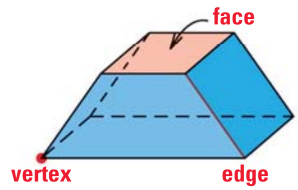


12.1 Explore Solids

Before	You identified polygons.
Now	You will identify solids.
Why	So you can analyze the frame of a house, as in Example 2.

Key Vocabulary
- **polyhedron**
  face, edge, vertex
- **base**
- **regular polyhedron**
- **convex polyhedron**
- **Platonic solids**
- **cross section**

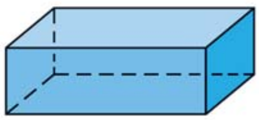
A **polyhedron** is a solid that is bounded by polygons, called **faces**, that enclose a single region of space. An **edge** of a polyhedron is a line segment formed by the intersection of two faces. A **vertex** of a polyhedron is a point where three or more edges meet. The plural of polyhedron is *polyhedra* or *polyhedrons*.



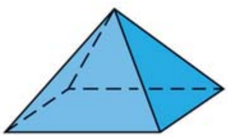
KEY CONCEPT — *For Your Notebook*

Types of Solids

Polyhedra

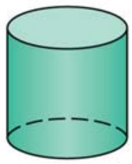


Prism

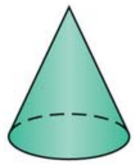


Pyramid

Not Polyhedra



Cylinder

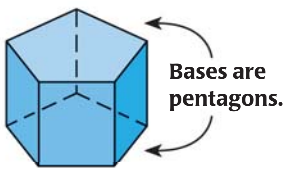


Cone

Sphere

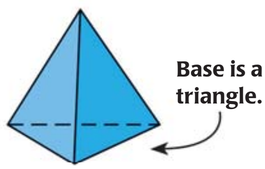
CLASSIFYING SOLIDS Of the five solids above, the prism and the pyramid are polyhedra. To name a prism or a pyramid, use the shape of the *base*.

Pentagonal prism



The two **bases** of a prism are congruent polygons in parallel planes.

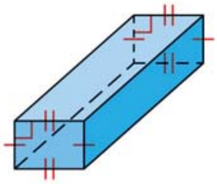
Triangular pyramid

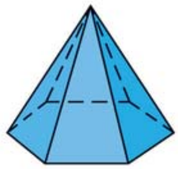


The **base** of a pyramid is a polygon.

EXAMPLE 1 Identify and name polyhedra

Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges.

a. 

b. 

c. 

Solution

a. The solid is formed by polygons, so it is a polyhedron. The two bases are congruent rectangles, so it is a rectangular prism. It has 6 faces, 8 vertices, and 12 edges.

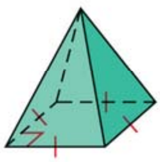
b. The solid is formed by polygons, so it is a polyhedron. The base is a hexagon, so it is a hexagonal pyramid. It has 7 faces, consisting of 1 base, 3 visible triangular faces, and 3 non-visible triangular faces. The polyhedron has 7 faces, 7 vertices, and 12 edges.

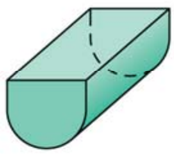
c. The cone has a curved surface, so it is not a polyhedron.

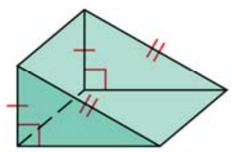
AnimatedGeometry at classzone.com

✓ GUIDED PRACTICE for Example 1

Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges.

1. 

2. 

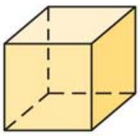
3. 

EULER'S THEOREM Notice in Example 1 that the sum of the number of faces and vertices of the polyhedra is two more than the number of edges. This suggests the following theorem, proved by the Swiss mathematician Leonhard Euler (pronounced "oi′-ler"), who lived from 1707 to 1783.

THEOREM *For Your Notebook*

THEOREM 12.1 Euler's Theorem

The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula $F + V = E + 2$.



$F = 6$, $V = 8$, $E = 12$
$6 + 8 = 12 + 2$

EXAMPLE 2 Use Euler's Theorem in a real-world situation

HOUSE CONSTRUCTION **Find the number of edges on the frame of the house.**

Solution

The frame has one face as its foundation, four that make up its walls, and two that make up its roof, for a total of 7 faces.

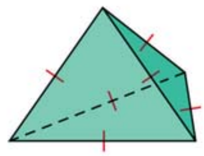
To find the number of vertices, notice that there are 5 vertices around each pentagonal wall, and there are no other vertices. So, the frame of the house has 10 vertices.

Use Euler's Theorem to find the number of edges.

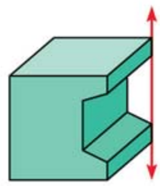
$F + V = E + 2$	Euler's Theorem
$7 + 10 = E + 2$	Substitute known values.
$15 = E$	Solve for E.

▶ The frame of the house has 15 edges.

REGULAR POLYHEDRA A polyhedron is **regular** if all of its faces are congruent regular polygons. A polyhedron is **convex** if any two points on its surface can be connected by a segment that lies entirely inside or on the polyhedron. If this segment goes outside the polyhedron, then the polyhedron is nonconvex, or *concave*.



regular, convex

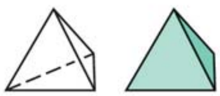


nonregular, concave

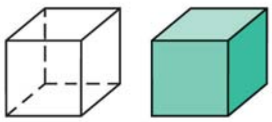
There are five regular polyhedra, called **Platonic solids** after the Greek philosopher Plato (c. 427 B.C.–347 B.C.). The five Platonic solids are shown.

READ VOCABULARY

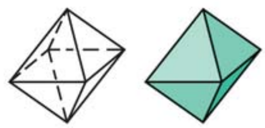
Notice that the names of four of the Platonic solids end in "hedron." *Hedron* is Greek for "side" or "face." Sometimes a cube is called a regular *hexahedron*.



Regular tetrahedron
4 faces



Cube
6 faces



Regular octahedron
8 faces

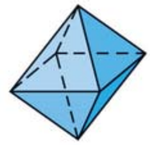
Regular dodecahedron
12 faces

Regular icosahedron
20 faces

There are only five regular polyhedra because the sum of the measures of the angles that meet at a vertex of a convex polyhedron must be less than 360°. This means that the only possible combinations of regular polygons at a vertex that will form a polyhedron are 3, 4, or 5 triangles, 3 squares, and 3 pentagons.

EXAMPLE 3 Use Euler's Theorem with Platonic solids

Find the number of faces, vertices, and edges of the regular octahedron. Check your answer using Euler's Theorem.



ANOTHER WAY

An octahedron has 8 faces, each of which has 3 vertices and 3 edges. Each vertex is shared by 4 faces; each edge is shared by 2 faces. They should only be counted once.

$V = \frac{8 \cdot 3}{4} = 6$

$E = \frac{8 \cdot 3}{2} = 12$

Solution

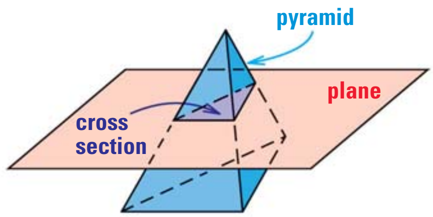
By counting on the diagram, the octahedron has 8 faces, 6 vertices, and 12 edges. Use Euler's Theorem to check.

$F + V = E + 2$ — Euler's Theorem

$8 + 6 = 12 + 2$ — Substitute.

$14 = 14$ ✓ — This is a true statement. So, the solution checks.

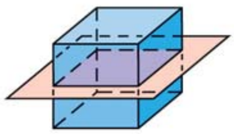
CROSS SECTIONS Imagine a plane slicing through a solid. The intersection of the plane and the solid is called a **cross section**. For example, the diagram shows that an intersection of a plane and a triangular pyramid is a triangle.



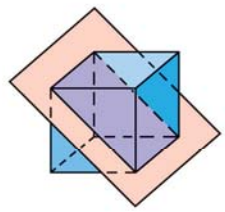
EXAMPLE 4 Describe cross sections

Describe the shape formed by the intersection of the plane and the cube.

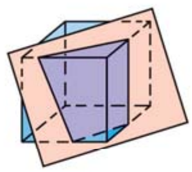
a.



b.



c.



Solution

a. The cross section is a square.

b. The cross section is a rectangle.

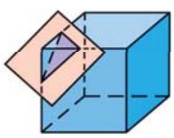
c. The cross section is a trapezoid.

✓ GUIDED PRACTICE for Examples 2, 3, and 4

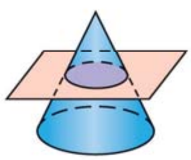
4. Find the number of faces, vertices, and edges of the regular dodecahedron on page 796. Check your answer using Euler's Theorem.

Describe **the shape formed by the intersection of the plane and the solid.**

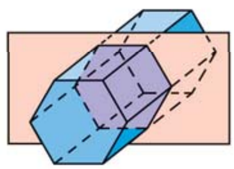
5.



6.



7.



12.1 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 25, and 35

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 21, 28, 30, 31, 39, and 41

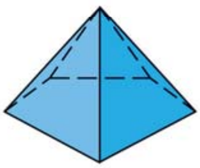
SKILL PRACTICE

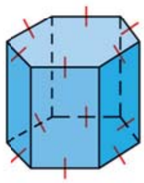
1. **VOCABULARY** Name the five Platonic solids and give the number of faces for each.

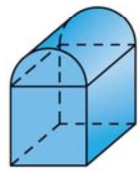
2. ★ **WRITING** State Euler's Theorem in words.

EXAMPLE 1 on p. 795 for Exs. 3–10

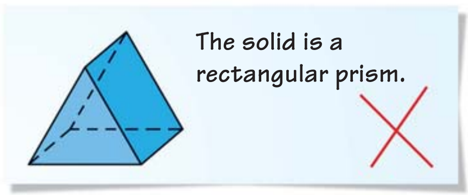
IDENTIFYING POLYHEDRA **Determine whether the solid is a polyhedron. If it is, name the polyhedron. *Explain* your reasoning.**

3. 

4. 

5. 

6. **ERROR ANALYSIS** *Describe* and correct the error in identifying the solid.



SKETCHING POLYHEDRA **Sketch the polyhedron.**

7. Rectangular prism
8. Triangular prism
9. Square pyramid
10. Pentagonal pyramid

EXAMPLES 2 and 3 on pp. 796–797 for Exs. 11–24

APPLYING EULER'S THEOREM **Use Euler's Theorem to find the value of n.**

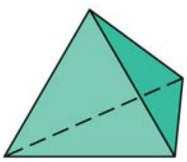
11. Faces: n
Vertices: 12
Edges: 18

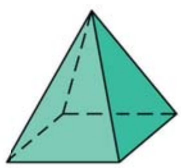
12. Faces: 5
Vertices: n
Edges: 8

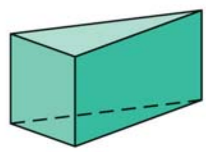
13. Faces: 10
Vertices: 16
Edges: n

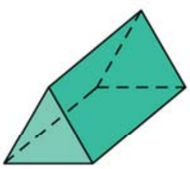
14. Faces: n
Vertices: 12
Edges: 30

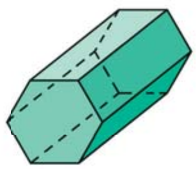
APPLYING EULER'S THEOREM **Find the number of faces, vertices, and edges of the polyhedron. Check your answer using Euler's Theorem.**

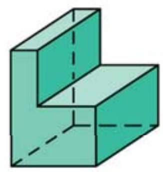
15. 

16. 

17. 

18. 

19. 

20. 

21. ★ **WRITING** *Explain* why a cube is also called a regular hexahedron.

PUZZLES **Determine whether the solid puzzle is *convex* or *concave*.**

22.

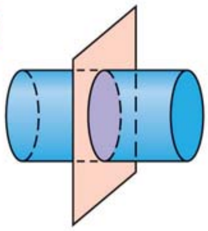
23.

24.

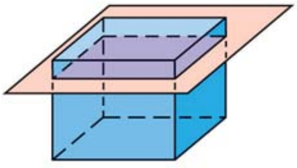
EXAMPLE 4
on p. 797
for Exs. 25–28

CROSS SECTIONS **Draw and *describe* the cross section formed by the intersection of the plane and the solid.**

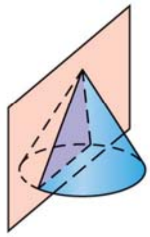
25.



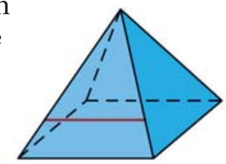
26.



27.



28. ★ **MULTIPLE CHOICE** What is the shape of the cross section formed by the plane parallel to the base that intersects the red line drawn on the square pyramid?



Ⓐ Square Ⓑ Triangle

Ⓒ Kite Ⓓ Trapezoid

29. **ERROR ANALYSIS** *Describe* and correct the error in determining that a tetrahedron has 4 faces, 4 edges, and 6 vertices.

30. ★ **MULTIPLE CHOICE** Which two solids have the same number of faces?

Ⓐ A triangular prism and a rectangular prism

Ⓑ A triangular pyramid and a rectangular prism

Ⓒ A triangular prism and a square pyramid

Ⓓ A triangular pyramid and a square pyramid

31. ★ **MULTIPLE CHOICE** How many faces, vertices, and edges does an octagonal prism have?

Ⓐ 8 faces, 6 vertices, and 12 edges

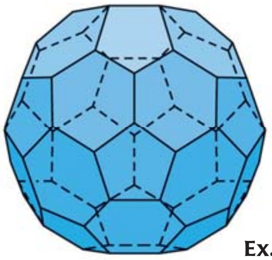
Ⓑ 8 faces, 12 vertices, and 18 edges

Ⓒ 10 faces, 12 vertices, and 20 edges

Ⓓ 10 faces, 16 vertices, and 24 edges

32. **EULER'S THEOREM** The solid shown has 32 faces and 90 edges. How many vertices does the solid have? *Explain* your reasoning.

33. **CHALLENGE** *Describe* how a plane can intersect a cube to form a hexagonal cross section.



Ex. 32

PROBLEM SOLVING

EXAMPLE 2
on p. 796
for Exs. 34–35

34. MUSIC The speaker shown at the right has 7 faces. Two faces are pentagons and 5 faces are rectangles.

a. Find the number of vertices.

b. Use Euler's Theorem to determine how many edges the speaker has.

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35. CRAFT BOXES The box shown at the right is a hexagonal prism. It has 8 faces. Two faces are hexagons and 6 faces are squares. Count the edges and vertices. Use Euler's Theorem to check your answer.

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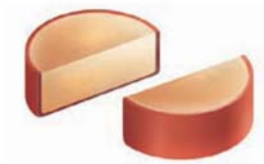
FOOD ***Describe*** **the shape that is formed by the cut made in the food shown.**

36. Watermelon

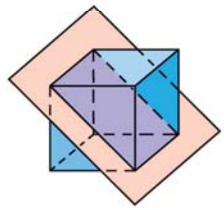
37. Bread

38. Cheese



39. ★ **SHORT RESPONSE** Name a polyhedron that has 4 vertices and 6 edges. Can you draw a polyhedron that has 4 vertices, 6 edges, and a different number of faces? *Explain* your reasoning.

40. MULTI-STEP PROBLEM The figure at the right shows a plane intersecting a cube through four of its vertices. An edge length of the cube is 6 inches.

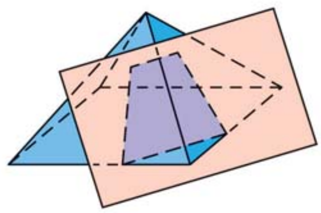


a. *Describe* the shape formed by the cross section.

b. What is the perimeter of the cross section?

c. What is the area of the cross section?

41. ★ **EXTENDED RESPONSE** Use the diagram of the square pyramid intersected by a plane.



a. *Describe* the shape of the cross section shown.

b. Can a plane intersect the pyramid at a point? If so, sketch the intersection.

c. *Describe* the shape of the cross section when the pyramid is sliced by a plane parallel to its base.

d. Is it possible to have a pentagon as a cross section of this pyramid? If so, draw the cross section.

42. PLATONIC SOLIDS Make a table of the number of faces, vertices, and edges for the five Platonic solids. Use Euler's Theorem to check each answer.

○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

REASONING **Is it possible for a cross section of a cube to have the given shape? If yes, *describe* or sketch how the plane intersects the cube.**

43. Circle **44.** Pentagon **45.** Rhombus

46. Isosceles triangle **47.** Regular hexagon **48.** Scalene triangle

49. **CUBE** *Explain* how the numbers of faces, vertices, and edges of a cube change when you cut off each feature.

a. A corner **b.** An edge **c.** A face **d.** 3 corners

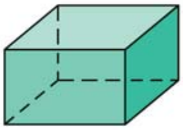
50. **TETRAHEDRON** *Explain* how the numbers of faces, vertices, and edges of a regular tetrahedron change when you cut off each feature.

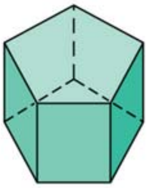
a. A corner **b.** An edge **c.** A face **d.** 2 edges

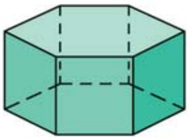
51. **CHALLENGE** The *angle defect D* at a vertex of a polyhedron is defined as follows:

$D = 360° -$ (sum of all angle measures at the vertex)

Verify that for the figures with regular bases below, $DV = 720$ where V is the number of vertices.



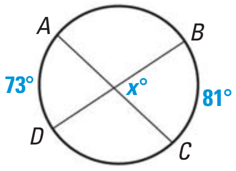




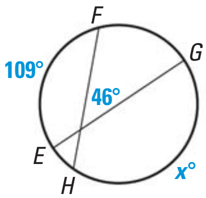
MIXED REVIEW

Find the value of x. *(p. 680)*

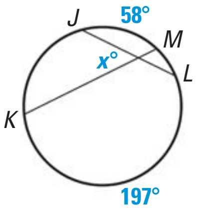
52.



53.



54.



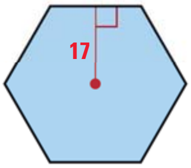
PREVIEW
Prepare for Lesson 12.2 in Exs. 55–60.

Use the given radius r or diameter d to find the circumference and area of the circle. Round your answers to two decimal places. *(p. 755)*

55. $r = 11$ cm **56.** $d = 28$ in. **57.** $d = 15$ ft

Find the perimeter and area of the regular polygon. Round your answers to two decimal places. *(p. 762)*

58.



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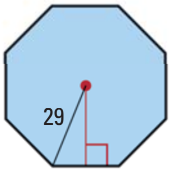
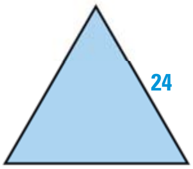


60.



12.2 Investigate Surface Area

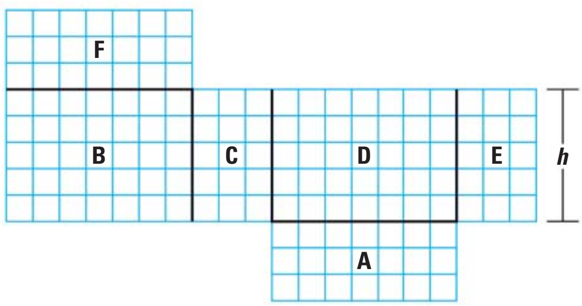
MATERIALS • graph paper • scissors • tape

QUESTION How can you find the surface area of a polyhedron?

A *net* is a pattern that can be folded to form a polyhedron. To find the *surface area* of a polyhedron, you can find the area of its net.

EXPLORE Create a polyhedron using a net

STEP 1 ***Draw a net*** Copy the net below on graph paper. Be sure to label the sections of the net.



STEP 2 ***Create a polyhedron*** Cut out the net and fold it along the black lines to form a polyhedron. Tape the edges together. Describe the polyhedron. Is it regular? Is it convex?

STEP 3 ***Find surface area*** The *surface area* of a polyhedron is the sum of the areas of its faces. Find the surface area of the polyhedron you just made. (Each square on the graph paper measures 1 unit by 1 unit.)

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Lay the net flat again and find the following measures.

   A: the area of Rectangle A

   P: the perimeter of Rectangle A

   h: the height of Rectangles B, C, D, and E

2. Use the values from Exercise 1 to find $2A + Ph$. *Compare* this value to the surface area you found in Step 3 above. What do you notice?

3. Make a conjecture about the surface area of a rectangular prism.

4. Use graph paper to draw the net of another rectangular prism. Fold the net to make sure that it forms a rectangular prism. Use your conjecture from Exercise 3 to calculate the surface area of the prism.

12.2 Surface Area of Prisms and Cylinders

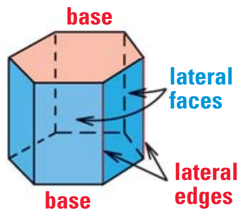
Before You found areas of polygons.

Now You will find the surface areas of prisms and cylinders.

Why? So you can find the surface area of a drum, as in Ex. 22.

Key Vocabulary
- **prism**
  lateral faces, lateral edges
- **surface area**
- **lateral area**
- **net**
- **right prism**
- **oblique prism**
- **cylinder**
- **right cylinder**

A **prism** is a polyhedron with two congruent faces, called *bases*, that lie in parallel planes. The other faces, called **lateral faces**, are parallelograms formed by connecting the corresponding vertices of the bases. The segments connecting these vertices are **lateral edges**. Prisms are classified by the shapes of their bases.



The **surface area** of a polyhedron is the sum of the areas of its faces. The **lateral area** of a polyhedron is the sum of the areas of its lateral faces.

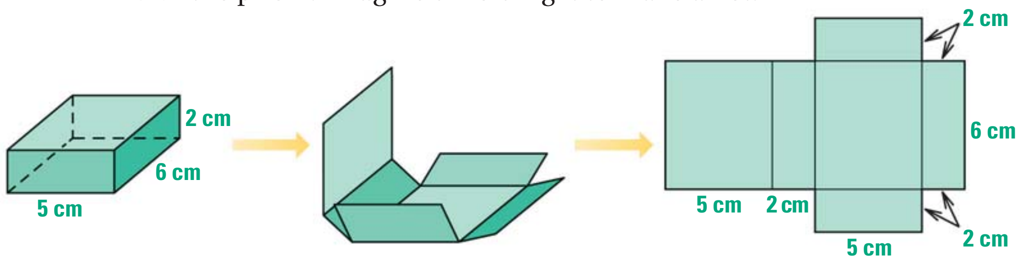
Imagine that you cut some edges of a polyhedron and unfold it. The two-dimensional representation of the faces is called a **net**. As you saw in the Activity on page 802, the surface area of a prism is equal to the area of its net.

EXAMPLE 1 Use the net of a prism

Find the surface area of a rectangular prism with height 2 centimeters, length 5 centimeters, and width 6 centimeters.

Solution

STEP 1 **Sketch** the prism. Imagine unfolding it to make a net.



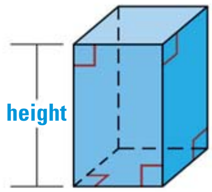
STEP 2 **Find** the areas of the rectangles that form the faces of the prism.

Congruent faces	Dimensions	Area of each face
Left and right faces	6 cm by 2 cm	$6 \cdot 2 = 12 \text{ cm}^2$
Front and back faces	5 cm by 2 cm	$5 \cdot 2 = 10 \text{ cm}^2$
Top and bottom faces	6 cm by 5 cm	$6 \cdot 5 = 30 \text{ cm}^2$

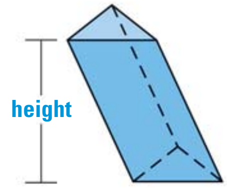
STEP 3 **Add** the areas of all the faces to find the surface area.

▶ The surface area of the prism is $S = 2(12) + 2(10) + 2(30) = 104 \text{ cm}^2$.

RIGHT PRISMS The height of a prism is the perpendicular distance between its bases. In a **right prism**, each lateral edge is perpendicular to both bases. A prism with lateral edges that are not perpendicular to the bases is an **oblique prism**.



Right rectangular prism



Oblique triangular prism

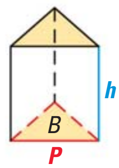
THEOREM — *For Your Notebook*

THEOREM 12.2 Surface Area of a Right Prism

The surface area S of a right prism is

$S = 2B + Ph = aP + Ph,$

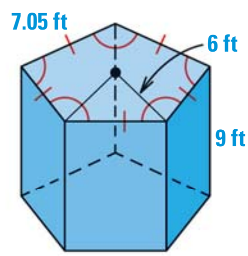
where a is the apothem of the base, B is the area of a base, P is the perimeter of a base, and h is the height.



$S = 2B + Ph = aP + Ph$

EXAMPLE 2 Find the surface area of a right prism

Find the surface area of the right pentagonal prism.



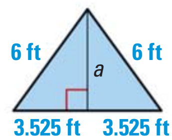
Solution

STEP 1 **Find** the perimeter and area of a base of the prism.

Each base is a regular pentagon.

Perimeter $P = 5(7.05) = 35.25$

Apothem $a = \sqrt{6^2 - 3.525^2} \approx 4.86$



REVIEW APOTHEM
For help with finding the apothem, see p. 762.

STEP 2 **Use** the formula for the surface area that uses the apothem.

$S = aP + Ph$	**Surface area of a right prism**
$\approx (4.86)(35.25) + (35.25)(9)$	**Substitute known values.**
≈ 488.57	**Simplify.**

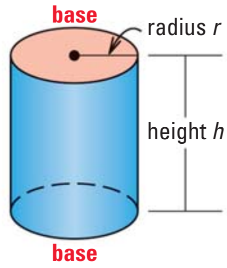
▶ The surface area of the right pentagonal prism is about 488.57 square feet.

✓ GUIDED PRACTICE for Examples 1 and 2

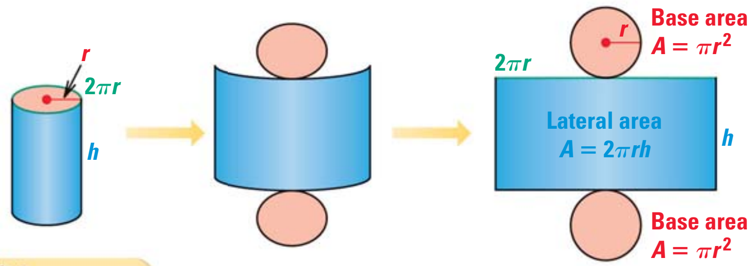
1. Draw a net of a triangular prism.

2. Find the surface area of a right rectangular prism with height 7 inches, length 3 inches, and width 4 inches using (a) a net and (b) the formula for the surface area of a right prism.

CYLINDERS A **cylinder** is a solid with congruent circular bases that lie in parallel planes. The height of a cylinder is the perpendicular distance between its bases. The radius of a base is the *radius* of the cylinder. In a **right cylinder**, the segment joining the centers of the bases is perpendicular to the bases.



The lateral area of a cylinder is the area of its curved surface. It is equal to the product of the circumference and the height, or $2\pi rh$. The surface area of a cylinder is equal to the sum of the lateral area and the areas of the two bases.



AnimatedGeometry at classzone.com

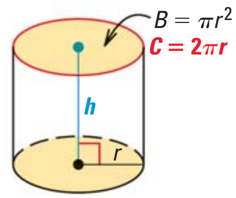
THEOREM — For Your Notebook

THEOREM 12.3 Surface Area of a Right Cylinder

The surface area S of a right cylinder is

$$S = 2B + Ch = 2\pi r^2 + 2\pi rh,$$

where B is the area of a base, C is the circumference of a base, r is the radius of a base, and h is the height.



$S = 2B + Ch = 2\pi r^2 + 2\pi rh$

EXAMPLE 3 Find the surface area of a cylinder

COMPACT DISCS You are wrapping a stack of 20 compact discs using a shrink wrap. Each disc is cylindrical with height 1.2 millimeters and radius 60 millimeters. What is the minimum amount of shrink wrap needed to cover the stack of 20 discs?

Solution

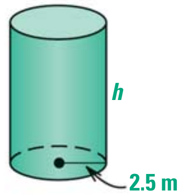
The 20 discs are stacked, so the height of the stack will be $20(1.2) = 24$ mm. The radius is 60 millimeters. The minimum amount of shrink wrap needed will be equal to the surface area of the stack of discs.

$S = 2\pi r^2 + 2\pi rh$	Surface area of a cylinder
$= 2\pi(60)^2 + 2\pi(60)(24)$	Substitute known values.
$\approx 31{,}667$	Use a calculator.

▶ You will need at least 31,667 square millimeters, or about 317 square centimeters of shrink wrap.

EXAMPLE 4 Find the height of a cylinder

Find the height of the right cylinder shown, which has a surface area of 157.08 square meters.



Solution

Substitute known values in the formula for the surface area of a right cylinder and solve for the height h.

$S = 2\pi r^2 + 2\pi rh$	Surface area of a cylinder
$157.08 = 2\pi(2.5)^2 + 2\pi(2.5)h$	Substitute known values.
$157.08 = 12.5\pi + 5\pi h$	Simplify.
$157.08 - 12.5\pi = 5\pi h$	Subtract 12.5π from each side.
$117.81 \approx 5\pi h$	Simplify. Use a calculator.
$7.5 \approx h$	Divide each side by 5π.

▶ The height of the cylinder is about 7.5 meters.

✓ GUIDED PRACTICE for Examples 3 and 4

3. Find the surface area of a right cylinder with height 18 centimeters and radius 10 centimeters. Round your answer to two decimal places.

4. Find the radius of a right cylinder with height 5 feet and surface area 208π square feet.

12.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 9, and 23

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 24, 25, and 26

SKILL PRACTICE

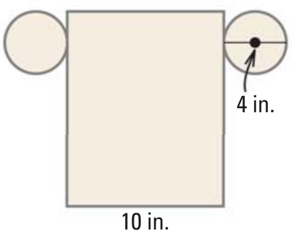
1. **VOCABULARY** Sketch a triangular prism. Identify its *bases*, *lateral faces*, and *lateral edges*.

2. ★ **WRITING** *Explain* how the formula $S = 2B + Ph$ applies to finding the surface area of both a right prism and a right cylinder.

EXAMPLE 1 on p. 803 for Exs. 3–5

USING NETS Find the surface area of the solid formed by the net. Round your answer to two decimal places.

3.



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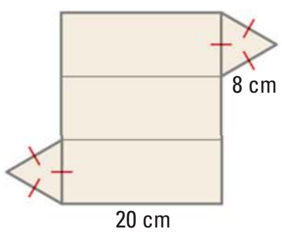

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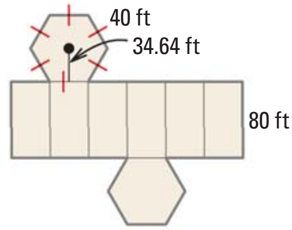
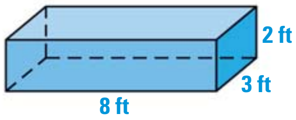


EXAMPLE 2
on p. 804
for Exs. 6–8

SURFACE AREA OF A PRISM **Find the surface area of the right prism. Round your answer to two decimal places.**

6.



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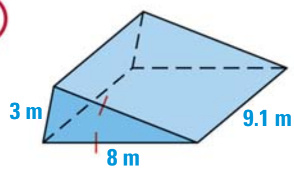

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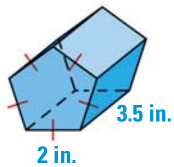


EXAMPLE 3
on p. 805
for Exs. 9–12

SURFACE AREA OF A CYLINDER **Find the surface area of the right cylinder using the given radius r and height h. Round your answer to two decimal places.**

9.

$r = 0.8$ in.
$h = 2$ in.

10.

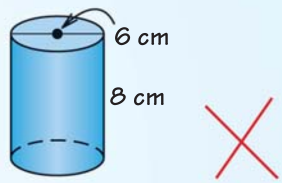
$r = 12$ mm
$h = 40$ mm

11.

$r = 8$ in.
$h = 8$ in.

12. ERROR ANALYSIS *Describe* and correct the error in finding the surface area of the right cylinder.

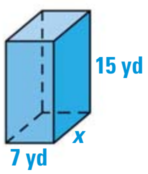
$$S = 2\pi(6^2) + 2\pi(6)(8)$$
$$= 2\pi(36) + 2\pi(48)$$
$$= 168\pi$$
$$\approx 528 \text{ cm}^2$$



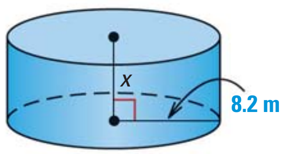
EXAMPLE 4
on p. 806
for Exs. 13–15

xy ALGEBRA **Solve for x given the surface area S of the right prism or right cylinder. Round your answer to two decimal places.**

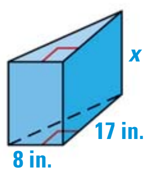
13. $S = 606 \text{ yd}^2$



14. $S = 1097 \text{ m}^2$



15. $S = 616 \text{ in.}^2$



16. SURFACE AREA OF A PRISM A triangular prism with a right triangular base has leg length 9 units and hypotenuse length 15 units. The height of the prism is 8 units. Sketch the prism and find its surface area.

17. ★ MULTIPLE CHOICE The length of each side of a cube is multiplied by 3. What is the change in the surface area of the cube?

Ⓐ The surface area is 3 times the original surface area.

Ⓑ The surface area is 6 times the original surface area.

Ⓒ The surface area is 9 times the original surface area.

Ⓓ The surface area is 27 times the original surface area.

18. SURFACE AREA OF A CYLINDER The radius and height of a right cylinder are each divided by $\sqrt{5}$. What is the change in surface area of the cylinder?

19. **SURFACE AREA OF A PRISM** Find the surface area of a right hexagonal prism with all edges measuring 10 inches.

20. **HEIGHT OF A CYLINDER** Find the height of a cylinder with a surface area of 108π square meters. The radius of the cylinder is twice the height.

21. **CHALLENGE** The *diagonal* of a cube is a segment whose endpoints are vertices that are not on the same face. Find the surface area of a cube with diagonal length 8 units. Round your answer to two decimal places.

PROBLEM SOLVING

EXAMPLE 3
on p. 805
for Ex. 22

22. **BASS DRUM** A bass drum has a diameter of 20 inches and a depth of 8 inches. Find the surface area of the drum.

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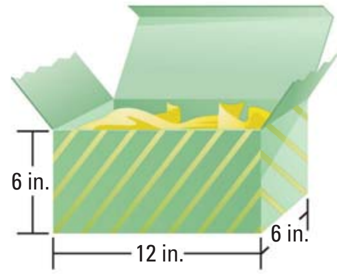
23. **GIFT BOX** An open gift box is shown at the right. When the gift box is closed, it has a length of 12 inches, a width of 6 inches, and a height of 6 inches.



a. What is the minimum amount of wrapping paper needed to cover the closed gift box?

b. Why is the area of the net of the box larger than the amount of paper found in part (a)?

c. When wrapping the box, why would you want more paper than the amount found in part (a)?

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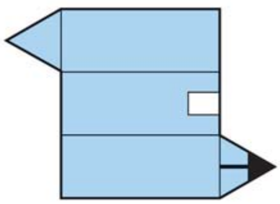
24. ★ **EXTENDED RESPONSE** A right cylinder has a radius of 4 feet and height of 10 feet.

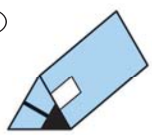
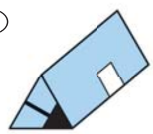
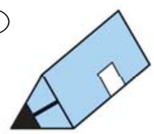
a. Find the surface area of the cylinder.

b. Suppose you can either *double the radius* or *double the height*. Which do you think will create a greater surface area?

c. Check your answer in part (b) by calculating the new surface areas.

25. ★ **MULTIPLE CHOICE** Which three-dimensional figure does the net represent?



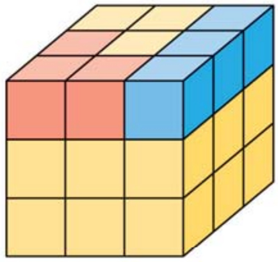
Ⓐ 
Ⓑ 
Ⓒ 
Ⓓ 

○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

26. ★ **SHORT RESPONSE** A company makes two types of recycling bins. One type is a right rectangular prism with length 14 inches, width 12 inches, and height 36 inches. The other type is a right cylinder with radius 6 inches and height 36 inches. Both types of bins are missing a base, so the bins have one open end. Which recycle bin requires more material to make? *Explain*.

27. **MULTI-STEP PROBLEM** Consider a cube that is built using 27 unit cubes as shown at the right.

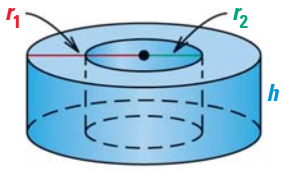


a. Find the surface area of the solid formed when the red unit cubes are removed from the solid shown.

b. Find the surface area of the solid formed when the blue unit cubes are removed from the solid shown.

c. Why are your answers different in parts (a) and (b)? *Explain*.

28. **SURFACE AREA OF A RING** The ring shown is a right cylinder of radius r_1 with a cylindrical hole of r_2. The ring has height h.

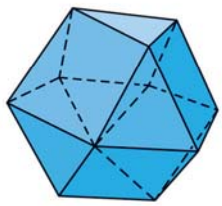


a. Find the surface area of the ring if r_1 is 12 meters, r_2 is 6 meters, and h is 8 meters. Round your answer to two decimal places.

b. Write a formula that can be used to find the surface area S of any cylindrical ring where $0 < r_2 < r_1$.

29. **DRAWING SOLIDS** A cube with edges 1 foot long has a cylindrical hole with diameter 4 inches drilled through one of its faces. The hole is drilled perpendicular to the face and goes completely through to the other side. Draw the figure and find its surface area.

30. **CHALLENGE** A cuboctahedron has 6 square faces and 8 equilateral triangle faces, as shown. A cuboctahedron can be made by slicing off the corners of a cube.



a. Sketch a net for the cuboctahedron.

b. Each edge of a cuboctahedron has a length of 5 millimeters. Find its surface area.

MIXED REVIEW

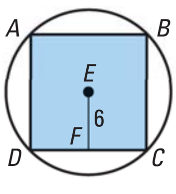
The sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides. *(p. 507)*

31. 1260° 32. 1080° 33. 720° 34. 1800°

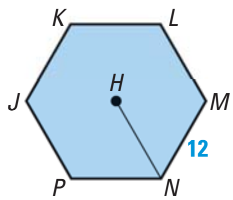
PREVIEW
Prepare for Lesson 12.3 in Exs. 35–37.

Find the area of the regular polygon. *(p. 762)*

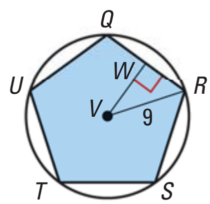
35.



36.



37.



12.3 Surface Area of Pyramids and Cones

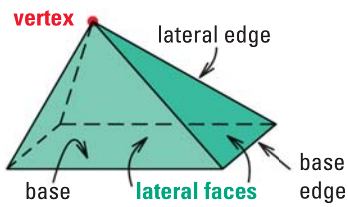
Before You found surface areas of prisms and cylinders.

Now You will find surface areas of pyramids and cones.

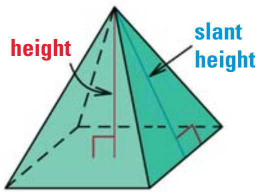
Why? So you can find the surface area of a volcano, as in Ex. 33.

Key Vocabulary
- pyramid
- vertex of a pyramid
- regular pyramid
- slant height
- cone
- vertex of a cone
- right cone
- lateral surface

A **pyramid** is a polyhedron in which the base is a polygon and the lateral faces are triangles with a common vertex, called the **vertex of the pyramid**. The intersection of two lateral faces is a *lateral edge*. The intersection of the base and a lateral face is a *base edge*. The height of the pyramid is the perpendicular distance between the base and the vertex.



Pyramid



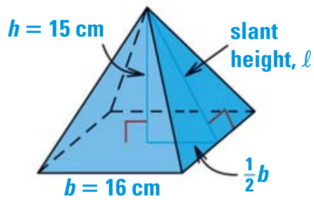
Regular pyramid

NAME PYRAMIDS
Pyramids are classified by the shapes of their bases.

A **regular pyramid** has a regular polygon for a base and the segment joining the vertex and the center of the base is perpendicular to the base. The lateral faces of a regular pyramid are congruent isosceles triangles. The **slant height** of a regular pyramid is the height of a lateral face of the regular pyramid. A nonregular pyramid does not have a slant height.

EXAMPLE 1 Find the area of a lateral face of a pyramid

A regular square pyramid has a height of 15 centimeters and a base edge length of 16 centimeters. Find the area of each lateral face of the pyramid.



Solution

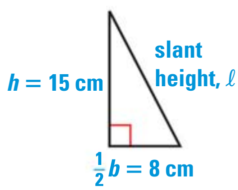
Use the Pythagorean Theorem to find the slant height ℓ.

$\ell^2 = h^2 + \left(\frac{1}{2}b\right)^2$ **Write formula.**

$\ell^2 = 15^2 + 8^2$ **Substitute for h and $\frac{1}{2}b$.**

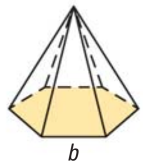
$\ell^2 = 289$ **Simplify.**

$\ell = 17$ **Find the positive square root.**

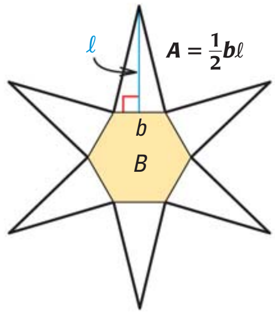


▸ The area of each triangular face is $A = \frac{1}{2}b\ell = \frac{1}{2}(16)(17) = 136$ square centimeters.

SURFACE AREA A regular hexagonal pyramid and its net are shown at the right. Let b represent the length of a base edge, and let ℓ represent the slant height of the pyramid.



The area of each lateral face is $\frac{1}{2}b\ell$ and the perimeter of the base is $P = 6b$. So, the surface area S is as follows.



$S = \text{(Area of base)} + 6\text{(Area of lateral face)}$

$S = B + 6\left(\frac{1}{2}b\ell\right)$ **Substitute.**

$S = B + \frac{1}{2}(6b)\ell$ **Rewrite $6\left(\frac{1}{2}b\ell\right)$ as $\frac{1}{2}(6b)\ell$.**

$S = B + \frac{1}{2}P\ell$ **Substitute P for $6b$.**

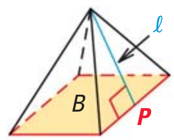
THEOREM *For Your Notebook*

THEOREM 12.4 Surface Area of a Regular Pyramid

The surface area S of a regular pyramid is

$$S = B + \frac{1}{2}P\ell,$$

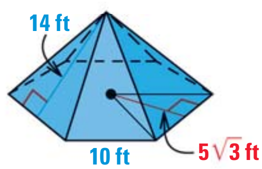
where B is the area of the base, P is the perimeter of the base, and ℓ is the slant height.



$S = B + \frac{1}{2}P\ell$

EXAMPLE 2 Find the surface area of a pyramid

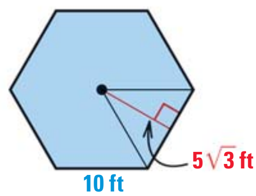
Find the surface area of the regular hexagonal pyramid.



Solution

REVIEW AREA
For help with finding the area of regular polygons, see p. 762.

First, find the area of the base using the formula for the area of a regular polygon, $\frac{1}{2}aP$. The apothem a of the hexagon is $5\sqrt{3}$ feet and the perimeter P is $6 \cdot 10 = 60$ feet. So, the area of the base B is $\frac{1}{2}(5\sqrt{3})(60) = 150\sqrt{3}$ square feet. Then, find the surface area.



$S = B + \frac{1}{2}P\ell$ **Formula for surface area of regular pyramid**

$= 150\sqrt{3} + \frac{1}{2}(60)(14)$ **Substitute known values.**

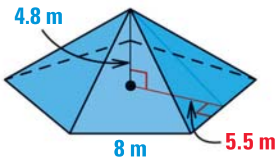
$= 150\sqrt{3} + 420$ **Simplify.**

≈ 679.81 **Use a calculator.**

▶ The surface area of the regular hexagonal pyramid is about 679.81 ft^2.

GUIDED PRACTICE for Examples 1 and 2

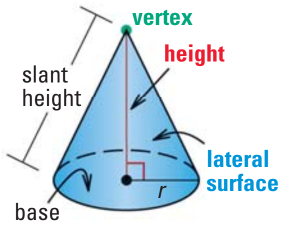
1. Find the area of each lateral face of the regular pentagonal pyramid shown.
2. Find the surface area of the regular pentagonal pyramid shown.



CONES A **cone** has a circular base and a **vertex** that is not in the same plane as the base. The radius of the base is the *radius* of the cone. The height is the perpendicular distance between the vertex and the base.

In a **right cone**, the segment joining the vertex and the center of the base is perpendicular to the base and the slant height is the distance between the vertex and a point on the base edge.

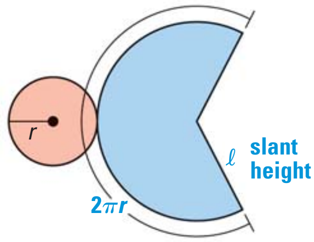
The **lateral surface** of a cone consists of all segments that connect the vertex with points on the base edge.



Right cone

SURFACE AREA When you cut along the slant height and base edge and lay a right cone flat, you get the net shown at the right.

The circular base has an area of πr^2 and the lateral surface is the sector of a circle. You can use a proportion to find the area of the sector, as shown below.



$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Arc length}}{\text{Circumference of circle}}$$ **Set up proportion.**

$$\frac{\text{Area of sector}}{\pi\ell^2} = \frac{2\pi r}{2\pi\ell}$$ **Substitute.**

$$\text{Area of sector} = \pi\ell^2 \cdot \frac{2\pi r}{2\pi\ell}$$ **Multiply each side by $\pi\ell^2$.**

$$\text{Area of sector} = \pi r\ell$$ **Simplify.**

The surface area of a cone is the sum of the base area, πr^2, and the lateral area, $\pi r\ell$. Notice that the quantity $\pi r\ell$ can be written as $\frac{1}{2}(2\pi r)\ell$, or $\frac{1}{2}C\ell$.

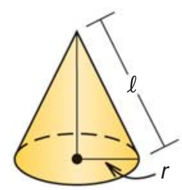
THEOREM *For Your Notebook*

THEOREM 12.5 Surface Area of a Right Cone

The surface area S of a right cone is

$$S = B + \frac{1}{2}C\ell = \pi r^2 + \pi r\ell,$$

where B is the area of the base, C is the circumference of the base, r is the radius of the base, and ℓ is the slant height.



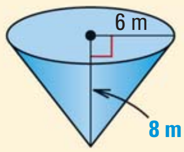
$S = B + \frac{1}{2}C\ell = \pi r^2 + \pi r\ell$

EXAMPLE 3 Standardized Test Practice

What is the surface area of the right cone?

(A) $72\pi \text{ m}^2$ (B) $96\pi \text{ m}^2$

(C) $132\pi \text{ m}^2$ (D) $136\pi \text{ m}^2$



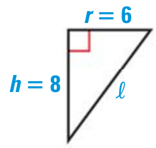
Solution

To find the slant height ℓ of the right cone, use the Pythagorean Theorem.

$\ell^2 = h^2 + r^2$ **Write formula.**

$\ell^2 = 8^2 + 6^2$ **Substitute.**

$\ell = 10$ **Find positive square root.**



ANOTHER WAY
You can use a Pythagorean triple to find ℓ.
$6 = 2 \cdot 3$ and $8 = 2 \cdot 4$, so $\ell = 2 \cdot 5 = 10$.

Use the formula for the surface area of a right cone.

$S = \pi r^2 + \pi r\ell$ **Formula for surface area of a right cone**

$= \pi(6^2) + \pi(6)(10)$ **Substitute.**

$= 96\pi$ **Simplify.**

▶ The correct answer is B. (A) (B) (C) (D)

EXAMPLE 4 Find the lateral area of a cone

TRAFFIC CONE The traffic cone can be approximated by a right cone with radius 5.7 inches and height 18 inches. Find the approximate lateral area of the traffic cone.

Solution

To find the slant height ℓ, use the Pythagorean Theorem.

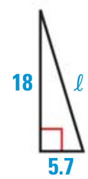
$\ell^2 = 18^2 + (5.7)^2$, so $\ell \approx 18.9$ inches.

Find the lateral area.

Lateral area $= \pi r\ell$ **Write formula.**

$= \pi(5.7)(18.9)$ **Substitute known values.**

≈ 338.4 **Simplify and use a calculator.**

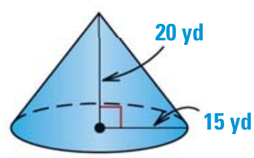


▶ The lateral area of the traffic cone is about 338.4 square inches.

✓ GUIDED PRACTICE for Examples 3 and 4

3. Find the lateral area of the right cone shown.

4. Find the surface area of the right cone shown.



12.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 11, and 29

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, and 31

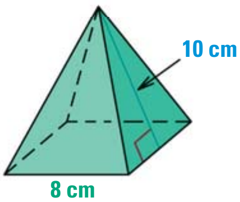
SKILL PRACTICE

1. **VOCABULARY** Draw a regular square pyramid. Label its *height*, *slant height*, and *base*.

2. ★ **WRITING** *Compare* the height and slant height of a right cone.

EXAMPLE 1
on p. 810
for Exs. 3–5

AREA OF A LATERAL FACE **Find the area of each lateral face of the regular pyramid.**

3.


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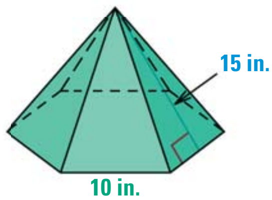
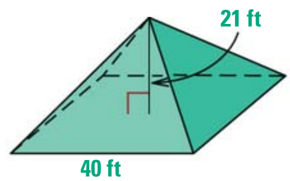
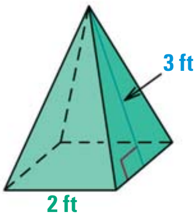
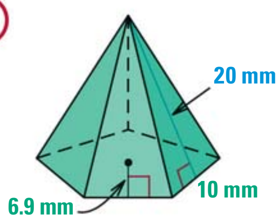


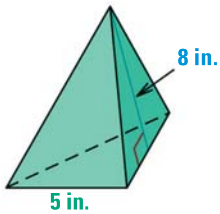
5.


EXAMPLE 2
on p. 811
for Exs. 6–9

SURFACE AREA OF A PYRAMID **Find the surface area of the regular pyramid. Round your answer to two decimal places.**

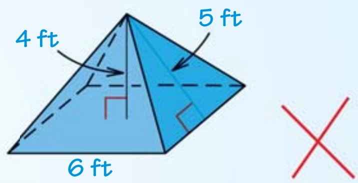
6.


7.


8.


9. **ERROR ANALYSIS** *Describe* and correct the error in finding the surface area of the regular pyramid.

$$S = B + \frac{1}{2}P\ell$$
$$= 6^2 + \frac{1}{2}(24)(4)$$
$$= 84 \text{ ft}^2$$



EXAMPLES 3 and 4
on p. 813
for Exs. 10–17

LATERAL AREA OF A CONE **Find the lateral area of the right cone. Round your answer to two decimal places.**

10.


$r = 7.5$ cm
$h = 25$ cm

11.


$r = 1$ in.
$h = 4$ in.

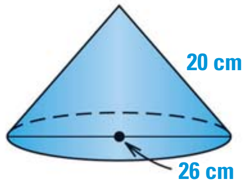
12.


$d = 7$ in.
$h = 1$ ft

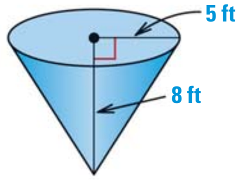
SURFACE AREA OF A CONE **Find the surface area of the right cone. Round your answer to two decimal places.**

13.

14.

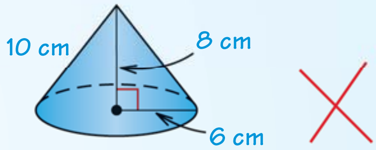


15.



16. **ERROR ANALYSIS** *Describe* and correct the error in finding the surface area of the right cone.

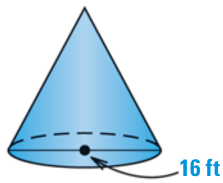
$$S = \pi(r^2) + \pi r^2 \ell$$
$$= \pi(36) + \pi(36)(10)$$
$$= 396\pi \text{ cm}^2$$



17. ★ **MULTIPLE CHOICE** The surface area of the right cone is 200π square feet. What is the slant height of the cone?

Ⓐ 10.5 ft Ⓑ 17 ft

Ⓒ 23 ft Ⓓ 24 ft



VISUAL REASONING **In Exercises 18–21, sketch the described solid and find its surface area. Round your answer to two decimal places.**

18. A right cone has a radius of 15 feet and a slant height of 20 feet.

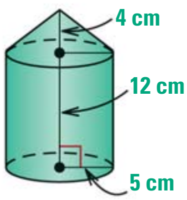
19. A right cone has a diameter of 16 meters and a height of 30 meters.

20. A regular pyramid has a slant height of 24 inches. Its base is an equilateral triangle with a base edge length of 10 inches.

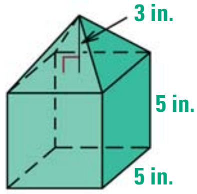
21. A regular pyramid has a hexagonal base with a base edge length of 6 centimeters and a slant height of 9 centimeters.

COMPOSITE SOLIDS **Find the surface area of the solid. The pyramids are regular and the cones are right. Round your answers to two decimal places, if necessary.**

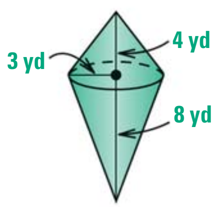
22.



23.



24.



25. **TETRAHEDRON** Find the surface area of a regular tetrahedron with edge length 4 centimeters.

26. **CHALLENGE** A right cone with a base of radius 4 inches and a regular pyramid with a square base both have a slant height of 5 inches. Both solids have the same surface area. Find the length of a base edge of the pyramid. Round your answer to the nearest hundredth of an inch.

PROBLEM SOLVING

EXAMPLE 2 on p. 811 for Ex. 27

27. CANDLES A candle is in the shape of a regular square pyramid with base edge length 6 inches. Its height is 4 inches. Find its surface area.

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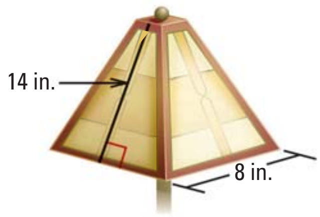
28. LAMPSHADE A glass lampshade is shaped like a regular square pyramid.



a. Approximate the lateral area of the lampshade shown.

b. *Explain* why your answer to part (a) is not the exact lateral area.

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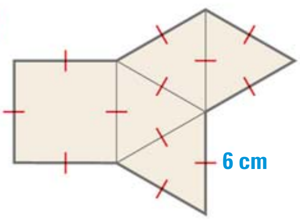
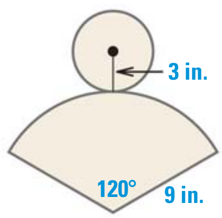
USING NETS **Name the figure that is represented by the net. Then find its surface area. Round your answer to two decimal places.**

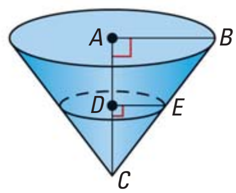
29.



30.



31. ★ **SHORT RESPONSE** In the figure, $AC = 4$, $AB = 3$, and $DC = 2$.

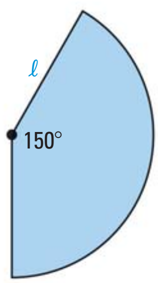


a. Prove $\triangle ABC \sim \triangle DEC$.

b. Find BC, DE, and EC.

c. Find the surface areas of the larger cone and the smaller cone in terms of π. *Compare* the surface areas using a percent.

32. MULTI-STEP PROBLEM The sector shown can be rolled to form the lateral surface of a right cone. The lateral surface area of the cone is 20 square meters.



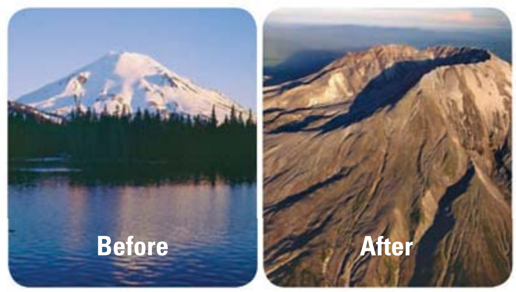
a. Write the formula for the area of a sector.

b. Use the formula in part (a) to find the slant height of the cone. *Explain* your reasoning.

c. Find the radius and height of the cone.

33. VOLCANOES Before 1980, Mount St. Helens was a conic volcano with a height from its base of about 1.08 miles and a base radius of about 3 miles. In 1980, the volcano erupted, reducing its height to about 0.83 mile.

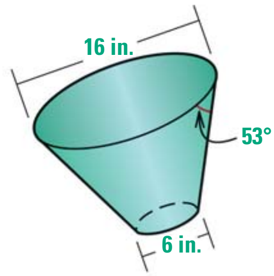
Approximate the lateral area of the volcano after 1980. (*Hint:* The ratio of the radius of the destroyed cone-shaped top to its height is the same as the ratio of the radius of the original volcano to its height.)



○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

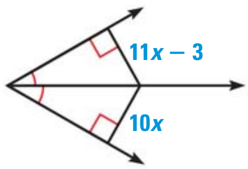
34. **CHALLENGE** An *Elizabethan collar* is used to prevent an animal from irritating a wound. The angle between the opening with a 16 inch diameter and the side of the collar is 53°. Find the surface area of the collar shown.



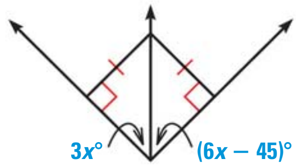
MIXED REVIEW

Find the value of x. *(p. 310)*

35.



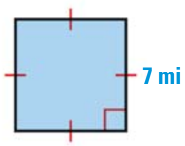
36.



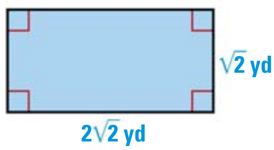
PREVIEW
Prepare for Lesson 12.4 in Exs. 37–39.

In Exercises 37–39, find the area of the polygon. *(pp. 720, 730)*

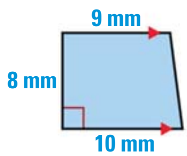
37.



38.



39.

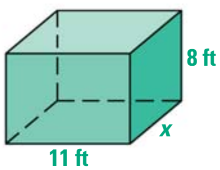


QUIZ *for Lessons 12.1–12.3*

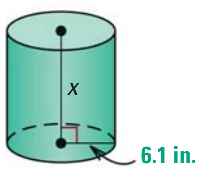
1. A polyhedron has 8 vertices and 12 edges. How many faces does the polyhedron have? *(p. 794)*

Solve for x given the surface area S of the right prism or right cylinder. Round your answer to two decimal places. *(p. 803)*

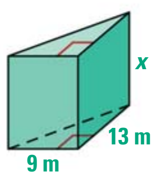
2. $S = 366\text{ ft}^2$



3. $S = 717\text{ in.}^2$

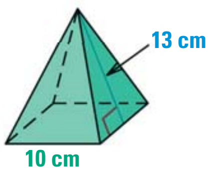


4. $S = 567\text{ m}^2$

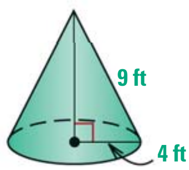


Find the surface area of the regular pyramid or right cone. Round your answer to two decimal places. *(p. 810)*

5.



6.



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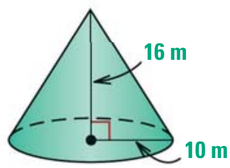

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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Lessons 12.1–12.3

1. **SHORT RESPONSE** Using Euler's Theorem, *explain* why it is not possible for a polyhedron to have 6 vertices and 7 edges.

2. **SHORT RESPONSE** *Describe* two methods of finding the surface area of a rectangular solid.

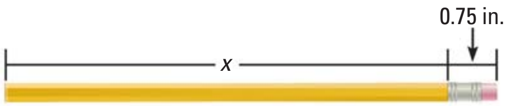
3. **EXTENDED RESPONSE** Some pencils are made from slats of wood that are machined into right regular hexagonal prisms.

   a. The formula for the surface area of a new unsharpened pencil without an eraser is

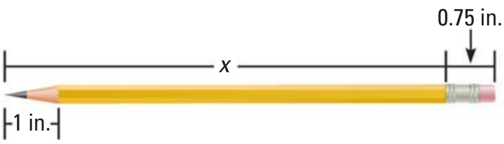
   $$S = 3\sqrt{3}r^2 + 6rh.$$

   Tell what each variable in this formula represents.

   b. After a pencil is painted, a metal band that holds an eraser is wrapped around one end. Write a formula for the surface area of the visible portion of the pencil, shown below.

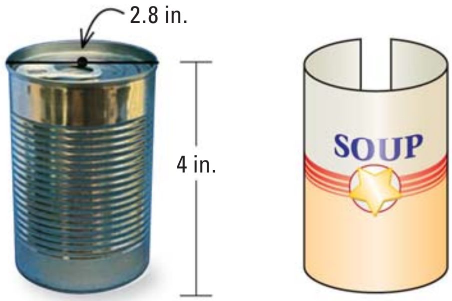


   c. After a pencil is sharpened, the end is shaped like a cone. Write a formula to find the surface area of the visible portion of the pencil, shown below.



   d. Use your formulas from parts (b) and (c) to write a formula for the difference of the surface areas of the two pencils. Define any variables in your formula.

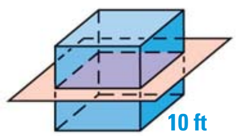
4. **GRIDDED ANSWER** The amount of paper needed for a soup can label is approximately equal to the lateral area of the can. Find the lateral area of the soup can in square inches. Round your answer to two decimal places.



5. **SHORT RESPONSE** If you know the diameter d and slant height ℓ of a right cone, how can you find the surface area of the cone?

6. **OPEN-ENDED** Identify an object in your school or home that is a rectangular prism. Measure its length, width, and height to the nearest quarter inch. Then approximate the surface area of the object.

7. **MULTI-STEP PROBLEM** The figure shows a plane intersecting a cube parallel to its base. The cube has a side length of 10 feet.



   a. Describe the shape formed by the cross section.

   b. Find the perimeter and area of the cross section.

   c. When the cross section is cut along its diagonal, what kind of triangles are formed?

   d. Find the area of one of the triangles formed in part (c).

8. **SHORT RESPONSE** A cone has a base radius of $3x$ units and a height of $4x$ units. The surface area of the cone is 1944π square units. Find the value of x. *Explain* your steps.

12.4 Volume of Prisms and Cylinders

Before You found surface areas of prisms and cylinders.

Now You will find volumes of prisms and cylinders.

Why So you can determine volume of water in an aquarium, as in Ex. 33.

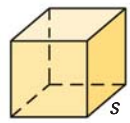
Key Vocabulary
• **volume**

The **volume** of a solid is the number of cubic units contained in its interior. Volume is measured in cubic units, such as cubic centimeters (cm^3).

POSTULATES *For Your Notebook*

POSTULATE 27 Volume of a Cube Postulate

The volume of a cube is the cube of the length of its side.



$V = s^3$

POSTULATE 28 Volume Congruence Postulate

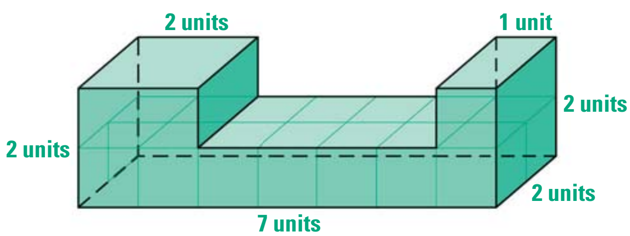
If two polyhedra are congruent, then they have the same volume.

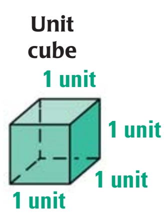
POSTULATE 29 Volume Addition Postulate

The volume of a solid is the sum of the volumes of all its nonoverlapping parts.

EXAMPLE 1 Find the number of unit cubes

3-D PUZZLE **Find the volume of the puzzle piece in cubic units.**





Solution

To find the volume, find the number of unit cubes it contains. Separate the piece into three rectangular boxes as follows:

The *base* is 7 units by 2 units. So, it contains 7 • 2, or 14 unit cubes.

The *upper left box* is 2 units by 2 units. So, it contains 2 • 2, or 4 unit cubes.

The *upper right box* is 1 unit by 2 units. So, it contains 1 • 2, or 2 unit cubes.

▶ By the Volume Addition Postulate, the total volume of the puzzle piece is $14 + 4 + 2 = 20$ cubic units.

VOLUME FORMULAS The volume of any right prism or right cylinder can be found by multiplying the area of its base by its height.

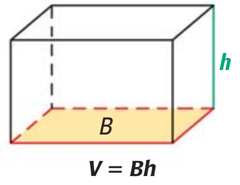
THEOREMS — *For Your Notebook*

THEOREM 12.6 Volume of a Prism

The volume V of a prism is

$V = Bh,$

where B is the area of a base and h is the height.

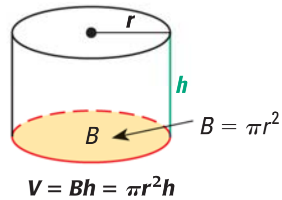


THEOREM 12.7 Volume of a Cylinder

The volume V of a cylinder is

$V = Bh = \pi r^2 h,$

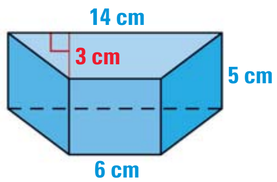
where B is the area of a base, h is the height, and r is the radius of a base.



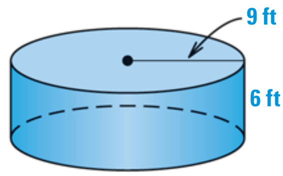
EXAMPLE 2 Find volumes of prisms and cylinders

Find the volume of the solid.

a. Right trapezoidal prism



b. Right cylinder



Solution

REVIEW AREA For help with finding the area of a trapezoid, see p. 730.

a. The area of a base is $\frac{1}{2}(3)(6 + 14) = 30 \text{ cm}^2$ and $h = 5$ cm.

$V = Bh = 30(5) = 150 \text{ cm}^3$

b. The area of the base is $\pi \cdot 9^2$, or $81\pi \text{ ft}^2$. Use $h = 6$ ft to find the volume.

$V = Bh = 81\pi(6) = 486\pi \approx 1526.81 \text{ ft}^3$

EXAMPLE 3 Use volume of a prism

xy ALGEBRA The volume of the cube is 90 cubic inches. Find the value of *x*.

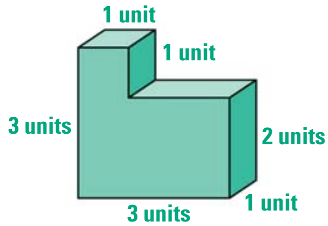
Solution

A side length of the cube is x inches.

$V = x^3$	**Formula for volume of a cube**
$90 \text{ in.}^3 = x^3$	**Substitute for *V*.**
$4.48 \text{ in.} \approx x$	**Find the cube root.**

✓ GUIDED PRACTICE for Examples 1, 2, and 3

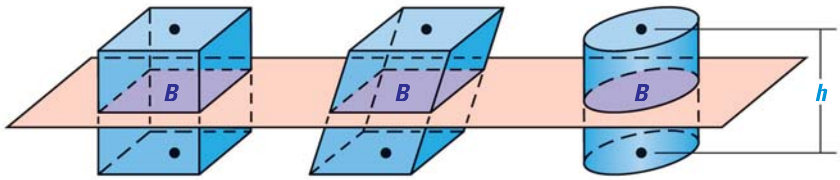
1. Find the volume of the puzzle piece shown in cubic units.



2. Find the volume of a square prism that has a base edge length of 5 feet and a height of 12 feet.

3. The volume of a right cylinder is 684π cubic inches and the height is 18 inches. Find the radius.

USING CAVALIERI'S PRINCIPLE Consider the solids below. All three have equal heights h and equal cross-sectional areas B. Mathematician Bonaventura Cavalieri (1598–1647) claimed that all three of the solids have the same volume. This principle is stated below.



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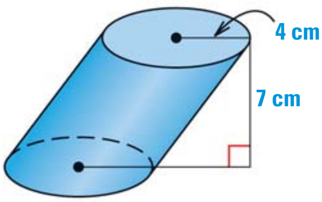
THEOREM — For Your Notebook

THEOREM 12.8 Cavalieri's Principle

If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

EXAMPLE 4 Find the volume of an oblique cylinder

Find the volume of the oblique cylinder.



Solution

Cavalieri's Principle allows you to use Theorem 12.7 to find the volume of the oblique cylinder.

$V = \pi r^2 h$	Formula for volume of a cylinder
$= \pi(4^2)(7)$	Substitute known values.
$= 112\pi$	Simplify.
≈ 351.86	Use a calculator.

▶ The volume of the oblique cylinder is about 351.86 cm^3.

APPLY THEOREMS
Cavalieri's Principle tells you that the volume formulas on page 820 work for oblique prisms and cylinders.

EXAMPLE 5 Solve a real-world problem

SCULPTURE The sculpture is made up of 13 beams. In centimeters, suppose the dimensions of each beam are 30 by 30 by 90. Find its volume.

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ANOTHER WAY
For alternative methods for solving the problem in Example 5, turn to page 826 for the **Problem Solving Workshop**.

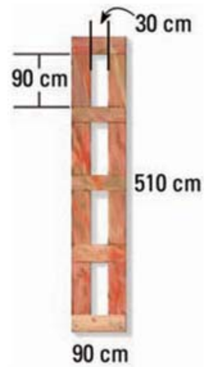
Solution

The area of the base B can be found by subtracting the area of the small rectangles from the area of the large rectangle.

B = Area of large rectangle − 4 • Area of small rectangle

$= 90 \cdot 510 - 4(30 \cdot 90)$

$= 35{,}100 \text{ cm}^2$



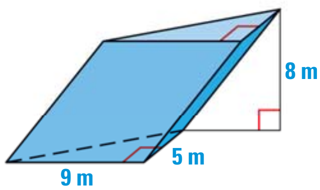
Use the formula for the volume of a prism.

$V = Bh$	**Formula for volume of a prism**
$= 35{,}100(30)$	**Substitute.**
$= 1{,}053{,}000 \text{ cm}^3$	**Simplify.**

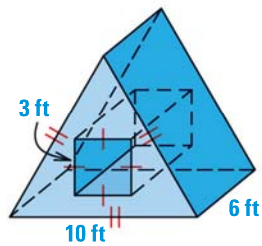
▶ The volume of the sculpture is $1{,}053{,}000 \text{ cm}^3$, or 1.053 m^3.

✓ GUIDED PRACTICE for Examples 4 and 5

4. Find the volume of the oblique prism shown below.



5. Find the volume of the solid shown below.



12.4 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 11, and 29
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 3, 21, and 33

SKILL PRACTICE

1. VOCABULARY In what type of units is the volume of a solid measured?

2. ★ WRITING Two solids have the same surface area. Do they have the same volume? *Explain* your reasoning.

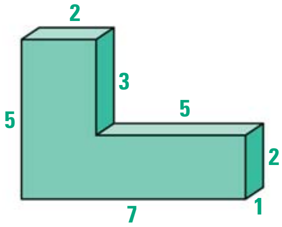
EXAMPLE 1
on p. 819 for Exs. 3–6

3. ★ MULTIPLE CHOICE How many 3 inch cubes can fit completely in a box that is 15 inches long, 9 inches wide, and 3 inches tall?

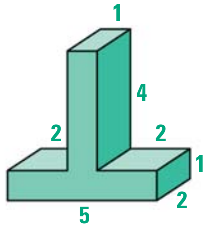
Ⓐ 15 Ⓑ 45 Ⓒ 135 Ⓓ 405

USING UNIT CUBES **Find the volume of the solid by determining how many unit cubes are contained in the solid.**

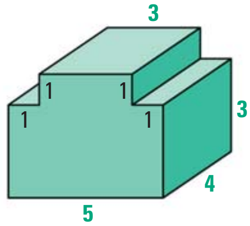
4.



5.



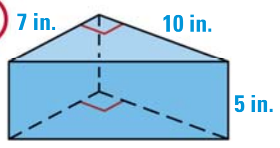
6.



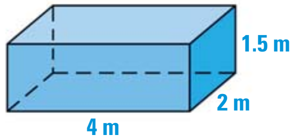
EXAMPLE 2
on p. 820
for Exs. 7–13

FINDING VOLUME **Find the volume of the right prism or right cylinder. Round your answer to two decimal places.**

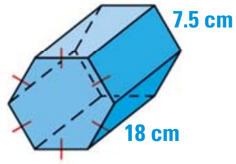
7.



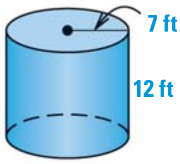
8.



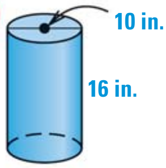
9.



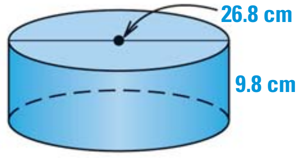
10.



11.



12.



13. **ERROR ANALYSIS** *Describe* and correct the error in finding the volume of a right cylinder with radius 4 feet and height 3 feet.

$$V = 2\pi rh$$
$$= 2\pi(4)(3)$$
$$= 24\pi \text{ ft}^3$$

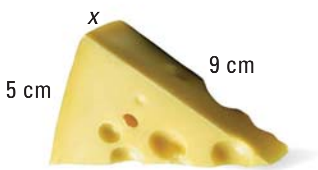
14. **FINDING VOLUME** Sketch a rectangular prism with height 3 feet, width 11 inches, and length 7 feet. Find its volume.

EXAMPLE 3
on p. 820
for Exs. 15–17

ALGEBRA **Find the length x using the given volume V.**

15. $V = 1000 \text{ in.}^3$

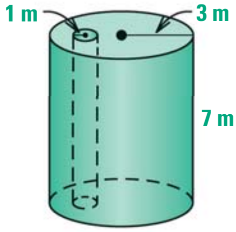
16. $V = 45 \text{ cm}^3$



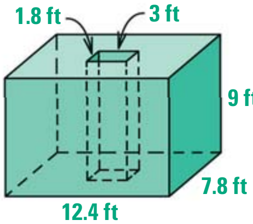
17. $V = 128\pi \text{ in.}^3$

COMPOSITE SOLIDS **Find the volume of the solid. The prisms and cylinders are right. Round your answer to two decimal places, if necessary.**

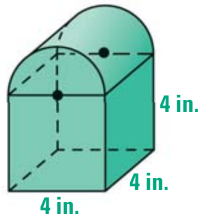
18.



19.



20.



21. ★ **MULTIPLE CHOICE** What is the height of a cylinder with radius 4 feet and volume 64π cubic feet?

(A) 4 feet (B) 8 feet (C) 16 feet (D) 256 feet

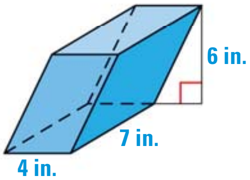
22. **FINDING HEIGHT** The bases of a right prism are right triangles with side lengths of 3 inches, 4 inches, and 5 inches. The volume of the prism is 96 cubic inches. What is the height of the prism?

23. **FINDING DIAMETER** A cylinder has height 8 centimeters and volume 1005.5 cubic centimeters. What is the diameter of the cylinder?

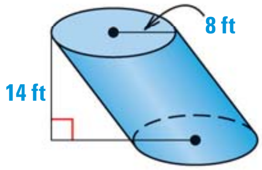
EXAMPLE 4 on p. 821 for Exs. 24–26

VOLUME OF AN OBLIQUE SOLID Use Cavalieri's Principle to find the volume of the oblique prism or cylinder. Round your answer to two decimal places.

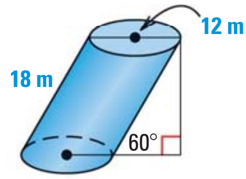
24.



25.



26.



27. **CHALLENGE** The bases of a right prism are rhombuses with diagonals 12 meters and 16 meters long. The height of the prism is 8 meters. Find the lateral area, surface area, and volume of the prism.

PROBLEM SOLVING

EXAMPLE 5 on p. 822 for Exs. 28–30

28. **JEWELRY** The bead at the right is a rectangular prism of length 17 millimeters, width 9 millimeters, and height 5 millimeters. A 3 millimeter wide hole is drilled through the smallest face. Find the volume of the bead.

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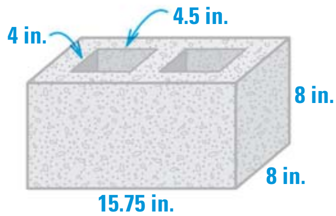
29. **MULTI-STEP PROBLEM** In the concrete block shown, the holes are 8 inches deep.



a. Find the volume of the block using the Volume Addition Postulate.

b. Find the volume of the block using the formula in Theorem 12.6.

c. *Compare* your answers in parts (a) and (b).

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30. **OCEANOGRAPHY** The Blue Hole is a cylindrical trench located on Lighthouse Reef Atoll, an island off the coast of Central America. It is approximately 1000 feet wide and 400 feet deep.

a. Find the volume of the Blue Hole.

b. About how many gallons of water does the Blue Hole contain? ($1 \text{ ft}^3 = 7.48$ gallons)

31. **ARCHITECTURE** A cylindrical column in the building shown has circumference 10 feet and height 20 feet. Find its volume. Round your answer to two decimal places.

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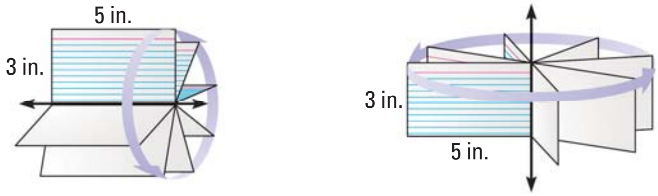
32. **ROTATIONS** A 3 inch by 5 inch index card is rotated around a horizontal line and a vertical line to produce two different solids, as shown. Which solid has a greater volume? *Explain* your reasoning.



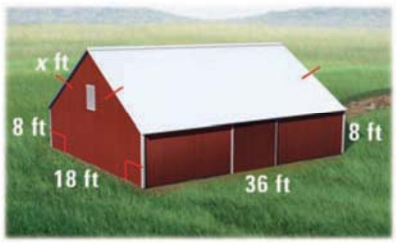
33. ★ **EXTENDED RESPONSE** An aquarium shaped like a rectangular prism has length 30 inches, width 10 inches, and height 20 inches.

a. **Calculate** You fill the aquarium $\frac{3}{4}$ full with water. What is the volume of the water?

b. **Interpret** When you submerge a rock in the aquarium, the water level rises 0.25 inch. Find the volume of the rock.

c. **Interpret** How many rocks of the same size as the rock in part (b) can you place in the aquarium before water spills out?

34. **CHALLENGE** A barn is in the shape of a pentagonal prism with the dimensions shown. The volume of the barn is 9072 cubic feet. Find the dimensions of each half of the roof.

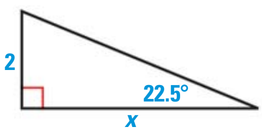


MIXED REVIEW

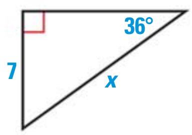
PREVIEW
Prepare for Lesson 12.5 in Exs. 35–40.

Find the value of x. Round your answer to two decimal places. *(pp. 466, 473)*

35.



36.



37.

Find the area of the figure described. Round your answer to two decimal places. *(pp. 755, 762)*

38. A circle with radius 9.5 inches

39. An equilateral triangle with perimeter 78 meters and apothem 7.5 meters

40. A regular pentagon with radius 10.6 inches

PROBLEM SOLVING WORKSHOP
LESSON 12.4

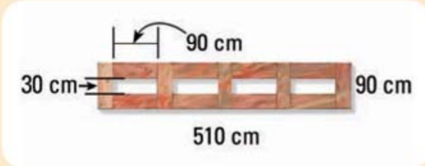
Using ALTERNATIVE METHODS

Another Way to Solve Example 5, page 822

MULTIPLE REPRESENTATIONS In Lesson 12.4, you used volume postulates and theorems to find volumes of prisms and cylinders. Now, you will learn two different ways to solve Example 5 on page 822.

PROBLEM

SCULPTURE The sculpture is made up of 13 beams. In centimeters, suppose the dimensions of each beam are 30 by 30 by 90. Find its volume.



METHOD 1

Finding Volume by Subtracting Empty Spaces One alternative approach is to compute the volume of the prism formed if the holes in the sculpture were filled. Then, to get the correct volume, you must subtract the volume of the four holes.

STEP 1 **Read** the problem. In centimeters, each beam measures 30 by 30 by 90.

The dimensions of the entire sculpture are 30 by 90 by $(4 \cdot 90 + 5 \cdot 30)$, or 30 by 90 by 510.

The dimensions of each hole are equal to the dimensions of one beam.

STEP 2 **Apply** the Volume Addition Postulate. The volume of the sculpture is equal to the volume of the larger prism minus 4 times the volume of a hole.

$$\begin{aligned}\text{Volume } V \text{ of sculpture} &= \text{Volume of larger prism} - \text{Volume of 4 holes}\\ &= 30 \cdot 90 \cdot 510 - 4(30 \cdot 30 \cdot 90)\\ &= 1{,}377{,}000 - 4 \cdot 81{,}000\\ &= 1{,}377{,}000 - 324{,}000\\ &= 1{,}053{,}000\end{aligned}$$

▶ The volume of the sculpture is 1,053,000 cubic centimeters, or 1.053 cubic meters.

STEP 3 **Check** page 822 to verify your new answer, and confirm that it is the same.

METHOD 2 **Finding Volume of Pieces** Another alternative approach is to use the dimensions of each beam.

STEP 1 **Look** at the sculpture. Notice that the sculpture consists of 13 beams, each with the same dimensions. Therefore, the volume of the sculpture will be 13 times the volume of one beam.

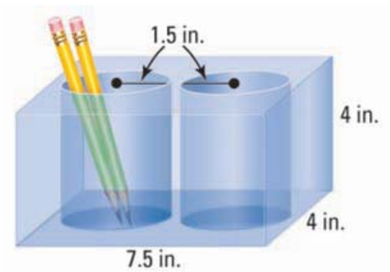
STEP 2 **Write** an expression for the volume of the sculpture and find the volume.

$$\begin{aligned}\text{Volume of sculpture} &= 13(\text{Volume of one beam})\\ &= 13(30 \cdot 30 \cdot 90)\\ &= 13 \cdot 81{,}000\\ &= 1{,}053{,}000\end{aligned}$$

▶ The volume of the sculpture is 1,053,000 cm^3, or 1.053 m^3.

PRACTICE

1. **PENCIL HOLDER** The pencil holder has the dimensions shown.



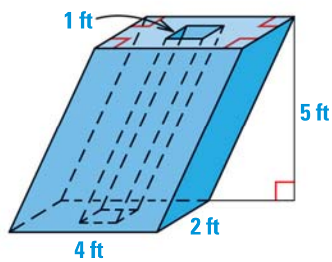
   a. Find its volume using the Volume Addition Postulate.

   b. Use its base area to find its volume.

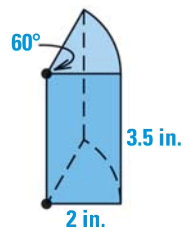
2. **ERROR ANALYSIS** A student solving Exercise 1 claims that the surface area is found by subtracting four times the base area of the cylinders from the surface area of the rectangular prism. *Describe* and correct the student's error.

3. **REASONING** You drill a circular hole of radius r through the base of a cylinder of radius R. Assume the hole is drilled completely through to the other base. You want the volume of the hole to be half the volume of the cylinder. Express r as a function of R.

4. **FINDING VOLUME** Find the volume of the solid shown below. Assume the hole has square cross sections.



5. **FINDING VOLUME** Find the volume of the solid shown to the right.



6. **SURFACE AREA** Refer to the diagram of the sculpture on page 826.

   a. *Describe* a method to find the surface area of the sculpture.

   b. *Explain* why adding the individual surface areas of the beams will give an incorrect result for the total surface area.

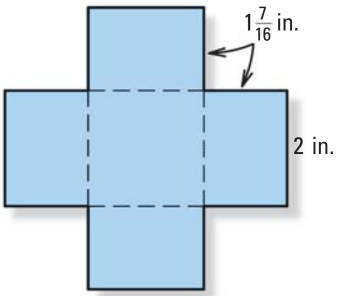
12.5 Investigate the Volume of a Pyramid

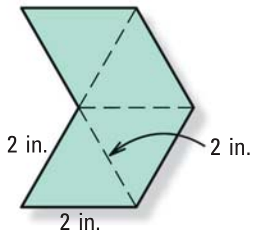
MATERIALS • ruler • poster board • scissors • tape • uncooked rice

QUESTION **How is the volume of a pyramid related to the volume of a prism with the same base and height?**

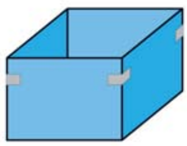
EXPLORE **Compare the volume of a prism and a pyramid using nets**

STEP 1 ***Draw nets*** Use a ruler to draw the two nets shown below on poster board. (Use $1\frac{7}{16}$ inches to approximate $\sqrt{2}$ inches.)





STEP 2 ***Create an open prism and an open pyramid*** Cut out the nets. Fold along the dotted lines to form an open prism and an open pyramid, as shown below. Tape each solid to hold it in place, making sure that the edges do not overlap.



STEP 3 ***Compare volumes*** Fill the pyramid with uncooked rice and pour it into the prism. Repeat this as many times as needed to fill the prism. How many times did you fill the pyramid? What does this tell you about the volume of the solids?

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. *Compare* the area of the base of the pyramid to the area of the base of the prism. Placing the pyramid inside the prism will help. What do you notice?
2. *Compare* the heights of the solids. What do you notice?
3. Make a conjecture about the ratio of the volumes of the solids.
4. Use your conjecture to write a formula for the volume of a pyramid that uses the formula for the volume of a prism.

12.5 Volume of Pyramids and Cones

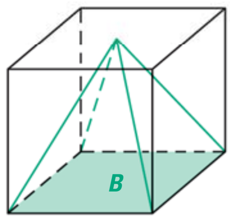
Before You found surface areas of pyramids and cones.

Now You will find volumes of pyramids and cones.

Why? So you can find the edge length of a pyramid, as in Example 2.

Key Vocabulary
- **pyramid,** *p. 810*
- **cone,** *p. 812*
- **volume,** *p. 819*

Recall that the volume of a prism is Bh, where B is the area of a base and h is the height. In the figure at the right, you can see that the volume of a pyramid must be less than the volume of a prism with the same base area and height. As suggested by the Activity on page 828, the volume of a pyramid is one third the volume of a prism.



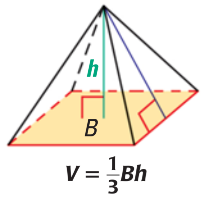
THEOREMS — *For Your Notebook*

THEOREM 12.9 Volume of a Pyramid

The volume V of a pyramid is

$$V = \frac{1}{3}Bh,$$

where B is the area of the base and h is the height.

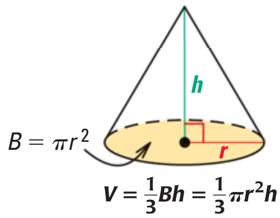


THEOREM 12.10 Volume of a Cone

The volume V of a cone is

$$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h,$$

where B is the area of the base, h is the height, and r is the radius of the base.

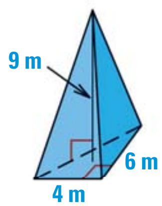


EXAMPLE 1 Find the volume of a solid

APPLY FORMULAS
The formulas given in Theorems 12.9 and 12.10 apply to right and oblique pyramids and cones. This follows from Cavalieri's Principle, stated on page 821.

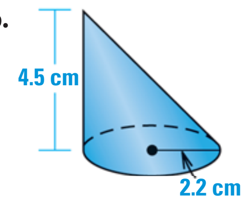
Find the volume of the solid.

a.



$$V = \frac{1}{3}Bh$$
$$= \frac{1}{3}\left(\frac{1}{2} \cdot 4 \cdot 6\right)(9)$$
$$= 36 \text{ m}^3$$

b.



$$V = \frac{1}{3}Bh$$
$$= \frac{1}{3}(\pi r^2)h$$
$$= \frac{1}{3}(\pi \cdot 2.2^2)(4.5)$$
$$= 7.26\pi$$
$$\approx 22.81 \text{ cm}^3$$

EXAMPLE 2 Use volume of a pyramid

xy ALGEBRA Originally, the pyramid had height 144 meters and volume 2,226,450 cubic meters. Find the side length of the square base.

Khafre's Pyramid, Egypt

Solution

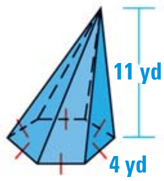
$V = \frac{1}{3}Bh$	**Write formula.**
$2{,}226{,}450 = \frac{1}{3}(x^2)(144)$	**Substitute.**
$6{,}679{,}350 = 144x^2$	**Multiply each side by 3.**
$46{,}384 \approx x^2$	**Divide each side by 144.**
$215 \approx x$	**Find the positive square root.**

▶ Originally, the side length of the base was about 215 meters.

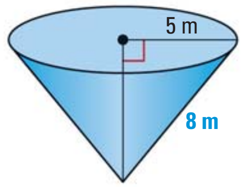
✓ GUIDED PRACTICE for Examples 1 and 2

Find the volume of the solid. Round your answer to two decimal places, if necessary.

1. Hexagonal pyramid



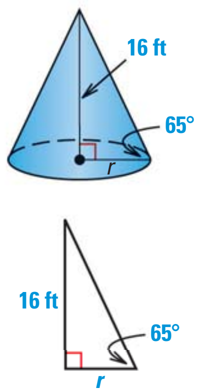
2. Right cone



3. The volume of a right cone is 1350π cubic meters and the radius is 18 meters. Find the height of the cone.

EXAMPLE 3 Use trigonometry to find the volume of a cone

Find the volume of the right cone.



Solution

To find the radius r of the base, use trigonometry.

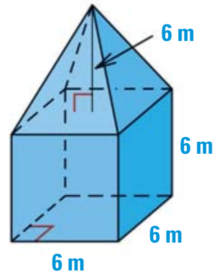
$\tan 65° = \frac{\text{opp.}}{\text{adj.}}$	**Write ratio.**
$\tan 65° = \frac{16}{r}$	**Substitute.**
$r = \frac{16}{\tan 65°} \approx 7.46$	**Solve for *r*.**

Use the formula for the volume of a cone.

$$V = \frac{1}{3}(\pi r^2)h \approx \frac{1}{3}\pi(7.46^2)(16) \approx 932.45 \text{ ft}^3$$

EXAMPLE 4 Find volume of a composite solid

Find the volume of the solid shown.



Solution

Volume of solid	=	Volume of cube	+	Volume of pyramid

$= s^3 + \frac{1}{3}Bh$ **Write formulas.**

$= 6^3 + \frac{1}{3}(6)^2 \cdot 6$ **Substitute.**

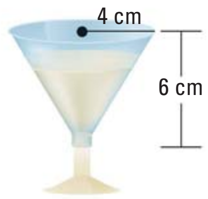
$= 216 + 72$ **Simplify.**

$= 288$ **Add.**

▶ The volume of the solid is 288 cubic meters.

EXAMPLE 5 Solve a multi-step problem

SCIENCE You are using the funnel shown to measure the coarseness of a particular type of sand. It takes 2.8 seconds for the sand to empty out of the funnel. Find the flow rate of the sand in milliliters per second. (1 mL = 1 cm^3)



Solution

STEP 1 **Find** the volume of the funnel using the formula for the volume of a cone.

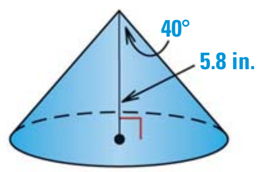
$$V = \frac{1}{3}(\pi r^2)h = \frac{1}{3}\pi(4^2)(6) \approx 101 \text{ cm}^3 = 101 \text{ mL}$$

STEP 2 **Divide** the volume of the funnel by the time it takes the sand to empty out of the funnel.

$$\frac{101 \text{ mL}}{2.8 \text{ s}} \approx 36.07 \text{ mL/s}$$

▶ The flow rate of the sand is about 36.07 milliliters per second.

✓ GUIDED PRACTICE for Examples 3, 4, and 5



4. Find the volume of the cone at the right. Round your answer to two decimal places.

5. A right cylinder with radius 3 centimeters and height 10 centimeters has a right cone on top of it with the same base and height 5 centimeters. Find the volume of the solid. Round your answer to two decimal places.

6. **WHAT IF?** In Example 5, suppose a different type of sand is used that takes 3.2 seconds to empty out of the funnel. Find its flow rate.

12.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 17, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 11, 18, and 35

◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

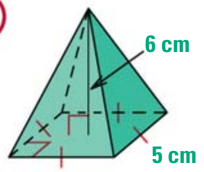
SKILL PRACTICE

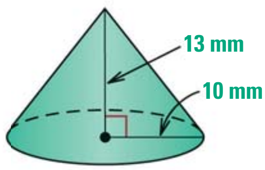
1. **VOCABULARY** *Explain* the difference between a *triangular prism* and a *triangular pyramid*. Draw an example of each.

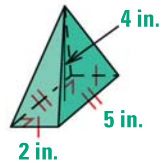
2. ★ **WRITING** *Compare* the volume of a square pyramid to the volume of a square prism with the same base and height as the pyramid.

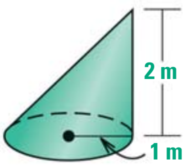
EXAMPLE 1 on p. 829 for Exs. 3–11

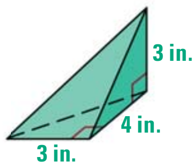
VOLUME OF A SOLID **Find the volume of the solid. Round your answer to two decimal places.**

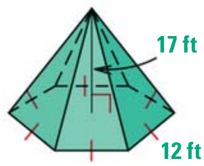
3. 

4. 

5. 

6. 

7. 

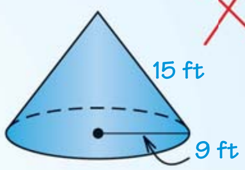
8. 

ERROR ANALYSIS *Describe* **and correct the error in finding the volume of the right cone or pyramid.**

9. $V = \frac{1}{3}\pi(9^2)(15)$

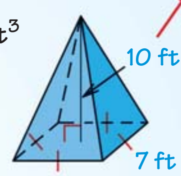
$= 405\pi$

$\approx 1272 \text{ ft}^3$



10. $V = \frac{1}{2}(49)(10)$

$= 245 \text{ ft}^3$



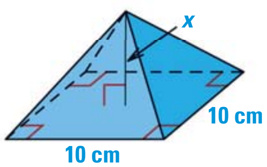
11. ★ **MULTIPLE CHOICE** The volume of a pyramid is 45 cubic feet and the height is 9 feet. What is the area of the base?

Ⓐ 3.87 ft^2 Ⓑ 5 ft^2 Ⓒ 10 ft^2 Ⓓ 15 ft^2

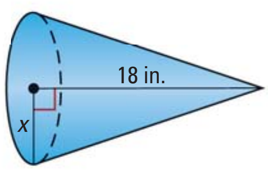
EXAMPLE 2 on p. 830 for Exs. 12–14

xy ALGEBRA **Find the value of x.**

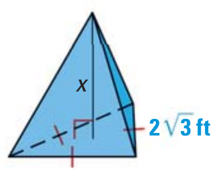
12. Volume $= 200 \text{ cm}^3$



13. Volume $= 216\pi \text{ in.}^3$



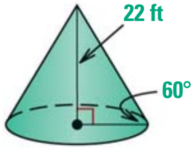
14. Volume $= 7\sqrt{3} \text{ ft}^3$



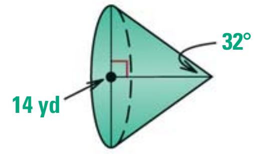
EXAMPLE 3
on p. 830
for Exs. 15–19

VOLUME OF A CONE **Find the volume of the right cone. Round your answer to two decimal places.**

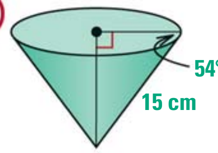
15.



16.



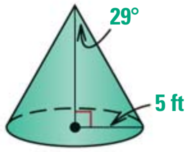
17.



18. ★ **MULTIPLE CHOICE** What is the approximate volume of the cone?

(A) 47.23 ft^3 (B) 236.14 ft^3

(C) 269.92 ft^3 (D) 354.21 ft^3

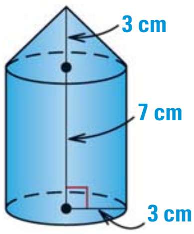


19. **HEIGHT OF A CONE** A cone with a diameter of 8 centimeters has volume 143.6 cubic centimeters. Find the height of the cone. Round your answer to two decimal places.

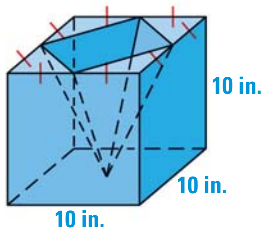
EXAMPLE 4
on p. 831
for Exs. 20–25

COMPOSITE SOLIDS **Find the volume of the solid. The prisms, pyramids, and cones are right. Round your answer to two decimal places.**

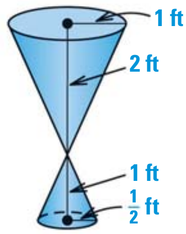
20.



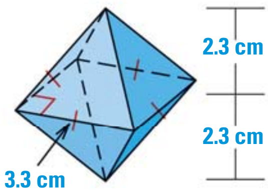
21.



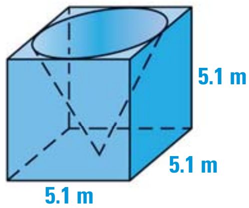
22.



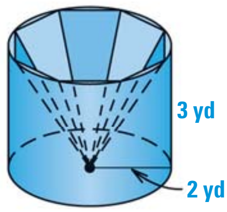
23.



24.



25.



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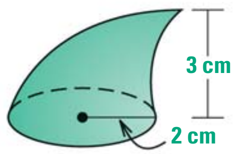
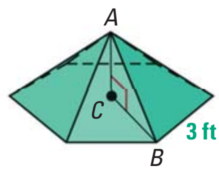
26. **FINDING VOLUME** The figure at the right is a cone that has been warped but whose cross sections still have the same area as a right cone with equal base area and height. Find the volume of this solid.



27. **FINDING VOLUME** Sketch a regular square pyramid with base edge length 5 meters inscribed in a cone with height 7 meters. Find the volume of the cone. *Explain* your reasoning.

28. **CHALLENGE** Find the volume of the regular hexagonal pyramid. Round your answer to the nearest hundredth of a cubic foot. In the diagram, $m\angle ABC = 35°$.



PROBLEM SOLVING

EXAMPLE 5
on p. 831
for Ex. 30

29. CAKE DECORATION A pastry bag filled with frosting has height 12 inches and radius 4 inches. A cake decorator can make 15 flowers using one bag of frosting.

a. How much frosting is in the pastry bag? Round your answer to the nearest cubic inch.

b. How many cubic inches of frosting are used to make each flower?

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POPCORN A snack stand serves a small order of popcorn in a cone-shaped cup and a large order of popcorn in a cylindrical cup.

30. Find the volume of the small cup.

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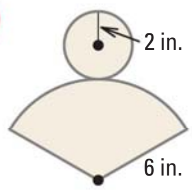
31. How many small cups of popcorn do you have to buy to equal the amount of popcorn in a large container? Do not perform any calculations. *Explain*.

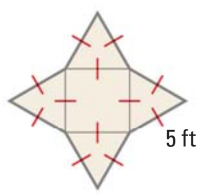
32. Which container gives you more popcorn for your money? *Explain*.

USING NETS In Exercises 33 and 34, use the net to sketch the solid. Then find the volume of the solid. Round your answer to two decimal places.

33.



34.



35. ★ EXTENDED RESPONSE A pyramid has height 10 feet and a square base with side length 7 feet.

a. How does the volume of the pyramid change if the base stays the same and the height is doubled?

b. How does the volume of the pyramid change if the height stays the same and the side length of the base is doubled?

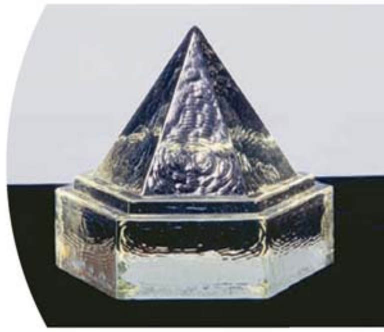
c. *Explain* why your answers to parts (a) and (b) are true for any height and side length.

36. AUTOMATIC FEEDER Assume the automatic pet feeder is a right cylinder on top of a right cone of the same radius. (1 cup = 14.4 in.^{3})

a. Calculate the amount of food in cups that can be placed in the feeder.

b. A cat eats one third of a cup of food, twice per day. How many days will the feeder have food without refilling it?

37. **NAUTICAL PRISMS** The nautical deck prism shown is composed of the following three solids: a regular hexagonal prism with edge length 3.5 inches and height 1.5 inches, a regular hexagonal prism with edge length 3.25 inches and height 0.25 inch, and a regular hexagonal pyramid with edge length 3 inches and height 3 inches. Find the volume of the deck prism.



38. **MULTI-STEP PROBLEM** Calculus can be used to show that the average value of r^2 of a circular cross section of a cone is $\frac{r_b^2}{3}$, where r_b is the radius of the base.

    a. Find the average area of a circular cross section of a cone whose base has radius R.

    b. Show that the volume of the cone can be expressed as follows: V_{cone} = (Average area of a circular cross section) • (Height of cone)

39. **MULTIPLE REPRESENTATIONS** Water flows into a reservoir shaped like a right cone at the rate of 1.8 cubic meters per minute. The height and diameter of the reservoir are equal.

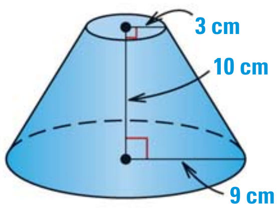
    a. **Using Algebra** As the water flows into the reservoir, the relationship $h = 2r$ is always true. Using this fact, show that $V = \frac{\pi h^3}{12}$.

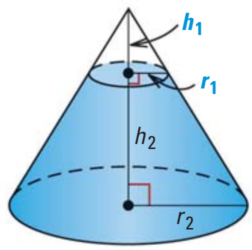
    b. **Making a Table** Make a table that gives the height h of the water after 1, 2, 3, 4, and 5 minutes.

    c. **Drawing a Graph** Make a graph of height versus time. Is there a linear relationship between the height of the water and time? *Explain.*

FRUSTUM A frustum of a cone is the part of the cone that lies between the base and a plane parallel to the base, as shown. Use the information to complete Exercises 40 and 41.

One method for calculating the volume of a frustum is to add the areas of the two bases to their geometric mean, then multiply the result by $\frac{1}{3}$ the height.





40. Use the measurements in the diagram at the left above to calculate the volume of the frustum.

41. Complete parts (a) and (b) below to write a formula for the volume of a frustum that has bases with radii r_1 and r_2 and a height h_2.

    a. Use similar triangles to find the value of h_1 in terms of h_2, r_1, and r_2.

    b. Write a formula in terms of h_2, r_1, and r_2 for V_{frustum} = (Original volume) − (Removed volume).

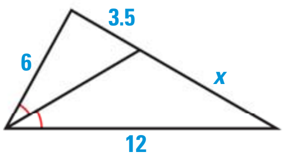
    c. Show that your formula in part (b) is equivalent to the formula involving geometric mean described above.

42. **CHALLENGE** A square pyramid is inscribed in a right cylinder so that the base of the pyramid is on a base of the cylinder, and the vertex of the pyramid is on the other base of the cylinder. The cylinder has radius 6 feet and height 12 feet. Find the volume of the pyramid. Round your answer to two decimal places.

MIXED REVIEW

In Exercises 43–45, find the value of *x*. *(p. 397)*

43.



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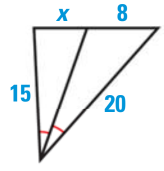

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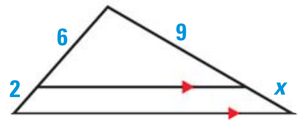
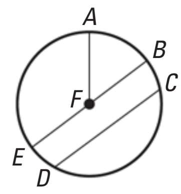


PREVIEW
Prepare for Lesson 12.6 in Exs. 46–52.

46. Copy the diagram at the right. Name a radius, diameter, and chord. *(p. 651)*

47. Name a minor arc of $\odot F$. *(p. 659)*

48. Name a major arc of $\odot F$. *(p. 659)*



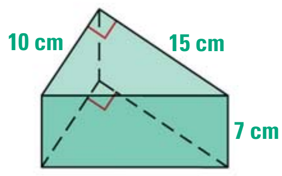
Find the area of the circle with the given radius *r*, diameter *d*, or circumference *C*. *(p. 755)*

49. $r = 3$ m **50.** $d = 7$ mi **51.** $r = 0.4$ cm **52.** $C = 8\pi$ in.

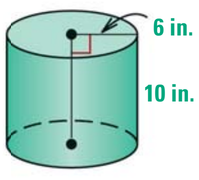
QUIZ *for Lessons 12.4–12.5*

Find the volume of the figure. Round your answer to two decimal places, if necessary. *(pp. 819, 829)*

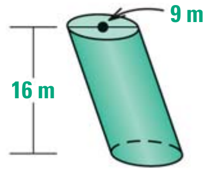
1.



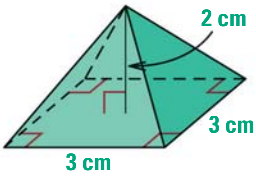
2.



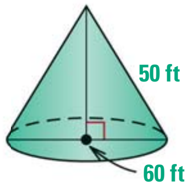
3.



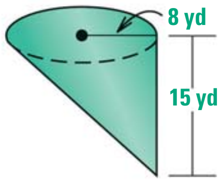
4.



5.



6.



7. Suppose you fill up a cone-shaped cup with water. You then pour the water into a cylindrical cup with the same radius. Both cups have a height of 6 inches. Without doing any calculation, determine how high the water level will be in the cylindrical cup once all of the water is poured into it. *Explain* your reasoning. *(p. 829)*

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Keystrokes

12.5 Minimize Surface Area

MATERIALS • computer

QUESTION How can you find the minimum surface area of a solid with a given volume?

A manufacturer needs a cylindrical container with a volume of 72 cubic centimeters. You have been asked to find the dimensions of such a container so that it has a minimum surface area.

EXAMPLE Use a spreadsheet

STEP 1 ***Make a table*** Make a table with the four column headings shown in Step 4. The first column is for the given volume V. In cell A2, enter 72. In cell A3, enter the formula "=A2".

STEP 2 ***Enter radius*** The second column is for the radius r. Cell B2 stores the starting value for r. So, enter 2 into cell B2. In cell B3, use the formula "=B2 + 0.05" to increase r in increments of 0.05 centimeter.

STEP 3 ***Enter formula for height*** The third column is for the height. In cell C2, enter the formula "=A2/(PI()*B2^2)". *Note:* Your spreadsheet might use a different expression for π.

STEP 4 ***Enter formula for surface area*** The fourth column is for the surface area. In cell D2, enter the formula "=2*PI()*B2^2+2*PI()*B2*C2".

	A	B	C	D
1	Volume V	Radius r	Height $=V/(\pi r^2)$	Surface area $S=2\pi r^2+2\pi r$
2	72.00	2.00	=A2/(PI()*B2^2)	=2*PI()*B2^2+2*PI()*B2*C2
3	=A2	=B2+0.05		

STEP 5 ***Create more rows*** Use the *Fill Down* feature to create more rows. Rows 3 and 4 of your spreadsheet should resemble the one below.

	A	B	C	D
...				
3	72.00	2.05	5.45	96.65
4	72.00	2.10	5.20	96.28

PRACTICE

1. From the data in your spreadsheet, which dimensions yield a minimum surface area for the given volume? *Explain* how you know.

2. **WHAT IF?** Find the dimensions that give the minimum surface area if the volume of a cylinder is instead 200π cubic centimeters.

12.6 Surface Area and Volume of Spheres

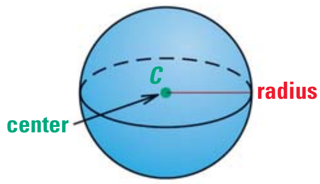
Before You found surface areas and volumes of polyhedra.

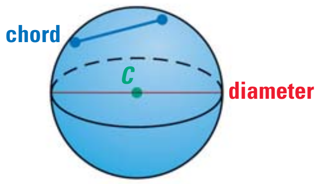
Now You will find surface areas and volumes of spheres.

Why? So you can find the volume of a tennis ball, as in Ex. 33.

Key Vocabulary
- **sphere**
  center, radius, chord, diameter
- **great circle**
- **hemispheres**

A **sphere** is the set of all points in space equidistant from a given point. This point is called the **center** of the sphere. A **radius** of a sphere is a segment from the center to a point on the sphere. A **chord** of a sphere is a segment whose endpoints are on the sphere. A **diameter** of a sphere is a chord that contains the center.





As with circles, the terms radius and diameter also represent distances, and the diameter is twice the radius.

THEOREM *For Your Notebook*

THEOREM 12.11 Surface Area of a Sphere

The surface area S of a sphere is

$$S = 4\pi r^2,$$

where r is the radius of the sphere.

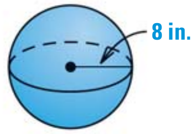
USE FORMULAS
If you understand how a formula is derived, then it will be easier for you to remember the formula.

SURFACE AREA FORMULA To understand how the formula for the surface area of a sphere is derived, think of a baseball. The surface area of a baseball is sewn from two congruent shapes, each of which resembles two joined circles, as shown.

So, the entire covering of the baseball consists of four circles, each with radius r. The area A of a circle with radius r is $A = \pi r^2$. So, the area of the covering can be approximated by $4\pi r^2$. This is the formula for the surface area of a sphere.

EXAMPLE 1 Find the surface area of a sphere

Find the surface area of the sphere.



Solution

$S = 4\pi r^2$ — Formula for surface area of a sphere

$= 4\pi(8^2)$ — Substitute 8 for *r*.

$= 256\pi$ — Simplify.

≈ 804.25 — Use a calculator.

▶ The surface area of the sphere is about 804.25 square inches.

EXAMPLE 2 Standardized Test Practice

The surface area of the sphere is 20.25π square centimeters. What is the diameter of the sphere?

Ⓐ 2.25 cm   Ⓑ 4.5 cm

Ⓒ 5.5 cm   Ⓓ 20.25 cm

Solution

$S = 4\pi r^2$ — Formula for surface area of a sphere

$20.25\pi = 4\pi r^2$ — Substitute 20.25π for *S*.

$5.0625 = r^2$ — Divide each side by 4π.

$2.25 = r$ — Find the positive square root.

The diameter of the sphere is $2r = 2 \cdot 2.25 = 4.5$ centimeters.

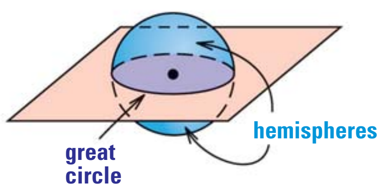
▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

AVOID ERRORS
Be sure to multiply the value of *r* by 2 to find the diameter.

✓ GUIDED PRACTICE for Examples 1 and 2

1. The diameter of a sphere is 40 feet. Find the surface area of the sphere.
2. The surface area of a sphere is 30π square meters. Find the radius of the sphere.

GREAT CIRCLES If a plane intersects a sphere, the intersection is either a single point or a circle. If the plane contains the center of the sphere, then the intersection is a **great circle** of the sphere. The circumference of a great circle is the circumference of the sphere. Every great circle of a sphere separates the sphere into two congruent halves called **hemispheres**.



EXAMPLE 3 Use the circumference of a sphere

EXTREME SPORTS In a sport called *sphereing*, a person rolls down a hill inside an inflatable ball surrounded by another ball. The diameter of the outer ball is 12 feet. Find the surface area of the outer ball.

Solution

The diameter of the outer sphere is 12 feet, so the radius is $\frac{12}{2} = 6$ feet.

Use the formula for the surface area of a sphere.

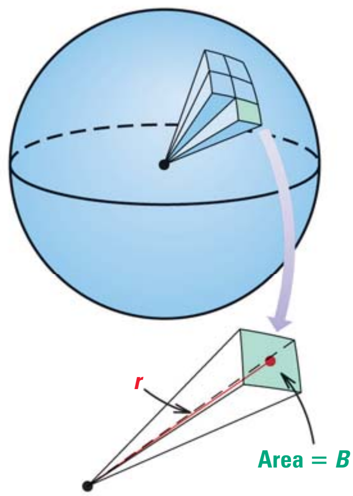
$$S = 4\pi r^2 = 4\pi(6^2) = 144\pi$$

▶ The surface area of the outer ball is 144π, or about 452.39 square feet.

✓ GUIDED PRACTICE for Example 3

3. In Example 3, the circumference of the inner ball is 6π feet. Find the surface area of the inner ball. Round your answer to two decimal places.

VOLUME FORMULA Imagine that the interior of a sphere with radius r is approximated by n pyramids, each with a base area of B and a height of r. The volume of each pyramid is $\frac{1}{3}Br$ and the sum of the base areas is nB. The surface area of the sphere is approximately equal to nB, or $4\pi r^2$. So, you can approximate the volume V of the sphere as follows.



$V \approx n\left(\frac{1}{3}Br\right)$	Each pyramid has a volume of $\frac{1}{3}Br$.
$\approx \frac{1}{3}(nB)r$	Regroup factors.
$= \frac{1}{3}(4\pi r^2)r$	Substitute $4\pi r^2$ for nB.
$= \frac{4}{3}\pi r^3$	Simplify.

THEOREM — For Your Notebook

THEOREM 12.12 Volume of a Sphere

The volume V of a sphere is

$$V = \frac{4}{3}\pi r^3,$$

where r is the radius of the sphere.

$V = \frac{4}{3}\pi r^3$

EXAMPLE 4 Find the volume of a sphere

The soccer ball has a diameter of 9 inches. Find its volume.

Solution

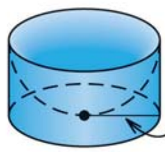
The diameter of the ball is 9 inches, so the radius is $\frac{9}{2} = 4.5$ inches.

$V = \frac{4}{3}\pi r^3$	**Formula for volume of a sphere**
$= \frac{4}{3}\pi(4.5)^3$	**Substitute.**
$= 121.5\pi$	**Simplify.**
≈ 381.70	**Use a calculator.**

▶ The volume of the soccer ball is 121.5π, or about 381.70 cubic inches.

EXAMPLE 5 Find the volume of a composite solid

Find the volume of the composite solid.



Solution

Volume of solid = Volume of cylinder − Volume of hemisphere

$= \pi r^2 h - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$	**Formulas for volume**
$= \pi(2)^2(2) - \frac{2}{3}\pi(2)^3$	**Substitute.**
$= 8\pi - \frac{2}{3}(8\pi)$	**Multiply.**
$= \frac{24}{3}\pi - \frac{16}{3}\pi$	**Rewrite fractions using least common denominator.**
$= \frac{8}{3}\pi$	**Simplify.**

▶ The volume of the solid is $\frac{8}{3}\pi$, or about 8.38 cubic inches.

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✓ GUIDED PRACTICE for Examples 4 and 5

4. The radius of a sphere is 5 yards. Find the volume of the sphere. Round your answer to two decimal places.

5. A solid consists of a hemisphere of radius 1 meter on top of a cone with the same radius and height 5 meters. Find the volume of the solid. Round your answer to two decimal places.

12.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 13, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 20, 28, 33, and 34

SKILL PRACTICE

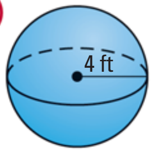
1. VOCABULARY What are the formulas for finding the surface area of a sphere and the volume of a sphere?

2. ★ WRITING When a plane intersects a sphere, what point in the sphere must the plane contain for the intersection to be a great circle? *Explain.*

EXAMPLE 1 on p. 839 for Exs. 3–5

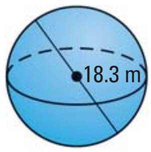
FINDING SURFACE AREA **Find the surface area of the sphere. Round your answer to two decimal places.**

3.



4.

5.



EXAMPLE 2 on p. 839 for Ex. 6

6. ★ MULTIPLE CHOICE What is the approximate radius of a sphere with surface area 32π square meters?

Ⓐ 2 meters Ⓑ 2.83 meters Ⓒ 4.90 meters Ⓓ 8 meters

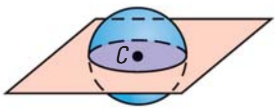
EXAMPLE 3 on p. 840 for Exs. 7–11

USING A GREAT CIRCLE **In Exercises 7–9, use the sphere below. The center of the sphere is C and its circumference is 9.6π inches.**

7. Find the radius of the sphere.

8. Find the diameter of the sphere.

9. Find the surface area of one hemisphere.



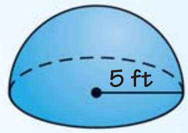
10. ERROR ANALYSIS *Describe* and correct the error in finding the surface area of a hemisphere with radius 5 feet.

$S = 4\pi r^2$

$= 4\pi(5)^2$

$= 100\pi$

$\approx 314.16 \text{ ft}^2$



11. GREAT CIRCLE The circumference of a great circle of a sphere is 48.4π centimeters. What is the surface area of the sphere?

EXAMPLE 4 on p. 841 for Exs. 12–15

FINDING VOLUME **Find the volume of the sphere using the given radius r or diameter d. Round your answer to two decimal places.**

12. $r = 6$ in.

13. $r = 40$ mm

14. $d = 5$ cm

15. ERROR ANALYSIS *Describe* and correct the error in finding the volume of a sphere with diameter 16 feet.

$$V = \frac{4}{3}\pi r^2$$
$$= \frac{4}{3}\pi(8)^2$$
$$= 85.33\pi \approx 268.08 \text{ ft}^2$$

USING VOLUME In Exercises 16–18, find the radius of a sphere with the given volume *V*. Round your answers to two decimal places.

16. $V = 1436.76 \text{ m}^3$

17. $V = 91.95 \text{ cm}^3$

18. $V = 20{,}814.37 \text{ in.}^3$

19. FINDING A DIAMETER The volume of a sphere is 36π cubic feet. What is the diameter of the sphere?

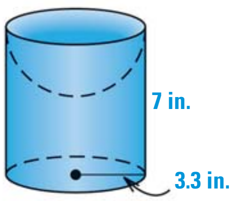
20. ★ **MULTIPLE CHOICE** Let V be the volume of a sphere, S be the surface area of the sphere, and r be the radius of the sphere. Which equation represents the relationship between these three measures?

Ⓐ $V = \frac{rS}{3}$ Ⓑ $V = \frac{r^2S}{3}$ Ⓒ $V = \frac{3}{2}rS$ Ⓓ $V = \frac{3}{2}r^2S$

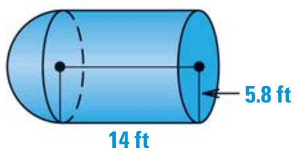
EXAMPLE 5
on p. 841
for Exs. 21–23

COMPOSITE SOLIDS Find the surface area and the volume of the solid. The cylinders and cones are right. Round your answer to two decimal places.

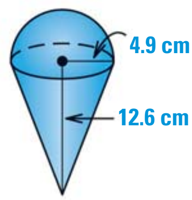
21.



22.



23.



USING A TABLE Copy and complete the table below. Leave your answers in terms of π.

	Radius of sphere	Circumference of great circle	Surface area of sphere	Volume of sphere
24.	10 ft	?	?	?
25.	?	26π in.	?	?
26.	?	?	$2500\pi \text{ cm}^2$	?
27.	?	?	?	$12{,}348\pi \text{ m}^3$

28. ★ **MULTIPLE CHOICE** A sphere is inscribed in a cube of volume 64 cubic centimeters. What is the surface area of the sphere?

Ⓐ $4\pi \text{ cm}^2$ Ⓑ $\frac{32}{3}\pi \text{ cm}^2$ Ⓒ $16\pi \text{ cm}^2$ Ⓓ $64\pi \text{ cm}^2$

29. CHALLENGE The volume of a right cylinder is the same as the volume of a sphere. The radius of the sphere is 1 inch.

a. Give three possibilities for the dimensions of the cylinder.

b. Show that the surface area of the cylinder is sometimes greater than the surface area of the sphere.

PROBLEM SOLVING

EXAMPLE 5 on p. 841 for Ex. 30

30. GRAIN SILO A grain silo has the dimensions shown. The top of the silo is a hemispherical shape. Find the volume of the grain silo.

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31. GEOGRAPHY The circumference of Earth is about 24,855 miles. Find the surface area of the Western Hemisphere of Earth.

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32. MULTI-STEP PROBLEM A ball has volume 1427.54 cubic centimeters.

a. Find the radius of the ball. Round your answer to two decimal places.

b. Find the surface area of the ball. Round your answer to two decimal places.

33. ★ SHORT RESPONSE Tennis balls are stored in a cylindrical container with height 8.625 inches and radius 1.43 inches.

a. The circumference of a tennis ball is 8 inches. Find the volume of a tennis ball.

b. There are 3 tennis balls in the container. Find the amount of space within the cylinder not taken up by the tennis balls.

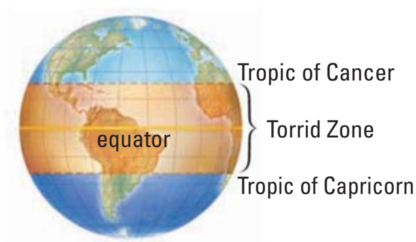
34. ★ EXTENDED RESPONSE A partially filled balloon has circumference 27π centimeters. Assume the balloon is a sphere.

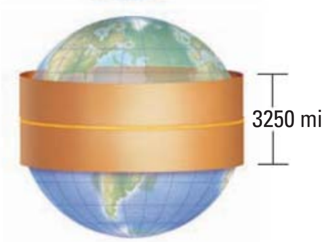
a. Calculate Find the volume of the balloon.

b. Predict Suppose you double the radius by increasing the air in the balloon. *Explain* what you expect to happen to the volume.

c. Justify Find the volume of the balloon with the radius doubled. Was your prediction from part (b) correct? What is the ratio of this volume to the original volume?

35. GEOGRAPHY The Torrid Zone on Earth is the area between the Tropic of Cancer and the Tropic of Capricorn, as shown. The distance between these two tropics is about 3250 miles. You can think of this distance as the height of a cylindrical belt around Earth at the equator, as shown.





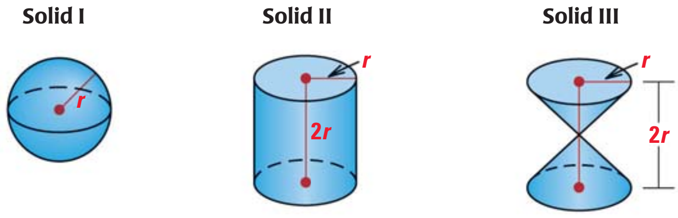
a. Estimate the surface area of the Torrid Zone and the surface area of Earth. (Earth's radius is about 3963 miles at the equator.)

b. A meteorite is equally likely to hit anywhere on Earth. Estimate the probability that a meteorite will land in the Torrid Zone.

○ = WORKED-OUT SOLUTIONS on p. WS1

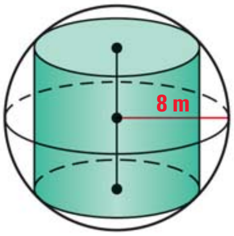
★ = STANDARDIZED TEST PRACTICE

36. **REASONING** List the following three solids in order of (a) surface area, and (b) volume, from least to greatest.



37. **ROTATION** A circle with diameter 18 inches is rotated about its diameter. Find the surface area and the volume of the solid formed.

38. **TECHNOLOGY** A cylinder with height $2x$ is inscribed in a sphere with radius 8 meters. The center of the sphere is the midpoint of the altitude that joins the centers of the bases of the cylinder.



a. Show that the volume V of the cylinder is $2\pi x(64 - x^2)$.

b. Use a graphing calculator to graph $V = 2\pi x(64 - x^2)$ for values of x between 0 and 8. Find the value of x that gives the maximum value of V.

c. Use the value for x from part (b) to find the maximum volume of the cylinder.

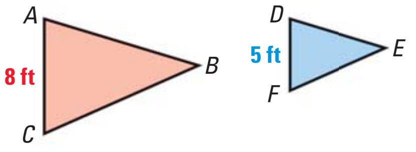
39. **CHALLENGE** A sphere with radius 2 centimeters is inscribed in a right cone with height 6 centimeters. Find the surface area and the volume of the cone.

MIXED REVIEW

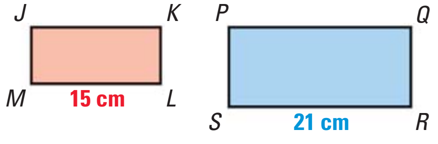
PREVIEW
Prepare for Lesson 12.7 in Exs. 40–41.

In Exercises 40 and 41, the polygons are similar. Find the ratio (red to blue) of their areas. Find the unknown area. Round your answer to two decimal places. *(p. 737)*

40. Area of $\triangle ABC = 42\text{ ft}^2$
Area of $\triangle DEF = \underline{\ ?\ }$

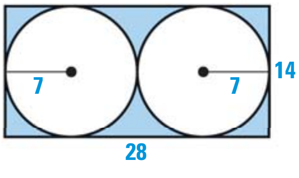


41. Area of $PQRS = 195\text{ cm}^2$
Area of $JKLM = \underline{\ ?\ }$

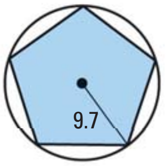


Find the probability that a randomly chosen point in the figure lies in the shaded region. *(p. 771)*

42.



43.



44. A cone is inscribed in a right cylinder with volume 330 cubic units. Find the volume of the cone. *(pp. 819, 829)*

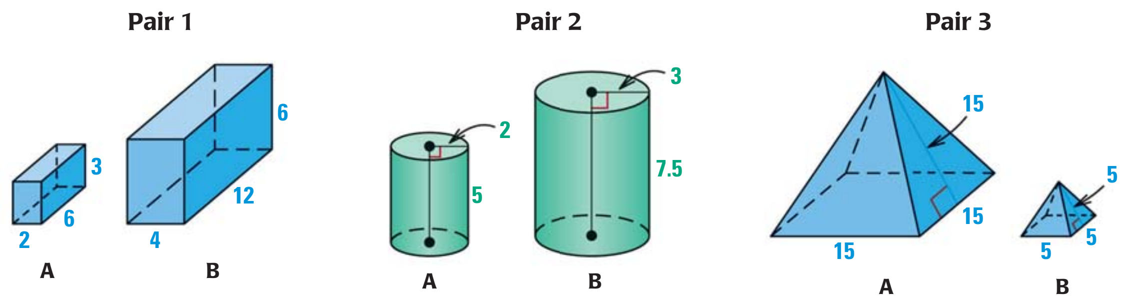
12.7 Investigate Similar Solids

MATERIALS • paper • pencil

QUESTION How are the surface areas and volumes of similar solids related?

EXPLORE Compare the surface areas and volumes of similar solids

The solids shown below are *similar*.



STEP 1 ***Make a table*** Copy and complete the table below.

	Scale factor of Solid A to Solid B	Surface area of Solid A, S_A	Surface area of Solid B, S_B	$\frac{S_A}{S_B}$
Pair 1	$\frac{1}{2}$	?	?	?
Pair 2	?	?	63π	?
Pair 3	?	?	?	$\frac{9}{1}$

STEP 2 ***Insert columns*** Insert columns for V_A, V_B, and $\frac{V_A}{V_B}$. Use the dimensions of the solids to find V_A, the volume of Solid A, and V_B, the volume of Solid B. Then find the ratio of these volumes.

STEP 3 ***Compare ratios*** Compare the ratios $\frac{S_A}{S_B}$ and $\frac{V_A}{V_B}$ to the scale factor.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Make a conjecture about how the surface areas and volumes of similar solids are related to the scale factor.
2. Use your conjecture to write a ratio of surface areas and volumes if the dimensions of two similar rectangular prisms are ℓ, w, h, and $k\ell$, kw, kh.

12.7 Explore Similar Solids

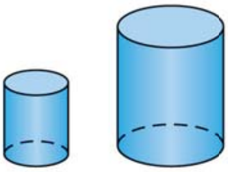
Before You used properties of similar polygons.

Now You will use properties of similar solids.

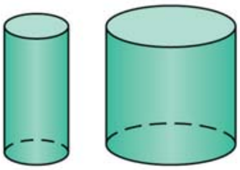
Why So you can determine a ratio of volumes, as in Ex. 26.

Key Vocabulary
- **similar solids**

Two solids of the same type with equal ratios of corresponding linear measures, such as heights or radii, are called **similar solids**. The common ratio is called the *scale factor* of one solid to the other solid. Any two cubes are similar, as well as any two spheres.



Similar cylinders

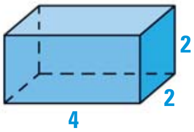


Nonsimilar cylinders

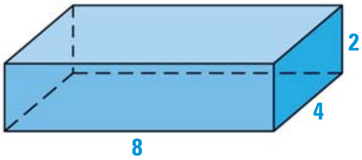
The green cylinders shown above are not similar. Their heights are equal, so they have a 1 : 1 ratio. The radii are different, however, so there is no common ratio.

EXAMPLE 1 Identify similar solids

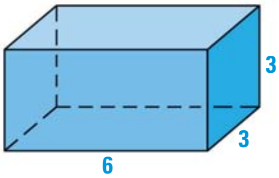
Tell whether the given right rectangular prism is similar to the right rectangular prism shown at the right.



a.



b.



Solution

a. Lengths $\frac{4}{8} = \frac{1}{2}$ Widths $\frac{2}{4} = \frac{1}{2}$ Heights $\frac{2}{2} = \frac{1}{1}$

▸ The prisms are not similar because the ratios of corresponding linear measures are not all equal.

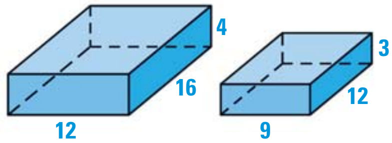
b. Lengths $\frac{4}{6} = \frac{2}{3}$ Widths $\frac{2}{3}$ Heights $\frac{2}{3}$

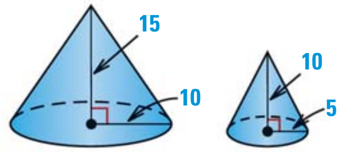
▸ The prisms are similar because the ratios of corresponding linear measures are all equal. The scale factor is 2 : 3.

COMPARE RATIOS
To compare the ratios of corresponding side lengths, write the ratios as fractions in simplest form.

✓ GUIDED PRACTICE for Example 1

Tell whether the pair of right solids is similar. *Explain* your reasoning.

1. 

2. 

SIMILAR SOLIDS THEOREM The surface areas S and volumes V of the similar solids in Example 1, part (b), are as follows.

Prism	Dimensions	Surface area, $S = 2B + Ph$	Volume, $V = Bh$
Smaller	4 by 2 by 2	$S = 2(8) + 12(2) = 40$	$V = 8(2) = 16$
Larger	6 by 3 by 3	$S = 2(18) + 18(3) = 90$	$V = 18(3) = 54$

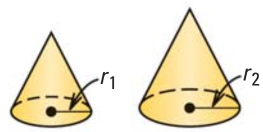
The ratio of side lengths is 2 : 3. Notice that the ratio of surface areas is 40 : 90, or 4 : 9, which can be written as $2^2 : 3^2$, and the ratio of volumes is 16 : 54, or 8 : 27, which can be written as $2^3 : 3^3$. This leads to the following theorem.

READ VOCABULARY
In Theorem 12.13, areas can refer to any pair of corresponding areas in the similar solids, such as lateral areas, base areas, and surface areas.

THEOREM — *For Your Notebook*

THEOREM 12.13 Similar Solids Theorem

If two similar solids have a scale factor of $a : b$, then corresponding areas have a ratio of $a^2 : b^2$, and corresponding volumes have a ratio of $a^3 : b^3$.



$\frac{r_1}{r_2} = \frac{a}{b}, \quad \frac{S_1}{S_2} = \frac{a^2}{b^2}, \quad \frac{V_1}{V_2} = \frac{a^3}{b^3}$

EXAMPLE 2 Use the scale factor of similar solids

PACKAGING The cans shown are similar with a scale factor of 87 : 100. Find the surface area and volume of the larger can.

$S = 51.84$ in.^{2}
$V = 28.27$ in.^{3}

Solution

Use Theorem 12.13 to write and solve two proportions.

$$\frac{\text{Surface area of I}}{\text{Surface area of II}} = \frac{a^2}{b^2} \qquad \frac{\text{Volume of I}}{\text{Volume of II}} = \frac{a^3}{b^3}$$

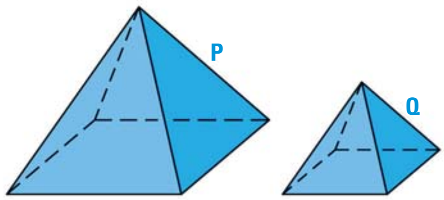
$$\frac{51.84}{\text{Surface area of II}} = \frac{87^2}{100^2} \qquad \frac{28.27}{\text{Volume of II}} = \frac{87^3}{100^3}$$

$$\text{Surface area of II} \approx 68.49 \qquad \text{Volume of II} \approx 42.93$$

▶ The surface area of the larger can is about 68.49 square inches, and the volume of the larger can is about 42.93 cubic inches.

EXAMPLE 3 Find the scale factor

The pyramids are similar. Pyramid P has a volume of 1000 cubic inches and Pyramid Q has a volume of 216 cubic inches. Find the scale factor of Pyramid P to Pyramid Q.



Solution

Use Theorem 12.13 to find the ratio of the two volumes.

$$\frac{a^3}{b^3} = \frac{1000}{216}$$ **Write ratio of volumes.**

$$\frac{a}{b} = \frac{10}{6}$$ **Find cube roots.**

$$\frac{a}{b} = \frac{5}{3}$$ **Simplify.**

▶ The scale factor of Pyramid P to Pyramid Q is 5 : 3.

EXAMPLE 4 Compare similar solids

CONSUMER ECONOMICS A store sells balls of yarn in two different sizes. The diameter of the larger ball is twice the diameter of the smaller ball. If the balls of yarn cost \$7.50 and \$1.50, respectively, which ball of yarn is the better buy?

Solution

STEP 1 **Compute** the ratio of volumes using the diameters.

$$\frac{\text{Volume of large ball}}{\text{Volume of small ball}} = \frac{2^3}{1^3} = \frac{8}{1}, \text{ or } 8:1$$

STEP 2 **Find** the ratio of costs.

$$\frac{\text{Price of large ball}}{\text{Volume of small ball}} = \frac{\$7.50}{\$1.50} = \frac{5}{1}, \text{ or } 5:1$$

STEP 3 **Compare** the ratios in Steps 1 and 2.

If the ratios were the same, neither ball would be a better buy. Comparing the smaller ball to the larger one, the price increase is less than the volume increase. So, you get more yarn for your dollar if you buy the larger ball of yarn.

▶ The larger ball of yarn is the better buy.

✓ GUIDED PRACTICE for Examples 2, 3, and 4

3. Cube C has a surface area of 54 square units and Cube D has a surface area of 150 square units. Find the scale factor of C to D. Find the edge length of C, and use the scale factor to find the volume of D.

4. **WHAT IF?** In Example 4, calculate a new price for the larger ball of yarn so that neither ball would be a better buy than the other.

12.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 9, and 27

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 16, 28, 31, and 33

◆ = **MULTIPLE REPRESENTATIONS** Ex. 34

SKILL PRACTICE

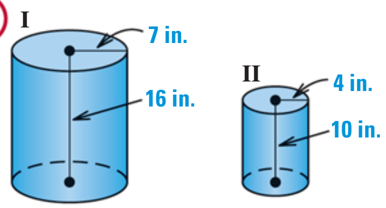
1. **VOCABULARY** What does it mean for two solids to be similar?

2. ★ **WRITING** How are the volumes of similar solids related?

EXAMPLE 1 on p. 847 for Exs. 3–7

IDENTIFYING SIMILAR SOLIDS **Tell whether the pair of right solids is similar. *Explain* your reasoning.**

3.



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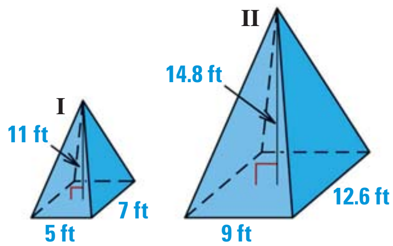

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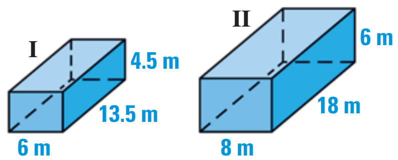

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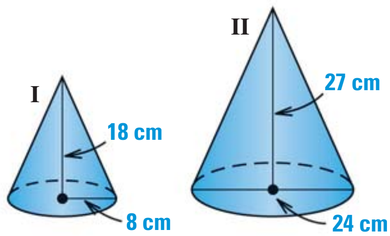
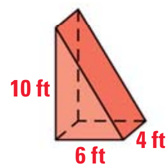


7. ★ **MULTIPLE CHOICE** Which set of dimensions corresponds to a triangular prism that is similar to the prism shown?

Ⓐ 2 feet by 1 foot by 5 feet Ⓑ 4 feet by 2 feet by 8 feet

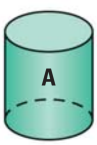
Ⓒ 9 feet by 6 feet by 20 feet Ⓓ 15 feet by 10 feet by 25 feet



EXAMPLE 2 on p. 848 for Exs. 8–11

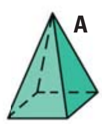
USING SCALE FACTOR **Solid A (shown) is similar to Solid B (not shown) with the given scale factor of A to B. Find the surface area and volume of Solid B.**

8. Scale factor of 1 : 2



$S = 150\pi$ in.^{2}
$V = 250\pi$ in.^{3}

9. Scale factor of 3 : 1



$S = 1500$ m^2
$V = 3434.6$ m^3

10. Scale factor of 5 : 2

$S = 2356.2$ cm^2
$V = 7450.9$ cm^3

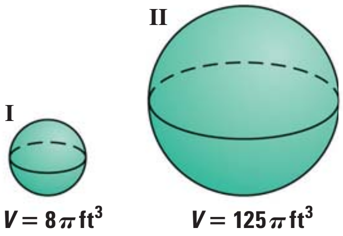
11. **ERROR ANALYSIS** The scale factor of two similar solids is 1 : 4. The volume of the smaller Solid A is 500π. *Describe* and correct the error in writing an equation to find the volume of the larger Solid B.

$$\frac{500\pi}{\text{Volume of B}} = \frac{1^2}{4^2}$$

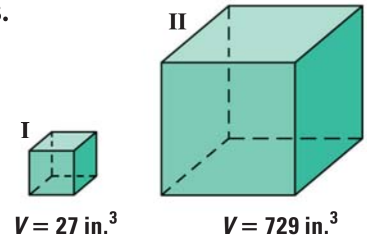
EXAMPLE 3
on p. 849
for Exs. 12–18

FINDING SCALE FACTOR In Exercises 12–15, Solid I is similar to Solid II. Find the scale factor of Solid I to Solid II.

12.



13.



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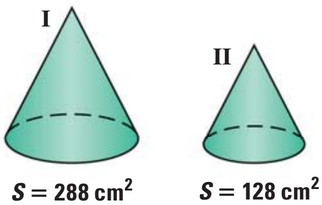

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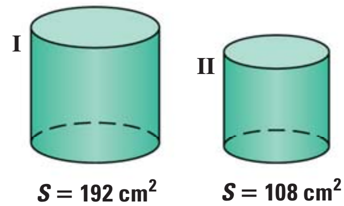


16. ★ **MULTIPLE CHOICE** The volumes of two similar cones are 8π and 27π. What is the ratio of the lateral areas of the cones?

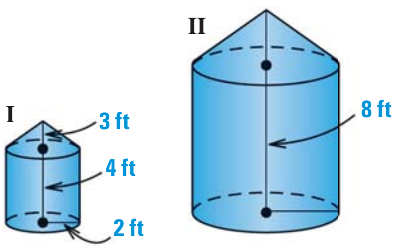
Ⓐ $\frac{8}{27}$ Ⓑ $\frac{1}{3}$ Ⓒ $\frac{4}{9}$ Ⓓ $\frac{2}{3}$

17. FINDING A RATIO Two spheres have volumes 2π cubic feet and 16π cubic feet. What is the ratio of the surface area of the smaller sphere to the surface area of the larger sphere?

18. FINDING SURFACE AREA Two cylinders have a scale factor of 2 : 3. The smaller cylinder has a surface area of 78π square meters. Find the surface area of the larger cylinder.

COMPOSITE SOLIDS In Exercises 19–22, Solid I is similar to Solid II. Find the surface area and volume of Solid II.

19.



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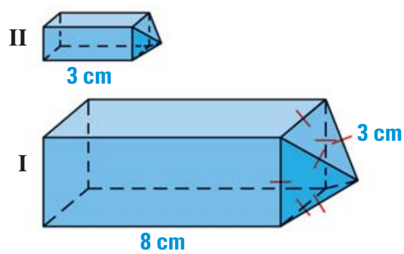

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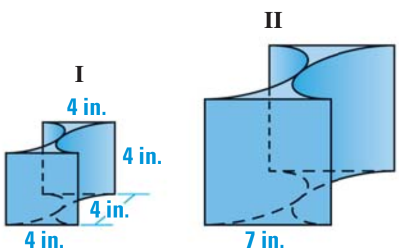

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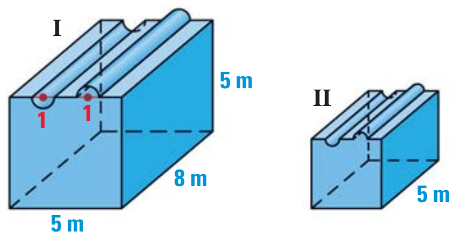


23. **ALGEBRA** Two similar cylinders have surface areas of 54π square feet and 384π square feet. The height of each cylinder is equal to its diameter. Find the radius and height of both cylinders.

24. **CHALLENGE** A plane parallel to the base of a cone divides the cone into two pieces with the dimensions shown. Find each ratio described.

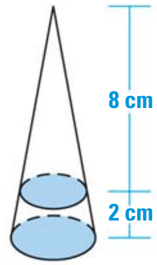
a. The area of the top shaded circle to the area of the bottom shaded circle

b. The slant height of the top part of the cone to the slant height of the whole cone

c. The lateral area of the top part of the cone to the lateral area of the whole cone

d. The volume of the top part of the cone to the volume of the whole cone

e. The volume of the top part of the cone to the volume of the bottom part



PROBLEM SOLVING

EXAMPLE 4
on p. 849
for Exs. 25–27

25. **COFFEE MUGS** The heights of two similar coffee mugs are 3.5 inches and 4 inches. The larger mug holds 12 fluid ounces. What is the capacity of the smaller mug?

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26. **ARCHITECTURE** You have a pair of binoculars that is similar in shape to the structure on page 847. Your binoculars are 6 inches high, and the height of the structure is 45 feet. Find the ratio of the volume of your binoculars to the volume of the structure.

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27. **PARTY PLANNING** Two similar punch bowls have a scale factor of 3 : 4. The amount of lemonade to be added is proportional to the volume. How much lemonade does the smaller bowl require if the larger bowl requires 64 fluid ounces?

28. ★ **OPEN-ENDED MATH** Using the scale factor 2 : 5, sketch a pair of solids in the correct proportions. Label the dimensions of the solids.

29. **MULTI-STEP PROBLEM** Two oranges are both spheres with diameters 3.2 inches and 4 inches. The skin on both oranges has an average thickness of $\frac{1}{8}$ inch.

a. Find the volume of each unpeeled orange.

b. *Compare* the ratio of the diameters to the ratio of the volumes.

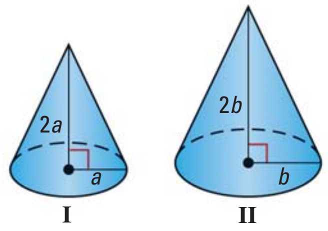
c. Find the diameter of each orange after being peeled.

d. *Compare* the ratio of surface areas of the peeled oranges to the ratio of the volumes of the peeled oranges.

Animated Geometry at classzone.com

○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE ◆ = MULTIPLE REPRESENTATIONS

30. **ALGEBRA** Use the two similar cones shown.



a. What is the scale factor of Cone I to Cone II? What should the ratio of the volume of Cone I to the volume of Cone II be?

b. Write an expression for the volume of each solid.

c. Write and simplify an expression for the ratio of the volume of Cone I to the volume of Cone II. Does your answer agree with your answer to part (a)? *Explain.*

31. ★ **EXTENDED RESPONSE** The scale factor of the model car at the right to the actual car is 1 : 18.

a. The model has length 8 inches. What is the length of the actual car?

b. Each tire of the model has a surface area of 12.1 square inches. What is the surface area of each tire of the actual car?

c. The actual car's engine has volume 8748 cubic inches. Find the volume of the model car's engine.

32. **USING VOLUMES** Two similar cylinders have volumes 16π and 432π. The larger cylinder has lateral area 72π. Find the lateral area of the smaller cylinder.

33. ★ **SHORT RESPONSE** A snow figure is made using three balls of snow with diameters 25 centimeters, 35 centimeters, and 45 centimeters. The smallest weighs about 1.2 kilograms. Find the total weight of the snow used to make the snow figure. *Explain* your reasoning.

34. **MULTIPLE REPRESENTATIONS** A gas is enclosed in a cubical container with side length s in centimeters. Its temperature remains constant while the side length varies. By the *Ideal Gas Law*, the pressure P in atmospheres (atm) of the gas varies inversely with its volume.

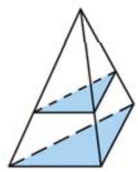
a. **Writing an Equation** Write an equation relating P and s. You will need to introduce a constant of variation k.

b. **Making a Table** Copy and complete the table below for various side lengths. Express the pressure P in terms of the constant k.

Side length s (cm)	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
Pressure P (atm)	?	$8k$	k	?	?

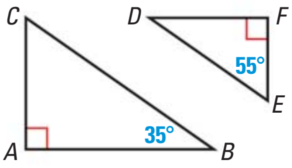
c. **Drawing a Graph** For this particular gas, $k = 1$. Use your table to sketch a graph of P versus s. Place P on the vertical axis and s on the horizontal axis. Does the graph show a linear relationship? *Explain.*

35. **CHALLENGE** A plane parallel to the base of a pyramid separates the pyramid into two pieces with equal volumes. The height of the pyramid is 12 feet. Find the height of the top piece.



MIXED REVIEW

Determine whether the triangles are similar. If they are, write a similarity statement. *(p. 381)*

36.


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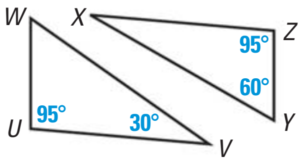

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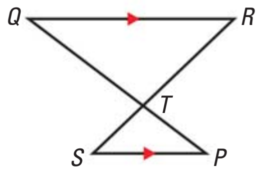


The sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides. *(p. 507)*

39. 900° **40.** 180° **41.** 540° **42.** 1080°

Write a standard equation of the circle with the given center and radius. *(p. 699)*

43. Center (2, 5), radius 4 **44.** Center (−3, 2), radius 6

Sketch the described solid and find its surface area. Round your answer to two decimal places, if necessary. *(p. 803)*

45. Right rectangular prism with length 8 feet, width 6 feet, and height 3 feet

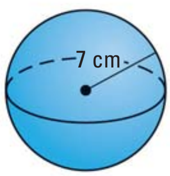
46. Right regular pentagonal prism with all edges measuring 12 millimeters

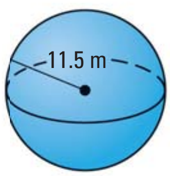
47. Right cylinder with radius 4 inches and height 4 inches

48. Right cylinder with diameter 9 centimeters and height 7 centimeters

QUIZ *for Lessons 12.6–12.7*

Find the surface area and volume of the sphere. Round your answers to two decimal places. *(p. 838)*

1.


2.


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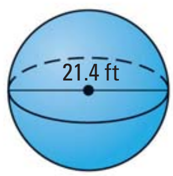
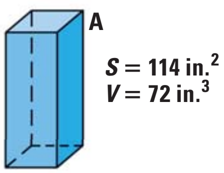
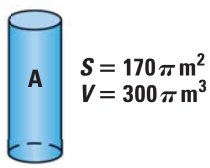


Solid A (shown) is similar to Solid B (not shown) with the given scale factor of A to B. Find the surface area *S* and volume *V* of Solid B. *(p. 847)*

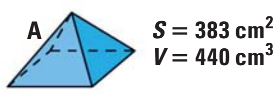
4. Scale factor of 1 : 3



5. Scale factor of 2 : 3



6. Scale factor of 5 : 4



7. Two similar cones have volumes 729π cubic feet and 343π cubic feet. What is the scale factor of the larger cone to the smaller cone? *(p. 847)*

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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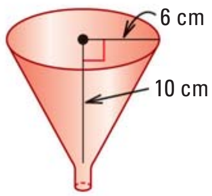
Lessons 12.4–12.7

1. **MULTI-STEP PROBLEM** You have a container in the shape of a right rectangular prism with inside dimensions of length 24 inches, width 16 inches, and height 20 inches.
   a. Find the volume of the inside of the container.
   b. You are going to fill the container with boxes of cookies that are congruent right rectangular prisms. Each box has length 8 inches, width 2 inches, and height 3 inches. Find the volume of one box of cookies.
   c. How many boxes of cookies will fit inside the cardboard container?

2. **SHORT RESPONSE** You have a cup in the shape of a cylinder with inside dimensions of diameter 2.5 inches and height 7 inches.
   a. Find the volume of the inside of the cup.
   b. You have an 18 ounce bottle of orange juice that you want to pour into the cup. Will all of the juice fit? *Explain* your reasoning. ($1 \text{ in.}^3 \approx 0.554$ fluid ounces)

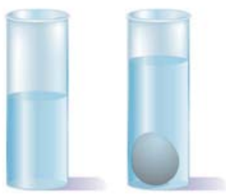
3. **EXTENDED RESPONSE** You have a funnel with the dimensions shown.



   a. Find the approximate volume of the funnel.
   b. You are going to use the funnel to put oil in a car. Oil flows out of the funnel at a rate of 45 milliliters per second. How long will it take to empty the funnel when it is full of oil? $(1 \text{ mL} = 1 \text{ cm}^3)$
   c. How long would it take to empty a funnel with radius 10 cm and height 6 cm?
   d. *Explain* why you can claim that the time calculated in part (c) is greater than the time calculated in part (b) without doing any calculations.

4. **EXTENDED RESPONSE** An official men's basketball has circumference 29.5 inches. An official women's basketball has circumference 28.5 inches.
   a. Find the surface area and volume of the men's basketball.
   b. Find the surface area and volume of the women's basketball using the formulas for surface area and volume of a sphere.
   c. Use your answers in part (a) and the Similar Solids Theorem to find the surface area and volume of the women's basketball. Do your results match your answers in part (b)?

5. **GRIDDED ANSWER** To accurately measure the radius of a spherical rock, you place the rock into a cylindrical glass containing water. When you do so, the water level rises $\frac{9}{64}$ inch. The radius of the glass is 2 inches. What is the radius of the rock?



6. **SHORT RESPONSE** Sketch a rectangular prism and label its dimensions. Change the dimensions of the prism so that its surface area increases and its volume decreases.

7. **SHORT RESPONSE** A hemisphere and a right cone have the same radius and the height of the cone is equal to the radius. *Compare* the volumes of the solids.

8. **SHORT RESPONSE** *Explain* why the height of a right cone is always less than its slant height. Include a diagram in your answer.

12 CHAPTER SUMMARY

BIG IDEAS

For Your Notebook

Big Idea 1

Exploring Solids and Their Properties

Euler's Theorem is useful when finding the number of faces, edges, or vertices on a polyhedron, especially when one of those quantities is difficult to count by hand.

For example, suppose you want to find the number of edges on a regular icosahedron, which has 20 faces. You count 12 vertices on the solid. To calculate the number of edges, use Euler's Theorem:

$F + V = E + 2$ **Write Euler's Theorem.**

$20 + 12 = E + 2$ **Substitute known values.**

$30 = E$ **Solve for *E*.**

Big Idea 2

Solving Problems Using Surface Area and Volume

Figure	Surface Area	Volume
Right prism	$S = 2B + Ph$	$V = Bh$
Right cylinder	$S = 2B + Ch$	$V = Bh$
Regular pyramid	$S = B + \frac{1}{2}P\ell$	$V = \frac{1}{3}Bh$
Right cone	$S = B + \frac{1}{2}C\ell$	$V = \frac{1}{3}Bh$
Sphere	$S = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

The volume formulas for prisms, cylinders, pyramids, and cones can be used for oblique solids.

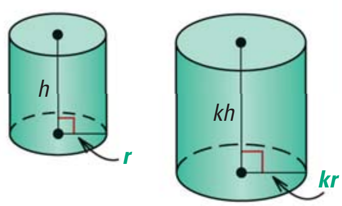
While many of the above formulas can be written in terms of more detailed variables, it is more important to remember the more general formulas for a greater understanding of why they are true.

Big Idea 3

Connecting Similarity to Solids

The similarity concepts learned in Chapter 6 can be extended to 3-dimensional figures as well.

Suppose you have a right cylindrical can whose surface area and volume are known. You are then given a new can whose linear dimensions are k times the dimensions of the original can. If the surface area of the original can is S and the volume of the original can is V, then the surface area and volume of the new can can be expressed as k^2S and k^3V, respectively.



12 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- polyhedron, *p. 794*
  face, edge, vertex, base
- regular polyhedron, *p. 796*
- convex polyhedron, *p. 796*
- Platonic solids, *p. 796*
- tetrahedron, *p. 796*
- cube, *p. 796*
- octahedron, *p. 796*
- dodecahedron, *p. 796*
- icosahedron, *p. 796*
- cross section, *p. 797*
- prism, *p. 803*
  lateral faces, lateral edges
- surface area, *p. 803*
- lateral area, *p. 803*
- net, *p. 803*
- right prism, *p. 804*
- oblique prism, *p. 804*
- cylinder, *p. 805*
- right cylinder, *p. 805*
- pyramid, *p. 810*
- vertex of a pyramid, *p. 810*
- regular pyramid, *p. 810*
- slant height, *p. 810*
- cone, *p. 812*
- vertex of a cone, *p. 812*
- right cone, *p. 812*
- lateral surface, *p. 812*
- volume, *p. 819*
- sphere, *p. 838*
  center, radius, chord, diameter
- great circle, *p. 839*
- hemisphere, *p. 839*
- similar solids, *p. 847*

VOCABULARY EXERCISES

1. Copy and complete: A _?_ is the set of all points in space equidistant from a given point.

2. **WRITING** Sketch a right rectangular prism and an oblique rectangular prism. *Compare* the prisms.

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 12.

12.1 Explore Solids *pp. 794–801*

EXAMPLE

A polyhedron has 16 vertices and 24 edges. How many faces does the polyhedron have?

$F + V = E + 2$	**Euler's Theorem**
$F + 16 = 24 + 2$	**Substitute known values.**
$F = 10$	**Solve for *F*.**

▶ The polyhedron has 10 faces.

EXERCISES

EXAMPLES 2 and 3 on pp. 796–797 for Exs. 3–5

Use Euler's Theorem to find the value of n.

3. Faces: 20
Vertices: n
Edges: 30

4. Faces: n
Vertices: 6
Edges: 12

5. Faces: 14
Vertices: 24
Edges: n

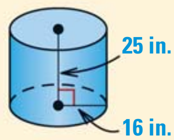
12 CHAPTER REVIEW

12.2 Surface Area of Prisms and Cylinders

pp. 803–809

EXAMPLE

Find the surface area of the right cylinder.



$S = 2\pi r^2 + 2\pi rh$ — **Write formula.**

$= 2\pi(16)^2 + 2\pi(16)(25)$ — **Substitute for r and h.**

$= 1312\pi$ — **Simplify.**

≈ 4121.77 — **Use a calculator.**

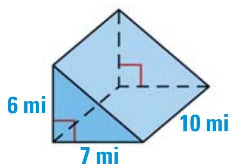
▶ The surface area of the cylinder is about 4121.77 square inches.

EXERCISES

EXAMPLES 2, 3, and 4 on pp. 804–806 for Exs. 6–9

Find the surface area of the right prism or right cylinder. Round your answer to two decimal places, if necessary.

6.



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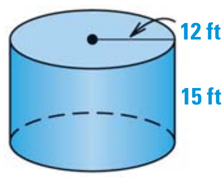

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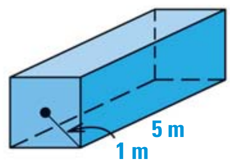

9. A cylinder has a surface area of 44π square meters and a radius of 2 meters. Find the height of the cylinder.

12.3 Surface Area of Pyramids and Cones

pp. 810–817

EXAMPLE

Find the lateral area of the right cone.

Lateral area $= \pi r\ell$ — **Write formula.**

$= \pi(6)(16)$ — **Substitute for r and ℓ.**

$= 96\pi$ — **Simplify.**

≈ 301.59 — **Use a calculator.**

▶ The lateral area of the cone is about 301.59 square centimeters.

EXERCISES

EXAMPLES 1, 2, and 4 on pp. 810–813 for Exs. 10–12

10. Find the surface area of a right square pyramid with base edge length 2 feet and height 5 feet.

11. The surface area of a cone with height 15 centimeters is 500π square centimeters. Find the radius of the base of the cone. Round your answer to two decimal places.

12. Find the surface area of a right octagonal pyramid with height 2.5 yards, and its base has apothem length 1.5 yards.

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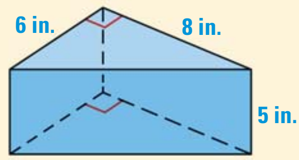
Chapter Review Practice

12.4 Volume of Prisms and Cylinders

pp. 819–825

EXAMPLE

Find the volume of the right triangular prism.



The area of the base is $B = \frac{1}{2}(6)(8) = 24$ square inches. Use $h = 5$ to find the volume.

$V = Bh$ **Write formula.**

$= 24(5)$ **Substitute for B and h.**

$= 120$ **Simplify.**

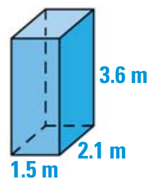
▶ The volume of the prism is 120 cubic inches.

EXERCISES

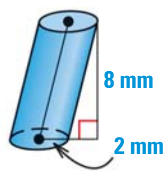
EXAMPLES 2 and 4
on pp. 820–821
for Exs. 13–15

Find the volume of the right prism or oblique cylinder. Round your answer to two decimal places.

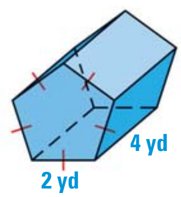
13.



14.



15.

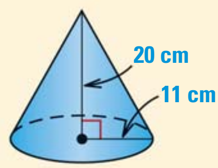


12.5 Volume of Pyramids and Cones

pp. 829–836

EXAMPLE

Find the volume of the right cone.



The area of the base is $B = \pi r^2 = \pi(11)^2 \approx 380.13 \text{ cm}^2$.
Use $h = 20$ to find the volume.

$V = \frac{1}{3}Bh$ **Write formula.**

$\approx \frac{1}{3}(380.13)(20)$ **Substitute for B and h.**

≈ 2534.20 **Simplify.**

▶ The volume of the cone is about 2534.20 cubic centimeters.

EXERCISES

EXAMPLES 1 and 2
on pp. 829–830
for Exs. 16–17

16. A cone with diameter 16 centimeters has height 15 centimeters. Find the volume of the cone. Round your answer to two decimal places.

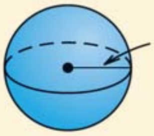
17. The volume of a pyramid is 60 cubic inches and the height is 15 inches. Find the area of the base.

12 CHAPTER REVIEW

12.6 Surface Area and Volume of Spheres pp. 838–845

EXAMPLE

Find the surface area of the sphere.



$S = 4\pi r^2$ — **Write formula.**

$= 4\pi(7)^2$ — **Substitute 7 for *r*.**

$= 196\pi$ — **Simplify.**

▸ The surface area of the sphere is 196π, or about 615.75 square meters.

EXERCISES

EXAMPLES 1, 4, and 5 on pp. 839, 841 for Exs. 18–19

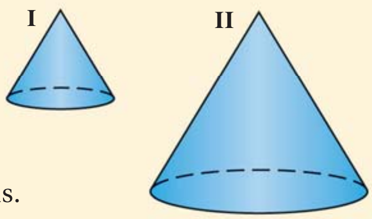
18. ASTRONOMY The shape of Pluto can be approximated as a sphere of diameter 2390 kilometers. Find the surface area and volume of Pluto. Round your answer to two decimal places.

19. A solid is composed of a cube with side length 6 meters and a hemisphere with diameter 6 meters. Find the volume of the solid. Round your answer to two decimal places.

12.7 Explore Similar Solids pp. 847–854

EXAMPLE

The cones are similar with a scale factor of 1 : 2. Find the surface area and volume of Cone II given that the surface area of Cone I is 384π square inches and the volume of Cone I is 768π cubic inches.



Use Theorem 12.13 to write and solve two proportions.

$$\frac{\text{Surface area of I}}{\text{Surface area of II}} = \frac{a^2}{b^2} \qquad \frac{\text{Volume of I}}{\text{Volume of II}} = \frac{a^3}{b^3}$$

$$\frac{384\pi}{\text{Surface area of II}} = \frac{1^2}{2^2} \qquad \frac{768\pi}{\text{Volume of II}} = \frac{1^3}{2^3}$$

$$\text{Surface area of II} = 1536\pi \text{ in.}^2 \qquad \text{Volume of II} = 6144\pi \text{ in.}^3$$

▸ The surface area of Cone II is 1536π, or about 4825.48 square inches, and the volume of Cone II is 6144π, or about 19,301.93 cubic inches.

EXERCISES

EXAMPLE 2 on p. 848 for Exs. 20–22

Solid A is similar to Solid B with the given scale factor of A to B. The surface area and volume of Solid A are given. Find the surface area and volume of Solid B.

20. Scale factor of 1 : 4
$S = 62 \text{ cm}^2$
$V = 30 \text{ cm}^3$

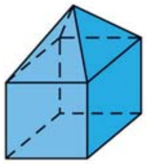
21. Scale factor of 1 : 3
$S = 112\pi \text{ m}^2$
$V = 160\pi \text{ m}^3$

22. Scale factor of 2 : 5
$S = 144\pi \text{ yd}^2$
$V = 288\pi \text{ yd}^3$

12 CHAPTER TEST

Find the number of faces, vertices, and edges of the polyhedron. Check your answer using Euler's Theorem.

1.



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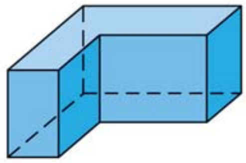

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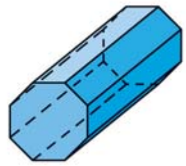
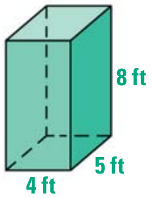


Find the surface area of the solid. The prisms, pyramids, cylinders, and cones are right. Round your answer to two decimal places, if necessary.

4.



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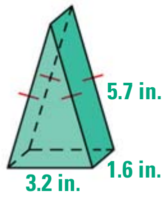

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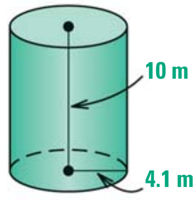

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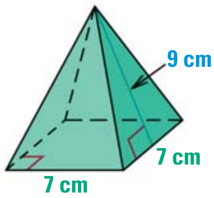

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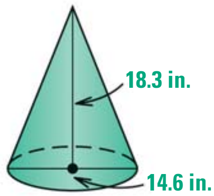

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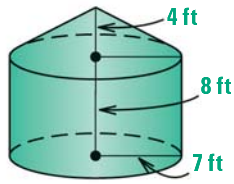
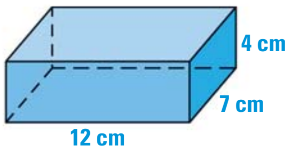


Find the volume of the right prism or right cylinder. Round your answer to two decimal places, if necessary.

10.



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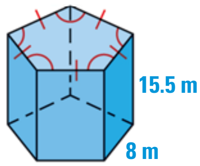
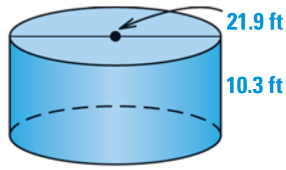
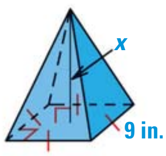


12.

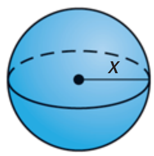


In Exercises 13–15, solve for x.

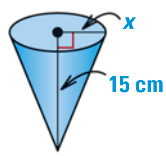
13. Volume $= 324 \text{ in.}^3$



14. Volume $= \frac{32\pi}{3} \text{ ft}^3$



15. Volume $= 180\pi \text{ cm}^3$



16. MARBLES The diameter of the marble shown is 35 millimeters. Find the surface area and volume of the marble.

17. PACKAGING Two similar cylindrical cans have a scale factor of 2 : 3. The smaller can has surface area 308π square inches and volume 735π cubic inches. Find the surface area and volume of the larger can.

CONTEXT-BASED MULTIPLE CHOICE QUESTIONS

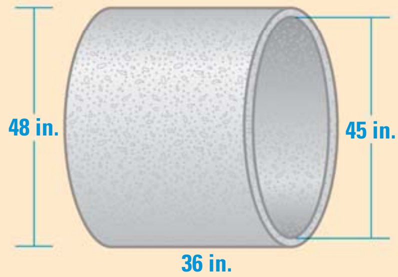
Some of the information you need to solve a context-based multiple choice question may appear in a table, a diagram, or a graph.

PROBLEM 1

One cubic foot of concrete weighs about 150 pounds. What is the approximate weight of the cylindrical section of concrete pipe shown?

Ⓐ 145 lb Ⓑ 686 lb

Ⓒ 2738 lb Ⓓ 5653 lb



Plan

INTERPRET THE DIAGRAM The pipe is a cylinder with length 36 inches and diameter 48 inches. The hollow center is also a cylinder with length 36 inches and diameter 45 inches. Find the volume of concrete used (in cubic feet). Then multiply by 150 pounds per cubic foot to find the weight of the concrete.

Solution

STEP 1 Find the volume of concrete used in the pipe.

Find the volume of a cylinder with diameter 48 inches and height 36 inches.

$$V = \pi r^2 h = \pi(24^2)(36) \approx 65{,}144 \text{ in.}^3$$

Find the volume of a cylinder with diameter 45 inches and height 36 inches.

$$V = \pi r^2 h = \pi(22.5^2)(36) \approx 57{,}256 \text{ in.}^3$$

To find the volume of concrete used in the pipe, subtract the smaller volume from the larger volume.

$$\text{Volume of concrete used in pipe} \approx 65{,}144 - 57{,}256 = 7889 \text{ in.}^3$$

STEP 2 Convert the volume to cubic feet.

Use unit analysis to convert 7889 cubic inches to cubic feet. There are 12 inches in 1 foot, so there are $12^3 = 1728$ cubic inches in 1 cubic foot.

$$7889 \cancel{\text{in.}^3} \cdot \frac{1 \text{ ft}^3}{1728 \cancel{\text{in.}^3}} \approx 4.57 \text{ ft}^3$$

STEP 3 Find the weight of the pipe.

To find the weight of the pipe, multiply the volume of the concrete used in the pipe by the weight of one cubic foot of concrete.

$$\text{Weight of pipe} \approx 4.57 \cancel{\text{ft}^3} \cdot \frac{150 \text{ lb}}{1 \cancel{\text{ft}^3}} = 685.5 \text{ lb}$$

The weight of the pipe is about 686 pounds.

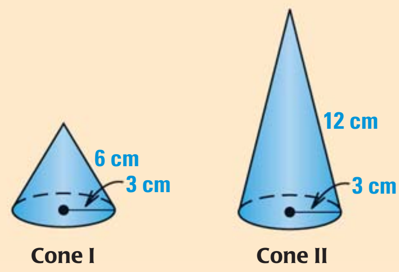
The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

Problem 2

What is the ratio of the surface area of Cone I to the surface area of Cone II?

Ⓐ 1 : 2 Ⓑ 1 : 4

Ⓒ 3 : 5 Ⓓ 3 : 8



Plan

INTERPRET THE DIAGRAM The diagram shows that the cones have the same radius, but different slant heights. Find and compare the surface areas.

Solution

STEP 1 Find the surface area of each cone.

Use the formula for the surface area of a cone.

$$\text{Surface area of Cone I} = \pi r^2 + \pi r\ell = \pi(3^2) + \pi(3)(6) = 9\pi + 18\pi = 27\pi$$

$$\text{Surface area of Cone II} = \pi r^2 + \pi r\ell = \pi(3^2) + \pi(3)(12) = 9\pi + 36\pi = 45\pi$$

STEP 2 Compare the surface areas.

Write a ratio.

$$\frac{\text{Surface area of Cone I}}{\text{Surface area of Cone II}} = \frac{27\pi}{45\pi} = \frac{3}{5}, \text{ or } 3:5$$

The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

PRACTICE

1. The amount a cannister can hold is proportional to its volume. The large cylindrical cannister in the table holds 2 kilograms of flour. About how many kilograms does the similar small cannister hold?

Size	Diameter
Small	24 cm
Medium	30 cm
Large	37.5 cm

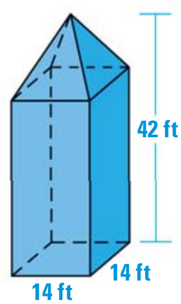
Ⓐ 0.5 kg Ⓑ 1 kg

Ⓒ 1.3 kg Ⓓ 1.6 kg

2. The solid shown is made of a rectangular prism and a square pyramid. The height of the pyramid is one third the height of the prism. What is the volume of the solid?

Ⓐ $457\frac{1}{3}$ ft^3 Ⓑ $6402\frac{2}{3}$ ft^3

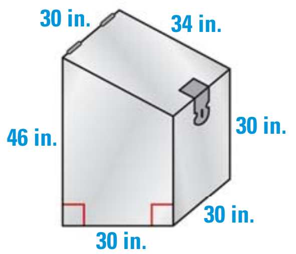
Ⓒ 6860 ft^3 Ⓓ 10,976 ft^3



12 ★ Standardized TEST PRACTICE

MULTIPLE CHOICE

In Exercises 1 and 2, use the diagram, which shows a bin for storing wood.



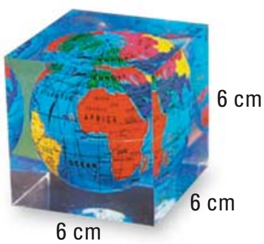
1. The bin is a prism. What is the shape of the base of the prism?

   (A) Triangle (B) Rectangle
   (C) Square (D) Trapezoid

2. What is the surface area of the bin?

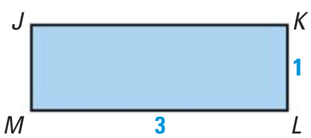
   (A) 3060 in.^2 (B) 6480 in.^2
   (C) 6960 in.^2 (D) 8760 in.^2

3. In the paperweight shown, a sphere with diameter 5 centimeters is embedded in a glass cube. What percent of the volume of the paperweight is taken up by the sphere?



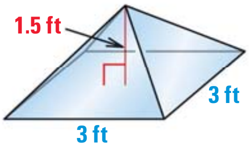
   (A) About 30% (B) About 40%
   (C) About 50% (D) About 60%

4. What is the volume of the solid formed when rectangle $JKLM$ is rotated 360° about $\overline{KL}$?



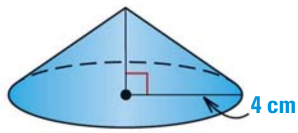
   (A) π (B) 3π
   (C) 6π (D) 9π

5. The skylight shown is made of four glass panes that are congruent isosceles triangles. One square foot of the glass used in the skylight weighs 3.25 pounds. What is the approximate total weight of the glass used in the four panes?



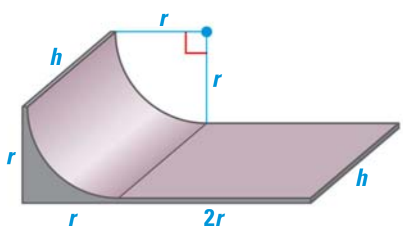
   (A) 10 lb (B) 15 lb
   (C) 29 lb (D) 41 lb

6. The volume of the right cone shown below is 16π cubic centimeters. What is the surface area of the cone?



   (A) $12\pi \text{ cm}^2$ (B) $18\pi \text{ cm}^2$
   (C) $36\pi \text{ cm}^2$ (D) $72\pi \text{ cm}^2$

7. The shaded surface of the skateboard ramp shown is divided into a flat rectangular portion and a curved portion. The curved portion is one fourth of a cylinder with radius r feet and height h feet. Which equation can be used to find the area of the top surface of the ramp?



   (A) $2rh + 2\pi r^2$ (B) $2rh + 2\pi rh$
   (C) $2rh + \frac{1}{4}\pi r^2$ (D) $2rh + \frac{1}{2}\pi rh$

GRIDDED ANSWER

8. The scale factor of two similar triangular prisms is 3 : 5. The volume of the larger prism is 175 cubic inches. What is the volume (in cubic inches) of the smaller prism?

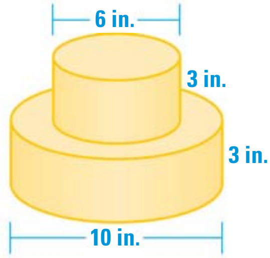
9. Two identical octagonal pyramids are joined together at their bases. The resulting polyhedron has 16 congruent triangular faces and 10 vertices. How many edges does it have?

10. The surface area of Sphere A is 27 square meters. The surface area of Sphere B is 48 square meters. What is the ratio of the diameter of Sphere A to the diameter of Sphere B, expressed as a decimal?

11. The volume of a square pyramid is 54 cubic meters. The height of the pyramid is 2 times the length of a side of its base. What is the height (in meters) of the pyramid?

SHORT RESPONSE

12. Two cake layers are right cylinders, as shown. The top and sides of each layer will be frosted, including the portion of the top of the larger layer that is under the smaller layer. One can of frosting covers 100 square inches. How many cans do you need to frost the cake?



13. The height of Cylinder B is twice the height of Cylinder A. The diameter of Cylinder B is half the diameter of Cylinder A. Let r be the radius and let h be the height of Cylinder A. Write expressions for the radius and height of Cylinder B. Which cylinder has a greater volume? *Explain*.

EXTENDED RESPONSE

14. A cylindrical oil tank for home use has the dimensions shown.

a. Find the volume of the tank to the nearest tenth of a cubic foot.

b. Use the fact that 1 cubic foot = 7.48 gallons to find how many gallons of oil are needed to fill the tank.

c. A homeowner uses about 1000 gallons of oil in a year. Assuming the tank is empty each time it was filled, how many times does the tank need to be filled during the year?

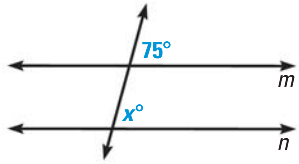
15. A manufacturer is deciding whether to package a product in a container shaped like a prism or one shaped like a cylinder. The manufacturer wants to use the least amount of material possible. The prism is 4 inches tall and has a square base with side length 3 inches. The height of the cylinder is 5 inches, and its radius is 1.6 inches.

a. Find the surface area and volume of each container. If necessary, round to the nearest tenth.

b. For each container, find the ratio of the volume to the surface area. *Explain* why the manufacturer should compare the ratios before making a decision.

CUMULATIVE REVIEW Chapters 1–12

Find the value of x that makes $m \parallel n$. *(p. 161)*

1.


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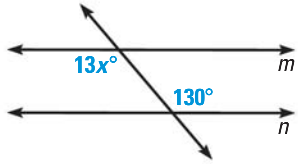

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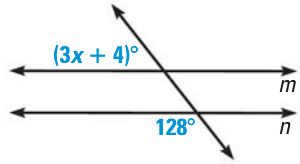
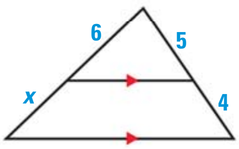


Find the value of the variable. *(p. 397)*

4.


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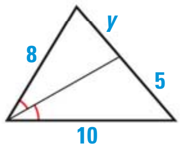

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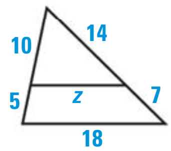
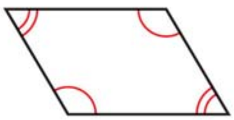


Explain **how you know that the quadrilateral is a parallelogram.** *(p. 522)*

7.


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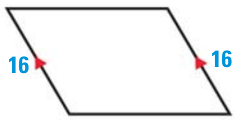
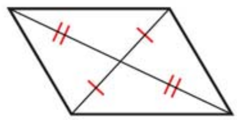
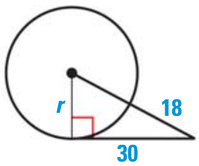


9.


Find the value of the variable. *(pp. 651, 672, 690)*

10.


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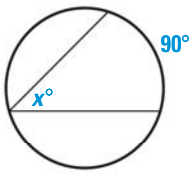

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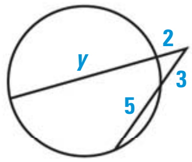
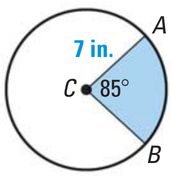


Find the area of the shaded region. *(p. 755)*

13.


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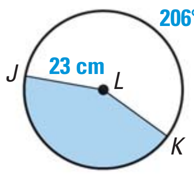
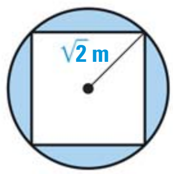
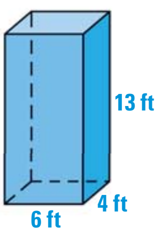
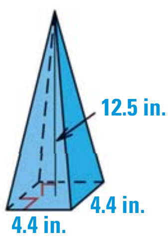
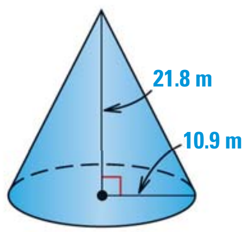


15.


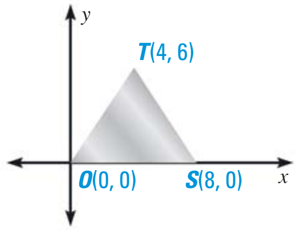
Find the surface area and volume of the right solid. Round your answer to two decimal places. *(pp. 803, 810, 819, 829)*

16.


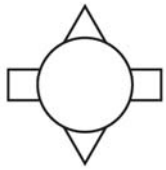
17.


18.


19. **PHYSICS** Find the coordinates of point P that will allow the triangular plate of uniform thickness to be balanced on a point. *(p. 319)*



20. **SYMMETRY** Copy the figure on the right. Determine whether the figure has *line symmetry* and whether it has *rotational symmetry*. Identify all lines of symmetry and angles of rotation that map the figure onto itself. *(p. 619)*

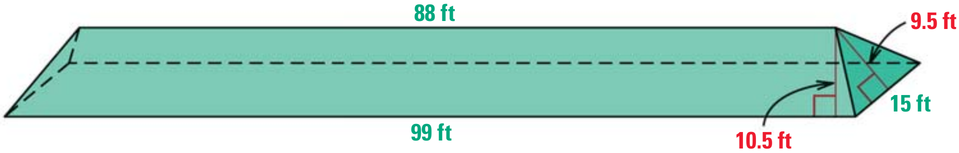


21. **TWO-WAY RADIOS** You and your friend want to test a pair of two-way radios. The radios are expected to transmit voices up to 6 miles. Your location is identified by the point $(-2, 4)$ on a coordinate plane where units are measured in miles. *(p. 699)*

   a. Write an inequality that represents the area expected to be covered by the radios.

   b. Determine whether your friend should be able to hear your voice when your friend is located at $(2, 0)$, $(3, 9)$, $(-6, -1)$, $(-6, 8)$, and $(-7, 5)$. *Explain* your reasoning.

22. **COVERED BRIDGE** A covered bridge has a roof with the dimensions shown. The top ridge of the roof is parallel to the base of the roof. The hidden back and left sides are the same as the front and right sides. Find the total area of the roof. *(pp. 720, 730)*



23. **CANDLES** The candle shown has diameter 2 inches and height 5.5 inches. *(pp. 803, 819)*

   a. Find the surface area and volume of the candle. Round your answer to two decimal places.

   b. The candle has a burning time of about 30 hours. Find the approximate volume of the candle after it has burned for 18 hours.

24. **GEOGRAPHY** The diameter of Earth is about 7920 miles. If approximately 70 percent of Earth's surface is covered by water, how many square miles of water are on Earth's surface? Round your answer to two decimal places. *(p. 838)*