3 Parallel and Perpendicular Lines

Before

In previous chapters, you learned the following skills, which you'll use in Chapter 3: describing angle pairs, using properties and postulates, using angle pair relationships, and sketching a diagram.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

1. Adjacent angles share a common ___?___.
2. Two angles are ___?___ angles if the sum of their measures is 180°.

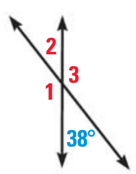
SKILLS AND ALGEBRA CHECK

The midpoint of $\overline{AB}$ is M. Find AB. *(Review p. 15 for 3.2.)*

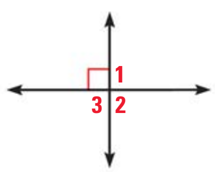
3. $AM = 5x - 2$, $MB = 2x + 7$
4. $AM = 4z + 1$, $MB = 6z - 11$

Find the measure of each numbered angle. *(Review p. 124 for 3.2, 3.3.)*

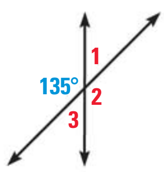
5.



6.



7.



Sketch a diagram for each statement. *(Review pp. 2, 96 for 3.3.)*

8. $\overleftrightarrow{QR}$ is perpendicular to $\overleftrightarrow{WX}$.
9. Lines m and n intersect at point P.

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 3, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 201. You will also use the key vocabulary listed below.

Big Ideas

1. Using properties of parallel and perpendicular lines
2. Proving relationships using angle measures
3. Making connections to lines in algebra

KEY VOCABULARY

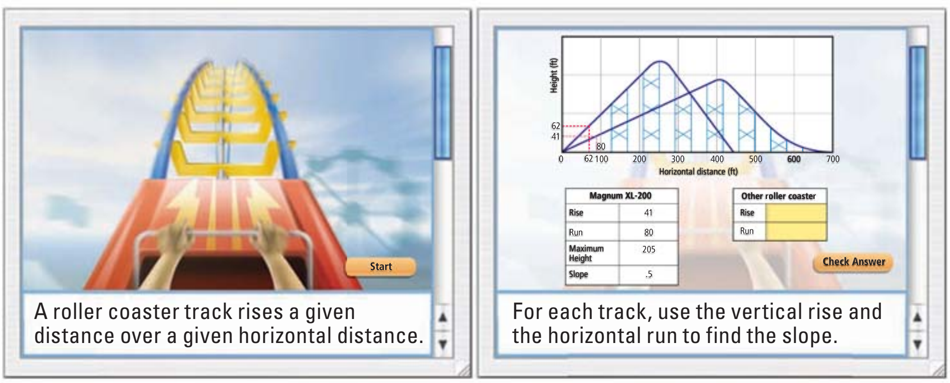
- parallel lines, *p. 147*
- skew lines, *p. 147*
- parallel planes, *p. 147*
- transversal, *p. 149*
- corresponding angles, *p. 149*
- alternate interior angles, *p. 149*
- alternate exterior angles, *p. 149*
- consecutive interior angles, *p. 149*
- paragraph proof, *p. 163*
- slope, *p. 171*
- slope-intercept form, *p. 180*
- standard form, *p. 182*
- distance from a point to a line, *p. 192*

Why?

You can use slopes of lines to determine steepness of lines. For example, you can compare the slopes of roller coasters to determine which is steeper.

Animated Geometry

The animation illustrated below for Example 5 on page 174 helps you answer this question: How steep is a roller coaster?



A roller coaster track rises a given distance over a given horizontal distance.

For each track, use the vertical rise and the horizontal run to find the slope.

Geometry at classzone.com

Animated Geometry at classzone.com

Other animations for Chapter 3: pages 148, 155, 163, and 181

Investigating Geometry ACTIVITY Use before Lesson 3.1

3.1 Draw and Interpret Lines

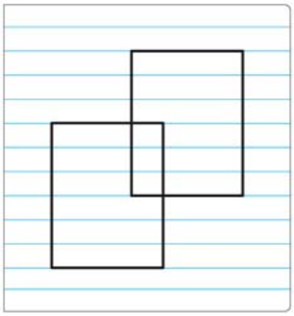
MATERIALS • pencil • straightedge • lined paper

QUESTION How are lines related in space?

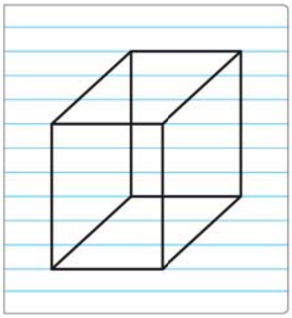
You can use a straightedge to draw a representation of a three-dimensional figure to explore lines in space.

EXPLORE Draw lines in space

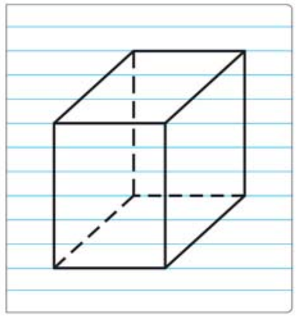
STEP 1 ***Draw rectangles***
Use a straightedge to draw two identical rectangles.



STEP 2 ***Connect corners***
Connect the corresponding corners of the rectangles.

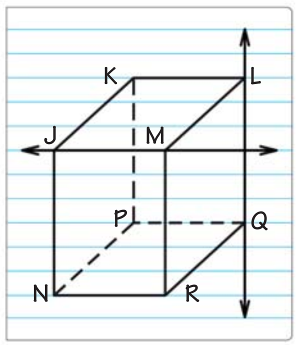


STEP 3 ***Erase parts***
Erase parts of "hidden" lines to form dashed lines.



DRAW CONCLUSIONS Use your observations to complete these exercises

Using your sketch from the steps above, label the corners as shown at the right. Then extend $\overline{JM}$ and $\overline{LQ}$. Add lines to the diagram if necessary.



1. Will $\overleftrightarrow{JM}$ and $\overleftrightarrow{LQ}$ ever intersect in space? (Lines that intersect on the page do not necessarily intersect in space.)

2. Will the pair of lines intersect in space?
   a. $\overleftrightarrow{JK}$ and $\overleftrightarrow{NR}$ **b.** $\overleftrightarrow{QR}$ and $\overleftrightarrow{MR}$
   c. $\overleftrightarrow{LM}$ and $\overleftrightarrow{MR}$ **d.** $\overleftrightarrow{KL}$ and $\overleftrightarrow{NQ}$

3. Does the pair of lines lie in one plane?
   a. $\overleftrightarrow{JK}$ and $\overleftrightarrow{QR}$ **b.** $\overleftrightarrow{QR}$ and $\overleftrightarrow{MR}$
   c. $\overleftrightarrow{JN}$ and $\overleftrightarrow{LR}$ **d.** $\overleftrightarrow{JL}$ and $\overleftrightarrow{NQ}$

4. Do pairs of lines that intersect in space also lie in the same plane? *Explain* your reasoning.

5. Draw a rectangle that is not the same as the one you used in the Explore. Repeat the three steps of the Explore. Will any of your answers to Exercises 1–3 change?

3.1 Identify Pairs of Lines and Angles

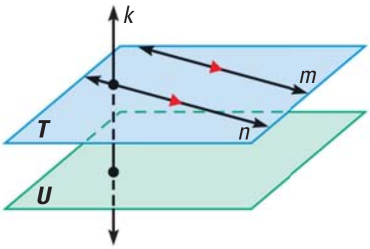
Before You identified angle pairs formed by two intersecting lines.

Now You will identify angle pairs formed by three intersecting lines.

Why? So you can classify lines in a real-world situation, as in Exs. 40–42.

Key Vocabulary
- parallel lines
- skew lines
- parallel planes
- transversal
- corresponding angles
- alternate interior angles
- alternate exterior angles
- consecutive interior angles

Two lines that do not intersect are either *parallel lines* or *skew lines*. Two lines are **parallel lines** if they do not intersect and are coplanar. Two lines are **skew lines** if they do not intersect and are not coplanar. Also, two planes that do not intersect are **parallel planes**.



Lines *m* and *n* are parallel lines ($m \parallel n$).

Lines *m* and *k* are skew lines.

Planes *T* and *U* are parallel planes ($T \parallel U$).

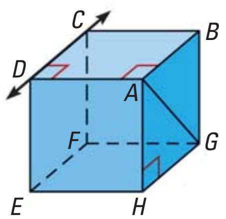
Lines *k* and *n* are intersecting lines, and there is a plane (not shown) containing them.

Small directed triangles, as shown on lines *m* and *n* above, are used to show that lines are parallel. The symbol $\parallel$ means "is parallel to," as in $m \parallel n$.

Segments and rays are parallel if they lie in parallel lines. A line is parallel to a plane if the line is in a plane parallel to the given plane. In the diagram above, line *n* is parallel to plane *U*.

EXAMPLE 1 Identify relationships in space

Think of each segment in the figure as part of a line. Which line(s) or plane(s) in the figure appear to fit the description?



a. Line(s) parallel to $\overleftrightarrow{CD}$ and containing point *A*

b. Line(s) skew to $\overleftrightarrow{CD}$ and containing point *A*

c. Line(s) perpendicular to $\overleftrightarrow{CD}$ and containing point *A*

d. Plane(s) parallel to plane *EFG* and containing point *A*

Solution

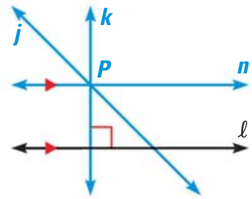
a. $\overleftrightarrow{AB}$, $\overleftrightarrow{HG}$, and $\overleftrightarrow{EF}$ all appear parallel to $\overleftrightarrow{CD}$, but only $\overleftrightarrow{AB}$ contains point *A*.

b. Both $\overleftrightarrow{AG}$ and $\overleftrightarrow{AH}$ appear skew to $\overleftrightarrow{CD}$ and contain point *A*.

c. $\overleftrightarrow{BC}$, $\overleftrightarrow{AD}$, $\overleftrightarrow{DE}$, and $\overleftrightarrow{FC}$ all appear perpendicular to $\overleftrightarrow{CD}$, but only $\overleftrightarrow{AD}$ contains point *A*.

d. Plane *ABC* appears parallel to plane *EFG* and contains point *A*.

PARALLEL AND PERPENDICULAR LINES Two lines in the same plane are either parallel or intersect in a point.



Through a point not on a line, there are infinitely many lines. Exactly one of these lines is parallel to the given line, and exactly one of them is perpendicular to the given line.

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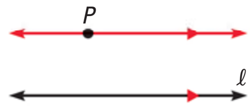
POSTULATES — *For Your Notebook*

POSTULATE 13 Parallel Postulate

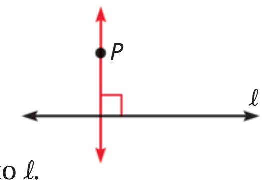
If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.



There is exactly one line through P parallel to ℓ.

POSTULATE 14 Perpendicular Postulate

If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.



There is exactly one line through P perpendicular to ℓ.

EXAMPLE 2 Identify parallel and perpendicular lines

PHOTOGRAPHY The given line markings show how the roads are related to one another.

a. Name a pair of parallel lines.

b. Name a pair of perpendicular lines.

c. Is $\overleftrightarrow{FE} \parallel \overleftrightarrow{AC}$? Explain.

Niagara Falls, New York

Solution

a. $\overleftrightarrow{MD} \parallel \overleftrightarrow{FE}$ **b.** $\overleftrightarrow{MD} \perp \overleftrightarrow{BF}$

c. $\overleftrightarrow{FE}$ is not parallel to $\overleftrightarrow{AC}$, because $\overleftrightarrow{MD}$ is parallel to $\overleftrightarrow{FE}$ and by the Parallel Postulate there is exactly one line parallel to $\overleftrightarrow{FE}$ through M.

✓ GUIDED PRACTICE for Examples 1 and 2

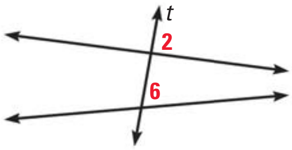
1. Look at the diagram in Example 1. Name the lines through point H that appear skew to $\overleftrightarrow{CD}$.

2. In Example 2, can you use the Perpendicular Postulate to show that $\overleftrightarrow{AC}$ is *not* perpendicular to $\overleftrightarrow{BF}$? *Explain* why or why not.

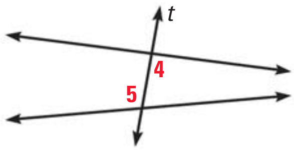
ANGLES AND TRANSVERSALS A **transversal** is a line that intersects two or more coplanar lines at different points.

KEY CONCEPT *For Your Notebook*

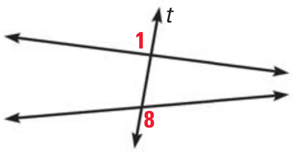
Angles Formed by Transversals



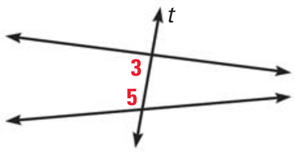
Two angles are **corresponding angles** if they have corresponding positions. For example, $\angle 2$ and $\angle 6$ are above the lines and to the right of the transversal *t*.



Two angles are **alternate interior angles** if they lie between the two lines and on opposite sides of the transversal.



Two angles are **alternate exterior angles** if they lie outside the two lines and on opposite sides of the transversal.



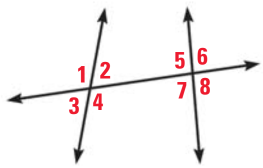
Two angles are **consecutive interior angles** if they lie between the two lines and on the same side of the transversal.

READ VOCABULARY
Another name for consecutive interior angles is **same-side interior angles**.

EXAMPLE 3 Identify angle relationships

Identify all pairs of angles of the given type.

a. Corresponding **b.** Alternate interior
c. Alternate exterior **d.** Consecutive interior



Solution

a. $\angle 1$ and $\angle 5$
$\angle 2$ and $\angle 6$
$\angle 3$ and $\angle 7$
$\angle 4$ and $\angle 8$

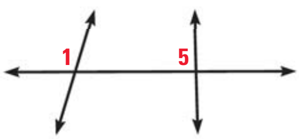
b. $\angle 2$ and $\angle 7$
$\angle 4$ and $\angle 5$

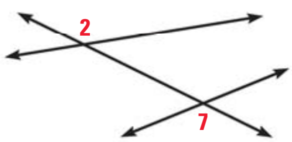
c. $\angle 1$ and $\angle 8$
$\angle 3$ and $\angle 6$

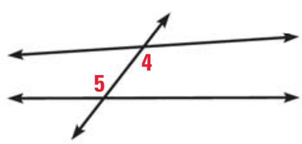
d. $\angle 2$ and $\angle 5$
$\angle 4$ and $\angle 7$

✓ GUIDED PRACTICE for Example 3

Classify the pair of numbered angles.

3.


4.


5.


3.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 25, and 35

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 28, 36, 37, and 39

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A line that intersects two other lines is a _?_.

2. ★ **WRITING** A table is set for dinner. Can the legs of the table and the top of the table lie in parallel planes? *Explain* why or why not.

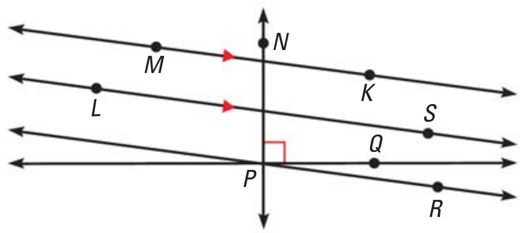
EXAMPLE 1 on p. 147 for Exs. 3–6

IDENTIFYING RELATIONSHIPS **Think of each segment in the diagram as part of a line. Which line(s) or plane(s) contain point *B* and appear to fit the description?**

3. Line(s) parallel to $\overleftrightarrow{CD}$
4. Line(s) perpendicular to $\overleftrightarrow{CD}$
5. Line(s) skew to $\overleftrightarrow{CD}$
6. Plane(s) parallel to plane *CDH*

EXAMPLE 2 on p. 148 for Exs. 7–10

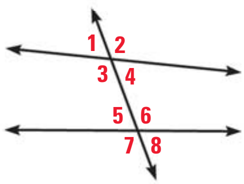
PARALLEL AND PERPENDICULAR LINES **Use the markings in the diagram.**



7. Name a pair of parallel lines.
8. Name a pair of perpendicular lines.
9. Is $\overleftrightarrow{PN} \parallel \overleftrightarrow{KM}$? *Explain.*
10. Is $\overleftrightarrow{PR} \perp \overleftrightarrow{NP}$? *Explain.*

EXAMPLE 3 on p. 149 for Exs. 11–15

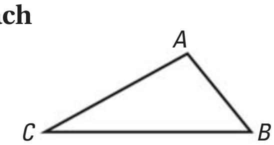
ANGLE RELATIONSHIPS **Identify all pairs of angles of the given type.**



11. Corresponding
12. Alternate interior
13. Alternate exterior
14. Consecutive interior

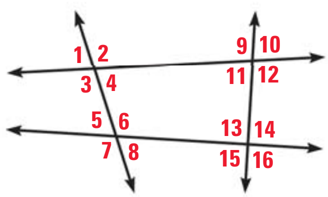
15. **ERROR ANALYSIS** *Describe* and correct the error in saying that $\angle 1$ and $\angle 8$ are corresponding angles in the diagram for Exercises 11–14.

APPLYING POSTULATES **How many lines can be drawn that fit each description? Copy the diagram and sketch all the lines.**



16. Lines through *B* and parallel to $\overleftrightarrow{AC}$
17. Lines through *A* and perpendicular to $\overleftrightarrow{BC}$

USING A DIAGRAM **Classify the angle pair as *corresponding, alternate interior, alternate exterior,* or *consecutive interior* angles.**



18. $\angle 5$ and $\angle 1$
19. $\angle 11$ and $\angle 13$
20. $\angle 6$ and $\angle 13$
21. $\angle 10$ and $\angle 15$
22. $\angle 2$ and $\angle 11$
23. $\angle 8$ and $\angle 4$

ANALYZING STATEMENTS **Copy and complete the statement with *sometimes*, *always*, or *never*. Sketch examples to *justify* your answer.**

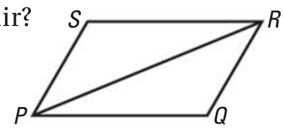
24. If two lines are parallel, then they are _?_ coplanar.

25. If two lines are not coplanar, then they _?_ intersect.

26. If three lines intersect at one point, then they are _?_ coplanar.

27. If two lines are skew to a third line, then they are _?_ skew to each other.

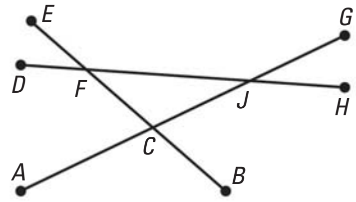
28. ★ **MULTIPLE CHOICE** $\angle RPQ$ and $\angle PRS$ are what type of angle pair?



Ⓐ Corresponding Ⓑ Alternate interior

Ⓒ Alternate exterior Ⓓ Consecutive interior

ANGLE RELATIONSHIPS **Copy and complete the statement. List all possible correct answers.**



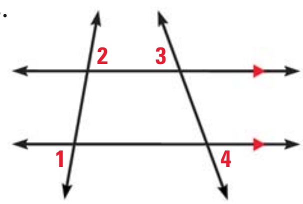
29. $\angle BCG$ and _?_ are corresponding angles.

30. $\angle BCG$ and _?_ are consecutive interior angles.

31. $\angle FCJ$ and _?_ are alternate interior angles.

32. $\angle FCA$ and _?_ are alternate exterior angles.

33. **CHALLENGE** Copy the diagram at the right and extend the lines.



  a. Measure $\angle 1$ and $\angle 2$.

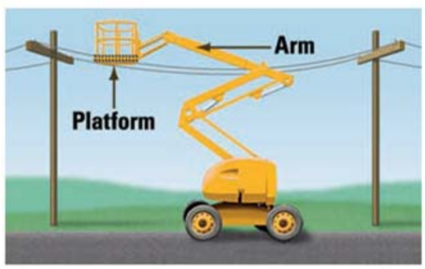
  b. Measure $\angle 3$ and $\angle 4$.

  c. Make a conjecture about alternate exterior angles formed when parallel lines are cut by transversals.

PROBLEM SOLVING

EXAMPLE 2
on p. 148
for Exs. 34–35

CONSTRUCTION **Use the picture of the cherry-picker for Exercises 34 and 35.**



34. Is the platform *perpendicular*, *parallel*, or *skew* to the ground?

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35. Is the arm *perpendicular*, *parallel*, or *skew* to a telephone pole?

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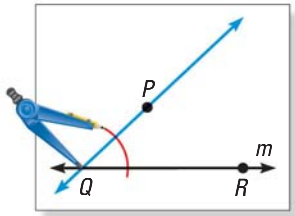
36. ★ **OPEN-ENDED MATH** *Describe* two lines in your classroom that are parallel, and two lines that are skew.

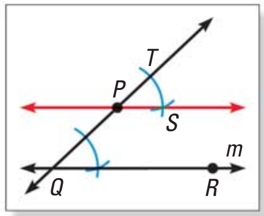
37. ★ **MULTIPLE CHOICE** What is the best description of the horizontal bars in the photo?

Ⓐ Parallel Ⓑ Perpendicular

Ⓒ Skew Ⓓ Intersecting

38. **CONSTRUCTION** Use these steps to construct a line through a given point P that is parallel to a given line m.



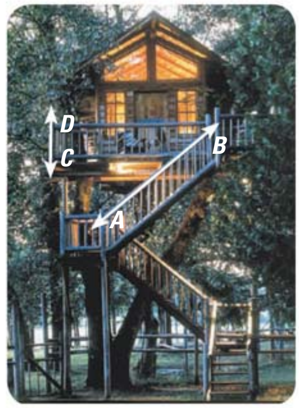


STEP 1 Draw points Q and R on m. Draw $\overleftrightarrow{PQ}$. Draw an arc with the compass point at Q so it crosses $\overleftrightarrow{QP}$ and $\overleftrightarrow{QR}$.

STEP 2 Copy $\angle PQR$ on $\overleftrightarrow{QP}$. Be sure the two angles are corresponding. Label the new angle $\angle TPS$. Draw $\overleftrightarrow{PS}$. $\overleftrightarrow{PS} \parallel \overleftrightarrow{QR}$.

39. ★ **SHORT RESPONSE** Two lines are cut by a transversal. Suppose the measure of a pair of alternate interior angles is 90°. *Explain* why the measure of all four interior angles must be 90°.

TREE HOUSE In Exercises 40–42, use the photo to decide whether the statement is *true* or *false*.



40. The plane containing the floor of the tree house is parallel to the ground.

41. All of the lines containing the railings of the staircase, such as $\overleftrightarrow{AB}$, are skew to the ground.

42. All of the lines containing the *balusters*, such as $\overleftrightarrow{CD}$, are perpendicular to the plane containing the floor of the tree house.

CHALLENGE Draw the figure described.

43. Lines ℓ and m are skew, lines ℓ and n are skew, and lines m and n are parallel.

44. Line ℓ is parallel to plane A, plane A is parallel to plane B, and line ℓ is not parallel to plane B.

MIXED REVIEW

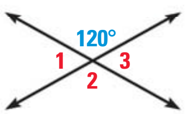
Use the Law of Detachment to make a valid conclusion. *(p. 87)*

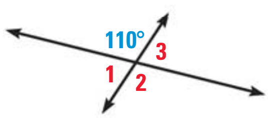
45. If the measure of an angle is less than 90°, then the angle is acute. The measure of $\angle A$ is 46°.

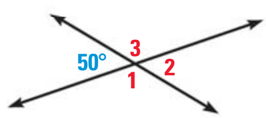
46. If a food has less than 140 milligrams of sodium per serving, then it is low sodium. A serving of soup has 90 milligrams of sodium per serving.

PREVIEW
Prepare for Lesson 3.2 in Exs. 47–49.

Find the measure of each numbered angle. *(p. 124)*

47.


48.


49.


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Keystrokes

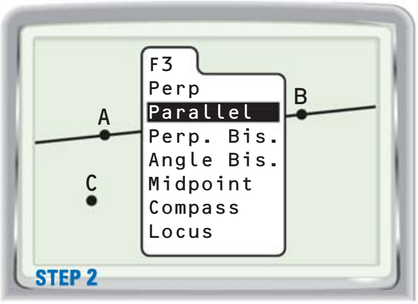
3.2 Parallel Lines and Angles

MATERIALS • graphing calculator or computer

QUESTION **What are the relationships among the angles formed by two parallel lines and a transversal?**

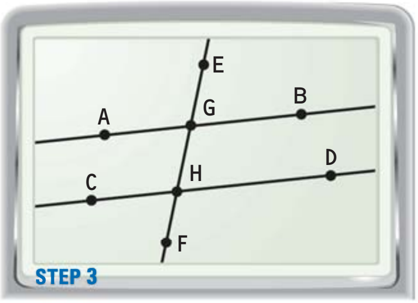
You can use geometry drawing software to explore parallel lines.

EXPLORE **Draw parallel lines and a transversal**



STEP 1 ***Draw line*** Draw and label two points A and B. Draw $\overleftrightarrow{AB}$.

STEP 2 ***Draw parallel line*** Draw a point not on $\overleftrightarrow{AB}$. Label it C. Choose Parallel from the F3 menu and select $\overleftrightarrow{AB}$. Then select C to draw a line through C parallel to $\overleftrightarrow{AB}$. Draw a point on the parallel line you constructed. Label it D.



STEP 3 ***Draw transversal*** Draw two points E and F outside the parallel lines. Draw transversal $\overleftrightarrow{EF}$. Find the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ by choosing Point from the F2 menu. Then choose Intersection. Label the intersection G. Find and label the intersection H of $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$.

STEP 4 ***Measure angle*** Measure all eight angles formed by the three lines by choosing Measure from the F5 menu, then choosing Angle.

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Record the angle measures from Step 4 in a table like the one shown. Which angles are congruent?

Angle	$\angle AGE$	$\angle EGB$	$\angle AGH$	$\angle BGH$	$\angle CHG$	$\angle GHD$	$\angle CHF$	$\angle DHF$
Measure 1	?	?	?	?	?	?	?	?

2. Drag point E or F to change the angle the transversal makes with the parallel lines. Be sure E and F stay outside the parallel lines. Record the new angle measures as row "Measure 2" in your table.

3. Make a conjecture about the measures of the given angles when two parallel lines are cut by a transversal.

a. Corresponding angles **b.** Alternate interior angles

4. REASONING Make and test a conjecture about the sum of the measures of two consecutive interior angles when two parallel lines are cut by a transversal.

3.2 Use Parallel Lines and Transversals

Before You identified angle pairs formed by a transversal.

Now You will use angles formed by parallel lines and transversals.

Why? So you can understand angles formed by light, as in Example 4.

Key Vocabulary
- **corresponding angles,** *p. 149*
- **alternate interior angles,** *p. 149*
- **alternate exterior angles,** *p. 149*
- **consecutive interior angles,** *p. 149*

ACTIVITY EXPLORE PARALLEL LINES

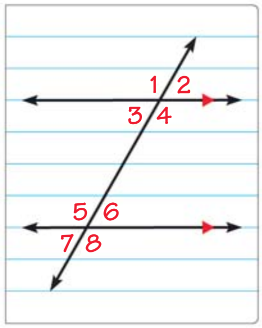
Materials: lined paper, tracing paper, straightedge

STEP 1 **Draw** a pair of parallel lines cut by a nonperpendicular transversal on lined paper. Label the angles as shown.

STEP 2 **Trace** your drawing onto tracing paper.

STEP 3 **Move** the tracing paper to position $\angle 1$ of the traced figure over $\angle 5$ of the original figure. Compare the angles. Are they congruent?

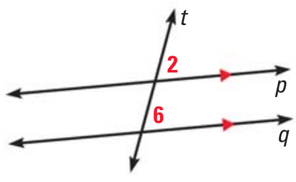
STEP 4 **Compare** the eight angles and list all the congruent pairs. What do you notice about the special angle pairs formed by the transversal?



POSTULATE *For Your Notebook*

POSTULATE 15 Corresponding Angles Postulate

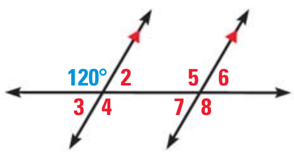
If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.



$\angle 2 \cong \angle 6$

EXAMPLE 1 Identify congruent angles

The measure of three of the numbered angles is 120°. Identify the angles. Explain your reasoning.



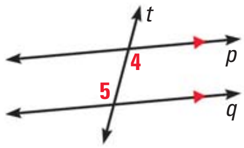
Solution

By the Corresponding Angles Postulate, $m\angle 5 = 120°$.
Using the Vertical Angles Congruence Theorem, $m\angle 4 = 120°$.
Because $\angle 4$ and $\angle 8$ are corresponding angles, by the Corresponding Angles Postulate, you know that $m\angle 8 = 120°$.

THEOREMS — *For Your Notebook*

THEOREM 3.1 Alternate Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

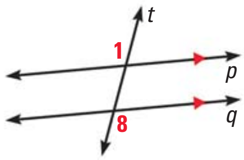


$\angle 4 \cong \angle 5$

Proof: Example 3, p. 156

THEOREM 3.2 Alternate Exterior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

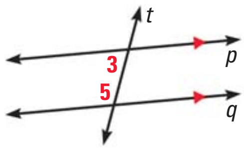


$\angle 1 \cong \angle 8$

Proof: Ex. 37, p. 159

THEOREM 3.3 Consecutive Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

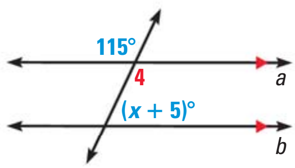


$\angle 3$ and $\angle 5$ are supplementary.

Proof: Ex. 41, p. 159

EXAMPLE 2 Use properties of parallel lines

ALGEBRA **Find the value of x.**



Solution

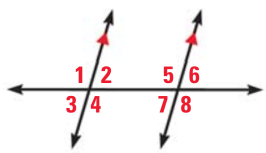
By the Vertical Angles Congruence Theorem, $m\angle 4 = 115°$. Lines a and b are parallel, so you can use the theorems about parallel lines.

$m\angle 4 + (x + 5)° = 180°$	Consecutive Interior Angles Theorem
$115° + (x + 5)° = 180°$	Substitute 115° for $m\angle 4$.
$x + 120 = 180$	Combine like terms.
$x = 60$	Subtract 120 from each side.

Animated Geometry at classzone.com

✓ GUIDED PRACTICE for Examples 1 and 2

Use the diagram at the right.



1. If $m\angle 1 = 105°$, find $m\angle 4$, $m\angle 5$, and $m\angle 8$. Tell which postulate or theorem you use in each case.
2. If $m\angle 3 = 68°$ and $m\angle 8 = (2x + 4)°$, what is the value of x? Show your steps.

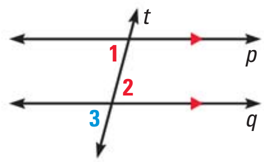
EXAMPLE 3 Prove the Alternate Interior Angles Theorem

Prove that if two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

WRITE PROOFS You can use the information from the diagram in your proof. Find any special angle pairs. Then decide what you know about those pairs.

Solution

Draw a diagram. Label a pair of alternate interior angles as $\angle 1$ and $\angle 2$. You are looking for an angle that is related to both $\angle 1$ and $\angle 2$. Notice that one angle is a vertical angle with $\angle 2$ and a corresponding angle with $\angle 1$. Label it $\angle 3$.



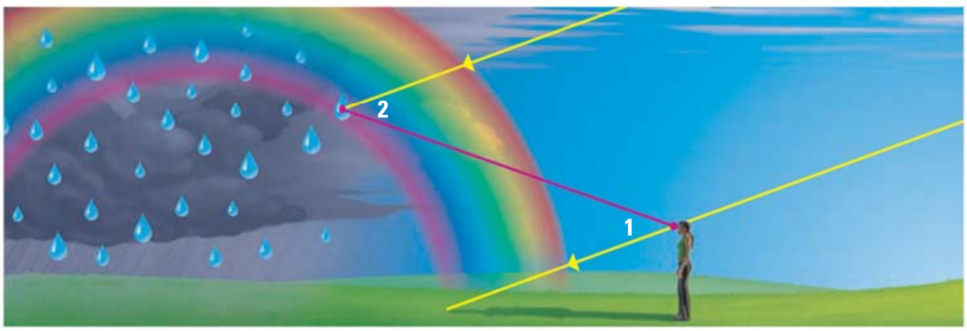
GIVEN ▶ $p \parallel q$

PROVE ▶ $\angle 1 \cong \angle 2$

STATEMENTS	REASONS
1. $p \parallel q$	**1.** Given
2. $\angle 1 \cong \angle 3$	**2.** Corresponding Angles Postulate
3. $\angle 3 \cong \angle 2$	**3.** Vertical Angles Congruence Theorem
4. $\angle 1 \cong \angle 2$	**4.** Transitive Property of Congruence

EXAMPLE 4 Solve a real-world problem

SCIENCE When sunlight enters a drop of rain, different colors of light leave the drop at different angles. This process is what makes a rainbow. For violet light, $m\angle 2 = 40°$. What is $m\angle 1$? How do you know?



Solution

Because the sun's rays are parallel, $\angle 1$ and $\angle 2$ are alternate interior angles. By the Alternate Interior Angles Theorem, $\angle 1 \cong \angle 2$. By the definition of congruent angles, $m\angle 1 = m\angle 2 = 40°$.

✓ GUIDED PRACTICE for Examples 3 and 4

3. In the proof in Example 3, if you use the third statement before the second statement, could you still prove the theorem? *Explain*.

4. **WHAT IF?** Suppose the diagram in Example 4 shows yellow light leaving a drop of rain. Yellow light leaves the drop at an angle of 41°. What is $m\angle 1$ in this case? How do you know?

3.2 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 9, and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 3, 21, 33, 39, and 40

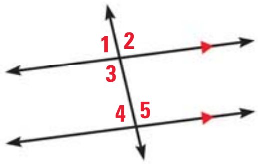
SKILL PRACTICE

EXAMPLES 1 and 2 on pp. 154–155 for Exs. 3–16

1. **VOCABULARY** Draw a pair of parallel lines and a transversal. Label a pair of *corresponding angles*.

2. ★ **WRITING** Two parallel lines are cut by a transversal. Which pairs of angles are congruent? Which pairs of angles are supplementary?

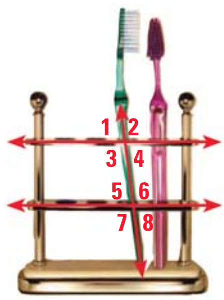
3. ★ **MULTIPLE CHOICE** In the figure at the right, which angle has the same measure as $\angle 1$?



Ⓐ $\angle 2$ Ⓑ $\angle 3$

Ⓒ $\angle 4$ Ⓓ $\angle 5$

USING PARALLEL LINES **Find the angle measure. Tell which postulate or theorem you use.**



4. If $m\angle 4 = 65°$, then $m\angle 1 = \underline{\ ?\ }$.

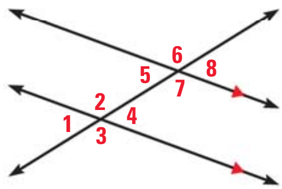
5. If $m\angle 7 = 110°$, then $m\angle 2 = \underline{\ ?\ }$.

6. If $m\angle 5 = 71°$, then $m\angle 4 = \underline{\ ?\ }$.

7. If $m\angle 3 = 117°$, then $m\angle 5 = \underline{\ ?\ }$.

8. If $m\angle 8 = 54°$, then $m\angle 1 = \underline{\ ?\ }$.

USING POSTULATES AND THEOREMS **What postulate or theorem justifies the statement about the diagram?**



9. $\angle 1 \cong \angle 5$

10. $\angle 4 \cong \angle 5$

11. $\angle 2 \cong \angle 7$

12. $\angle 2$ and $\angle 5$ are supplementary.

13. $\angle 3 \cong \angle 6$

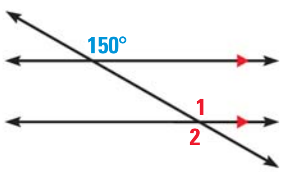
14. $\angle 3 \cong \angle 7$

15. $\angle 1 \cong \angle 8$

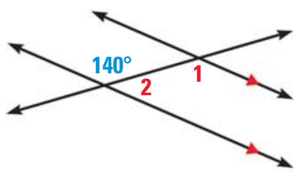
16. $\angle 4$ and $\angle 7$ are supplementary.

USING PARALLEL LINES **Find $m\angle 1$ and $m\angle 2$. *Explain* your reasoning.**

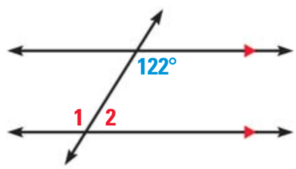
17.



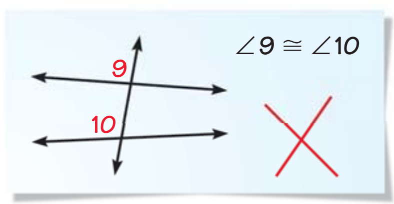
18.



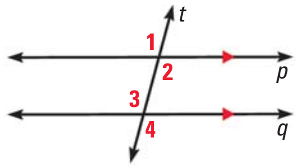
19.



20. **ERROR ANALYSIS** A student concludes that $\angle 9 \cong \angle 10$ by the Corresponding Angles Postulate. Describe and correct the error in this reasoning.

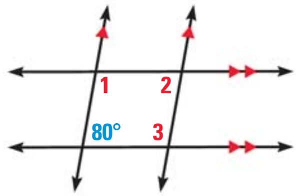


21. ★ **SHORT RESPONSE** Given $p \parallel q$, *describe* two methods you can use to show that $\angle 1 \cong \angle 4$.

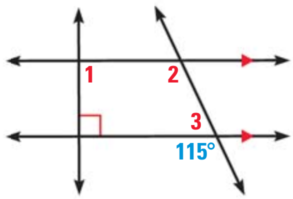


USING PARALLEL LINES **Find $m\angle 1$, $m\angle 2$, and $m\angle 3$. *Explain* your reasoning.**

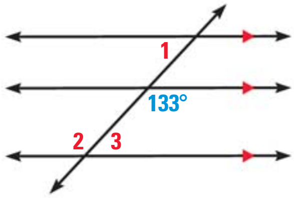
22.



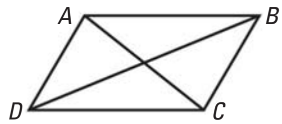
23.



24.



ANGLES **Use the diagram at the right.**

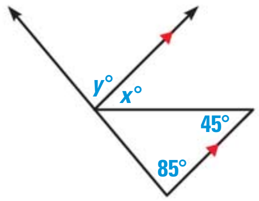


25. Name two pairs of congruent angles if $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ are parallel.

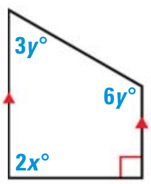
26. Name two pairs of supplementary angles if $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ are parallel.

xy **ALGEBRA** **Find the values of *x* and *y*.**

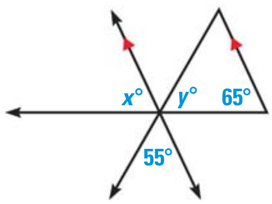
27.



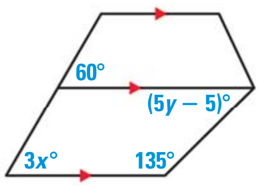
28.



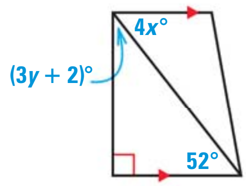
29.



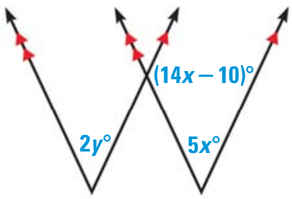
30.



31.



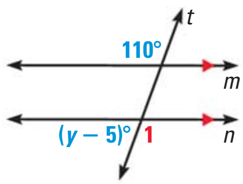
32.



33. ★ **MULTIPLE CHOICE** What is the value of *y* in the diagram?

Ⓐ 70 Ⓑ 75

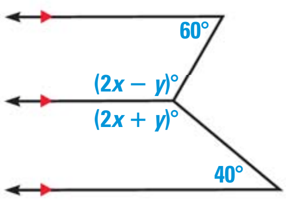
Ⓒ 110 Ⓓ 115



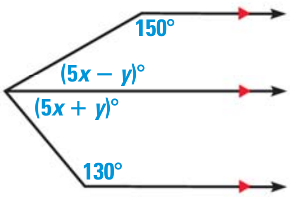
34. **DRAWING** Draw a four-sided figure with sides $\overline{MN}$ and $\overline{PQ}$, such that $\overline{MN} \parallel \overline{PQ}$, $\overline{MP} \parallel \overline{NQ}$, and $\angle MNQ$ is an acute angle. Which angle pairs formed are congruent? *Explain* your reasoning.

CHALLENGE **Find the values of *x* and *y*.**

35.



36.



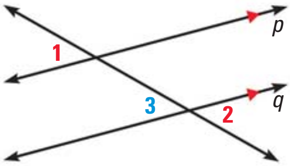
○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

EXAMPLE 3
on p. 156
for Ex. 37

37. **PROVING THEOREM 3.2** If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent. Use the steps below to write a proof of the Alternate Exterior Angles Theorem.



GIVEN ▶ $p \parallel q$

PROVE ▶ $\angle 1 \cong \angle 2$

a. Show that $\angle 1 \cong \angle 3$.

b. Then show that $\angle 1 \cong \angle 2$.

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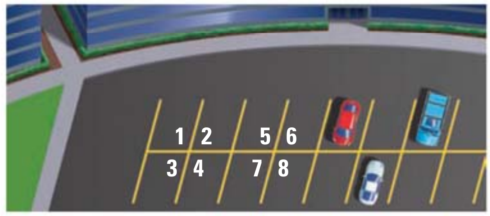
EXAMPLE 4
on p. 156
for Exs. 38–40

38. **PARKING LOT** In the diagram, the lines dividing parking spaces are parallel. The measure of $\angle 1$ is 110°.



a. Identify the angle(s) congruent to $\angle 1$.

b. Find $m\angle 6$.

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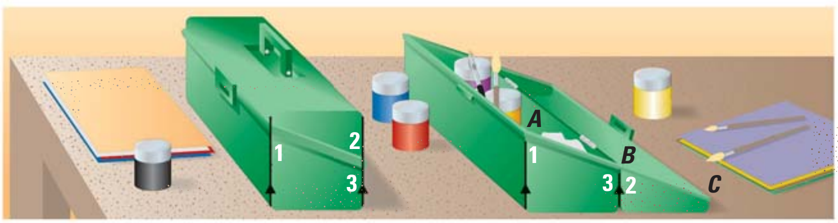
39. ★ **SHORT RESPONSE** The *Toddler*™ is a walking robot. Each leg of the robot has two parallel bars and a foot. When the robot walks, the leg bars remain parallel as the foot slides along the surface.

a. As the legs move, are there pairs of angles that are always congruent? always supplementary? If so, which angles?

b. *Explain* how having parallel leg bars allows the robot's foot to stay flat on the floor as it moves.

40. ★ **EXTENDED RESPONSE** You are designing a box like the one below.

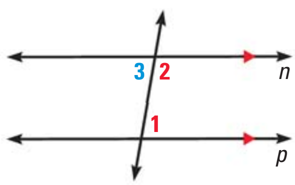


a. The measure of $\angle 1$ is 70°. What is $m\angle 2$? What is $m\angle 3$?

b. *Explain* why $\angle ABC$ is a straight angle.

c. **What If?** If $m\angle 1$ is 60°, will $\angle ABC$ still be a straight angle? Will the opening of the box be *more steep* or *less steep*? *Explain*.

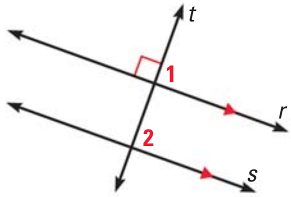
41. **PROVING THEOREM 3.3** If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary. Write a proof of the Consecutive Interior Angles Theorem.



GIVEN ▶ $n \parallel p$

PROVE ▶ $\angle 1$ and $\angle 2$ are supplementary.

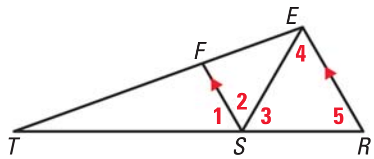
42. **PROOF** The Perpendicular Transversal Theorem (page 192) states that if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other. Write a proof of the Perpendicular Transversal Theorem.



GIVEN ▶ $t \perp r$, $r \parallel s$

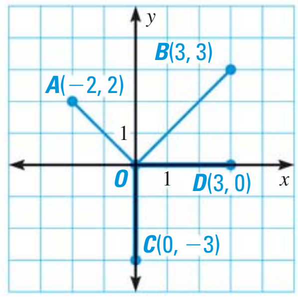
PROVE ▶ $t \perp s$

43. **CHALLENGE** In the diagram, $\angle 4 \cong \angle 5$. $\overline{SE}$ bisects $\angle RSF$. Find $m\angle 1$. *Explain* your reasoning.



MIXED REVIEW

44. Find the length of each segment in the coordinate plane at the right. Which segments are congruent? *(p. 15)*



Are angles with the given measures *complementary*, *supplementary*, or *neither*? *(p. 35)*

45. $m\angle 1 = 62°$, $m\angle 2 = 128°$

46. $m\angle 3 = 130°$, $m\angle 4 = 70°$

47. $m\angle 5 = 44°$, $m\angle 6 = 46°$

Find the perimeter of the equilateral figure with the given side length. *(pp. 42, 49)*

48. Pentagon, 20 cm

49. Octagon, 2.5 ft

50. Decagon, 33 in.

PREVIEW
Prepare for Lesson 3.3 in Exs. 51–52.

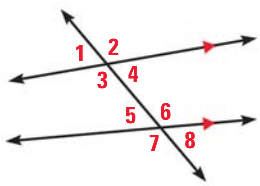
Write the converse of the statement. Is the converse true? *(p. 79)*

51. Three points are collinear if they lie on the same line.

52. If the measure of an angle is 119°, then the angle is obtuse.

QUIZ *for Lessons 3.1–3.2*

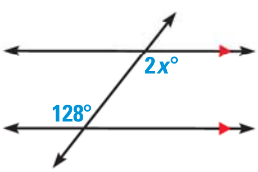
Copy and complete the statement. *(p. 147)*



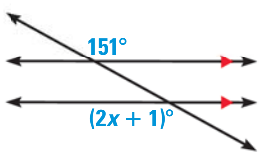
1. $\angle 2$ and ? are corresponding angles.
2. $\angle 3$ and ? are consecutive interior angles.
3. $\angle 3$ and ? are alternate interior angles.
4. $\angle 2$ and ? are alternate exterior angles.

Find the value of *x*. *(p. 154)*

5.



6.



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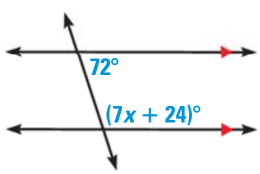


3.3 Prove Lines are Parallel

Before You used properties of parallel lines to determine angle relationships.

Now You will use angle relationships to prove that lines are parallel.

Why? So you can describe how sports equipment is arranged, as in Ex. 32.

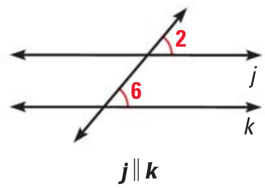
Key Vocabulary
- **paragraph proof**
- **converse,** *p. 80*
- **two-column proof,** *p. 112*

Postulate 16 below is the converse of Postulate 15 in Lesson 3.2. Similarly, the theorems in Lesson 3.2 have true converses. Remember that the converse of a true conditional statement is not necessarily true, so each converse of a theorem must be proved, as in Example 3.

POSTULATE *For Your Notebook*

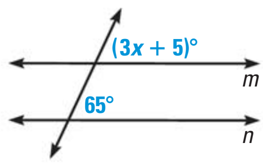
POSTULATE 16 Corresponding Angles Converse

If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.



EXAMPLE 1 Apply the Corresponding Angles Converse

ALGEBRA Find the value of x that makes $m \parallel n$.



Solution

Lines m and n are parallel if the marked corresponding angles are congruent.

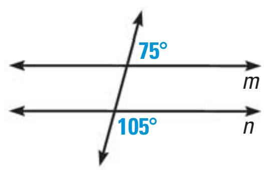
$(3x + 5)° = 65°$ **Use Postulate 16 to write an equation.**

$3x = 60$ **Subtract 5 from each side.**

$x = 20$ **Divide each side by 3.**

▶ The lines m and n are parallel when $x = 20$.

✓ GUIDED PRACTICE for Example 1



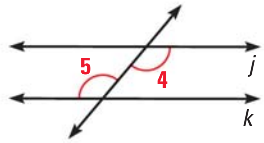
1. Is there enough information in the diagram to conclude that $m \parallel n$? *Explain*.
2. *Explain* why Postulate 16 is the converse of Postulate 15.

THEOREMS *For Your Notebook*

THEOREM 3.4 Alternate Interior Angles Converse

If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

Proof: Example 3, p. 163

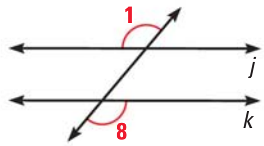


$j \parallel k$

THEOREM 3.5 Alternate Exterior Angles Converse

If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.

Proof: Ex. 36, p. 168

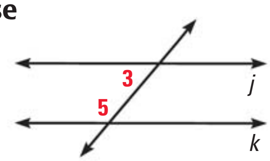


$j \parallel k$

THEOREM 3.6 Consecutive Interior Angles Converse

If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.

Proof: Ex. 37, p. 168



If $\angle 3$ and $\angle 5$ are supplementary, then $j \parallel k$.

EXAMPLE 2 Solve a real-world problem

SNAKE PATTERNS How can you tell whether the sides of the pattern are parallel in the photo of a diamond-back snake?

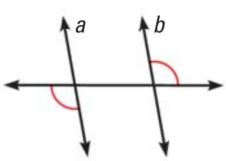
Solution

Because the alternate interior angles are congruent, you know that the sides of the pattern are parallel.

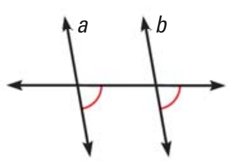
✓ GUIDED PRACTICE for Example 2

Can you prove that lines *a* and *b* are parallel? *Explain* why or why not.

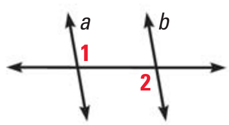
3.



4.



5. $m\angle 1 + m\angle 2 = 180°$

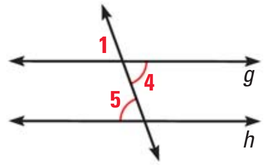


EXAMPLE 3 Prove the Alternate Interior Angles Converse

Prove that if two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

AVOID ERRORS
Before you write a proof, identify the GIVEN and PROVE statements for the situation described or for any diagram you draw.

Solution



GIVEN ▶ $\angle 4 \cong \angle 5$

PROVE ▶ $g \parallel h$

STATEMENTS	REASONS
1. $\angle 4 \cong \angle 5$	**1.** Given
2. $\angle 1 \cong \angle 4$	**2.** Vertical Angles Congruence Theorem
3. $\angle 1 \cong \angle 5$	**3.** Transitive Property of Congruence
4. $g \parallel h$	**4.** Corresponding Angles Converse

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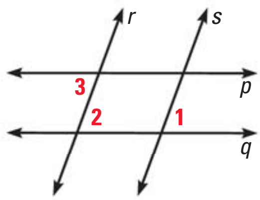
PARAGRAPH PROOFS A proof can also be written in paragraph form, called a **paragraph proof**. The statements and reasons in a paragraph proof are written in sentences, using words to explain the logical flow of the argument.

EXAMPLE 4 Write a paragraph proof

In the figure, $r \parallel s$ and $\angle 1$ is congruent to $\angle 3$. Prove $p \parallel q$.

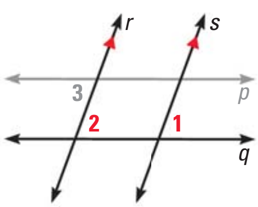


Solution

Look at the diagram to make a plan. The diagram suggests that you look at angles 1, 2, and 3. Also, you may find it helpful to focus on one pair of lines and one transversal at a time.

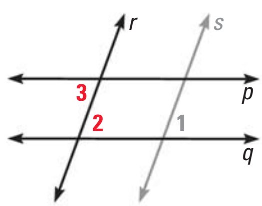
Plan for Proof

a. Look at $\angle 1$ and $\angle 2$.



$\angle 1 \cong \angle 2$ because $r \parallel s$.

b. Look at $\angle 2$ and $\angle 3$.



If $\angle 2 \cong \angle 3$, then $p \parallel q$.

TRANSITIONAL WORDS
In paragraph proofs, **transitional words** such as *so*, *then*, and *therefore* help to make the logic clear.

Plan in Action

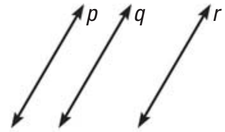
a. It is given that $r \parallel s$, **so** by the Corresponding Angles Postulate, $\angle 1 \cong \angle 2$.

b. It is also given that $\angle 1 \cong \angle 3$. **Then** $\angle 2 \cong \angle 3$ by the Transitive Property of Congruence for angles. **Therefore,** by the Alternate Interior Angles Converse, $p \parallel q$.

THEOREM — *For Your Notebook*

THEOREM 3.7 Transitive Property of Parallel Lines

If two lines are parallel to the same line, then they are parallel to each other.

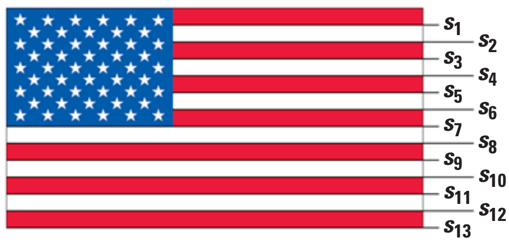


If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

Proofs: Ex. 38, p. 168; Ex. 38, p. 177

EXAMPLE 5 Use the Transitive Property of Parallel Lines

U.S. FLAG The flag of the United States has 13 alternating red and white stripes. Each stripe is parallel to the stripe immediately below it. Explain why the top stripe is parallel to the bottom stripe.



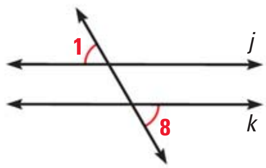
Solution

USE SUBSCRIPTS
When you name several similar items, you can use one variable with subscripts to keep track of the items.

The stripes from top to bottom can be named $s_1, s_2, s_3, \ldots, s_{13}$. Each stripe is parallel to the one below it, so $s_1 \parallel s_2$, $s_2 \parallel s_3$, and so on. Then $s_1 \parallel s_3$ by the Transitive Property of Parallel Lines. Similarly, because $s_3 \parallel s_4$, it follows that $s_1 \parallel s_4$. By continuing this reasoning, $s_1 \parallel s_{13}$. So, the top stripe is parallel to the bottom stripe.

✓ GUIDED PRACTICE for Examples 3, 4, and 5

6. If you use the diagram at the right to prove the Alternate Exterior Angles Converse, what GIVEN and PROVE statements would you use?



7. Copy and complete the following paragraph proof of the Alternate Interior Angles Converse using the diagram in Example 3.

It is given that $\angle 4 \cong \angle 5$. By the __?__, $\angle 1 \cong \angle 4$. Then by the Transitive Property of Congruence, __?__. So, by the __?__, $g \parallel h$.

8. Each step is parallel to the step immediately above it. The bottom step is parallel to the ground. *Explain* why the top step is parallel to the ground.

3.3 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 29, and 37
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 23, 24, 33, and 39

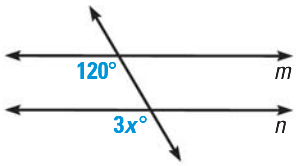
SKILL PRACTICE

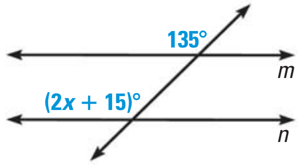
1. **VOCABULARY** Draw a pair of parallel lines with a transversal. Identify all pairs of *alternate exterior angles*.

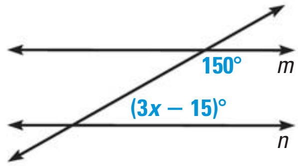
2. ★ **WRITING** Use the theorems from the previous lesson and the converses of those theorems in this lesson. Write three biconditionals about parallel lines and transversals.

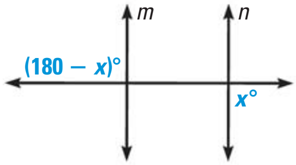
EXAMPLE 1 on p. 161 for Exs. 3–9

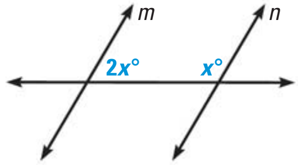
xy ALGEBRA Find the value of x that makes $m \parallel n$.

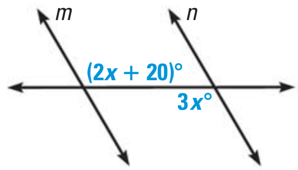
3.


4.


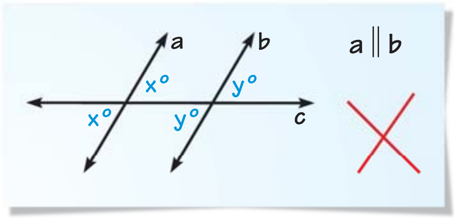
5.


6.


7.


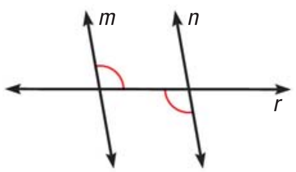
8.


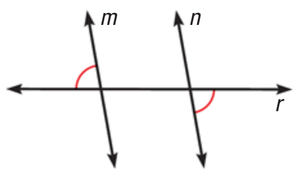
9. **ERROR ANALYSIS** A student concluded that lines *a* and *b* are parallel. *Describe* and correct the student's error.

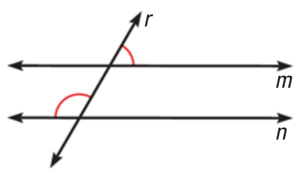


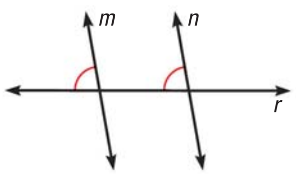
EXAMPLE 2 on p. 162 for Exs. 10–17

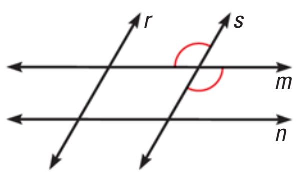
IDENTIFYING PARALLEL LINES Is there enough information to prove $m \parallel n$? If so, state the postulate or theorem you would use.

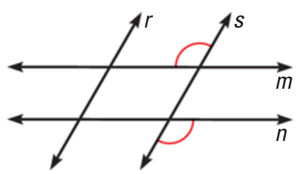
10.


11.


12.


13.


14.


15.


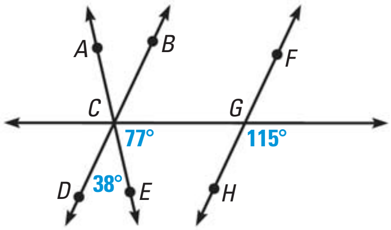
16. ★ **OPEN-ENDED MATH** Use lined paper to draw two parallel lines cut by a transversal. Use a protractor to measure one angle. Find the measures of the other seven angles without using the protractor. Give a theorem or postulate you use to find each angle measure.

17. MULTI-STEP PROBLEM Complete the steps below to determine whether $\overleftrightarrow{DB}$ and $\overleftrightarrow{HF}$ are parallel.

a. Find $m\angle DCG$ and $m\angle CGH$.

b. *Describe* the relationship between $\angle DCG$ and $\angle CGH$.

c. Are $\overleftrightarrow{DB}$ and $\overleftrightarrow{HF}$ parallel? *Explain* your reasoning.



EXAMPLE 3 on p. 163 for Ex. 18

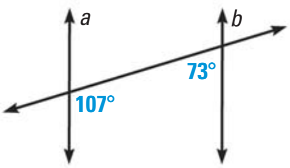
18. PLANNING A PROOF Use these steps to plan a proof of the Consecutive Interior Angles Converse, as stated on page 162.

a. Draw a diagram you can use in a proof of the theorem.

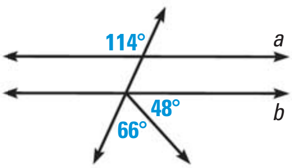
b. Write the GIVEN and PROVE statements.

REASONING Can you prove that lines *a* and *b* are parallel? If so, *explain* how.

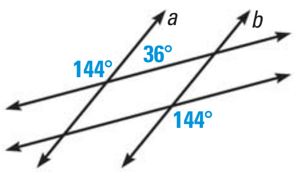
19.



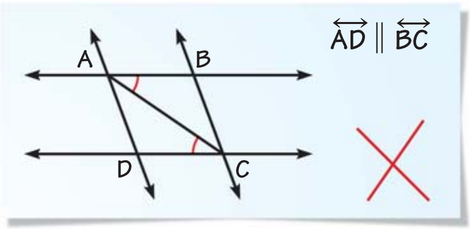
20.



21.



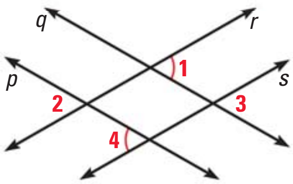
22. ERROR ANALYSIS A student decided that $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ based on the diagram below. *Describe* and correct the student's error.



23. ★ MULTIPLE CHOICE Use the diagram at the right. You know that $\angle 1 \cong \angle 4$. What can you conclude?

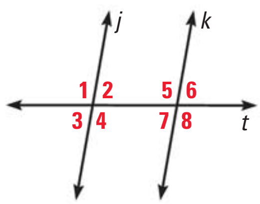
Ⓐ $p \parallel q$ Ⓑ $r \parallel s$

Ⓒ $\angle 2 \cong \angle 3$ Ⓓ None of the above



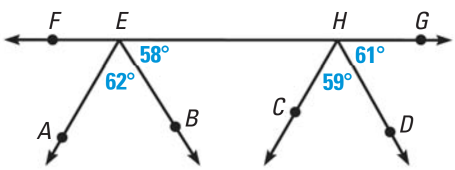
REASONING Use the diagram at the right for Exercises 24 and 25.

24. ★ SHORT RESPONSE In the diagram, assume $j \parallel k$. How many angle measures must be given in order to find the measure of every angle? *Explain* your reasoning.



25. PLANNING A PROOF In the diagram, assume $\angle 1$ and $\angle 7$ are supplementary. Write a plan for a proof showing that lines j and k are parallel.

26. REASONING Use the diagram at the right. Which rays are parallel? Which rays are not parallel? *Justify* your conclusions.



27. **VISUAL REASONING** A point R is not in plane ABC.

   a. How many lines through R are perpendicular to plane ABC?

   b. How many lines through R are parallel to plane ABC?

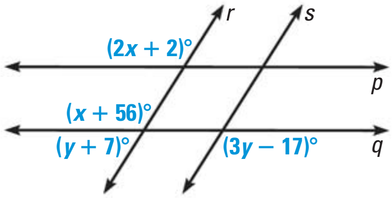
   c. How many planes through R are parallel to plane ABC?

28. **CHALLENGE** Use the diagram.

   a. Find x so that $p \parallel q$.

   b. Find y so that $r \parallel s$.

   c. Can r be parallel to s and p be parallel to q at the same time? *Explain.*



PROBLEM SOLVING

EXAMPLE 2
on p. 162
for Exs. 29–30

29. **PICNIC TABLE** How do you know that the top of the picnic table is parallel to the ground?

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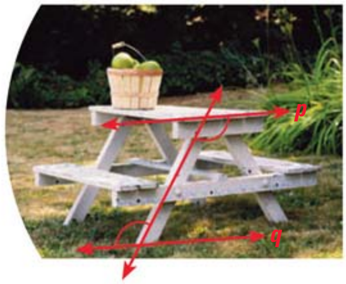
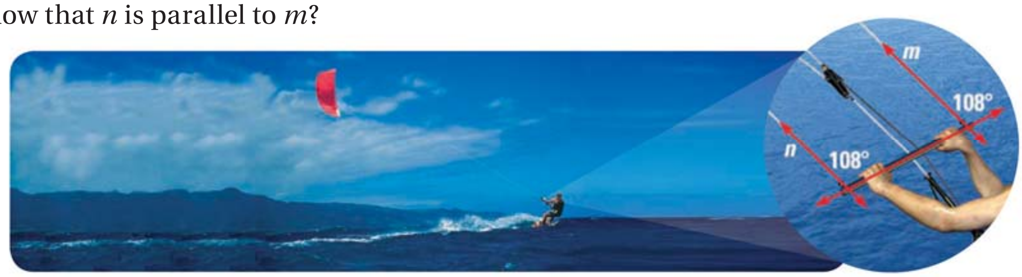


30. **KITEBOARDING** The diagram of the control bar of the kite shows the angles formed between the control bar and the kite lines. How do you know that n is parallel to m?



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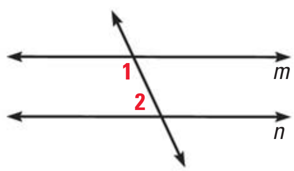
31. **DEVELOPING PROOF** Copy and complete the proof.

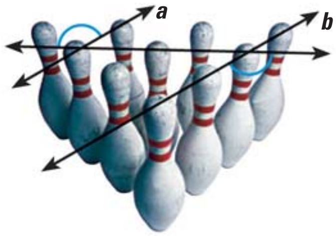
   GIVEN ▶ $m\angle 1 = 115°$, $m\angle 2 = 65°$

   PROVE ▶ $m \parallel n$



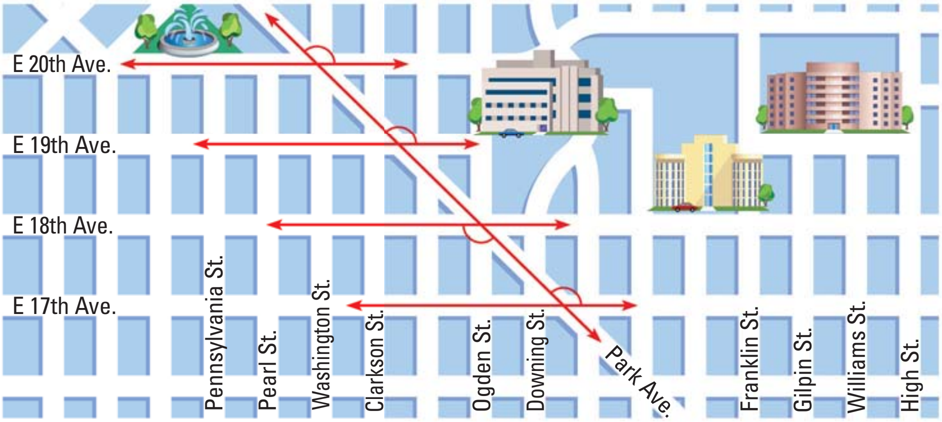
STATEMENTS	REASONS
1. $m\angle 1 = 115°$ and $m\angle 2 = 65°$	1. Given
2. $115° + 65° = 180°$	2. Addition
3. $m\angle 1 + m\angle 2 = 180°$	3. ___?___
4. $\angle 1$ and $\angle 2$ are supplementary.	4. ___?___
5. $m \parallel n$	5. ___?___

32. BOWLING PINS How do you know that the bowling pins are set up in parallel lines?



EXAMPLE 5
on p. 164
for Ex. 33

33. ★ SHORT RESPONSE The map shows part of Denver, Colorado. Use the markings on the map. Are the numbered streets parallel to one another? *Explain* how you can tell.

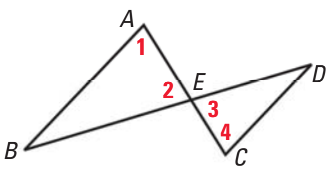


EXAMPLE 3
on p. 163
for Exs. 34–35

PROOF **Use the diagram and the given information to write a two-column or paragraph proof.**

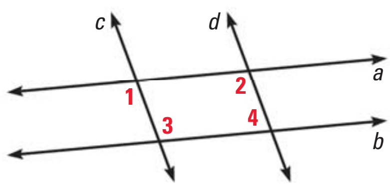
34. GIVEN ▶ $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

PROVE ▶ $\overline{AB} \parallel \overline{CD}$



35. GIVEN ▶ $a \parallel b$, $\angle 2 \cong \angle 3$

PROVE ▶ $c \parallel d$

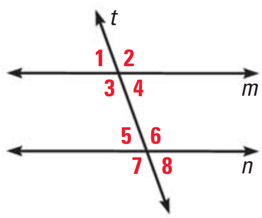


EXAMPLE 4
on p. 163
for Exs. 36–37

PROOF **In Exercises 36 and 37, use the diagram to write a paragraph proof.**

36. PROVING THEOREM 3.5 Prove the Alternate Exterior Angles Converse.

37. PROVING THEOREM 3.6 Prove the Consecutive Interior Angles Converse.



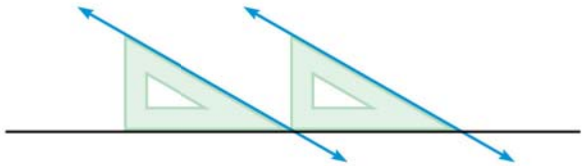
38. MULTI-STEP PROBLEM Use these steps to prove Theorem 3.7, the Transitive Property of Parallel Lines.

a. Copy the diagram in the Theorem box on page 164. Draw a transversal through all three lines.

b. Write the GIVEN and PROVE statements.

c. Use the properties of angles formed by parallel lines and transversals to prove the theorem.

39. ★ **EXTENDED RESPONSE** Architects and engineers make drawings using a plastic triangle with angle measures 30°, 60°, and 90°. The triangle slides along a fixed horizontal edge.



a. *Explain* why the blue lines shown are parallel.

b. *Explain* how the triangle can be used to draw vertical parallel lines.

REASONING **Use the diagram below in Exercises 40–44. How would you show that the given lines are parallel?**

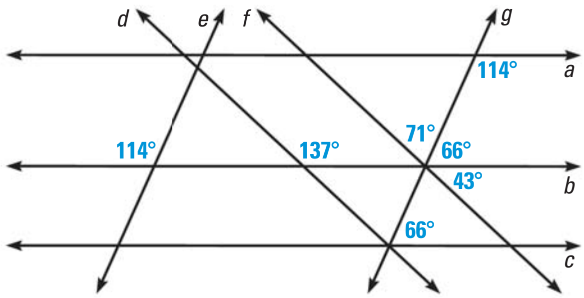
40. a and b

41. b and c

42. d and f

43. e and g

44. a and c



45. **CHALLENGE** Use these steps to investigate the angle bisectors of corresponding angles.

a. **Construction** Use a compass and straightedge or geometry drawing software to construct line ℓ, point P not on ℓ, and line n through P parallel to ℓ. Construct point Q on ℓ and construct $\overline{PQ}$. Choose a pair of alternate interior angles and construct their angle bisectors.

b. **Write a Proof** Are the angle bisectors parallel? Make a conjecture. Write a proof of your conjecture.

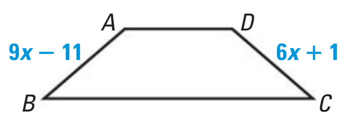
MIXED REVIEW

Solve the equation. *(p. 875)*

46. $\frac{3}{4}x = -1$ **47.** $\frac{-2}{3}x = -1$ **48.** $\frac{1}{5}x = -1$ **49.** $-6x = -1$

50. You can choose one of eight sandwich fillings and one of four kinds of bread. How many different sandwiches are possible? *(p. 891)*

51. Find the value of x if $\overline{AB} \cong \overline{AD}$ and $\overline{CD} \cong \overline{AD}$. *Explain* your steps. *(p. 112)*



PREVIEW
Prepare for Lesson 3.4 in Exs. 52–54.

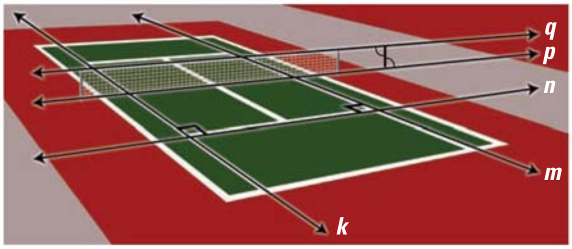
Simplify the expression.

52. $\frac{-7-2}{8-(-4)}$ *(p. 870)* **53.** $\frac{0-(-3)}{1-6}$ *(p. 870)* **54.** $\frac{3x-x}{-4x+2x}$ *(p. 139)*

MIXED REVIEW *of Problem Solving*

Lessons 3.1–3.3

1. **MULTI-STEP PROBLEM** Use the diagram of the tennis court below.

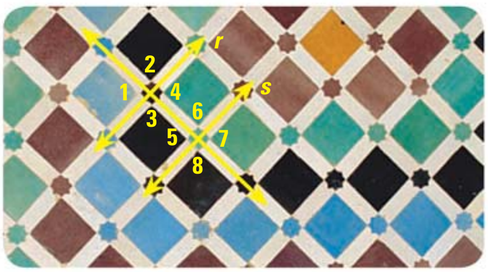


   a. Identify two pairs of parallel lines so each pair is on a different plane.

   b. Identify a pair of skew lines.

   c. Identify two pairs of perpendicular lines.

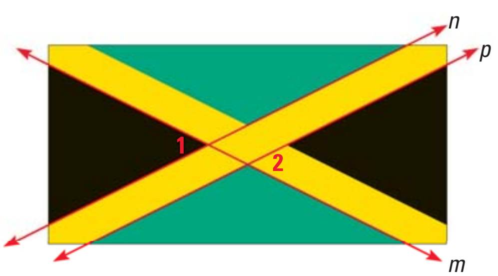
2. **MULTI-STEP PROBLEM** Use the picture of the tile floor below.



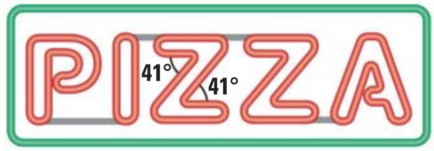
   a. Name the kind of angle pair each angle forms with $\angle 1$.

   b. Lines r and s are parallel. Name the angles that are congruent to $\angle 3$.

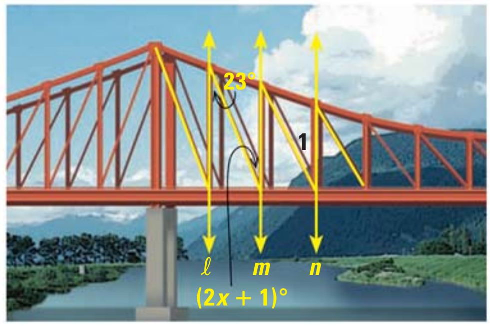
3. **OPEN-ENDED** The flag of Jamaica is shown. Given that $n \parallel p$ and $m\angle 1 = 53°$, determine the measure of $\angle 2$. *Justify* each step in your argument, labeling any angles needed for your justification.



4. **SHORT RESPONSE** A neon sign is shown below. Are the top and the bottom of the Z parallel? *Explain* how you know.



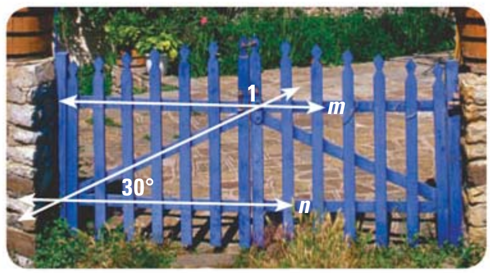
5. **EXTENDED RESPONSE** Use the diagram of the bridge below.



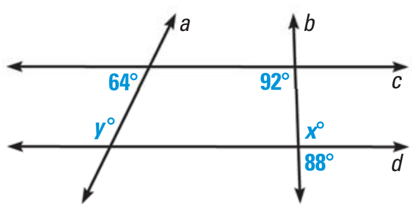
   a. Find the value of x that makes lines ℓ and m parallel.

   b. Suppose that $\ell \parallel m$ and $\ell \parallel n$. Find $m\angle 1$. *Explain* how you found your answer. Copy the diagram and label any angles you need for your explanation.

6. **GRIDDED ANSWER** In the photo of the picket fence, $m \parallel n$. What is $m\angle 1$ in degrees?



7. **SHORT RESPONSE** Find the values of x and y. *Explain* your steps.



3.4 Find and Use Slopes of Lines

Before You used properties of parallel lines to find angle measures.

Now You will find and compare slopes of lines.

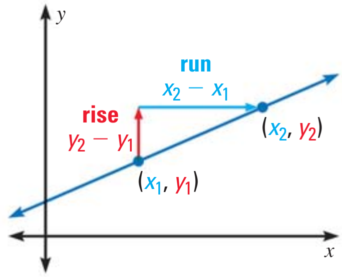
Why So you can compare rates of speed, as in Example 4.

Key Vocabulary
- **slope,** *p. 879*
- **rise,** *p. 879*
- **run,** *p. 879*

The **slope** of a nonvertical line is the ratio of vertical change (*rise*) to horizontal change (*run*) between any two points on the line.

If a line in the coordinate plane passes through points (x_1, y_1) and (x_2, y_2) then the slope m is

$$m = \frac{\textbf{rise}}{\textbf{run}} = \frac{\textbf{change in } y}{\textbf{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$



KEY CONCEPT — *For Your Notebook*

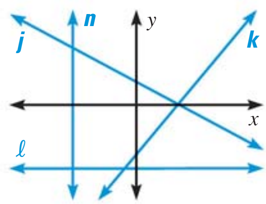
Slope of Lines in the Coordinate Plane

Negative slope: falls from left to right, as in line j

Positive slope: rises from left to right, as in line k

Zero slope (slope of 0): horizontal, as in line ℓ

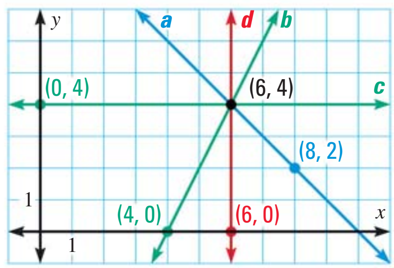
Undefined slope: vertical, as in line n



EXAMPLE 1 Find slopes of lines in a coordinate plane

REVIEW SLOPE
For more help with slope, see p. 879.

Find the slope of line *a* and line *d*.



Solution

Slope of line a: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - 2}{6 - 8} = \dfrac{2}{-2} = \mathbf{-1}$

Slope of line d: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - 0}{6 - 6} = \dfrac{\mathbf{4}}{\mathbf{0}}$, which is undefined.

✓ GUIDED PRACTICE for Example 1

Use the graph in Example 1. Find the slope of the line.

1. Line b

2. Line c

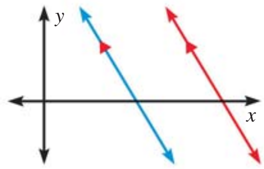
COMPARING SLOPES When two lines intersect in a coordinate plane, the steeper line has the slope with greater absolute value. You can also compare slopes to tell whether two lines are parallel or perpendicular.

POSTULATES — *For Your Notebook*

POSTULATE 17 Slopes of Parallel Lines

In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope.

Any two vertical lines are parallel.

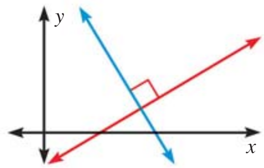


$m_1 = m_2$

POSTULATE 18 Slopes of Perpendicular Lines

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is −1.

Horizontal lines are perpendicular to vertical lines.



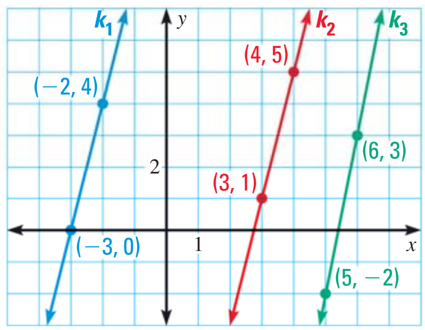
$m_1 \cdot m_2 = -1$

READ VOCABULARY

If the product of two numbers is −1, then the numbers are called *negative reciprocals*.

EXAMPLE 2 Identify parallel lines

Find the slope of each line. Which lines are parallel?



Solution

Find the slope of k_1 through (−2, 4) and (−3, 0).

$$m_1 = \frac{0-4}{-3-(-2)} = \frac{-4}{-1} = 4$$

Find the slope of k_2 through (4, 5) and (3, 1).

$$m_2 = \frac{1-5}{3-4} = \frac{-4}{-1} = 4$$

Find the slope of k_3 through (6, 3) and (5, −2).

$$m_3 = \frac{-2-3}{5-6} = \frac{-5}{-1} = 5$$

▶ Compare the slopes. Because k_1 and k_2 have the same slope, they are parallel. The slope of k_3 is different, so k_3 is not parallel to the other lines.

✓ GUIDED PRACTICE for Example 2

3. Line m passes through (−1, 3) and (4, 1). Line t passes through (−2, −1) and (3, −3). Are the two lines parallel? *Explain* how you know.

EXAMPLE 3 Draw a perpendicular line

Line h passes through (3, 0) and (7, 6). Graph the line perpendicular to h that passes through the point (2, 5).

Solution

STEP 1 **Find** the slope m_1 of line h through (3, 0) and (7, 6).

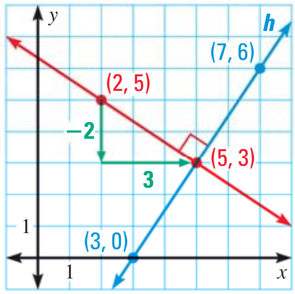
$$m_1 = \frac{6-0}{7-3} = \frac{6}{4} = \frac{3}{2}$$

STEP 2 **Find** the slope m_2 of a line perpendicular to h. Use the fact that the product of the slopes of two perpendicular lines is -1.

$$\frac{3}{2} \cdot m_2 = -1 \quad \text{Slopes of perpendicular lines}$$

$$m_2 = \frac{-2}{3} \quad \text{Multiply each side by } \frac{2}{3}.$$

STEP 3 **Use** the rise and run to graph the line.

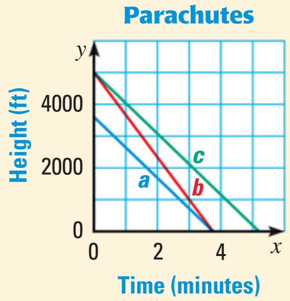


REVIEW GRAPHING

Given a point on a line and the line's slope, you can use the rise and run to find a second point and draw the line.

EXAMPLE 4 Standardized Test Practice

A skydiver made jumps with three parachutes. The graph shows the height of the skydiver from the time the parachute opened to the time of the landing for each jump. Which statement is true?



Ⓐ The parachute opened at the same height in jumps a and b.

Ⓑ The parachute was open for the same amount of time in jumps b and c.

Ⓒ The skydiver descended at the same rate in jumps a and b.

Ⓓ The skydiver descended at the same rate in jumps a and c.

Solution

The rate at which the skydiver descended is represented by the slope of the segments. The segments that have the same slope are a and c.

▸ The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

ELIMINATE CHOICES

The y-intercept represents the height when the parachute opened, so the heights in jumps a and b were not the same. So you can eliminate choice A.

✓ GUIDED PRACTICE for Examples 3 and 4

4. Line n passes through (0, 2) and (6, 5). Line m passes through (2, 4) and (4, 0). Is $n \perp m$? *Explain.*

5. In Example 4, which parachute is in the air for the longest time? *Explain.*

6. In Example 4, what do the x-intercepts represent in the situation? How can you use this to eliminate one of the choices?

EXAMPLE 5 Solve a real-world problem

ROLLER COASTERS During the climb on the Magnum XL-200 roller coaster, you move 41 feet upward for every 80 feet you move horizontally. At the crest of the hill, you have moved 400 feet forward.

a. **Making a Table** Make a table showing the height of the Magnum at every 80 feet it moves horizontally. How high is the roller coaster at the top of its climb?

b. **Calculating** Write a fraction that represents the height the Magnum climbs for each foot it moves horizontally. What does the numerator represent?

c. **Using a Graph** Another roller coaster, the Millenium Force, climbs at a slope of 1. At its crest, the horizontal distance from the starting point is 310 feet. Compare this climb to that of the Magnum. Which climb is steeper?

Solution

a.

Horizontal distance (ft)	80	160	240	320	400
Height (ft)	41	82	123	164	205

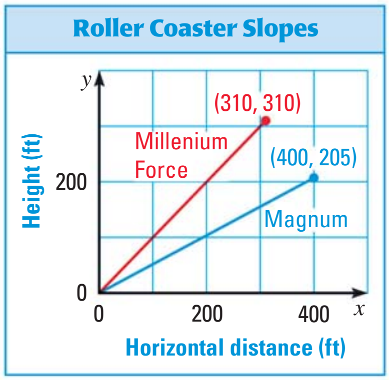
The Magnum XL-200 is 205 feet high at the top of its climb.

b. Slope of the Magnum $= \frac{\text{rise}}{\text{run}} = \frac{41}{80} = \frac{41 \div 80}{80 \div 80} = \frac{0.5125}{1}$

The numerator, 0.5125, represents the slope in decimal form.

c. Use a graph to compare the climbs. Let x be the horizontal distance and let y be the height. Because the slope of the Millenium Force is 1, the rise is equal to the run. So the highest point must be at (310, 310).

▶ The graph shows that the Millenium Force has a steeper climb, because the slope of its line is greater ($1 > 0.5125$).



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GUIDED PRACTICE for Example 5

7. Line q passes through the points (0, 0) and (−4, 5). Line t passes through the points (0, 0) and (−10, 7). Which line is steeper, q or t?

8. **WHAT IF?** Suppose a roller coaster climbed 300 feet upward for every 350 feet it moved horizontally. Is it *more steep* or *less steep* than the Magnum? than the Millenium Force?

3.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 13, and 35

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 34, 35, and 41

◆ = **MULTIPLE REPRESENTATIONS** Ex. 37

SKILL PRACTICE

1. **VOCABULARY** *Describe* what is meant by the slope of a nonvertical line.

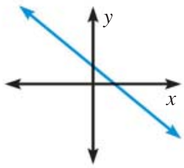
2. ★ **WRITING** What happens when you apply the slope formula to a horizontal line? What happens when you apply it to a vertical line?

EXAMPLE 1 on p. 171 for Exs. 3–12

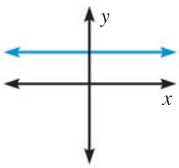
MATCHING **Match the description of the slope of a line with its graph.**

3. m is positive.　4. m is negative.　5. m is zero.　6. m is undefined.

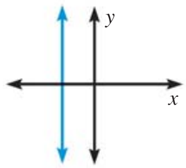
A.



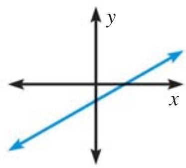
B.



C.



D.

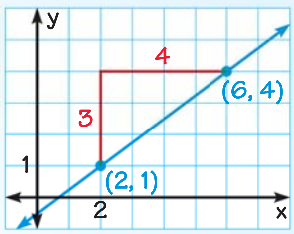


FINDING SLOPE **Find the slope of the line that passes through the points.**

7. (3, 5), (5, 6)　8. (−2, 2), (2, −6)　9. (−5, −1), (3, −1)　10. (2, 1), (0, 6)

ERROR ANALYSIS ***Describe*** **and correct the error in finding the slope of the line.**

11. $m = \frac{4}{3}$



12. Slope of the line through (2, 7) and (4, 5)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-5}{4-2} = \frac{2}{2} = 1$$

EXAMPLES 2 and 3 on pp. 172–173 for Exs. 13–18

TYPES OF LINES **Tell whether the lines through the given points are *parallel*, *perpendicular*, or *neither*. *Justify* your answer.**

13. **Line 1:** (1, 0), (7, 4)
Line 2: (7, 0), (3, 6)

14. **Line 1:** (−3, 1), (−7, −2)
Line 2: (2, −1), (8, 4)

15. **Line 1:** (−9, 3), (−5, 7)
Line 2: (−11, 6), (−7, 2)

GRAPHING **Graph the line through the given point with the given slope.**

16. $P(3, -2)$, slope $-\frac{1}{6}$　17. $P(-4, 0)$, slope $\frac{5}{2}$　18. $P(0, 5)$, slope $\frac{2}{3}$

EXAMPLES 4 and 5 on pp. 173–174 for Exs. 19–22

STEEPNESS OF A LINE **Tell which line through the given points is steeper.**

19. **Line 1:** (−2, 3), (3, 5)
Line 2: (3, 1), (6, 5)

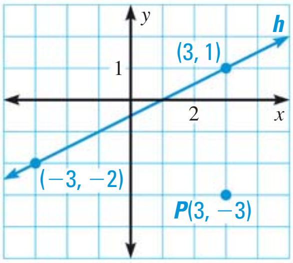
20. **Line 1:** (−2, −1), (1, −2)
Line 2: (−5, −3), (−1, −4)

21. **Line 1:** (−4, 2), (−3, 6)
Line 2: (1, 6), (3, 8)

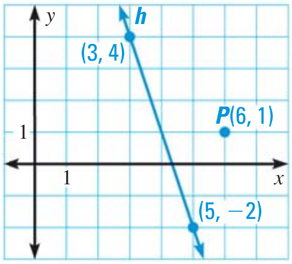
22. **REASONING** Use your results from Exercises 19–21. *Describe* a way to determine which of two lines is steeper without graphing them.

PERPENDICULAR LINES **Find the slope of line n perpendicular to line h and passing through point P. Then copy the graph and graph line n.**

23.



24.



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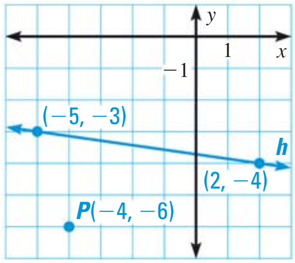

26. **REASONING** Use the concept of slope to decide whether the points $(-3, 3)$, $(1, -2)$, and $(4, 0)$ lie on the same line. *Explain* your reasoning and include a diagram.

GRAPHING **Graph a line with the given description.**

27. Through $(0, 2)$ and parallel to the line through $(-2, 4)$ and $(-5, 1)$

28. Through $(1, 3)$ and perpendicular to the line through $(-1, -1)$ and $(2, 0)$

29. Through $(-2, 1)$ and parallel to the line through $(3, 1)$ and $\left(4, -\frac{1}{2}\right)$

CHALLENGE **Find the unknown coordinate so the line through the points has the given slope.**

30. $(-3, 2), (0, y)$; slope -2 **31.** $(-7, -4), (x, 0)$; slope $\frac{1}{3}$ **32.** $(4, -3), (x, 1)$; slope -4

PROBLEM SOLVING

33. **WATER SLIDE** The water slide is 6 feet tall, and the end of the slide is 9 feet from the base of the ladder. About what slope does the slide have?

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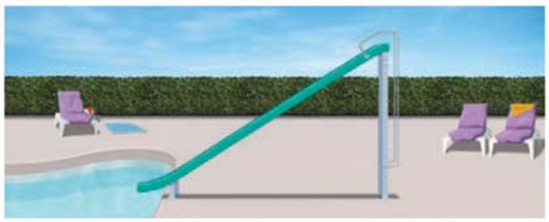

EXAMPLE 5
on p. 174
for Exs. 34–37

34. ★ **MULTIPLE CHOICE** Which car has better gas mileage?

Ⓐ A Ⓑ B

Ⓒ Same rate Ⓓ Cannot be determined

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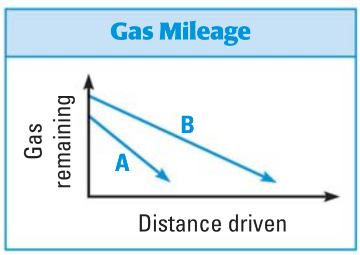
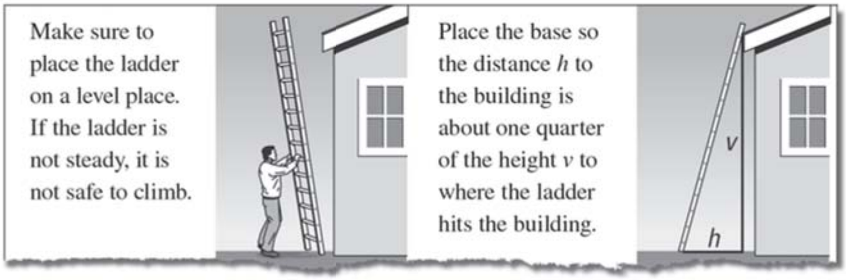


35. ★ **SHORT RESPONSE** *Compare* the graphs of the three lines described below. Which is most steep? Which is the least steep? Include a sketch in your answer.

Line a: through the point $(3, 0)$ with a y-intercept of 4
Line b: through the point $(3, 0)$ with a y-intercept greater than 4
Line c: through the point $(3, 0)$ with a y-intercept between 0 and 4

○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE ◆ = MULTIPLE REPRESENTATIONS

36. MULTI-STEP PROBLEM Ladder safety guidelines include the following recommendation about ladder placement. The horizontal distance h between the base of the ladder and the object the ladder is resting against should be about one quarter of the vertical distance v between the ground and where the ladder rests against the object.



a. Find the recommended slope for a ladder.

b. Suppose the base of a ladder is 6 feet away from a building. The ladder has the recommended slope. Find v.

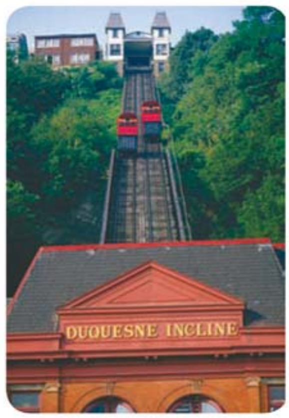
c. Suppose a ladder is 34 feet from the ground where it touches a building. The ladder has the recommended slope. Find h.

37. MULTIPLE REPRESENTATIONS The Duquesne (pronounced "du-KAYN") Incline was built in 1888 in Pittsburgh, Pennsylvania, to move people up and down a mountain there. On the incline, you move about 29 feet vertically for every 50 feet you move horizontally. When you reach the top of the hill, you have moved a horizontal distance of about 700 feet.

a. Making a Table Make a table showing the vertical distance that the incline moves for each 50 feet of horizontal distance during its climb. How high is the incline at the top?

b. Drawing a Graph Write a fraction that represents the slope of the incline's climb path. Draw a graph to show the climb path.

c. Comparing Slopes The Burgenstock Incline in Switzerland moves about 144 vertical feet for every 271 horizontal feet. Write a fraction to represent the slope of this incline's path. Which incline is steeper, the *Burgenstock* or the *Duquesne*?



38. PROVING THEOREM 3.7 Use slopes of lines to write a paragraph proof of the Transitive Property of Parallel Lines on page 164.

AVERAGE RATE OF CHANGE In Exercises 39 and 40, slope can be used to describe an *average rate of change*. To write an average rate of change, rewrite the slope fraction so the denominator is one.

39. BUSINESS In 2000, a business made a profit of \$8500. In 2006, the business made a profit of \$15,400. Find the average rate of change in dollars per year from 2000 to 2006.

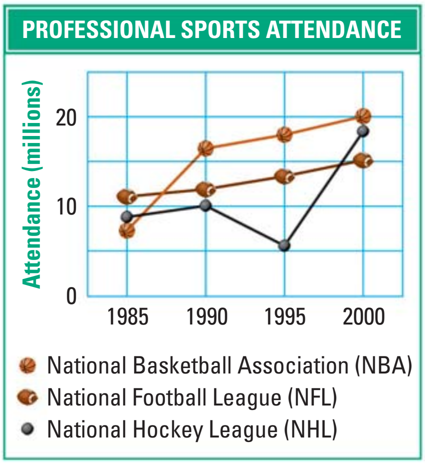
40. ROCK CLIMBING A rock climber begins climbing at a point 400 feet above sea level. It takes the climber 45 minutes to climb to the destination, which is 706 feet above sea level. Find the average rate of change in feet per minute for the climber from start to finish.

41. ★ **EXTENDED RESPONSE** The line graph shows the regular season attendance (in millions) for three professional sports organizations from 1985 to 2000.

a. During which five-year period did the NBA attendance increase the most? Estimate the rate of change for this five-year period in people per year.

b. During which five-year period did the NHL attendance increase the most? Estimate the rate of change for this five-year period in people per year.

c. **Interpret** The line graph for the NFL seems to be almost linear between 1985 and 2000. Write a sentence about what this means in terms of the real-world situation.



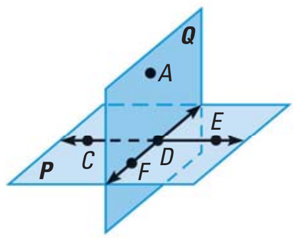
42. **CHALLENGE** Find two values of k such that the points $(-3, 1)$, $(0, k)$, and $(k, 5)$ are collinear. *Explain* your reasoning.

MIXED REVIEW

43. Is the point $(-1, -7)$ on the line $y = 2x - 5$? *Explain*. *(p. 878)*

44. Find the intercepts of the graph of $y = -3x + 9$. *(p. 879)*

Use the diagram to write two examples of each postulate. *(p. 96)*



45. Through any two points there exists exactly one line.

46. Through any three noncollinear points there exists exactly one plane.

PREVIEW
Prepare for Lesson 3.5 in Exs. 47–49.

Solve the equation for *y*. Write a reason for each step. *(p. 105)*

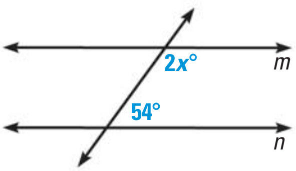
47. $6x + 4y = 40$

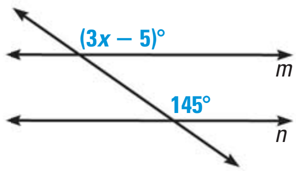
48. $\frac{1}{2}x - \frac{5}{4}y = -10$

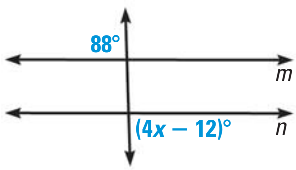
49. $16 - 3y = 24x$

QUIZ *for Lessons 3.3–3.4*

Find the value of x that makes $m \parallel n$. *(p. 161)*

1.


2.


3.


Find the slope of the line that passes through the given points. *(p. 171)*

4. $(1, -1)$, $(3, 3)$

5. $(1, 2)$, $(4, 5)$

6. $(-3, -2)$, $(-7, -6)$

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Keystrokes

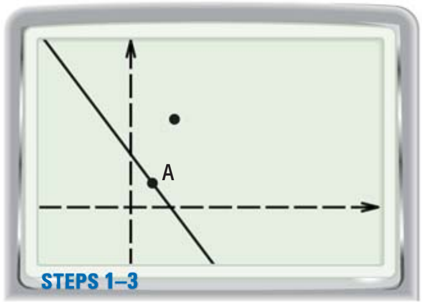
3.4 Investigate Slopes

MATERIALS • graphing calculator or computer

QUESTION **How can you verify the Slopes of Parallel Lines Postulate?**

You can verify the postulates you learned in Lesson 3.4 using geometry drawing software.

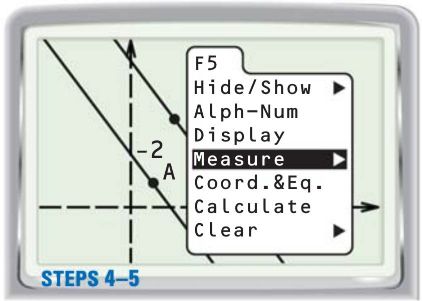
EXAMPLE **Verify the Slopes of Parallel Lines Postulate**



STEP 1 ***Show axes*** Show the x-axis and the y-axis by choosing Hide/Show Axes from the F5 menu.

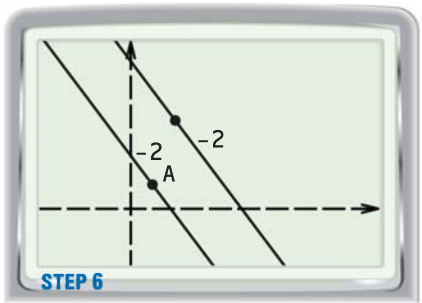
STEP 2 ***Draw line*** Draw a line by choosing Line from the F2 menu. Do not use one of the axes as your line. Choose a point on the line and label it A.

STEP 3 ***Graph point*** Graph a point not on the line by choosing Point from the F2 menu.



STEP 4 ***Draw parallel line*** Choose Parallel from the F3 menu and select the line. Then select the point not on the line.

STEP 5 ***Measure slopes*** Select one line and choose Measure Slope from the F5 menu. Repeat this step for the second line.



STEP 6 ***Move line*** Drag point A to move the line. What do you expect to happen?

PRACTICE

1. Use geometry drawing software to verify the Slopes of Perpendicular Lines Postulate.

a. Construct a line and a point not on that line. Use Steps 1–3 from the Example above.

b. Construct a line that is perpendicular to your original line and passes through the given point.

c. Measure the slopes of the two lines. Multiply the slopes. What do you expect the product of the slopes to be?

2. **WRITING** Use the arrow keys to move your line from Exercise 1. *Describe* what happens to the product of the slopes when one of the lines is vertical. *Explain* why this happens.

3.5 Write and Graph Equations of Lines

Before You found slopes of lines.

Now You will find equations of lines.

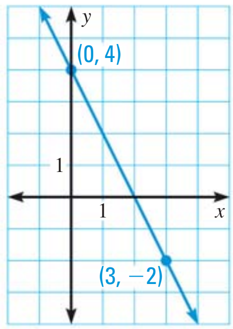
Why? So you can find monthly gym costs, as in Example 4.

Key Vocabulary
- **slope-intercept form**
- **standard form**
- ***x*-intercept,** *p. 879*
- ***y*-intercept,** *p. 879*

Linear equations may be written in different forms. The general form of a linear equation in **slope-intercept form** is $y = mx + b$, where m is the slope and b is the y-intercept.

EXAMPLE 1 Write an equation of a line from a graph

Write an equation of the line in slope-intercept form.



Solution

STEP 1 **Find** the slope. Choose two points on the graph of the line, (0, 4) and (3, −2).

$$m = \frac{4 - (-2)}{0 - 3} = \frac{6}{-3} = -2$$

STEP 2 **Find** the y-intercept. The line intersects the y-axis at the point (0, 4), so the y-intercept is 4.

STEP 3 **Write** the equation.

$y = mx + b$ **Use slope-intercept form.**

$y = -2x + 4$ **Substitute −2 for *m* and 4 for *b*.**

EXAMPLE 2 Write an equation of a parallel line

Write an equation of the line passing through the point (−1, 1) that is parallel to the line with the equation $y = 2x - 3$.

Solution

STEP 1 **Find** the slope m. The slope of a line parallel to $y = 2x - 3$ is the same as the given line, so the slope is 2.

STEP 2 **Find** the y-intercept b by using $m = 2$ and $(x, y) = (-1, 1)$.

$y = mx + b$ **Use slope-intercept form.**

$1 = 2(-1) + b$ **Substitute for *x*, *y*, and *m*.**

$3 = b$ **Solve for *b*.**

▶ Because $m = 2$ and $b = 3$, an equation of the line is $y = 2x + 3$.

LINEAR EQUATIONS
The graph of a linear equation represents all the solutions of the equation. So, the given point must be a solution of the equation.

CHECKING BY GRAPHING You can check that equations are correct by graphing. In Example 2, you can use a graph to check that $y = 2x - 3$ is parallel to $y = 2x + 3$.

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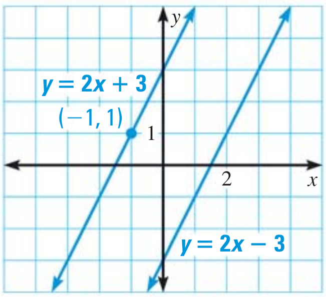


EXAMPLE 3 Write an equation of a perpendicular line

Write an equation of the line j passing through the point (2, 3) that is perpendicular to the line k with the equation $y = -2x + 2$.

Solution

STEP 1 **Find** the slope m of line j. Line k has a slope of -2.

$-2 \cdot m = -1$ The product of the slopes of $\perp$ lines is -1.

$m = \frac{1}{2}$ Divide each side by -2.

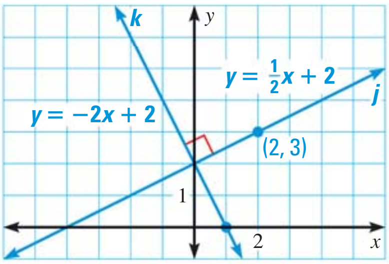
STEP 2 **Find** the y-intercept b by using $m = \frac{1}{2}$ and $(x, y) = (2, 3)$.

$y = mx + b$ Use slope-intercept form.

$3 = \frac{1}{2}(2) + b$ Substitute for *x*, *y*, and *m*.

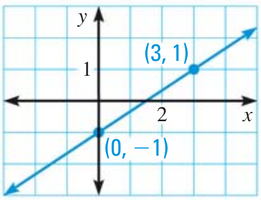
$2 = b$ Solve for *b*.

▶ Because $m = \frac{1}{2}$ and $b = 2$, an equation of line j is $y = \frac{1}{2}x + 2$. You can check that the lines j and k are perpendicular by graphing, then using a protractor to measure one of the angles formed by the lines.



GUIDED PRACTICE for Examples 1, 2, and 3

1. Write an equation of the line in the graph at the right.



2. Write an equation of the line that passes through (−2, 5) and (1, 2).

3. Write an equation of the line that passes through the point (1, 5) and is parallel to the line with the equation $y = 3x - 5$. Graph the lines to check that they are parallel.

4. How do you know the lines $x = 4$ and $y = 2$ are perpendicular?

EXAMPLE 4 Write an equation of a line from a graph

GYM MEMBERSHIP The graph models the total cost of joining a gym. Write an equation of the line. Explain the meaning of the slope and the y-intercept of the line.

Solution

STEP 1 **Find** the slope.

$$m = \frac{363 - 231}{5 - 2} = \frac{132}{3} = 44$$

STEP 2 **Find** the y-intercept. Use the slope and one of the points on the graph.

$y = mx + b$ **Use slope-intercept form.**

$231 = 44 \cdot 2 + b$ **Substitute for *x*, *y*, and *m*.**

$143 = b$ **Simplify.**

STEP 3 **Write** the equation. Because $m = 44$ and $b = 143$, an equation of the line is $y = 44x + 143$.

▶ The equation $y = 44x + 143$ models the cost. The slope is the monthly fee, \$44, and the y-intercept is the initial cost to join the gym, \$143.

STANDARD FORM Another form of a linear equation is *standard form*. In **standard form**, the equation is written as $Ax + By = C$, where A and B are not both zero.

EXAMPLE 5 Graph a line with equation in standard form

Graph $3x + 4y = 12$.

Solution

> **CHOOSE A METHOD**
> Another way you could graph the equation is to solve the equation for *y*. Then the equation will be in slope-intercept form. Use rise and run from the point where the line crosses the *y*-axis to find a second point. Then graph the line.

The equation is in standard form, so you can use the intercepts.

STEP 1 **Find** the intercepts.

To find the x-intercept, let $y = 0$.

$3x + 4y = 12$

$3x + 4(0) = 12$

$x = 4$

To find the y-intercept, let $x = 0$.

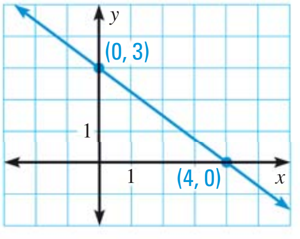
$3x + 4y = 12$

$3(0) + 4y = 12$

$y = 3$

STEP 2 **Graph** the line.

The intercepts are $(4, 0)$ and $(0, 3)$. Graph these points, then draw a line through the points.



✓ GUIDED PRACTICE for Examples 4 and 5

5. The equation $y = 50x + 125$ models the total cost of joining a climbing gym. What are the meaning of the slope and the y-intercept of the line?

Graph the equation.

6. $2x - 3y = 6$ **7.** $y = 4$ **8.** $x = -3$

WRITING EQUATIONS You can write linear equations to model real-world situations, such as comparing costs to find a better buy.

EXAMPLE 6 Solve a real-world problem

DVD RENTAL You can rent DVDs at a local store for \$4.00 each. An Internet company offers a flat fee of \$15.00 per month for as many rentals as you want. How many DVDs do you need to rent to make the online rental a better buy?

ANOTHER WAY
For alternative methods for solving the problem in Example 6, turn to page 188 for the **Problem Solving Workshop**.

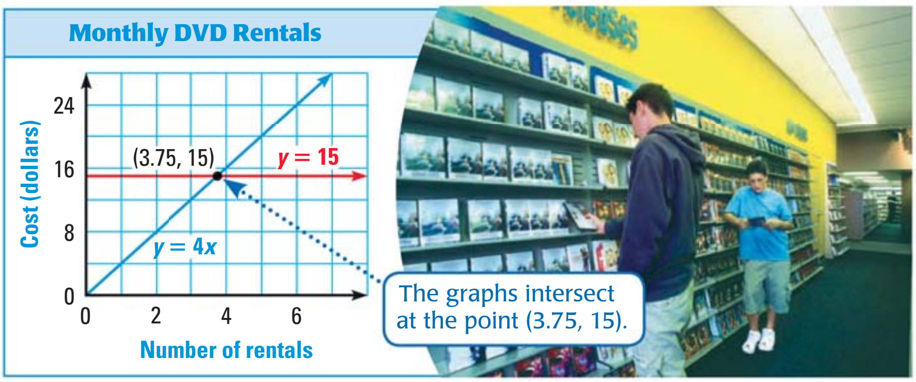
Solution

STEP 1 **Model** each rental with an equation.

Cost of one month's rental online: $y = 15$

Cost of one month's rental locally: $y = 4x$, where x represents the number of DVDs rented

STEP 2 **Graph** each equation.



READ VOCABULARY
The point at which the costs are the same is sometimes called the *break-even point.*

▶ The point of intersection is (3.75, 15). Using the graph, you can see that it is cheaper to rent locally if you rent 3 or fewer DVDs per month. If you rent 4 or more DVDs per month, it is cheaper to rent online.

✓ GUIDED PRACTICE for Example 6

9. **WHAT IF?** In Example 6, suppose the online rental is \$16.50 per month and the local rental is \$4 each. How many DVDs do you need to rent to make the online rental a better buy?

10. How would your answer to Exercise 9 change if you had a 2-for-1 coupon that you could use once at the local store?

3.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 17, 23, and 61

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 29, 64, and 65

SKILL PRACTICE

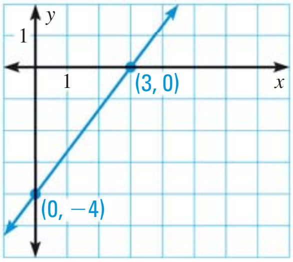
1. VOCABULARY What does *intercept* mean in the expression *slope-intercept form*?

2. ★ WRITING Explain how you can use the standard form of a linear equation to find the intercepts of a line.

EXAMPLE 1 on p. 180 for Exs. 3–22

WRITING EQUATIONS **Write an equation of the line shown.**

3.



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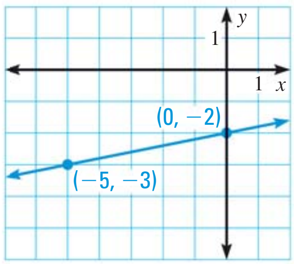

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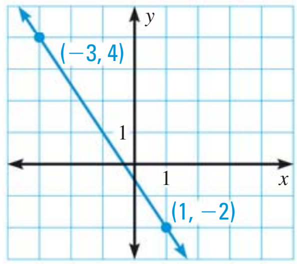

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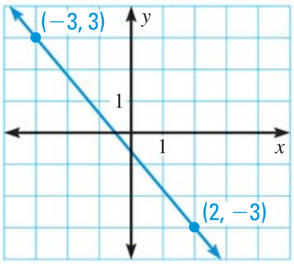

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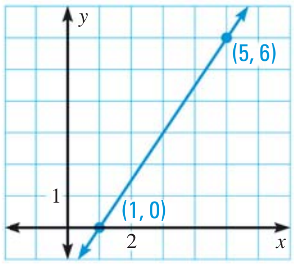

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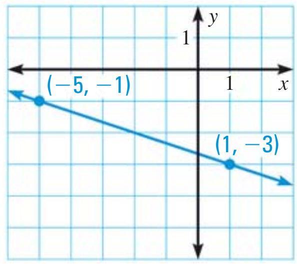
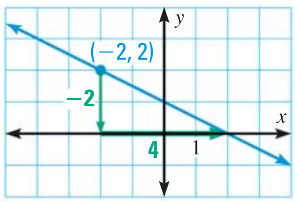


9. ★ MULTIPLE CHOICE Which equation is an equation of the line in the graph?



Ⓐ $y = -\frac{1}{2}x$ Ⓑ $y = -\frac{1}{2}x + 1$

Ⓒ $y = -2x$ Ⓓ $y = -2x + 1$

WRITING EQUATIONS **Write an equation of the line with the given slope *m* and *y*-intercept *b*.**

10. $m = -5, b = -12$ **11.** $m = 3, b = 2$ **12.** $m = 4, b = -6$

13. $m = -\frac{5}{2}, b = 0$ **14.** $m = \frac{4}{9}, b = -\frac{2}{9}$ **15.** $m = -\frac{11}{5}, b = -12$

WRITING EQUATIONS **Write an equation of the line that passes through the given point *P* and has the given slope *m*.**

16. $P(-1, 0), m = -1$ **17.** $P(5, 4), m = 4$ **18.** $P(6, -2), m = 3$

19. $P(-8, -2), m = -\frac{2}{3}$ **20.** $P(0, -3), m = -\frac{1}{6}$ **21.** $P(-13, 7), m = 0$

22. WRITING EQUATIONS Write an equation of a line with undefined slope that passes through the point $(3, -2)$.

EXAMPLE 2
on p. 180
for Exs. 23–29

PARALLEL LINES **Write an equation of the line that passes through point P and is parallel to the line with the given equation.**

23. $P(0, -1)$, $y = -2x + 3$ **24.** $P(-7, -4)$, $y = 16$ **25.** $P(3, 8)$, $y - 1 = \frac{1}{5}(x + 4)$

26. $P(-2, 6)$, $x = -5$ **27.** $P(-2, 1)$, $10x + 4y = -8$ **28.** $P(4, 0)$, $-x + 2y = 12$

29. ★ **MULTIPLE CHOICE** Line a passes through points $(-2, 1)$ and $(2, 9)$. Which equation is an equation of a line parallel to line a?

(A) $y = -2x + 5$ (B) $y = -\frac{1}{2}x + 5$ (C) $y = \frac{1}{2}x - 5$ (D) $y = 2x - 5$

EXAMPLE 3
on p. 181
for Exs. 30–35

PERPENDICULAR LINES **Write an equation of the line that passes through point P and is perpendicular to the line with the given equation.**

30. $P(0, 0)$, $y = -9x - 1$ **31.** $P(-1, 1)$, $y = \frac{7}{3}x + 10$ **32.** $P(4, -6)$, $y = -3$

33. $P(2, 3)$, $y - 4 = -2(x + 3)$ **34.** $P(0, -5)$, $x = 20$ **35.** $P(-8, 0)$, $3x - 5y = 6$

EXAMPLE 5
on p. 182
for Exs. 36–45

GRAPHING EQUATIONS **Graph the equation.**

36. $8x + 2y = -10$ **37.** $x + y = 1$ **38.** $4x - y = -8$

39. $-x + 3y = -9$ **40.** $y - 2 = -1$ **41.** $y + 2 = x - 1$

42. $x + 3 = -4$ **43.** $2y - 4 = -x + 1$ **44.** $3(x - 2) = -y - 4$

45. **ERROR ANALYSIS** *Describe* and correct the error in finding the x- and y-intercepts of the graph of $5x - 3y = -15$.

To find the x-intercept, let x = 0:	To find the y-intercept, let y = 0:
$5x - 3y = -15$	$5x - 3y = -15$
$5(0) - 3y = -15$	$5x - 3(0) = -15$
$y = 5$	$x = -3$

IDENTIFYING PARALLEL LINES **Which lines are parallel, if any?**

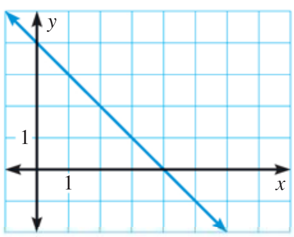
46. $y = 3x - 4$
$x + 3y = 6$
$3(x + 1) = y - 2$

47. $x + 2y = 9$
$y = 0.5x + 7$
$-x + 2y = -5$

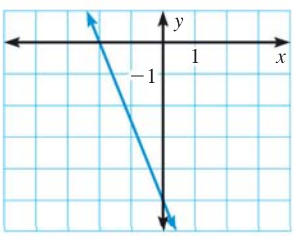
48. $x - 6y = 10$
$6x - y = 11$
$x + 6y = 12$

USING INTERCEPTS **Identify the x- and y-intercepts of the line. Use the intercepts to write an equation of the line.**

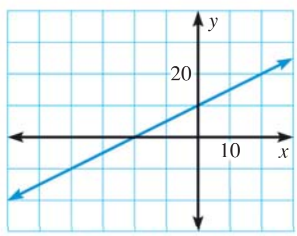
49.



50.



51.



52. **INTERCEPTS** A line passes through the points $(-10, -3)$ and $(6, 1)$. Where does the line intersect the x-axis? Where does the line intersect the y-axis?

SOLUTIONS TO EQUATIONS **Graph the linear equations. Then use the graph to estimate how many solutions the equations share.**

53. $y = 4x + 9$
$4x - y = 1$

54. $3y + 4x = 16$
$2x - y = 18$

55. $y = -5x + 6$
$10x + 2y = 12$

56. xy **ALGEBRA** Solve Exercises 53–55 algebraically. (For help, see Skills Review Handbook, p. 880.) Make a conjecture about how the solution(s) can tell you whether the lines intersect, are parallel, or are the same line.

57. xy **ALGEBRA** Find a value for k so that the line through $(-1, k)$ and $(-7, -2)$ is parallel to the line with equation $y = x + 1$.

58. xy **ALGEBRA** Find a value for k so that the line through $(k, 2)$ and $(7, 0)$ is perpendicular to the line with equation $y = x - \frac{28}{5}$.

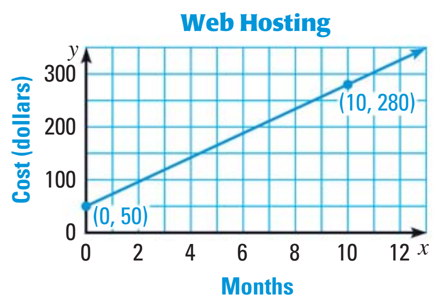
59. **CHALLENGE** Graph the points $R(-7, -3)$, $S(-2, 3)$, and $T(10, -7)$. Connect them to make $\triangle RST$. Write an equation of the line containing each side. *Explain* how you can use slopes to show that $\triangle RST$ has one right angle.

PROBLEM SOLVING

EXAMPLE 4
on p. 182
for Exs. 60–61

60. **WEB HOSTING** The graph models the total cost of using a web hosting service for several months. Write an equation of the line. Tell what the slope and y-intercept mean in this situation. Then find the total cost of using the web hosting service for one year.

@HomeTutor for problem solving help at classzone.com



61. **SCIENCE** Scientists believe that a Tyrannosaurus Rex weighed about 2000 kilograms by age 14. It then had a growth spurt for four years, gaining 2.1 kilograms per day. Write an equation to model this situation. What are the slope and y-intercept? Tell what the slope and y-intercept mean in this situation.

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Field Museum, Chicago, Illinois

EXAMPLE 6
on p. 183
for Exs. 62–65

62. **MULTI-STEP PROBLEM** A national park has two options: a \$50 pass for all admissions during the year, or a \$4 entrance fee each time you enter.

a. Model Write an equation to model the cost of going to the park for a year using a pass and another equation for paying a fee each time.

b. Graph Graph both equations you wrote in part (a).

c. Interpret How many visits do you need to make for the pass to be cheaper? *Explain*.

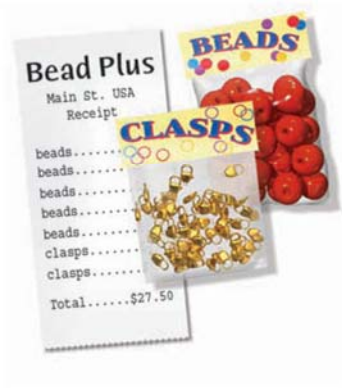
○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

63. **PIZZA COSTS** You are buying slices of pizza for you and your friends. A small slice costs \$2 and a large slice costs \$3. You have \$24 to spend. Write an equation in standard form $Ax + By = C$ that models this situation. What do the values of A, B, and C mean in this situation?

64. ★ **SHORT RESPONSE** You run at a rate of 4 miles per hour and your friend runs at a rate of 3.5 miles per hour. Your friend starts running 10 minutes before you, and you run for a half hour on the same path. Will you catch up to your friend? Use a graph to support your answer.

65. ★ **EXTENDED RESPONSE** Audrey and Sara are making jewelry. Audrey buys 2 bags of beads and 1 package of clasps for a total of \$13. Sara buys 5 bags of beads and 2 packages of clasps for a total of \$27.50.



a. Let b be the price of one bag of beads and let c be the price of one package of clasps. Write equations to represent the total cost for Audrey and the total cost for Sara.

b. Graph the equations from part (a).

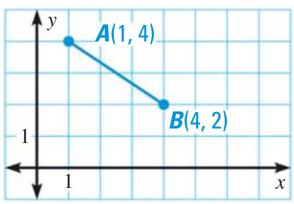
c. *Explain* the meaning of the intersection of the two lines in terms of the real-world situation.

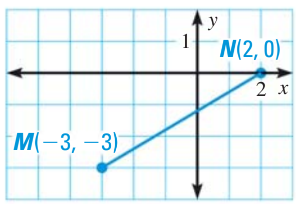
66. **CHALLENGE** Michael is deciding which gym membership to buy. Points (2, 112) and (4, 174) give the cost of gym membership at one gym after two and four months. Points (1, 62) and (3, 102) give the cost of gym membership at a second gym after one and three months. Write equations to model the cost of each gym membership. At what point do the graphs intersect, if they intersect? Which gym is cheaper? *Explain.*

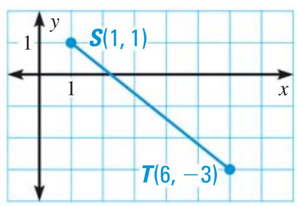
MIXED REVIEW

PREVIEW
Prepare for Lesson 3.6 in Exs. 67–69.

Find the length of each segment. Round to the nearest tenth of a unit. *(p. 15)*

67.


68.


69.


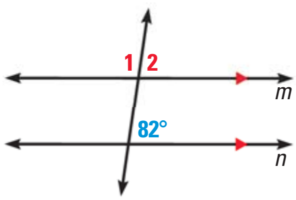
Describe **the pattern in the numbers. Write the next number in the pattern.** *(p. 72)*

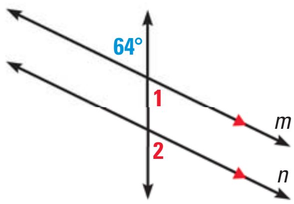
70. −2, −7, −12, −17, . . .

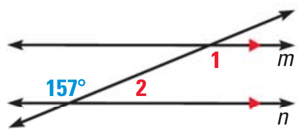
71. 4, 8, 16, 32, . . .

72. 101, 98, 95, 92, . . .

Find $m\angle 1$ and $m\angle 2$. *Explain* your reasoning. *(p. 154)*

73.


74.


75.


PROBLEM SOLVING WORKSHOP
LESSON 3.5

Using ALTERNATIVE METHODS

Another Way to Solve Example 6, page 183

MULTIPLE REPRESENTATIONS In Example 6 on page 183, you saw how to graph equations to solve a problem about renting DVDs. Another way you can solve the problem is *using a table*. Alternatively, you can use the equations to solve the problem *algebraically*.

PROBLEM

DVD RENTAL You can rent DVDs at a local store for $4.00 each. An Internet company offers a flat fee of $15.00 per month for as many rentals as you want. How many DVDs do you need to rent to make the online rental a better buy?

METHOD 1

Using a Table You can make a table to answer the question.

STEP 1 **Make** a table representing each rental option.

DVDs rented	Renting locally	Renting online
1	$4	$15
2	$8	$15

STEP 2 **Add** rows to your table until you see a pattern.

DVDs rented	Renting locally	Renting online
1	$4	$15
2	$8	$15
3	$12	$15
4	$16	$15
5	$20	$15
6	$24	$15

STEP 3 **Analyze** the table. Notice that the values in the second column (the cost of renting locally) are less than the values in the third column (the cost of renting online) for three or fewer DVDs. However, the values in the second column are greater than those in the third column for four or more DVDs.

▶ It is cheaper to rent locally if you rent 3 or fewer DVDs per month. If you rent 4 or more DVDs per month, it is cheaper to rent online.

METHOD 2

Using Algebra You can solve one of the equations for one of its variables. Then substitute that expression for the variable in the other equation.

STEP 1 **Write** an equation for each rental option.

Cost of one month's rental online: $y = 15$

Cost of one month's rental locally: $y = 4x$, where x represents the number of DVDs rented

STEP 2 **Substitute** the value of y from one equation into the other equation.

$y = 4x$

$15 = 4x$ **Substitute 15 for *y*.**

$3.75 = x$ **Divide each side by 4.**

STEP 3 **Analyze** the solution of the equation. If you could rent 3.75 DVDs, your cost for local and online rentals would be the same. However, you can only rent a whole number of DVDs. Look at what happens when you rent 3 DVDs and when you rent 4 DVDs, the whole numbers just less than and just greater than 3.75.

▸ It is cheaper to rent locally if you rent 3 or fewer DVDs per month. If you rent 4 or more DVDs per month, it is cheaper to rent online.

PRACTICE

1. **IN-LINE SKATES** You can rent in-line skates for \$5 per hour, or buy a pair of skates for \$130. How many hours do you need to skate for the cost of buying skates to be cheaper than renting them?

2. **WHAT IF?** Suppose the in-line skates in Exercise 1 also rent for \$12 per day. How many days do you need to skate for the cost of buying skates to be cheaper than renting them?

3. **BUTTONS** You buy a button machine for \$200 and supplies to make one hundred fifty buttons for \$30. Suppose you charge \$2 for a button. How many buttons do you need to sell to earn back what you spent?

4. **MANUFACTURING** A company buys a new widget machine for \$1200. It costs \$5 to make each widget. The company sells each widget for \$15. How many widgets do they need to sell to earn back the money they spent on the machine?

5. **WRITING** Which method(s) did you use to solve Exercises 1–4? *Explain* your choice(s).

6. **MONEY** You saved \$1000. If you put this money in a savings account, it will earn 1.5% annual interest. If you put the \$1000 in a certificate of deposit (CD), it will earn 3% annual interest. To earn the most money, does it ever make sense to put your money in the savings account? *Explain*.

3.6 Prove Theorems About Perpendicular Lines

Before You found the distance between points in the coordinate plane.

Now You will find the distance between a point and a line.

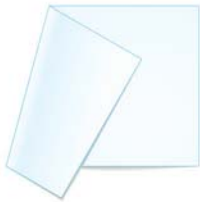
Why? So you can determine lengths in art, as in Example 4.

Key Vocabulary
- **distance from a point to a line**

ACTIVITY FOLD PERPENDICULAR LINES

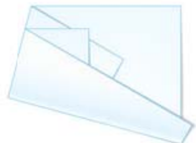
Materials: paper, protractor

STEP 1



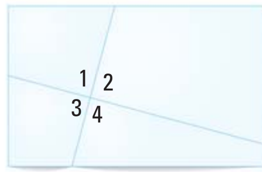
Fold a piece of paper.

STEP 2



Fold the paper again, so that the original fold lines up on itself.

STEP 3



Unfold the paper.

DRAW CONCLUSIONS

1. What type of angles appear to be formed where the fold lines intersect?
2. Measure the angles with a protractor. Which angles are congruent? Which angles are right angles?

The activity above suggests several properties of perpendicular lines.

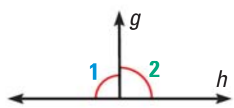
THEOREMS *For Your Notebook*

THEOREM 3.8

If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

If $\angle 1 \cong \angle 2$, then $g \perp h$.

Proof: Ex. 31, p. 196

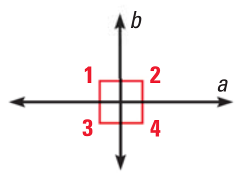


THEOREM 3.9

If two lines are perpendicular, then they intersect to form four right angles.

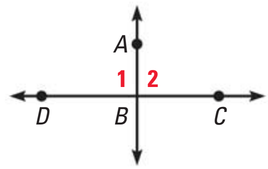
If $a \perp b$, then $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ are right angles.

Proof: Ex. 32, p. 196



EXAMPLE 1 Draw conclusions

In the diagram at the right, $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$. What can you conclude about $\angle 1$ and $\angle 2$?



Solution

$\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ are perpendicular, so by Theorem 3.9, they form four right angles. You can conclude that $\angle 1$ and $\angle 2$ are right angles, so $\angle 1 \cong \angle 2$.

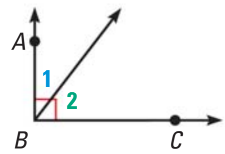
THEOREM *For Your Notebook*

THEOREM 3.10

If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

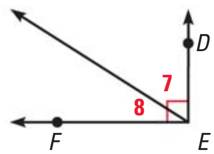
If $\overrightarrow{BA} \perp \overrightarrow{BC}$, then $\angle 1$ and $\angle 2$ are complementary.

Proof: Example 2, below



EXAMPLE 2 Prove Theorem 3.10

Prove that if two sides of two adjacent acute angles are perpendicular, then the angles are complementary.



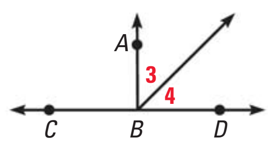
GIVEN ▶ $\overrightarrow{ED} \perp \overrightarrow{EF}$

PROVE ▶ $\angle 7$ and $\angle 8$ are complementary.

STATEMENTS	REASONS
1. $\overrightarrow{ED} \perp \overrightarrow{EF}$	**1.** Given
2. $\angle DEF$ is a right angle.	**2.** $\perp$ lines intersect to form 4 rt. $\angle$s. (Theorem 3.9)
3. $m\angle DEF = 90°$	**3.** Definition of a right angle
4. $m\angle 7 + m\angle 8 = m\angle DEF$	**4.** Angle Addition Postulate
5. $m\angle 7 + m\angle 8 = 90°$	**5.** Substitution Property of Equality
6. $\angle 7$ and $\angle 8$ are complementary.	**6.** Definition of complementary angles

✓ GUIDED PRACTICE for Examples 1 and 2

1. Given that $\angle ABC \cong \angle ABD$, what can you conclude about $\angle 3$ and $\angle 4$? *Explain* how you know.



2. Write a plan for proof for Theorem 3.9, that if two lines are perpendicular, then they intersect to form four right angles.

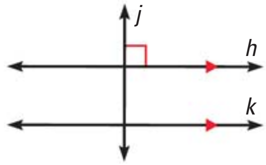
THEOREMS *For Your Notebook*

THEOREM 3.11 Perpendicular Transversal Theorem

If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other.

If $h \parallel k$ and $j \perp h$, then $j \perp k$.

Proof: Ex. 42, p. 160; Ex. 33, p. 196

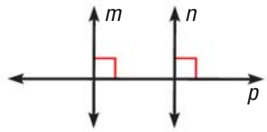


THEOREM 3.12 Lines Perpendicular to a Transversal Theorem

In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

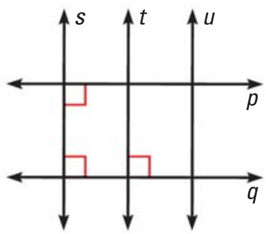
If $m \perp p$ and $n \perp p$, then $m \parallel n$.

Proof: Ex. 34, p. 196



EXAMPLE 3 Draw conclusions

Determine which lines, if any, must be parallel in the diagram. Explain your reasoning.



Solution

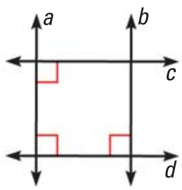
Lines p and q are both perpendicular to s, so by Theorem 3.12, $p \parallel q$. Also, lines s and t are both perpendicular to q, so by Theorem 3.12, $s \parallel t$.

✓ GUIDED PRACTICE for Example 3

Use the diagram at the right.

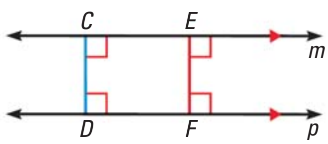
3. Is $b \parallel a$? *Explain* your reasoning.

4. Is $b \perp c$? *Explain* your reasoning.



DISTANCE FROM A LINE The **distance from a point to a line** is the length of the perpendicular segment from the point to the line. This perpendicular segment is the shortest distance between the point and the line. For example, the distance between point A and line k is AB. You will prove this in Chapter 5.

Distance from a point to a line

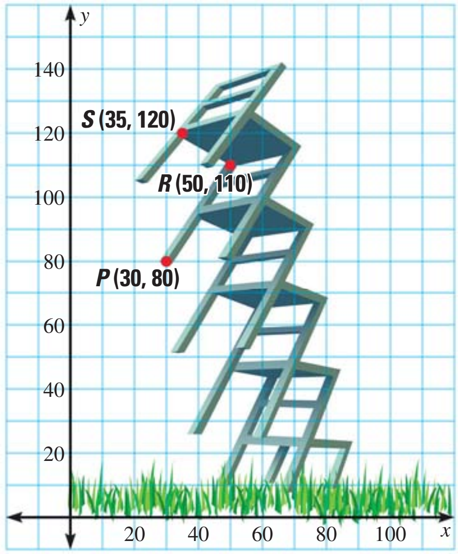


Distance between two parallel lines

The *distance between two parallel lines* is the length of any perpendicular segment joining the two lines. For example, the distance between line p and line m above is CD or EF.

EXAMPLE 4 Find the distance between two parallel lines

SCULPTURE The sculpture below is drawn on a graph where units are measured in inches. What is the approximate length of $\overline{SR}$, the depth of a seat?



Solution

You need to find the length of a perpendicular segment from a back leg to a front leg on one side of the chair.

Using the points $P(30, 80)$ and $R(50, 110)$, the slope of each leg is

$\frac{110 - 80}{50 - 30} = \frac{30}{20} = \frac{3}{2}.$

The segment SR has a slope of

$\frac{120 - 110}{35 - 50} = -\frac{10}{15} = -\frac{2}{3}.$

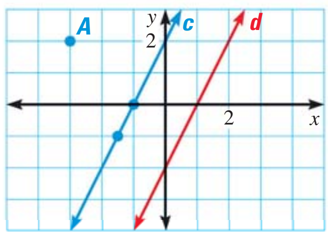
The segment $\overline{SR}$ is perpendicular to the leg so the distance SR is

$d = \sqrt{(35 - 50)^2 + (120 - 110)^2} \approx 18.0$ inches.

▶ The length of $\overline{SR}$ is about 18.0 inches.

GUIDED PRACTICE for Example 4

Use the graph at the right for Exercises 5 and 6.



5. What is the distance from point A to line c?
6. What is the distance from line c to line d?
7. Graph the line $y = x + 1$. What point on the line is the shortest distance from the point $(4, 1)$? What is the distance? Round to the nearest tenth.

3.6 EXERCISES

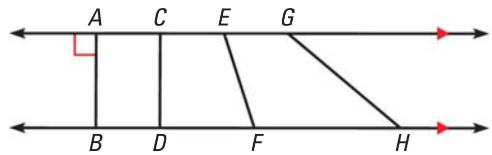
HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 19, 23, and 29

★ = **STANDARDIZED TEST PRACTICE** Exs. 11, 12, 21, 22, and 30

SKILL PRACTICE

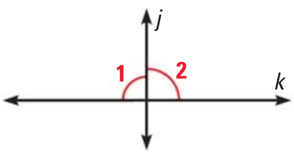
1. **VOCABULARY** The length of which segment shown is called the distance between the two parallel lines? *Explain*.



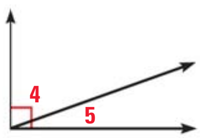
EXAMPLES 1 and 2 on p. 191 for Exs. 2–7

JUSTIFYING STATEMENTS **Write the theorem that justifies the statement.**

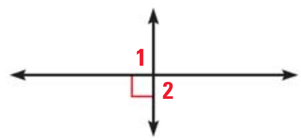
2. $j \perp k$



3. $\angle 4$ and $\angle 5$ are complementary.

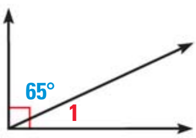


4. $\angle 1$ and $\angle 2$ are right angles.

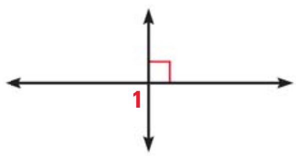


APPLYING THEOREMS **Find $m\angle 1$.**

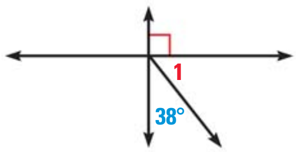
5.



6.



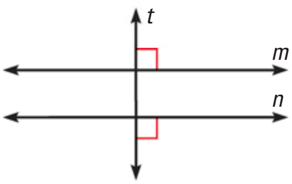
7.



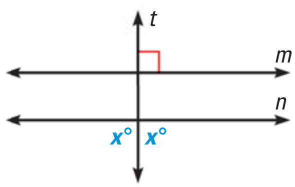
EXAMPLE 3 on p. 192 for Exs. 8–12

SHOWING LINES PARALLEL ***Explain*** **how you would show that $m \parallel n$.**

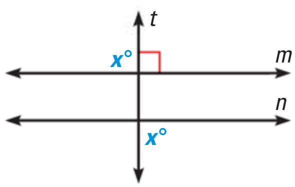
8.



9.



10.



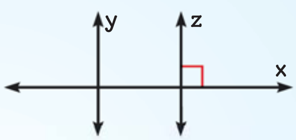
11. ★ **SHORT RESPONSE** *Explain* how to draw two parallel lines using only a straightedge and a protractor.

12. ★ **SHORT RESPONSE** *Describe* how you can fold a sheet of paper to create two parallel lines that are perpendicular to the same line.

EXAMPLES 3 and 4 on pp. 192–193 for Exs. 13–14

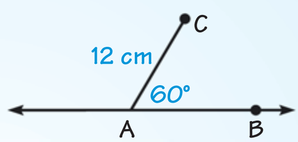
ERROR ANALYSIS ***Explain*** **why the statement about the figure is incorrect.**

13.



Lines y and z are parallel.

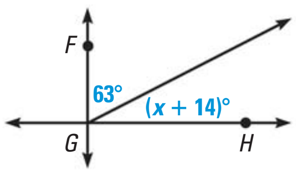
14.



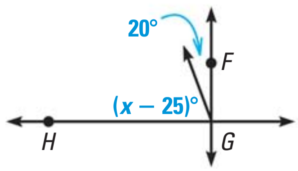
The distance from $\overleftrightarrow{AB}$ to point C is 12 cm.

FINDING ANGLE MEASURES **In the diagram, $\overleftrightarrow{FG} \perp \overleftrightarrow{GH}$. Find the value of x.**

15.



16.



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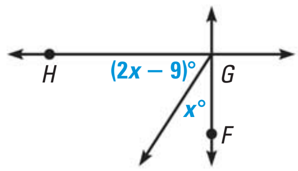
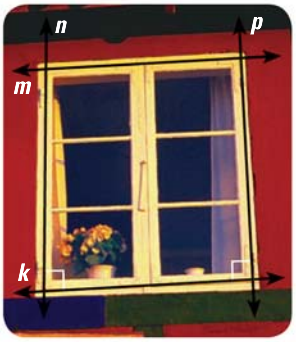


DRAWING CONCLUSIONS **Determine which lines, if any, must be parallel. *Explain* your reasoning.**

18.



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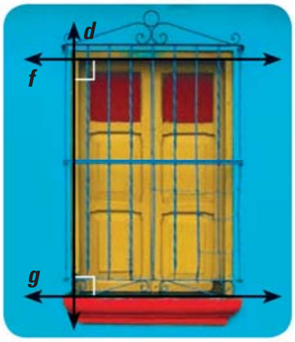
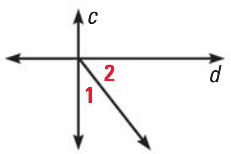


20.

21. ★ **MULTIPLE CHOICE** Which statement must be true if $c \perp d$?



Ⓐ $m\angle 1 + m\angle 2 = 90°$ Ⓑ $m\angle 1 + m\angle 2 < 90°$

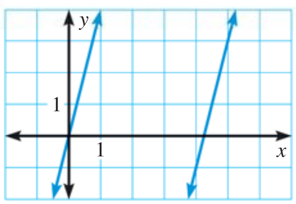
Ⓒ $m\angle 1 + m\angle 2 > 90°$ Ⓓ Cannot be determined

22. ★ **WRITING** *Explain* why the distance between two lines is only defined for parallel lines.

EXAMPLE 4
on p. 193
for Exs. 23–24

FINDING DISTANCES **Use the Distance Formula to find the distance between the two parallel lines. Round to the nearest tenth, if necessary.**

23.



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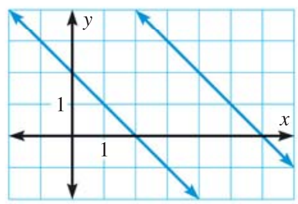
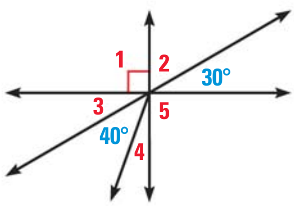


25. **CONSTRUCTION** You are given a line n and a point P not on n. Use a compass to find two points on n equidistant from P. Then use the steps for the construction of a segment bisector (page 33) to construct a line perpendicular to n through P.

26. **FINDING ANGLES** Find all the unknown angle measures in the diagram at the right. *Justify* your reasoning for each angle measure.

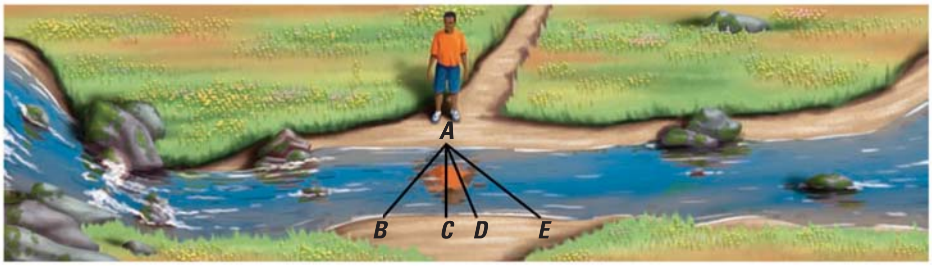


27. **FINDING DISTANCES** Find the distance between the lines with the equations $y = \frac{3}{2}x + 4$ and $-3x + 2y = -1$.

28. **CHALLENGE** *Describe* how you would find the distance from a point to a plane. Can you find the distance from a line to a plane? *Explain.*

PROBLEM SOLVING

29. **STREAMS** You are trying to cross a stream from point A. Which point should you jump to in order to jump the shortest distance? *Explain.*



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30. ★ **SHORT RESPONSE** The segments that form the path of a crosswalk are usually perpendicular to the crosswalk. Sketch what the segments would look like if they were perpendicular to the crosswalk. Which method requires less paint? *Explain.*

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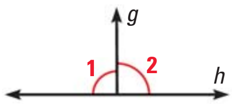
EXAMPLE 2 on p. 191 for Exs. 31–34

31. **PROVING THEOREM 3.8** Copy and complete the proof that if two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

GIVEN ▶ $\angle 1$ and $\angle 2$ are a linear pair.
$\angle 1 \cong \angle 2$

PROVE ▶ $g \perp h$



STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are a linear pair.	**1.** Given
2. $\angle 1$ and $\angle 2$ are supplementary.	**2.** ?
3. ?	**3.** Definition of supplementary angles
4. $\angle 1 \cong \angle 2$	**4.** Given
5. $m\angle 1 = m\angle 2$	**5.** ?
6. $m\angle 1 + m\angle 1 = 180°$	**6.** Substitution Property of Equality
7. $2(m\angle 1) = 180°$	**7.** Combine like terms.
8. $m\angle 1 = 90°$	**8.** ?
9. ?	**9.** Definition of a right angle
10. $g \perp h$	**10.** ?

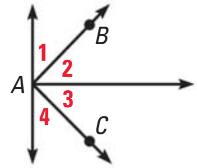
PROVING THEOREMS **Write a proof of the given theorem.**

32. Theorem 3.9

33. Theorem 3.11, Perpendicular Transversal Theorem

34. Theorem 3.12, Lines Perpendicular to a Transversal Theorem

CHALLENGE **Suppose the given statement is true. Determine whether $\overrightarrow{AB} \perp \overrightarrow{AC}$.**



35. $\angle 1$ and $\angle 2$ are congruent.

36. $\angle 3$ and $\angle 4$ are complementary.

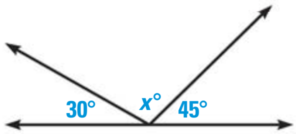
37. $m\angle 1 = m\angle 3$ and $m\angle 2 = m\angle 4$

38. $m\angle 1 = 40°$ and $m\angle 4 = 50°$

MIXED REVIEW

PREVIEW
Prepare for Lesson 4.1 in Exs. 39–41.

Find the value of x. *(p. 24)*

39.


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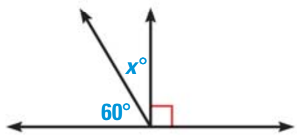

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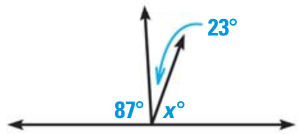
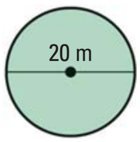


Find the circumference and area of the circle. Round to the nearest tenth. *(p. 49)*

42.


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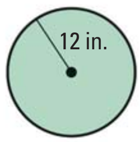

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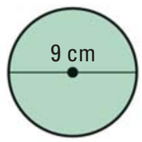
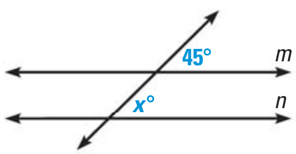


Find the value of x that makes $m \parallel n$. *(p. 161)*

45.


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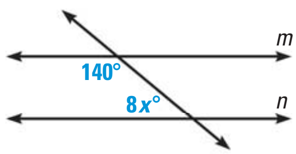

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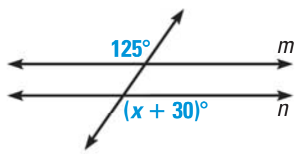


QUIZ for Lessons 3.5–3.6

Write an equation of the line that passes through point P and is parallel to the line with the given equation. *(p. 180)*

1. $P(0, 0), y = -3x + 1$

2. $P(-5, -6), y - 8 = 2x + 10$

3. $P(1, -2), x = 15$

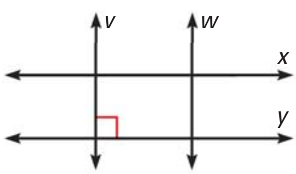
Write an equation of the line that passes through point P and is perpendicular to the line with the given equation. *(p. 180)*

4. $P(3, 4), y = 2x - 1$

5. $P(2, 5), y = -6$

6. $P(4, 0), 12x + 3y = 9$

Determine which lines, if any, must be parallel. *Explain.* *(p. 190)*

7.


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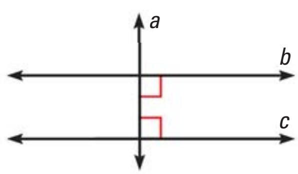

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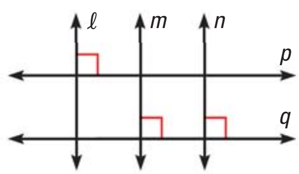


Extension
Use after Lesson 3.6

Taxicab Geometry

GOAL Find distances in a non-Euclidean geometry.

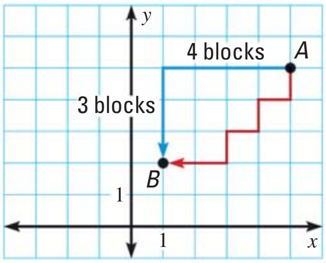
Key Vocabulary
- **taxicab geometry**

HISTORY NOTE
Euclidean geometry is named after a Greek mathematician. Euclid (circa third century B.C.) used postulates and deductive reasoning to prove the theorems you are studying in this book. Non-Euclidean geometries start by assuming different postulates, so they result in different theorems.

You have learned that the shortest distance between two points is the length of the straight line segment between them. This is true in the *Euclidean* geometry that you are studying. But think about what happens when you are in a city and want to get from point A to point B. You cannot walk through the buildings, so you have to go along the streets.

Taxicab geometry is the non-Euclidean geometry that a taxicab or a pedestrian must obey.

In taxicab geometry, you can travel either horizontally or vertically parallel to the axes. In this geometry, the distance between two points is the shortest number of *blocks* between them.

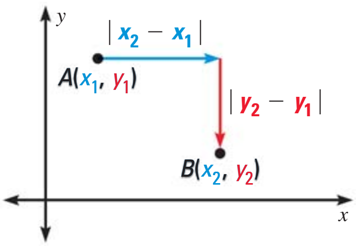


KEY CONCEPT — *For Your Notebook*

Taxicab Distance

The distance between two points is the sum of the differences in their coordinates.

$AB = |x_2 - x_1| + |y_2 - y_1|$



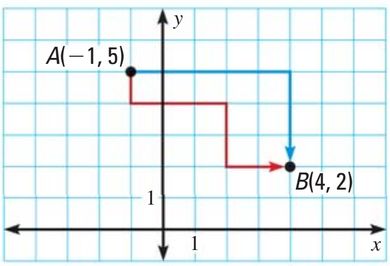
EXAMPLE 1 Find a taxicab distance

Find the taxicab distance from $A(-1, 5)$ to $B(4, 2)$. Draw two different shortest paths from A to B.

Solution

$$\begin{aligned} AB &= |x_2 - x_1| + |y_2 - y_1| \\ &= |4 - (-1)| + |2 - 5| \\ &= |5| + |-3| \\ &= 8 \end{aligned}$$

▶ The shortest path is 8 blocks. Two possible paths are shown.



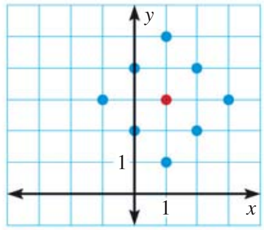
REVIEW ABSOLUTE VALUE
For help with absolute value, see p. 870.

CIRCLES In Euclidean geometry, a *circle* is all points that are the same distance from a fixed point, called the *center*. That distance is the *radius*. Taxicab geometry uses the same definition for a circle, but taxicab circles are not round.

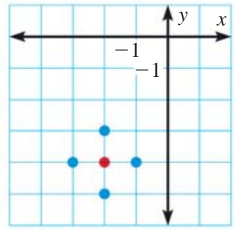
EXAMPLE 2 Draw a taxicab circle

Draw the taxicab circle with the given radius r and center C.

a. $r = 2$, $C(1, 3)$



b. $r = 1$, $C(-2, -4)$



PRACTICE

EXAMPLE 1 on p. 198 for Exs. 1–6

FINDING DISTANCE **Find the taxicab distance between the points.**

1. $(4, 2), (0, 0)$ **2.** $(3, 5), (6, 2)$ **3.** $(-6, 3), (8, 5)$

4. $(-1, -3), (5, -2)$ **5.** $(-3, 5), (-1, 5)$ **6.** $(-7, 3), (-7, -4)$

EXAMPLE 2 on p. 199 for Exs. 7–9

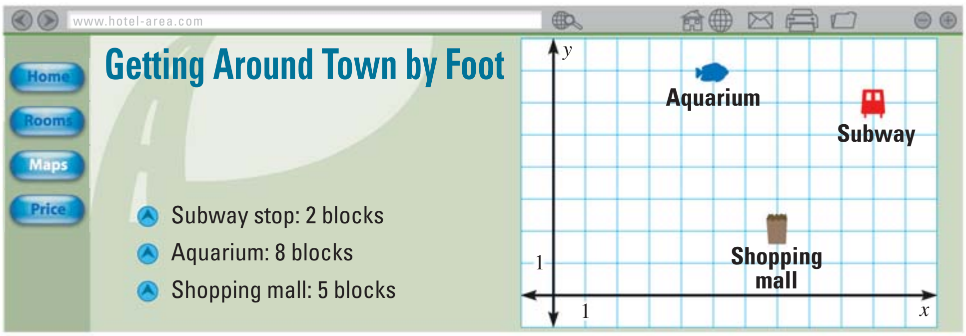
DRAWING CIRCLES **Draw the taxicab circle with radius r and center C.**

7. $r = 2$, $C(3, 4)$ **8.** $r = 4$, $C(0, 0)$ **9.** $r = 5$, $C(-1, 3)$

FINDING MIDPOINTS **A *midpoint* in taxicab geometry is a point where the distance to the endpoints are equal. Find all the midpoints of $\overline{AB}$.**

10. $A(2, 4), B(-2, -2)$ **11.** $A(1, -3), B(1, 3)$ **12.** $A(2, 2), B(-3, 0)$

13. TRAVEL PLANNING A hotel's website claims that the hotel is an easy walk to a number of sites of interest. What are the coordinates of the hotel?



14. REASONING The taxicab distance between two points is always greater than or equal to the Euclidean distance between the two points. *Explain* what must be true about the points for both distances to be equal.

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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Lessons 3.4–3.6

1. MULTI-STEP PROBLEM You are planning a party. You would like to have the party at a roller skating rink or bowling alley. The table shows the total cost to rent the facilities by number of hours.

Hours	Roller skating rink cost ($)	Bowling alley cost ($)
1	35	20
2	70	40
3	105	60
4	140	80
5	175	100

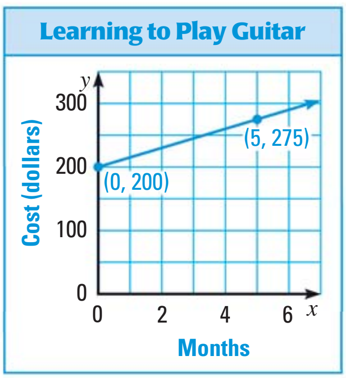
a. Use the data in the table. Write and graph two equations to represent the total cost y to rent the facilities, where x is the number of hours you rent the facility.

b. Are the lines from part (a) parallel? *Explain* why or why not.

c. What is the meaning of the slope in each equation from part (a)?

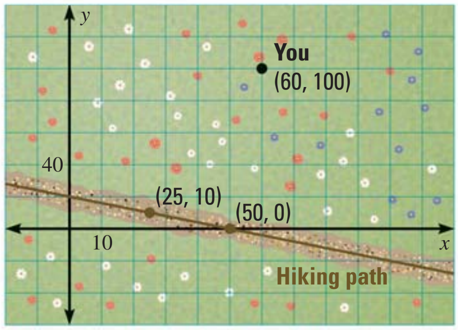
d. Suppose the bowling alley charges an extra $25 set-up fee. Write and graph an equation to represent this situation. Is this line parallel to either of the lines from part (a)? *Explain* why or why not.

2. GRIDDED ANSWER The graph models the accumulated cost of buying a used guitar and taking lessons over the first several months. Find the slope of the line.



3. OPEN-ENDED Write an equation of a line parallel to $2x + 3y = 6$. Then write an equation of a line perpendicular to your line.

4. SHORT RESPONSE You are walking across a field to get to a hiking path. Use the graph below to find the shortest distance you can walk to reach the path. *Explain* how you know you have the shortest distance.



5. EXTENDED RESPONSE The Johnstown Inclined Plane in Johnstown, Pennsylvania, is a cable car that transports people up and down the side of a hill. During the cable car's climb, you move about 17 feet upward for every 25 feet you move forward. At the top of the incline, the horizontal distance from where you started is about 500 feet.

a. How high is the car at the top of its climb compared to its starting height?

b. Find the slope of the climb.

c. Another cable car incline in Pennsylvania, the Monongahela Incline, climbs at a slope of about 0.7 for a horizontal distance of about 517 feet. *Compare* this climb to that of the Johnstown Inclined Plane. Which is steeper? *Justify* your answer.

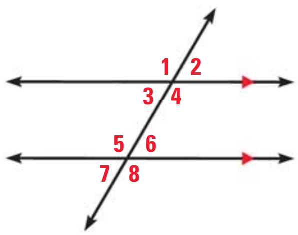
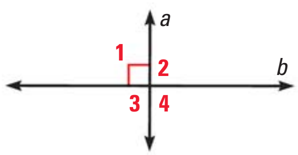
3 CHAPTER SUMMARY

BIG IDEAS — *For Your Notebook*

Big Idea 1

Using Properties of Parallel and Perpendicular Lines

When parallel lines are cut by a transversal, angle pairs are formed. Perpendicular lines form congruent right angles.

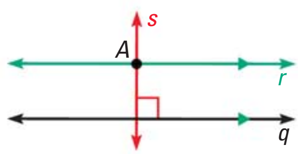
	$\angle 2$ and $\angle 6$ are corresponding angles, and they are congruent. $\angle 3$ and $\angle 6$ are alternate interior angles, and they are congruent. $\angle 1$ and $\angle 8$ are alternate exterior angles, and they are congruent. $\angle 3$ and $\angle 5$ are consecutive interior angles, and they are supplementary.
	If $a \perp b$, then $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are all right angles.

Big Idea 2

Proving Relationships Using Angle Measures

You can use the angle pairs formed by lines and a transversal to show that the lines are parallel. Also, if lines intersect to form a right angle, you know that the lines are perpendicular.

Through point A not on line q, there is only one line r parallel to q and one line s perpendicular to q.

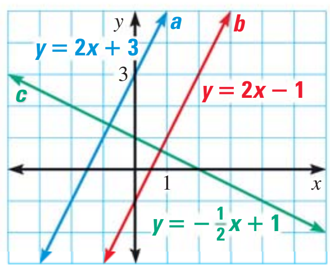


Big Idea 3

Making Connections to Lines in Algebra

In Algebra 1, you studied slope as a rate of change and linear equations as a way of modeling situations.

Slope and equations of lines are also a useful way to represent the lines and segments that you study in Geometry. For example, the slopes of parallel lines are the same ($a \parallel b$), and the product of the slopes of perpendicular lines is -1 ($a \perp c$, and $b \perp c$).



3 CHAPTER REVIEW

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• Multi-Language Glossary
• Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

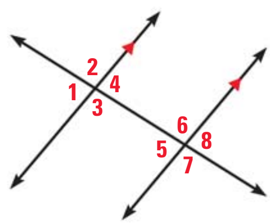
- parallel lines, *p. 147*
- skew lines, *p. 147*
- parallel planes, *p. 147*
- transversal, *p. 149*
- corresponding angles, *p. 149*
- alternate interior angles, *p. 149*
- alternate exterior angles, *p. 149*
- consecutive interior angles, *p. 149*
- paragraph proof, *p. 163*
- slope, *p. 171*
- slope-intercept form, *p. 180*
- standard form, *p. 182*
- distance from a point to a line, *p. 192*

VOCABULARY EXERCISES

1. Copy and complete: Two lines that do not intersect and are not coplanar are called _?_.

2. **WRITING** *Compare* alternate interior angle pairs and consecutive interior angle pairs.

Copy and complete the statement using the figure at the right.



3. $\angle 1$ and _?_ are corresponding angles.

4. $\angle 3$ and _?_ are alternate interior angles.

5. $\angle 4$ and _?_ are consecutive interior angles.

6. $\angle 7$ and _?_ are alternate exterior angles.

Identify the form of the equation as *slope-intercept form* or *standard form*.

7. $14x - 2y = 26$

8. $y = 7x - 13$

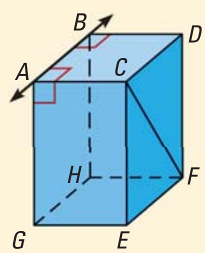
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 3.

3.1 Identify Pairs of Lines and Angles *pp. 147–152*

EXAMPLE

Think of each segment in the rectangular box at the right as part of a line.



a. $\overleftrightarrow{BD}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{BH}$, and $\overleftrightarrow{AG}$ appear perpendicular to $\overleftrightarrow{AB}$.

b. $\overleftrightarrow{CD}$, $\overleftrightarrow{GH}$, and $\overleftrightarrow{EF}$ appear parallel to $\overleftrightarrow{AB}$.

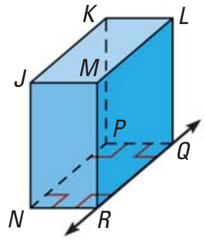
c. $\overleftrightarrow{CF}$ and $\overleftrightarrow{EG}$ appear skew to $\overleftrightarrow{AB}$.

d. Plane *EFG* appear parallel to plane *ABC*.

EXERCISES

EXAMPLE 1 on p. 147 for Exs. 9–12

Think of each segment in the diagram of a rectangular box as part of a line. Which line(s) or plane(s) contain point *N* and appear to fit the description?

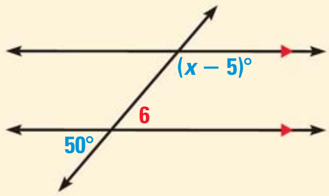


9. Line(s) perpendicular to $\overleftrightarrow{QR}$
10. Line(s) parallel to $\overleftrightarrow{QR}$
11. Line(s) skew to $\overleftrightarrow{QR}$
12. Plane(s) parallel to plane LMQ

3.2 Use Parallel Lines and Transversals *pp. 154–160*

EXAMPLE

Use properties of parallel lines to find the value of *x*.



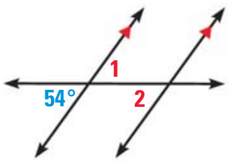
By the Vertical Angles Congruence Theorem, $m\angle 6 = 50°$.

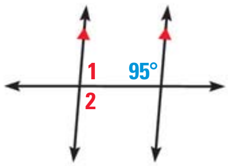
$(x - 5)° + m\angle 6 = 180°$	**Consecutive Interior Angles Theorem**
$(x - 5)° + 50° = 180°$	**Substitute 50° for $m\angle 6$.**
$x = 135$	**Solve for *x*.**

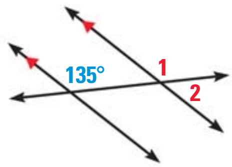
EXERCISES

EXAMPLES 1 and 2 on pp. 154–155 for Exs. 13–19

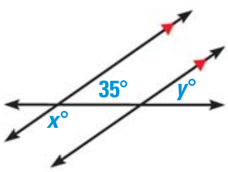
Find $m\angle 1$ and $m\angle 2$. *Explain* your reasoning.

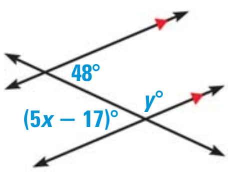
13. 

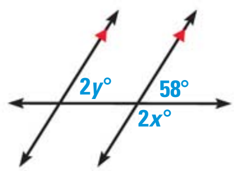
14. 

15. 

Find the values of *x* and *y*.

16. 

17. 

18. 

19. **FLAG OF PUERTO RICO** Sketch the rectangular flag of Puerto Rico as shown at the right. Find the measure of $\angle 1$ if $m\angle 3 = 55°$. *Justify* each step in your argument.

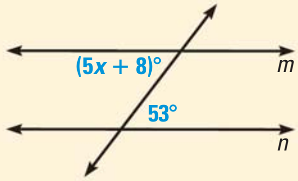
3 CHAPTER REVIEW

3.3 Prove Lines are Parallel
pp. 161–169

EXAMPLE

Find the value of x that makes $m \parallel n$.

Lines m and n are parallel when the marked corresponding angles are congruent.



$$(5x + 8)^\circ = 53^\circ$$
$$5x = 45$$
$$x = 9$$

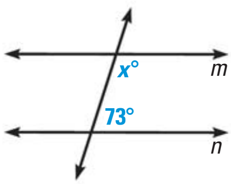
▶ The lines m and n are parallel when $x = 9$.

EXERCISES

EXAMPLE 1 on p. 161 for Exs. 20–22

Find the value of x that makes $m \parallel n$.

20.



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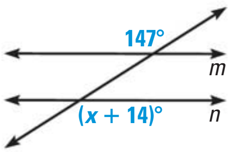

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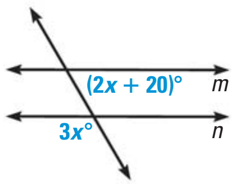
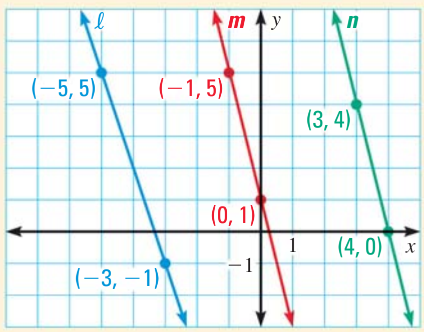


3.4 Find and Use Slopes of Lines
pp. 171–178

EXAMPLE

Find the slope of each line. Which lines are parallel?



$$\text{Slope of } \ell = \frac{-1 - 5}{-3 - (-5)} = \frac{-6}{2} = -3$$

$$\text{Slope of } m = \frac{1 - 5}{0 - (-1)} = \frac{-4}{1} = -4$$

$$\text{Slope of } n = \frac{0 - 4}{4 - 3} = \frac{-4}{1} = -4$$

▶ Because m and n have the same slope, they are parallel. The slope of ℓ is different, so ℓ is not parallel to the other lines.

EXERCISES

EXAMPLES 2 and 3 on pp. 172–173 for Exs. 23–24

Tell whether the lines through the given points are *parallel*, *perpendicular*, or *neither*.

23. Line 1: (8, 12), (7, −5)
Line 2: (−9, 3), (8, 2)

24. Line 1: (3, −4), (−1, 4)
Line 2: (2, 7), (5, 1)

3.5 Write and Graph Equations of Lines *pp. 180–187*

EXAMPLE

Write an equation of the line k passing through the point $(-4, 1)$ that is perpendicular to the line n with the equation $y = 2x - 3$.

First, find the slope of line k. Line n has a slope of 2.

$$2 \cdot m = -1$$

$$m = -\frac{1}{2}$$

Then, use the given point and the slope in the slope-intercept form to find the y-intercept.

$$y = mx + b$$

$$1 = -\frac{1}{2}(-4) + b$$

$$-1 = b$$

▶ An equation of line k is $y = -\frac{1}{2}x - 1$.

EXERCISES

EXAMPLES 2 and 3 on pp. 180–181 for Exs. 25–26

Write equations of the lines that pass through point P and are (a) parallel and (b) perpendicular to the line with the given equation.

25. $P(3, -1)$, $y = 6x - 4$

26. $P(-6, 5)$, $7y + 4x = 2$

3.6 Prove Theorems About Perpendicular Lines *pp. 190–197*

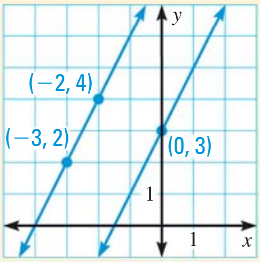
EXAMPLE

Find the distance between $y = 2x + 3$ and $y = 2x + 8$.

Find the length of a perpendicular segment from one line to the other. Both lines have a slope of 2, so the slope of a perpendicular segment to each line is $-\frac{1}{2}$.

The segment from $(0, 3)$ to $(-2, 4)$ has a slope of $\frac{4 - 3}{-2 - 0} = -\frac{1}{2}$. So, the distance between the lines is

$d = \sqrt{(-2 - 0)^2 + (4 - 3)^2} = \sqrt{5} \approx 2.2$ units.

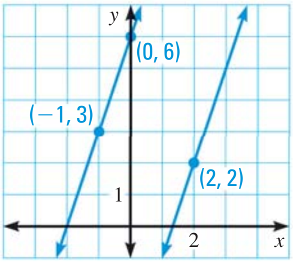


EXERCISES

EXAMPLE 4 on p. 193 for Exs. 27–28

Use the Distance Formula to find the distance between the two parallel lines. Round to the nearest tenth, if necessary.

27.



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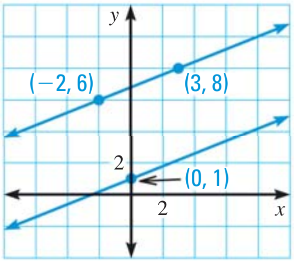
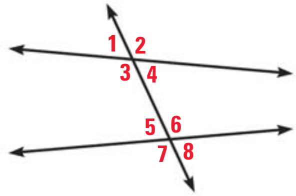


3 CHAPTER TEST

Classify the pairs of angles as *corresponding, alternate interior, alternate exterior,* or *consecutive interior.*

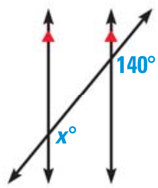


1. $\angle 1$ and $\angle 8$ **2.** $\angle 2$ and $\angle 6$ **3.** $\angle 3$ and $\angle 5$

4. $\angle 4$ and $\angle 5$ **5.** $\angle 3$ and $\angle 7$ **6.** $\angle 3$ and $\angle 6$

Find the value of x.

7.



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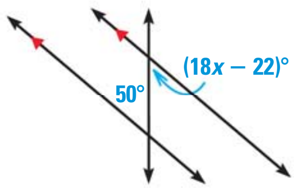

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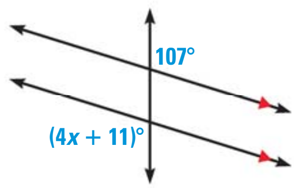
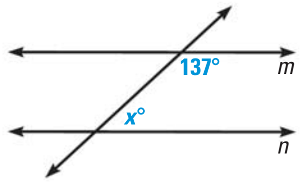


Find the value of x that makes $m \parallel n$.

10.



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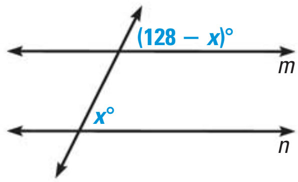

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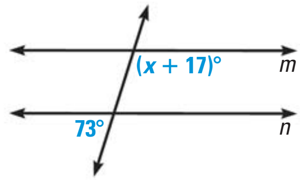


Find the slope of the line that passes through the points.

13. $(3, -1), (3, 4)$ **14.** $(2, 7), (-1, -3)$ **15.** $(0, 5), (-6, 12)$

Write an equation of the line that passes through the given point P and has the given slope m.

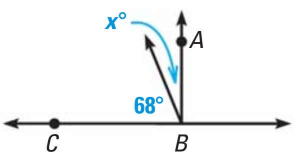
16. $P(-2, 4), m = 3$ **17.** $P(7, 12), m = -0.2$ **18.** $P(3, 5), m = -8$

Write an equation of the line that passes through point P and is perpendicular to the line with the given equation.

19. $P(1, 3), y = 2x - 1$ **20.** $P(0, 2), y = -x + 3$ **21.** $P(2, -3), x - y = 4$

In Exercises 22–24, $\overline{AB} \perp \overline{BC}$. Find the value of x.

22.



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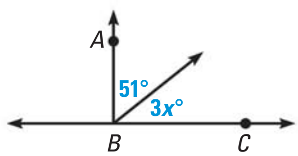

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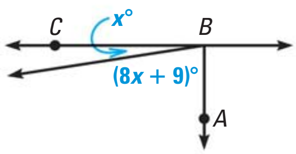
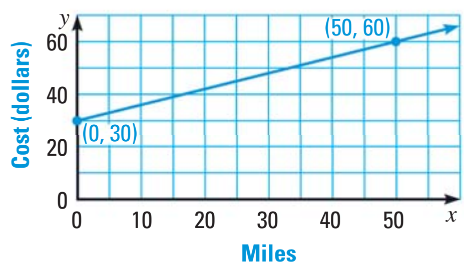


25. RENTAL COSTS The graph at the right models the cost of renting a moving van. Write an equation of the line. Then find the cost of renting the van for a 100 mile trip.



3 ALGEBRA REVIEW

Animated Algebra classzone.com

GRAPH AND SOLVE LINEAR INEQUALITIES

EXAMPLE 1 *Graph a linear inequality in two variables*

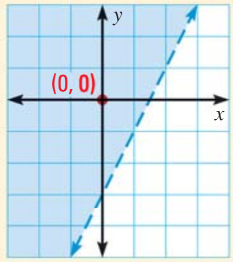
Graph the inequality $0 > 2x - 3 - y$.

Solution

Rewrite the inequality in slope-intercept form, $y > 2x - 3$.

The boundary line $y = 2x - 3$ is not part of the solution, so use a dashed line.

To decide where to shade, use a point not on the line, such as (0, 0), as a test point. Because $0 > 2 \cdot 0 - 3$, (0, 0) is a solution. Shade the half-plane that includes (0, 0).



EXAMPLE 2 *Use an inequality to solve a real-world problem*

SAVINGS Lily has saved \$49. She plans to save \$12 per week to buy a camera that costs \$124. In how many weeks will she be able to buy the camera?

Solution

Let w represent the number of weeks needed.

$49 + 12w \geq 124$ Write an algebraic model.

$12w \geq 75$ Subtract 49 from each side.

$w \geq 6.25$ Divide each side by 12.

▶ She must save for 7 weeks to be able to buy the camera.

EXERCISES

EXAMPLE 1 for Exs. 1–8

Graph the linear inequality.

1. $y > -2x + 3$
2. $y \leq 0.5x - 4$
3. $-2.5x + y \geq 1.5$
4. $x < 3$
5. $y < -2$
6. $5x - y > -5$
7. $2x + 3y \geq -18$
8. $3x - 4y \leq 6$

EXAMPLE 2 for Exs. 9–11

Solve.

9. **LOANS** Eric borrowed \$46 from his mother. He will pay her back at least \$8 each month. At most, how many months will it take him?

10. **GRADES** Manuel's quiz scores in history are 76, 81, and 77. What score must he get on his fourth quiz to have an average of at least 80?

11. **PHONE CALLS** Company A charges a monthly fee of \$5 and \$.07 per minute for phone calls. Company B charges no monthly fee, but charges \$.12 per minute. After how many minutes of calls is the cost of using Company A less than the cost of using Company B?

3 ★ Standardized TEST PREPARATION

MULTIPLE CHOICE QUESTIONS

If you have difficulty solving a multiple choice problem directly, you may be able to use another approach to eliminate incorrect answer choices and obtain the correct answer.

PROBLEM 1

Which ordered pair is a solution of the equations $y = 2x - 5$ and $4x + 3y = 45$?

Ⓐ (3, 11) Ⓑ (5, 5) Ⓒ (6, 7) Ⓓ (7, 6)

METHOD 1

SOLVE DIRECTLY Find the ordered pair that is the solution by using substitution.

Because the first equation is solved for y, substitute $y = 2x - 5$ into $4x + 3y = 45$.

$$\begin{aligned} 4x + 3y &= 45 \\ 4x + 3(2x - 5) &= 45 \\ 4x + 6x - 15 &= 45 \\ 10x - 15 &= 45 \\ 10x &= 60 \\ x &= 6 \end{aligned}$$

Solve for y by substituting 6 for x in the first equation.

$$\begin{aligned} y &= 2x - 5 \\ y &= 2(6) - 5 \\ y &= 12 - 5 \\ y &= 7 \end{aligned}$$

So, the solution of the linear system is (6, 7), which is choice C. Ⓐ Ⓑ Ⓒ Ⓓ

METHOD 2

ELIMINATE CHOICES Another method is to eliminate incorrect answer choices.

Substitute choice A into the equations.

$$\begin{aligned} y &= 2x - 5 \\ 11 &\stackrel{?}{=} 2(3) - 5 \\ 11 &\stackrel{?}{=} 6 - 5 \\ 11 &\neq 1 \ ✗ \end{aligned}$$

The point is not a solution of $y = 2x - 5$, so there is no need to check the other equation. You can eliminate choice A.

Substitute choice B into the equations.

$$\begin{aligned} y &= 2x - 5 & 4x + 3y &= 45 \\ 5 &\stackrel{?}{=} 2(5) - 5 & 4(5) + 3(5) &\stackrel{?}{=} 45 \\ 5 &\stackrel{?}{=} 10 - 5 & 20 + 15 &\stackrel{?}{=} 45 \\ 5 &= 5 \ ✓ & 35 &\neq 45 \ ✗ \end{aligned}$$

You can eliminate choice B.

Substitute choice C into the equations.

$$\begin{aligned} y &= 2x - 5 & 4x + 3y &= 45 \\ 7 &\stackrel{?}{=} 2(6) - 5 & 4(6) + 3(7) &\stackrel{?}{=} 45 \\ 7 &\stackrel{?}{=} 12 - 5 & 24 + 21 &\stackrel{?}{=} 45 \\ 7 &= 7 \ ✓ & 45 &= 45 \ ✓ \end{aligned}$$

Choice C makes both equations true so, the answer is choice C. Ⓐ Ⓑ Ⓒ Ⓓ

PROBLEM 2

Which equation is an equation of the line through the point $(-1, 1)$ and perpendicular to the line through the points $(2, 4)$ and $(-4, 6)$?

Ⓐ $y = -\frac{1}{3}x + \frac{2}{3}$ Ⓑ $y = 3x + 4$

Ⓒ $y = \frac{1}{3}x + \frac{4}{3}$ Ⓓ $y = 3x - 2$

METHOD 1

SOLVE DIRECTLY Find the slope of the line through the points $(2, 4)$ and $(-4, 6)$.

$$m = \frac{6-4}{-4-2} = \frac{2}{-6} = -\frac{1}{3}$$

The slope of the line perpendicular to this line is 3, because $3 \cdot \left(-\frac{1}{3}\right) = -1$. Use $y = 3x + b$ and the point $(-1, 1)$ to find b.

$1 = 3(-1) + b$, so $b = 4$.

The equation of the line is $y = 3x + 4$. The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

METHOD 2

ELIMINATE CHOICES Another method to consider is to eliminate choices based on the slope, then substitute the point to find the correct equation.

$$m = \frac{6-4}{-4-2} = -\frac{1}{3}$$

The slope of the line perpendicular to this line is 3. Choices A and C do not have a slope of 3, so you can eliminate these choices. Next, try substituting the point $(-1, 1)$ into answer choice B.

$1 \stackrel{?}{=} 3(-1) + 4$ ✓

This is a true statement.

The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

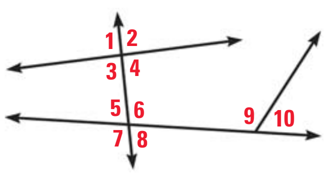
PRACTICE

Explain **why you can eliminate the highlighted answer choice.**

1. Use the diagram below. Which pair of angles are alternate exterior angles?

Ⓐ 4 and 5 Ⓑ 2 and 6

Ⓒ 1 and 8 Ⓓ ~~1 and 10~~



2. Which equation is an equation of the line parallel to the line through the points $(-1, 4)$ and $(1, 1)$?

Ⓐ $y = -\frac{3}{2}x - 3$ Ⓑ $y = \frac{3}{2}x - 3$

Ⓒ ~~$y = \frac{2}{3}x - 3$~~ Ⓓ $y = 3x - 3$

3 ★ Standardized TEST PRACTICE

MULTIPLE CHOICE

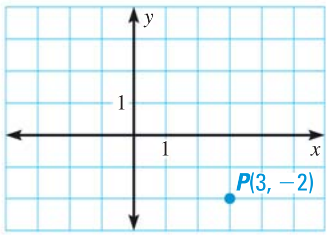
1. A line is to be drawn through point P in the graph so that it never crosses the y-axis. Through which point does it pass?

Ⓐ $(-2, 3)$

Ⓑ $(-3, -2)$

Ⓒ $(3, 2)$

Ⓓ $(-3, 2)$



2. Which equation is an equation of a line parallel to $-2x + 3y = 15$?

Ⓐ $y = -\frac{2}{3}x + 7$ Ⓑ $y = \frac{2}{3}x + 7$

Ⓒ $y = -\frac{3}{2}x + 7$ Ⓓ $y = -6x + 7$

3. Two trains, E and F, travel along parallel tracks. Each track is 110 miles long. They begin their trips at the same time. Train E travels at a rate of 55 miles per hour and train F travels at a rate of 22 miles per hour. How many miles will train F have left to travel after train E completes its trip?

Ⓐ 5 miles Ⓑ 33 miles

Ⓒ 60 miles Ⓓ 66 miles

4. A line segment is parallel to the y-axis and is 9 units long. The two endpoints are $(3, 6)$ and (a, b). What is a value of b?

Ⓐ -6 Ⓑ -3

Ⓒ 3 Ⓓ 6

5. Which equation is an equation of a line perpendicular to $y = 5x + 7$?

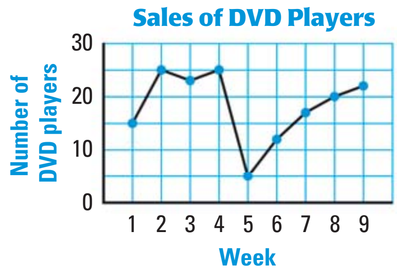
Ⓐ $y = -5x + 9$

Ⓑ $y = 5x + 16$

Ⓒ $y = \frac{1}{5}x + 7$

Ⓓ $y = -\frac{1}{5}x + 7$

6. According to the graph, which is the closest approximation of the decrease in sales between week 4 and week 5?



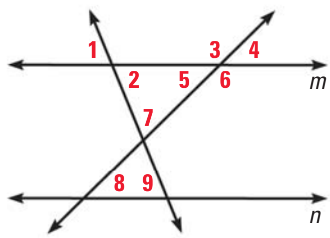
Ⓐ 24 DVD players

Ⓑ 20 DVD players

Ⓒ 18 DVD players

Ⓓ 15 DVD players

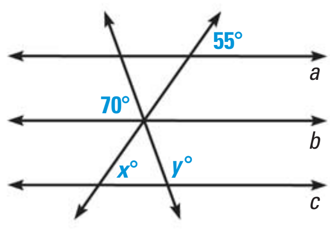
7. In the diagram, $m \parallel n$. Which pair of angles have equal measures?



Ⓐ $\angle 3$ and $\angle 5$ Ⓑ $\angle 4$ and $\angle 7$

Ⓒ $\angle 1$ and $\angle 9$ Ⓓ $\angle 2$ and $\angle 6$

8. Five lines intersect as shown in the diagram. Lines a, b, and c are parallel. What is the value of $x + y$?



Ⓐ 125 Ⓑ 165

Ⓒ 195 Ⓓ 235

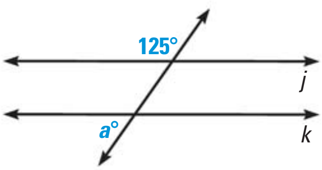
GRIDDED ANSWER

9. What is the slope of a line perpendicular to $5x - 3y = 9$?

10. What is the slope of the line passing through the points $(1, 1)$ and $(-2, -2)$?

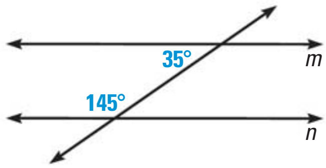
11. What is the y-intercept of the line that is parallel to the line $2x - y = 3$ and passes through the point $(-3, 4)$?

12. What is the value of a if line j is parallel to line k?



SHORT RESPONSE

13. *Explain* how you know that lines m and n are parallel to each other.



14. What is one possible value for the slope of a line passing through the point $(1, 1)$ and passing *between* the points $(-2, -2)$ and $(-2, -3)$ but not containing either one of them?

EXTENDED RESPONSE

15. Mrs. Smith needs a babysitter. Lauren who lives next door charges \$5 per hour for her services. Zachary who lives across town charges \$4 per hour plus \$3 for bus fare.

a. Using this information, write equations to represent Lauren and Zachary's babysitting fees. Let F represent their fees and h represent the number of hours.

b. Graph the equations you wrote in part (a).

c. Based on their fees, which babysitter would be a better choice for Mrs. Smith if she is going out for two hours? *Explain* your answer.

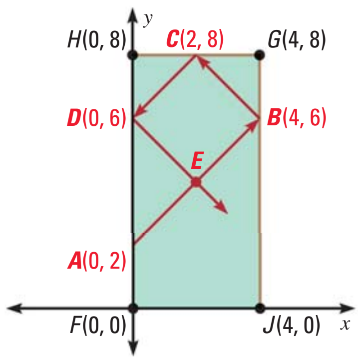
d. Mrs. Smith needs to go out for four hours. Which babysitter would be the less expensive option for her? *Justify* your response.

16. In a game of pool, a cue ball is hit from point A and follows the path of arrows as shown on the pool table at the right. In the diagram, $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{ED}$.

a. *Compare* the slopes of $\overline{AB}$ and $\overline{BC}$. What can you conclude about $\angle ABC$?

b. If $m\angle BCG = 45°$, what is $m\angle DCH$? *Explain* your reasoning.

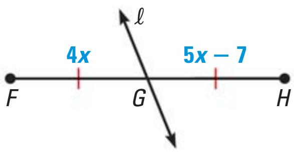
c. If the cue ball is hit harder, will it fall into Pocket F? *Justify* your answer.



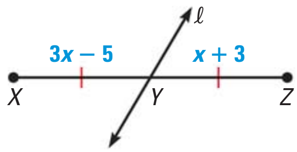
CUMULATIVE REVIEW Chapters 1–3

Line ℓ bisects the segment. Find the indicated lengths. *(p. 15)*

1. GH and FH



2. XY and XZ

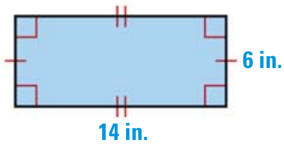


Classify the angle with the given measure as *acute*, *obtuse*, *right*, or *straight*. *(p. 24)*

3. $m\angle A = 28°$
4. $m\angle A = 113°$
5. $m\angle A = 79°$
6. $m\angle A = 90°$

Find the perimeter and area of the figure. *(p. 49)*

7.



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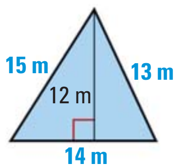

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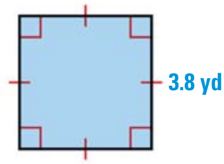


***Describe* the pattern in the numbers. Write the next number in the pattern.** *(p. 72)*

10. 1, 8, 27, 64, . . .
11. 128, 32, 8, 2, . . .
12. 2, −6, 18, −54, . . .

Use the Law of Detachment to make a valid conclusion. *(p. 87)*

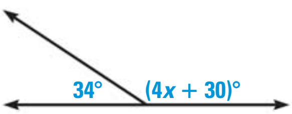
13. If $6x < 42$, then $x < 7$. The value of $6x$ is 24.
14. If an angle measure is greater than 90°, then it is an obtuse angle. The measure of $\angle A$ is 103°.
15. If a musician plays a violin, then the musician plays a stringed instrument. The musician is playing a violin.

Solve the equation. Write a reason for each step. *(p. 105)*

16. $3x - 14 = 34$
17. $-4(x + 3) = -28$
18. $43 - 9(x - 7) = -x - 6$

Find the value of the variable(s). *(pp. 124, 154)*

19.



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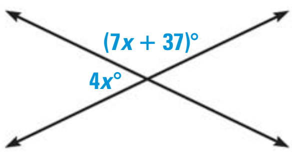

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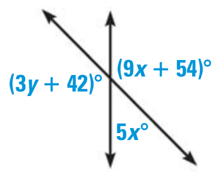

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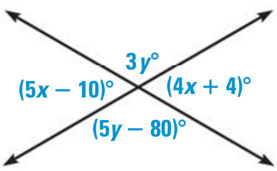

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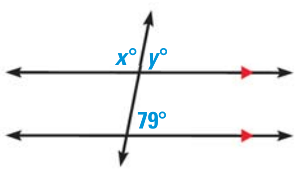

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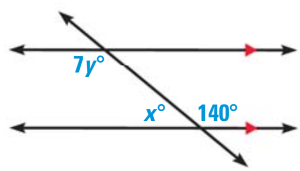


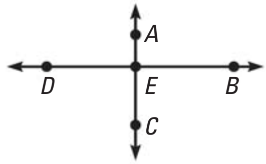
Find the slope of the line through the given points. ***(p. 171)***

25. $(5, -2), (7, -2)$ **26.** $(8, 3), (3, 14)$ **27.** $(-1, 2), (0, 4)$

Write equations of the lines that pass through point P and are (a) parallel and (b) perpendicular to the line with the given equation. ***(p. 180)***

28. $P(3, -2), y = 6x + 7$ **29.** $P(-2, 12), y = -x - 3$ **30.** $P(7, -1), 6y + 2x = 18$

31. Use the diagram at the right. If $\angle AEB \cong \angle AED$, is $\overleftrightarrow{AC} \perp \overleftrightarrow{DB}$? *Explain* how you know. ***(p. 190)***



EVERYDAY INTERSECTIONS **In Exercises 32–34, what kind of geometric intersection does the photograph suggest?** ***(p. 2)***

32.

33.

34.

35. MAPS The distance between Westville and Easton is 37 miles. The distance between Reading and Easton is 52 miles. How far is Westville from Reading? ***(p. 9)***

36. GARDENING A rectangular garden is 40 feet long and 25 feet wide. What is the area of the garden? ***(p. 49)***

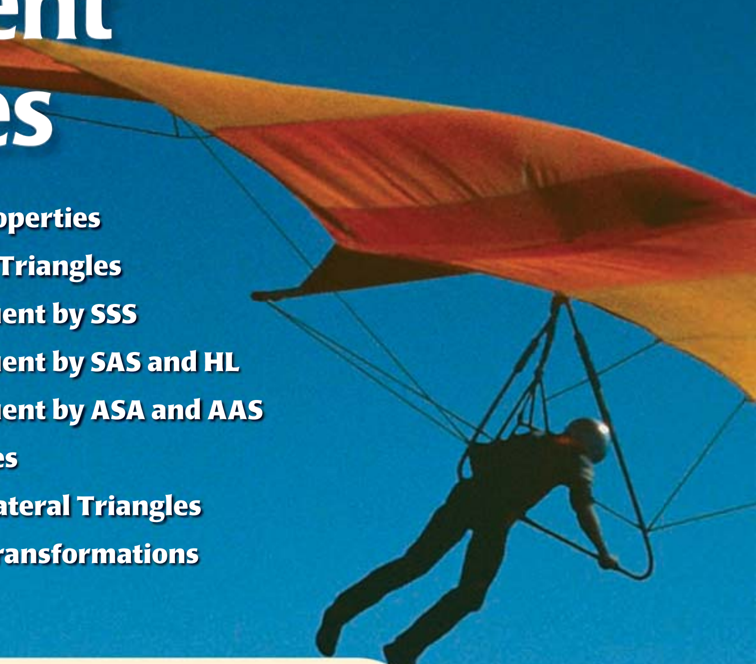
ADVERTISING **In Exercises 37 and 38, use the following advertising slogan: "Do you want the lowest prices on new televisions? Then come and see Matt's TV Warehouse."** ***(p. 79)***

37. Write the slogan in if-then form. What are the hypothesis and conclusion of the conditional statement?

38. Write the converse, inverse, and contrapositive of the conditional statement you wrote in Exercise 37.

39. CARPENTRY You need to cut eight wood planks that are the same size. You measure and cut the first plank. You cut the second piece using the first plank as a guide, as shown at the right. You use the second plank to cut the third plank. You continue this pattern. Is the last plank you cut the same length as the first? *Explain* your reasoning. ***(p. 112)***

4 Congruent Triangles



Before

In previous chapters, you learned the following skills, which you'll use in Chapter 4: classifying angles, solving linear equations, finding midpoints, and using angle relationships.

Prerequisite Skills

VOCABULARY CHECK

Classify the angle as *acute*, *obtuse*, *right*, or *straight*.

1. $m\angle A = 115°$ **2.** $m\angle B = 90°$ **3.** $m\angle C = 35°$ **4.** $m\angle D = 95°$

SKILLS AND ALGEBRA CHECK

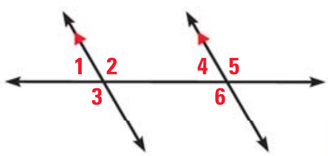
Solve the equation. *(Review p. 65 for 4.1, 4.2.)*

5. $70 + 2y = 180$ **6.** $2x = 5x - 54$ **7.** $40 + x + 65 = 180$

Find the coordinates of the midpoint of $\overline{PQ}$. *(Review p. 15 for 4.3.)*

8. $P(2, -5), Q(-1, -2)$ **9.** $P(-4, 7), Q(1, -5)$ **10.** $P(h, k), Q(h, 0)$

Name the theorem or postulate that justifies the statement about the diagram. *(Review p. 154 for 4.3–4.5.)*



11. $\angle 2 \cong \angle 3$ **12.** $\angle 1 \cong \angle 4$

13. $\angle 2 \cong \angle 6$ **14.** $\angle 3 \cong \angle 5$

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 4, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 281. You will also use the key vocabulary listed below.

Big Ideas

1. Classifying triangles by sides and angles
2. Proving that triangles are congruent
3. Using coordinate geometry to investigate triangle relationships

KEY VOCABULARY

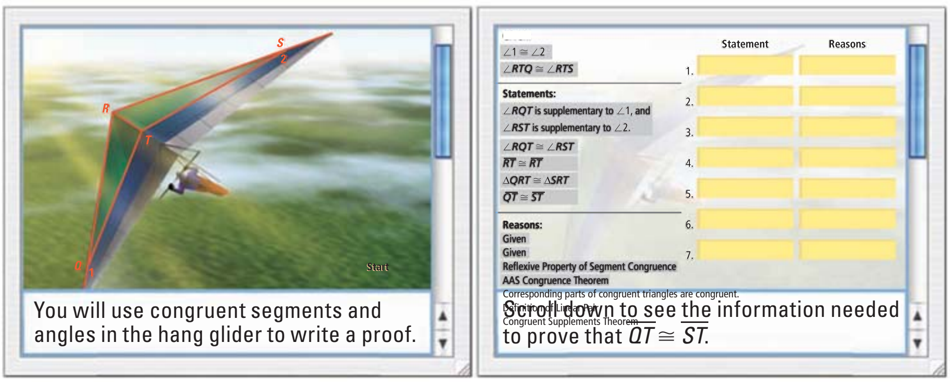
- triangle, *p. 217*
  scalene, isosceles, equilateral, acute, right, obtuse, equiangular
- interior angles, *p. 218*
- exterior angles, *p. 218*
- corollary, *p. 220*
- congruent figures, *p. 225*
- corresponding parts, *p. 225*
- right triangle, *p. 241*
  legs, hypotenuse
- flow proof, *p. 250*
- isosceles triangle, *p. 264*
  legs, vertex angle, base, base angles
- transformation, *p. 272*
  translation, reflection, rotation

Why?

Triangles are used to add strength to structures in real-world situations. For example, the frame of a hang glider involves several triangles.

Animated Geometry

The animation illustrated below for Example 1 on page 256 helps you answer this question: What must be true about $\overline{QT}$ and $\overline{ST}$ for the hang glider to fly straight?



Geometry at classzone.com

Animated Geometry at classzone.com

Other animations for Chapter 4: pages 234, 242, 250, 257, and 274

Investigating Geometry ACTIVITY *Use before Lesson 4.1*

4.1 Angle Sums in Triangles

MATERIALS • paper • pencil • scissors • ruler

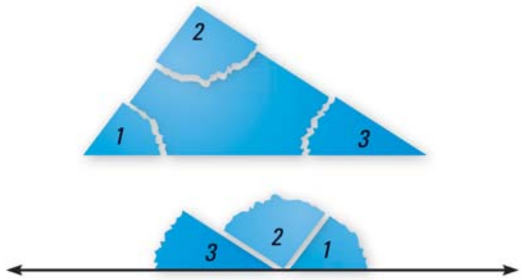
QUESTION **What are some relationships among the *interior angles* of a triangle and *exterior angles* of a triangle?**

EXPLORE 1 Find the sum of the measures of interior angles

STEP 1 ***Draw triangles*** Draw and cut out several different triangles.

STEP 2 ***Tear off corners*** For each triangle, tear off the three corners and place them next to each other, as shown in the diagram.

STEP 3 ***Make a conjecture*** Make a conjecture about the sum of the measures of the interior angles of a triangle.



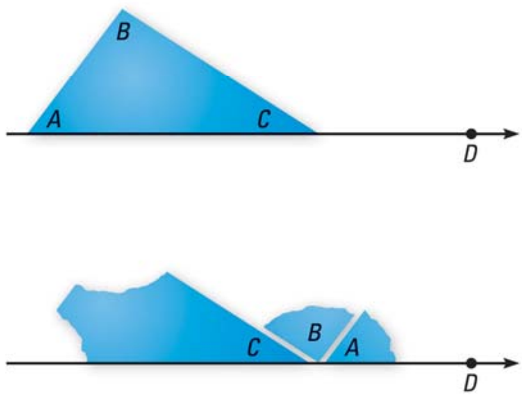
$\angle 1$, $\angle 2$, and $\angle 3$ are *interior angles.*

EXPLORE 2 Find the measure of an exterior angle of a triangle

STEP 1 ***Draw exterior angle*** Draw and cut out several different triangles. Place each triangle on a piece of paper and extend one side to form an *exterior angle*, as shown in the diagram.

STEP 2 ***Tear off corners*** For each triangle, tear off the corners that are not next to the exterior angle. Use them to fill the exterior angle, as shown.

STEP 3 ***Make a conjecture*** Make a conjecture about the relationship between the measure of an exterior angle of a triangle and the measures of the nonadjacent interior angles.



In the top figure, $\angle BCD$ is an *exterior angle.*

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Given the measures of two interior angles of a triangle, how can you find the measure of the third angle?
2. Draw several different triangles that each have one right angle. Show that the two acute angles of a right triangle are complementary.

4.1 Apply Triangle Sum Properties

Before You classified angles and found their measures.

Now You will classify triangles and find measures of their angles.

Why? So you can place actors on stage, as in Ex. 40.

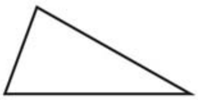
Key Vocabulary
- **triangle**
  scalene, isosceles, equilateral, acute, right, obtuse, equiangular
- **interior angles**
- **exterior angles**
- **corollary to a theorem**

A **triangle** is a polygon with three sides. A triangle with vertices *A*, *B*, and *C* is called "triangle *ABC*" or "$\triangle ABC$."

KEY CONCEPT *For Your Notebook*

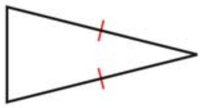
Classifying Triangles by Sides

Scalene Triangle



No congruent sides

Isosceles Triangle



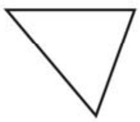
At least 2 congruent sides

Equilateral Triangle

3 congruent sides

Classifying Triangles by Angles

Acute Triangle

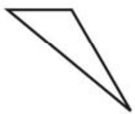


3 acute angles

Right Triangle

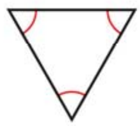
1 right angle

Obtuse Triangle



1 obtuse angle

Equiangular Triangle



3 congruent angles

READ VOCABULARY
Notice that an equilateral triangle is also isosceles. An equiangular triangle is also acute.

EXAMPLE 1 Classify triangles by sides and by angles

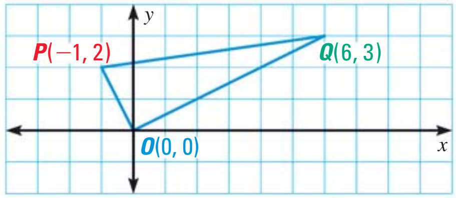
SUPPORT BEAMS Classify the triangular shape of the support beams in the diagram by its sides and by measuring its angles.

Solution

The triangle has a pair of congruent sides, so it is isosceles. By measuring, the angles are 55°, 55°, and 70°. It is an acute isosceles triangle.

EXAMPLE 2 Classify a triangle in a coordinate plane

Classify $\triangle PQO$ by its sides. Then determine if the triangle is a right triangle.



Solution

STEP 1 **Use** the distance formula to find the side lengths.

$$OP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{((-1) - 0)^2 + (2 - 0)^2} = \sqrt{5} \approx 2.2$$

$$OQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - 0)^2 + (3 - 0)^2} = \sqrt{45} \approx 6.7$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - (-1))^2 + (3 - 2)^2} = \sqrt{50} \approx 7.1$$

STEP 2 **Check** for right angles. The slope of $\overline{OP}$ is $\frac{2 - 0}{-1 - 0} = -2$. The slope of $\overline{OQ}$ is $\frac{3 - 0}{6 - 0} = \frac{1}{2}$. The product of the slopes is $-2\left(\frac{1}{2}\right) = -1$, so $\overline{OP} \perp \overline{OQ}$ and $\angle POQ$ is a right angle.

▶ Therefore, $\triangle PQO$ is a right scalene triangle.

✓ GUIDED PRACTICE for Examples 1 and 2

1. Draw an obtuse isosceles triangle and an acute scalene triangle.
2. Triangle ABC has the vertices $A(0, 0)$, $B(3, 3)$, and $C(-3, 3)$. Classify it by its sides. Then determine if it is a right triangle.

ANGLES When the sides of a polygon are extended, other angles are formed. The original angles are the **interior angles**. The angles that form linear pairs with the interior angles are the **exterior angles**.

READ DIAGRAMS
Each vertex has a *pair* of congruent exterior angles. However, it is common to show only *one* exterior angle at each vertex.

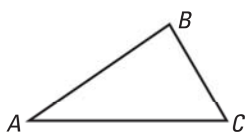
interior angles   exterior angles

THEOREM — *For Your Notebook*

THEOREM 4.1 Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.

Proof: p. 219; Ex. 53, p. 224



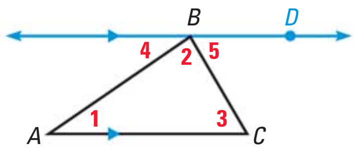
$m\angle A + m\angle B + m\angle C = 180°$

AUXILIARY LINES To prove certain theorems, you may need to add a line, a segment, or a ray to a given diagram. An *auxiliary* line is used in the proof of the Triangle Sum Theorem.

PROOF Triangle Sum Theorem

GIVEN ▶ $\triangle ABC$

PROVE ▶ $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



Plan for Proof

a. Draw an auxiliary line through B and parallel to $\overline{AC}$.

b. Show that $m\angle 4 + m\angle 2 + m\angle 5 = 180°$, $\angle 1 \cong \angle 4$, and $\angle 3 \cong \angle 5$.

c. By substitution, $m\angle 1 + m\angle 2 + m\angle 3 = 180°$.

Plan in Action

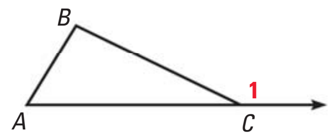
STATEMENTS	REASONS
a. 1. Draw $\overleftrightarrow{BD}$ parallel to $\overline{AC}$.	1. Parallel Postulate
b. 2. $m\angle 4 + m\angle 2 + m\angle 5 = 180°$	2. Angle Addition Postulate and definition of straight angle
3. $\angle 1 \cong \angle 4$, $\angle 3 \cong \angle 5$	3. Alternate Interior Angles Theorem
4. $m\angle 1 = m\angle 4$, $m\angle 3 = m\angle 5$	4. Definition of congruent angles
c. 5. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	5. Substitution Property of Equality

THEOREM *For Your Notebook*

THEOREM 4.2 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

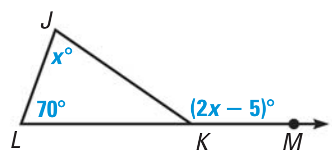
Proof: Ex. 50, p. 223



$m\angle 1 = m\angle A + m\angle B$

EXAMPLE 3 Find an angle measure

xy ALGEBRA **Find $m\angle JKM$.**



Solution

STEP 1 **Write** and solve an equation to find the value of x.

$(2x - 5)° = 70° + x°$ **Apply the Exterior Angle Theorem.**

$x = 75$ **Solve for x.**

STEP 2 **Substitute** 75 for x in $2x - 5$ to find $m\angle JKM$.

$2x - 5 = 2 \cdot 75 - 5 = 145$

▶ The measure of $\angle JKM$ is 145°.

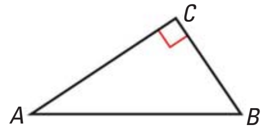
A **corollary to a theorem** is a statement that can be proved easily using the theorem. The corollary below follows from the Triangle Sum Theorem.

COROLLARY *For Your Notebook*

Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.

Proof: Ex. 48, p. 223



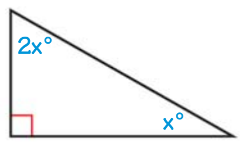
$m\angle A + m\angle B = 90°$

EXAMPLE 4 Find angle measures from a verbal description

ARCHITECTURE The tiled staircase shown forms a right triangle. The measure of one acute angle in the triangle is twice the measure of the other. Find the measure of each acute angle.

Solution

First, sketch a diagram of the situation. Let the measure of the smaller acute angle be $x°$. Then the measure of the larger acute angle is $2x°$. The Corollary to the Triangle Sum Theorem states that the acute angles of a right triangle are complementary.



Use the corollary to set up and solve an equation.

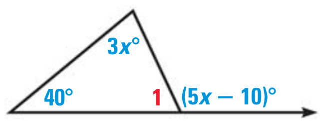
$x° + 2x° = 90°$ **Corollary to the Triangle Sum Theorem**

$x = 30$ **Solve for *x*.**

▶ So, the measures of the acute angles are $30°$ and $2(30°) = 60°$.

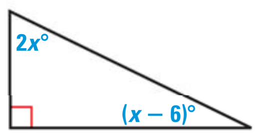
✓ GUIDED PRACTICE for Examples 3 and 4

3. Find the measure of $\angle 1$ in the diagram shown.



4. Find the measure of each interior angle of $\triangle ABC$, where $m\angle A = x°$, $m\angle B = 2x°$, and $m\angle C = 3x°$.

5. Find the measures of the acute angles of the right triangle in the diagram shown.



6. In Example 4, what is the measure of the obtuse angle formed between the staircase and a segment extending from the horizontal leg?

4.1 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 15, and 41

★ = **STANDARDIZED TEST PRACTICE** Exs. 7, 20, 31, 43, and 51

SKILL PRACTICE

VOCABULARY **Match the triangle description with the most specific name.**

1. Angle measures: 30°, 60°, 90° — **A.** Isosceles

2. Side lengths: 2 cm, 2 cm, 2 cm — **B.** Scalene

3. Angle measures: 60°, 60°, 60° — **C.** Right

4. Side lengths: 6 m, 3 m, 6 m — **D.** Obtuse

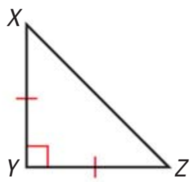
5. Side lengths: 5 ft, 7 ft, 9 ft — **E.** Equilateral

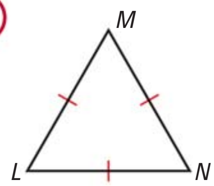
6. Angle measures: 20°, 125°, 35° — **F.** Equiangular

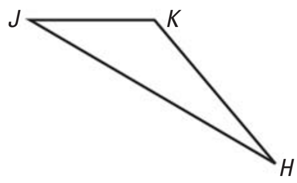
7. ★ **WRITING** Can a right triangle also be obtuse? *Explain* why or why not.

EXAMPLE 1 on p. 217 for Exs. 8–10

CLASSIFYING TRIANGLES **Copy the triangle and measure its angles. Classify the triangle by its sides and by its angles.**

8. 

9. 

10. 

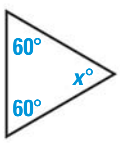
EXAMPLE 2 on p. 218 for Exs. 11–13

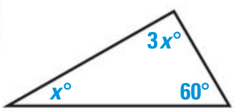
COORDINATE PLANE **A triangle has the given vertices. Graph the triangle and classify it by its sides. Then determine if it is a right triangle.**

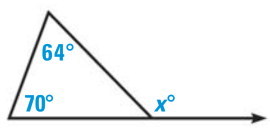
11. $A(2, 3), B(6, 3), C(2, 7)$ **12.** $A(3, 3), B(6, 9), C(6, -3)$ **13.** $A(1, 9), B(4, 8), C(2, 5)$

EXAMPLE 3 on p. 219 for Exs. 14–19

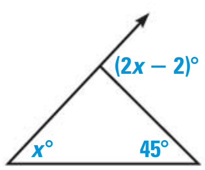
FINDING ANGLE MEASURES **Find the value of *x*. Then classify the triangle by its angles.**

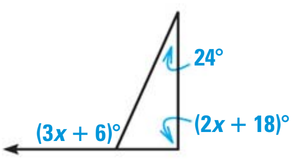
14. 

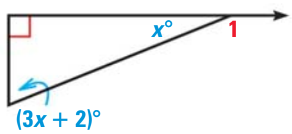
15. 

16. 

xy **ALGEBRA** **Find the measure of the exterior angle shown.**

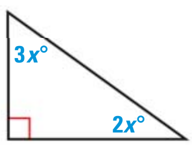
17. 

18. 

19. 

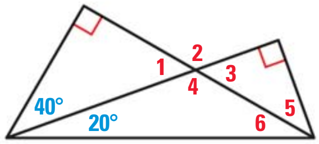
EXAMPLE 4 on p. 220 for Ex. 20

20. ★ **SHORT RESPONSE** *Explain* how to use the Corollary to the Triangle Sum Theorem to find the measure of each angle.



ANGLE RELATIONSHIPS **Find the measure of the numbered angle.**

21. $\angle 1$
22. $\angle 2$
23. $\angle 3$
24. $\angle 4$
25. $\angle 5$
26. $\angle 6$



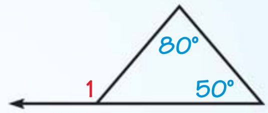
27. **ALGEBRA** In $\triangle PQR$, $\angle P \cong \angle R$ and the measure of $\angle Q$ is twice the measure of $\angle R$. Find the measure of each angle.

28. **ALGEBRA** In $\triangle EFG$, $m\angle F = 3(m\angle G)$, and $m\angle E = m\angle F - 30°$. Find the measure of each angle.

ERROR ANALYSIS **In Exercises 29 and 30, *describe* and correct the error.**

29. All equilateral triangles are also isosceles. So, if $\triangle ABC$ is isosceles, then it is equilateral as well.

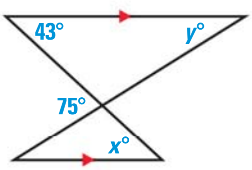
30. $m\angle 1 + 80° + 50° = 180°$

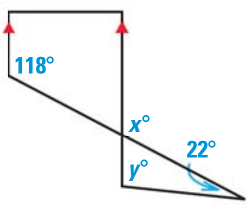


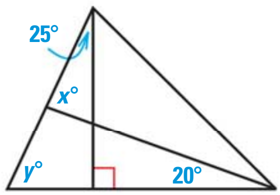
31. ★ **MULTIPLE CHOICE** Which of the following is not possible?

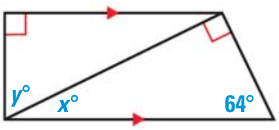
Ⓐ An acute scalene triangle
Ⓑ A triangle with two acute exterior angles
Ⓒ An obtuse isosceles triangle
Ⓓ An equiangular acute triangle

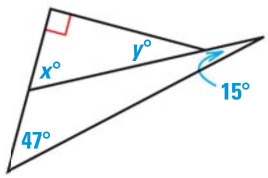
ALGEBRA **In Exercises 32–37, find the values of *x* and *y*.**

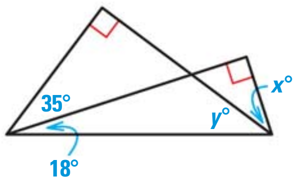
32.


33.


34.


35.


36.


37.


38. **VISUALIZATION** Is there an angle measure that is so small that any triangle with that angle measure will be an obtuse triangle? *Explain.*

39. **CHALLENGE** Suppose you have the equations $y = ax + b$, $y = cx + d$, and $y = ex + f$.

   a. When will these three lines form a triangle?

   b. Let $c = 1$, $d = 2$, $e = 4$, and $f = -7$. Find values of a and b so that no triangle is formed by the three equations.

   c. Draw the triangle formed when $a = \frac{4}{3}$, $b = \frac{1}{3}$, $c = -\frac{4}{3}$, $d = \frac{41}{3}$, $e = 0$, and $f = -1$. Then classify the triangle by its sides.

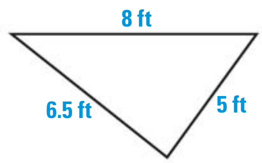
○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

PROBLEM SOLVING

EXAMPLE 1 on p. 217 for Ex. 40

40. THEATER Three people are standing on a stage. The distances between the three people are shown in the diagram. Classify the triangle formed by its sides. Then copy the triangle, measure the angles, and classify the triangle by its angles.



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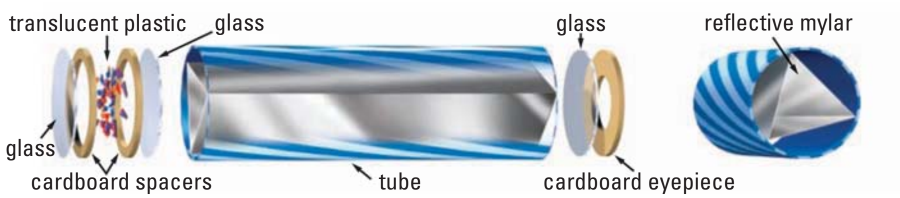
41. KALEIDOSCOPES You are making a kaleidoscope. The directions state that you are to arrange three pieces of reflective mylar in an equilateral and equiangular triangle. You must cut three strips from a piece of mylar 6 inches wide. What are the side lengths of the triangle used to form the kaleidoscope? What are the measures of the angles? *Explain*.



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42. SCULPTURE You are bending a strip of metal into an isosceles triangle for a sculpture. The strip of metal is 20 inches long. The first bend is made 6 inches from one end. *Describe* two ways you could complete the triangle.

43. ★ MULTIPLE CHOICE Which inequality describes the possible measures of an angle of a triangle?

(A) $0° \le x° \le 180°$ (B) $0° \le x° < 180°$ (C) $0° < x° < 180°$ (D) $0° < x° \le 180°$

SLING CHAIRS The brace of a sling chair forms a triangle with the seat and legs of the chair. Suppose $m\angle 2 = 50°$ and $m\angle 3 = 65°$.

44. Find $m\angle 6$.

45. Find $m\angle 5$.

46. Find $m\angle 1$.

47. Find $m\angle 4$.

48. PROOF Prove the Corollary to the Triangle Sum Theorem on page 220.

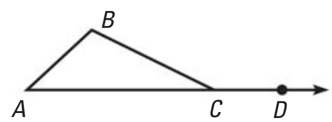
49. MULTI-STEP PROBLEM The measures of the angles of a triangle are $(2\sqrt{2x}°)$, $(5\sqrt{2x}°)$, and $(2\sqrt{2x}°)$.

a. Write an equation to show the relationship of the angles.

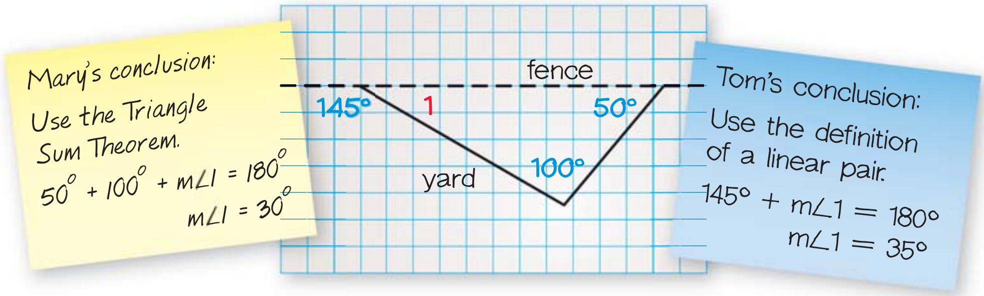
b. Find the measure of each angle.

c. Classify the triangle by its angles.

50. PROVING THEOREM 4.2 Prove the Exterior Angle Theorem. (*Hint:* Find two equations involving $m\angle ACB$.)



51. ★ **EXTENDED RESPONSE** The figure below shows an initial plan for a triangular flower bed that Mary and Tom plan to build along a fence. They are discussing what the measure of $\angle 1$ should be.



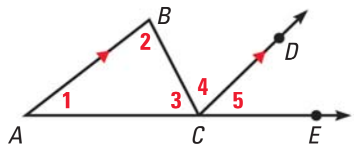
Did Mary and Tom both reason correctly? If not, who made a mistake and what mistake was made? If they did both reason correctly, what can you conclude about their initial plan? *Explain*.

52. **xy ALGEBRA** $\triangle ABC$ is isosceles. $AB = x$ and $BC = 2x - 4$.

a. Find two possible values for x if the perimeter of $\triangle ABC$ is 32.

b. How many possible values are there for x if the perimeter of $\triangle ABC$ is 12?

53. **CHALLENGE** Use the diagram to write a proof of the Triangle Sum Theorem. Your proof should be different than the proof of the Triangle Sum Theorem on page 219.



MIXED REVIEW

$\angle A$ and $\angle B$ are complementary. Find $m\angle A$ and $m\angle B$. *(p. 35)*

54. $m\angle A = (3x + 16)^\circ$
$m\angle B = (4x - 3)^\circ$

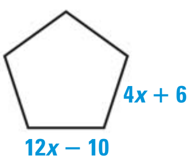
55. $m\angle A = (4x - 2)^\circ$
$m\angle B = (7x + 4)^\circ$

56. $m\angle A = (3x + 4)^\circ$
$m\angle B = (2x + 6)^\circ$

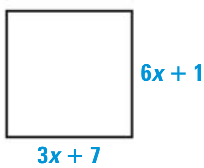
PREVIEW
Prepare for Lesson 4.2 in Exs. 57–59.

Each figure is a regular polygon. Find the value of x. *(p. 42)*

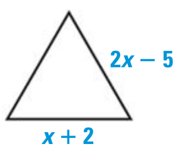
57.



58.

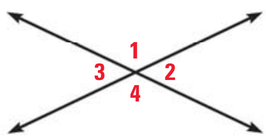


59.



60. Use the Symmetric Property of Congruence to complete the statement: If __?__ $\cong$ __?__, then $\angle DEF \cong \angle PQR$. *(p. 112)*

Use the diagram at the right. *(p. 124)*



61. If $m\angle 1 = 127^\circ$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

62. If $m\angle 4 = 170^\circ$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$.

63. If $m\angle 3 = 54^\circ$, find $m\angle 1$, $m\angle 2$, and $m\angle 4$.

4.2 Apply Congruence and Triangles

Before You identified congruent angles.

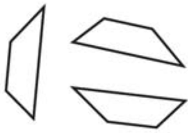
Now You will identify congruent figures.

Why? So you can determine if shapes are identical, as in Example 3.

Key Vocabulary
- **congruent figures**
- **corresponding parts**

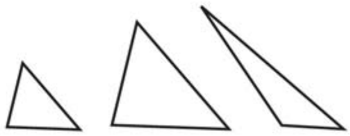
Two geometric figures are *congruent* if they have exactly the same size and shape. Imagine cutting out one of the congruent figures. You could then position the cut-out figure so that it fits perfectly onto the other figure.

Congruent



Same size and shape

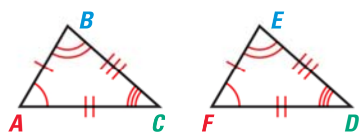
Not congruent



Different sizes or shapes

In two **congruent figures**, all the parts of one figure are congruent to the **corresponding parts** of the other figure. In congruent polygons, this means that the *corresponding sides* and the *corresponding angles* are congruent.

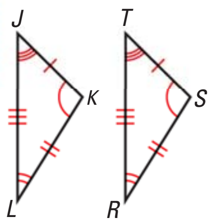
CONGRUENCE STATEMENTS When you write a congruence statement for two polygons, always list the corresponding vertices in the same order. You can write congruence statements in more than one way. Two possible congruence statements for the triangles at the right are $\triangle ABC \cong \triangle FED$ or $\triangle BCA \cong \triangle EDF$.



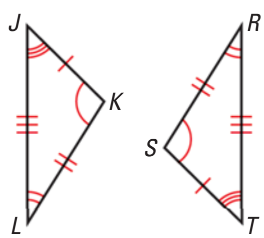
Corresponding angles	$\angle A \cong \angle F$	$\angle B \cong \angle E$	$\angle C \cong \angle D$
Corresponding sides	$\overline{AB} \cong \overline{FE}$	$\overline{BC} \cong \overline{ED}$	$\overline{AC} \cong \overline{FD}$

EXAMPLE 1 Identify congruent parts

VISUAL REASONING
To help you identify corresponding parts, turn $\triangle RST$.



Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts.



Solution

The diagram indicates that $\triangle JKL \cong \triangle TSR$.

Corresponding angles $\angle J \cong \angle T$, $\angle K \cong \angle S$, $\angle L \cong \angle R$

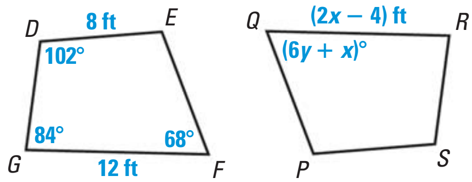
Corresponding sides $\overline{JK} \cong \overline{TS}$, $\overline{KL} \cong \overline{SR}$, $\overline{LJ} \cong \overline{RT}$

EXAMPLE 2 Use properties of congruent figures

In the diagram, *DEFG* ≅ *SPQR*.

a. Find the value of x.

b. Find the value of y.



Solution

a. You know that $\overline{FG} \cong \overline{QR}$.

$FG = QR$

$12 = 2x - 4$

$16 = 2x$

$8 = x$

b. You know that $\angle F \cong \angle Q$.

$m\angle F = m\angle Q$

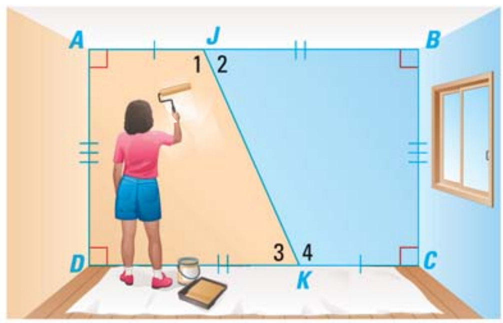
$68° = (6y + x)°$

$68 = 6y + 8$

$10 = y$

EXAMPLE 3 Show that figures are congruent

PAINTING If you divide the wall into orange and blue sections along $\overline{JK}$, will the sections of the wall be the same size and shape? *Explain.*



Solution

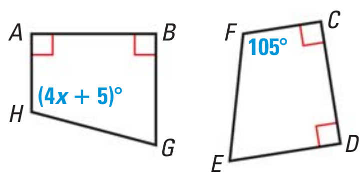
From the diagram, $\angle A \cong \angle C$ and $\angle D \cong \angle B$ because all right angles are congruent. Also, by the Lines Perpendicular to a Transversal Theorem, $\overline{AB} \parallel \overline{DC}$. Then, $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$ by the Alternate Interior Angles Theorem. So, all pairs of corresponding angles are congruent.

The diagram shows $\overline{AJ} \cong \overline{CK}$, $\overline{KD} \cong \overline{JB}$, and $\overline{DA} \cong \overline{BC}$. By the Reflexive Property, $\overline{JK} \cong \overline{KJ}$. All corresponding parts are congruent, so $AJKD \cong CKJB$.

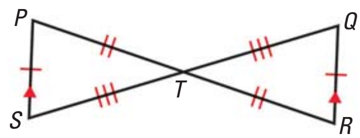
▶ Yes, the two sections will be the same size and shape.

✓ GUIDED PRACTICE for Examples 1, 2, and 3

In the diagram at the right, *ABGH* ≅ *CDEF*.



1. Identify all pairs of congruent corresponding parts.
2. Find the value of x and find $m\angle H$.
3. Show that $\triangle PTS \cong \triangle RTQ$.

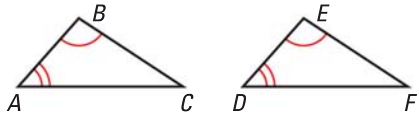


THEOREM — *For Your Notebook*

THEOREM 4.3 Third Angles Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

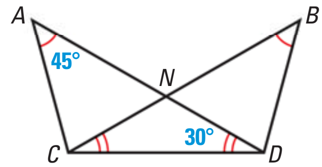
Proof: Ex. 28, p. 230



If $\angle A \cong \angle D$, and $\angle B \cong \angle E$, then $\angle C \cong \angle F$.

EXAMPLE 4 Use the Third Angles Theorem

Find $m\angle BDC$.



Solution

$\angle A \cong \angle B$ and $\angle ADC \cong \angle BCD$, so by the Third Angles Theorem, $\angle ACD \cong \angle BDC$. By the Triangle Sum Theorem, $m\angle ACD = 180° - 45° - 30° = 105°$.

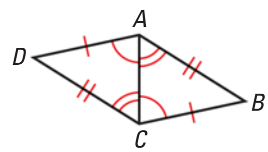
▶ So, $m\angle ACD = m\angle BDC = 105°$ by the definition of congruent angles.

ANOTHER WAY

For an alternative method for solving the problem in Example 4, turn to page 232 for the **Problem Solving Workshop**.

EXAMPLE 5 Prove that triangles are congruent

Write a proof.



GIVEN ▶ $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$, $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$

PROVE ▶ $\triangle ACD \cong \triangle CAB$

Plan for Proof

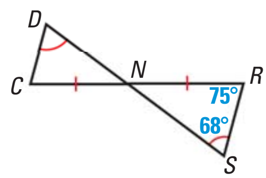
a. Use the Reflexive Property to show that $\overline{AC} \cong \overline{AC}$.

b. Use the Third Angles Theorem to show that $\angle B \cong \angle D$.

Plan in Action

	STATEMENTS	REASONS
	1. $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$	1. Given
a.	2. $\overline{AC} \cong \overline{AC}$	2. Reflexive Property of Congruence
	3. $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$	3. Given
b.	4. $\angle B \cong \angle D$	4. Third Angles Theorem
	5. $\triangle ACD \cong \triangle CAB$	5. Definition of $\cong$ ⚠

✓ GUIDED PRACTICE for Examples 4 and 5



4. In the diagram, what is $m\angle DCN$?

5. By the definition of congruence, what additional information is needed to know that $\triangle NDC \cong \triangle NSR$?

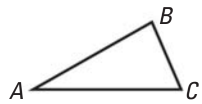
PROPERTIES OF CONGRUENT TRIANGLES The properties of congruence that are true for segments and angles are also true for triangles.

THEOREM *For Your Notebook*

THEOREM 4.4 Properties of Congruent Triangles

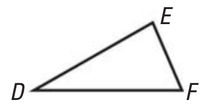
Reflexive Property of Congruent Triangles

For any triangle ABC, $\triangle ABC \cong \triangle ABC$.



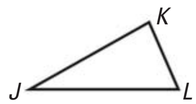
Symmetric Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.



Transitive Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.



4.2 EXERCISES

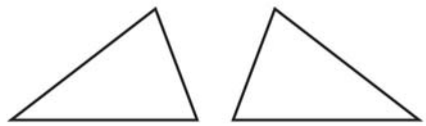
HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 15, and 25

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, 21, 24, 27, and 30

SKILL PRACTICE

1. **VOCABULARY** Copy the congruent triangles shown. Then label the vertices of the triangles so that $\triangle JKL \cong \triangle RST$. Identify all pairs of congruent *corresponding angles* and *corresponding sides*.

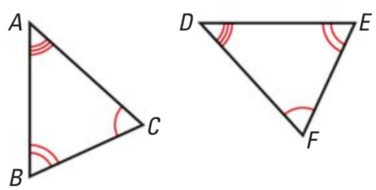


2. ★ **WRITING** Based on this lesson, what information do you need to prove that two triangles are congruent? *Explain.*

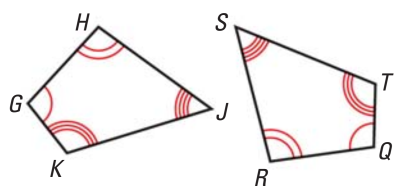
EXAMPLE 1 on p. 225 for Exs. 3–4

USING CONGRUENCE Identify all pairs of congruent corresponding parts. Then write another congruence statement for the figures.

3. $\triangle ABC \cong \triangle DEF$



4. $GHJK \cong QRST$



EXAMPLE 2 on p. 226 for Exs. 5–10

READING A DIAGRAM In the diagram, $\triangle XYZ \cong \triangle MNL$. Copy and complete the statement.

5. $m\angle Y =$ _?_

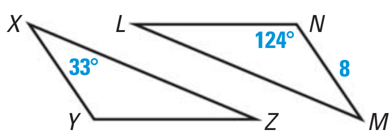
6. $m\angle M =$ _?_

7. $YX =$ _?_

8. $\overline{YZ} \cong$ _?_

9. $\triangle LNM \cong$ _?_

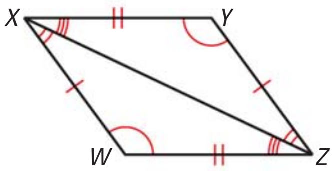
10. $\triangle YXZ \cong$ _?_



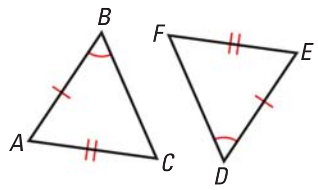
EXAMPLE 3
on p. 226
for Exs. 11–14

NAMING CONGRUENT FIGURES **Write a congruence statement for any figures that can be proved congruent. *Explain* your reasoning.**

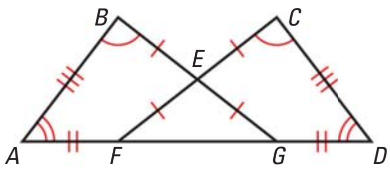
11.



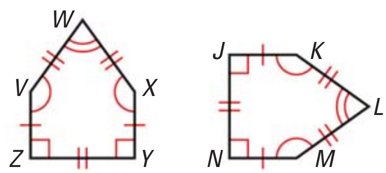
12.



13.



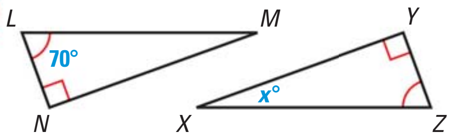
14.



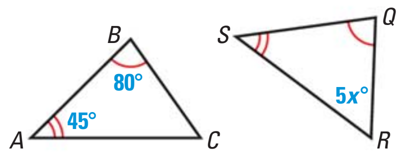
EXAMPLE 4
on p. 227
for Exs. 15–16

THIRD ANGLES THEOREM **Find the value of *x*.**

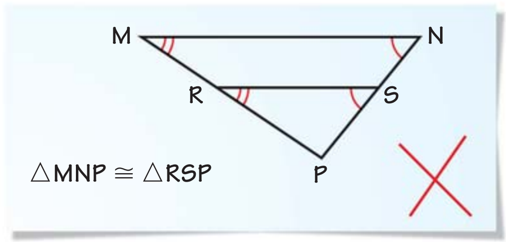
15.



16.



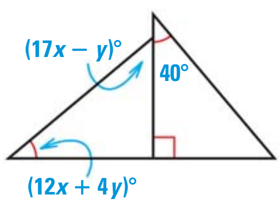
17. **ERROR ANALYSIS** A student says that $\triangle MNP \cong \triangle RSP$ because the corresponding angles of the triangles are congruent. *Describe* the error in this statement.



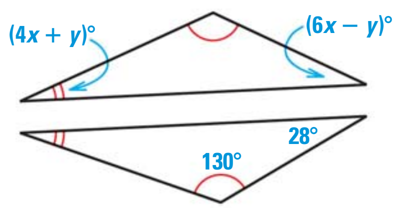
18. ★ **OPEN-ENDED MATH** Graph the triangle with vertices $L(3, 1)$, $M(8, 1)$, and $N(8, 8)$. Then graph a triangle congruent to $\triangle LMN$.

xy ALGEBRA **Find the values of *x* and *y*.**

19.



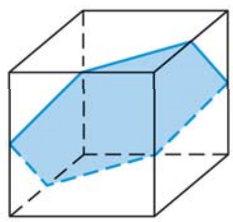
20.



21. ★ **MULTIPLE CHOICE** Suppose $\triangle ABC \cong \triangle EFD$, $\triangle EFD \cong \triangle GIH$, $m\angle A = 90°$, and $m\angle F = 20°$. What is $m\angle H$?

Ⓐ 20° Ⓑ 70° Ⓒ 90° Ⓓ Cannot be determined

22. **CHALLENGE** A hexagon is contained in a cube, as shown. Each vertex of the hexagon lies on the midpoint of an edge of the cube. This hexagon is equiangular. *Explain* why it is also regular.



PROBLEM SOLVING

23. **RUG DESIGNS** The rug design is made of congruent triangles. One triangular shape is used to make all of the triangles in the design. Which property guarantees that all the triangles are congruent?

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24. ★ **OPEN-ENDED MATH** Create a design for a rug made with congruent triangles that is different from the one in the photo above.

25. **CAR STEREO** A car stereo fits into a space in your dashboard. You want to buy a new car stereo, and it must fit in the existing space. What measurements need to be the same in order for the new stereo to be congruent to the old one?

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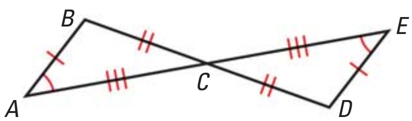
EXAMPLE 5
on p. 227
for Ex. 26

26. **PROOF** Copy and complete the proof.

GIVEN ▶ $\overline{AB} \cong \overline{ED}$, $\overline{BC} \cong \overline{DC}$, $\overline{CA} \cong \overline{CE}$, $\angle BAC \cong \angle DEC$

PROVE ▶ $\triangle ABC \cong \triangle EDC$



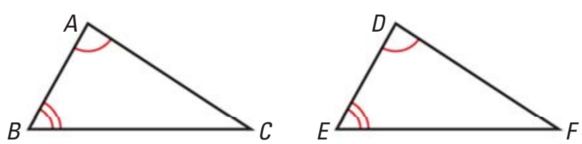
STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{ED}$, $\overline{BC} \cong \overline{DC}$, $\overline{CA} \cong \overline{CE}$, $\angle BAC \cong \angle DEC$	1. Given
2. $\angle BCA \cong \angle DCE$	2. __?__
3. __?__	3. Third Angles Theorem
4. $\triangle ABC \cong \triangle EDC$	4. __?__

27. ★ **SHORT RESPONSE** Suppose $\triangle ABC \cong \triangle DCB$, and the triangles share vertices at points B and C. Draw a figure that illustrates this situation. Is $\overline{AC} \parallel \overline{BD}$? *Explain.*

28. **PROVING THEOREM 4.3** Use the plan to prove the Third Angles Theorem.

GIVEN ▶ $\angle A \cong \angle D$, $\angle B \cong \angle E$

PROVE ▶ $\angle C \cong \angle F$



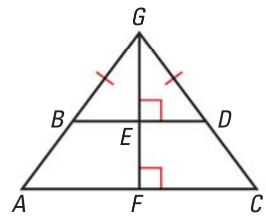
Plan for Proof Use the Triangle Sum Theorem to show that the sums of the angle measures are equal. Then use substitution to show $\angle C \cong \angle F$.

29. **REASONING** Given that $\triangle AFC \cong \triangle DFE$, must F be the midpoint of $\overline{AD}$ and $\overline{EC}$? Include a drawing with your answer.

30. ★ **SHORT RESPONSE** You have a set of tiles that come in two different shapes, as shown. You can put two of the triangular tiles together to make a quadrilateral that is the same size and shape as the quadrilateral tile.

Explain how you can find all of the angle measures of each tile by measuring only two angles.

31. **MULTI-STEP PROBLEM** In the diagram, quadrilateral $ABEF \cong$ quadrilateral $CDEF$.



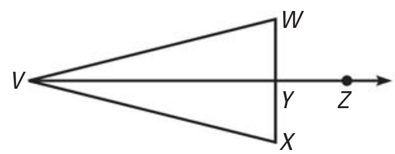
   a. *Explain* how you know that $\overline{BE} \cong \overline{DE}$ and $\angle ABE \cong \angle CDE$.

   b. *Explain* how you know that $\angle GBE \cong \angle GDE$.

   c. *Explain* how you know that $\angle GEB \cong \angle GED$.

   d. Do you have enough information to prove that $\triangle BEG \cong \triangle DEG$? *Explain*.

32. **CHALLENGE** Use the diagram to write a proof.



GIVEN ▶ $\overline{WX} \perp \overrightarrow{VZ}$ at Y, Y is the midpoint of $\overline{WX}$, $\overline{VW} \cong \overline{VX}$, and $\overrightarrow{VZ}$ bisects $\angle WVX$.

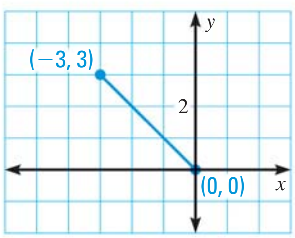
PROVE ▶ $\triangle VWY \cong \triangle VXY$

MIXED REVIEW

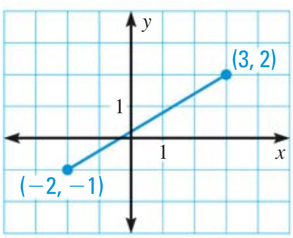
PREVIEW
Prepare for Lesson 4.3 in Exs. 33–35.

Use the Distance Formula to find the length of the segment. Round your answer to the nearest tenth of a unit. *(p. 15)*

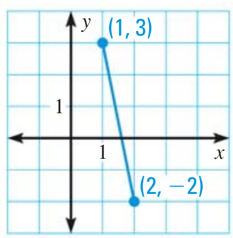
33.



34.

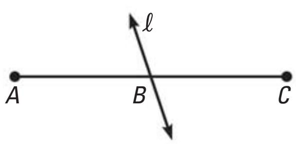


35.

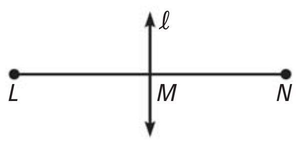


Line ℓ bisects the segment. Write a congruence statement. *(p. 15)*

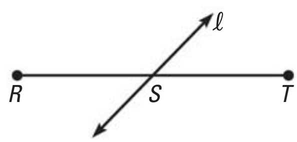
36.



37.



38.



Write the converse of the statement. *(p. 79)*

39. If three points are coplanar, then they lie in the same plane.

40. If the sky is cloudy, then it is raining outside.

PROBLEM SOLVING WORKSHOP LESSON 4.2

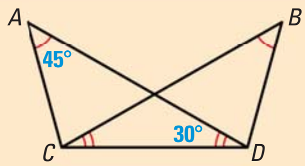
Using ALTERNATIVE METHODS

Another Way to Solve Example 4, page 227

MULTIPLE REPRESENTATIONS In Example 4 on page 227, you used congruencies in triangles that overlapped. When you solve problems like this, it may be helpful to redraw the art so that the triangles do not overlap.

PROBLEM

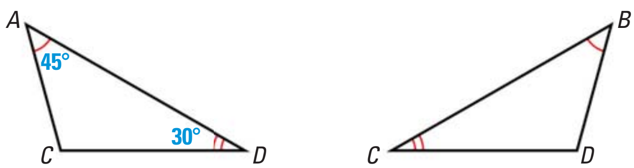
Find $m\angle BDC$.



METHOD

Drawing A Diagram

STEP 1 **Identify** the triangles that overlap. Then redraw them so that they are separate. Copy all labels and markings.



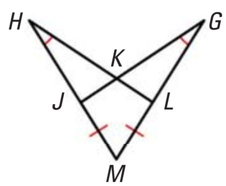
STEP 2 **Analyze** the situation. By the Triangle Sum Theorem, $m\angle ACD = 180° - 45° - 30° = 105°$.

Also, because $\angle A \cong \angle B$ and $\angle ADC \cong \angle BCD$, by the Third Angles Theorem, $\angle ACD \cong \angle BDC$, and $m\angle ACD = m\angle BDC = 105°$.

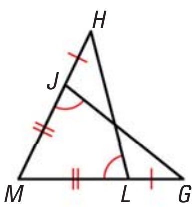
PRACTICE

1. **DRAWING FIGURES** Draw $\triangle HLM$ and $\triangle GJM$ so they do not overlap. Copy all labels and mark any known congruences.

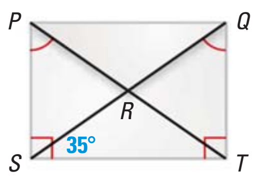
   a.



   b.



2. **ENVELOPE** Draw $\triangle PQS$ and $\triangle QPT$ so that they do not overlap. Find $m\angle PTS$.



4.3 Investigate Congruent Figures

MATERIALS • straws • string • ruler • protractor

QUESTION How much information is needed to tell whether two figures are congruent?

EXPLORE 1 Compare triangles with congruent sides

STEP 1

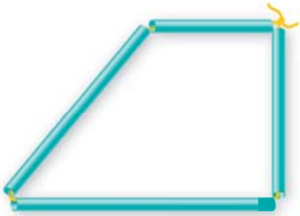
Make a triangle Cut straws to make side lengths of 8 cm, 10 cm, and 12 cm. Thread the string through the straws. Make a triangle by connecting the ends of the string.

STEP 2

Make another triangle Use the same length straws to make another triangle. If possible, make it different from the first. Compare the triangles. What do you notice?

EXPLORE 2 Compare quadrilaterals with congruent sides

STEP 1



Make a quadrilateral Cut straws to make side lengths of 5 cm, 7 cm, 9 cm, and 11 cm. Thread the string through the straws. Make a quadrilateral by connecting the string.

STEP 2

Make another quadrilateral Make a second quadrilateral using the same length straws. If possible, make it different from the first. Compare the quadrilaterals. What do you notice?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Can you make two triangles with the same side lengths that are different shapes? *Justify* your answer.
2. If you know that three sides of a triangle are congruent to three sides of another triangle, can you say the triangles are congruent? *Explain*.
3. Can you make two quadrilaterals with the same side lengths that are different shapes? *Justify* your answer.
4. If four sides of a quadrilateral are congruent to four sides of another quadrilateral, can you say the quadrilaterals are congruent? *Explain*.

4.3 Prove Triangles Congruent by SSS

Before You used the definition of congruent figures.

Now You will use the side lengths to prove triangles are congruent.

Why So you can determine if triangles in a tile floor are congruent, as in Ex. 22.

Key Vocabulary
- **congruent figures,** *p. 225*
- **corresponding parts,** *p. 225*

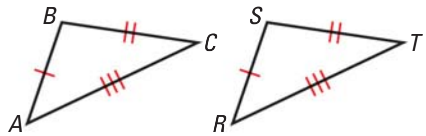
In the Activity on page 233, you saw that there is only one way to form a triangle given three side lengths. In general, any two triangles with the same three side lengths must be congruent.

POSTULATE *For Your Notebook*

POSTULATE 19 Side-Side-Side (SSS) Congruence Postulate

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If **S**ide $\overline{AB} \cong \overline{RS}$,
Side $\overline{BC} \cong \overline{ST}$, and
Side $\overline{CA} \cong \overline{TR}$,
then $\triangle ABC \cong \triangle RST$.

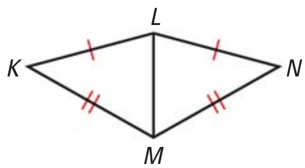


EXAMPLE 1 Use the SSS Congruence Postulate

Write a proof.

GIVEN ▶ $\overline{KL} \cong \overline{NL}$, $\overline{KM} \cong \overline{NM}$

PROVE ▶ $\triangle KLM \cong \triangle NLM$



Proof It is given that $\overline{KL} \cong \overline{NL}$ and $\overline{KM} \cong \overline{NM}$. By the Reflexive Property, $\overline{LM} \cong \overline{LM}$. So, by the SSS Congruence Postulate, $\triangle KLM \cong \triangle NLM$.

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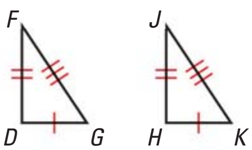
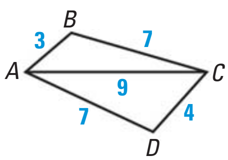
✓ GUIDED PRACTICE for Example 1

Decide whether the congruence statement is true. *Explain* your reasoning.

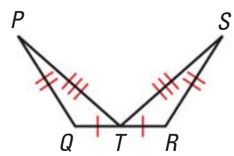
1. $\triangle DFG \cong \triangle HJK$



2. $\triangle ACB \cong \triangle CAD$



3. $\triangle QPT \cong \triangle RST$



EXAMPLE 2 Standardized Test Practice

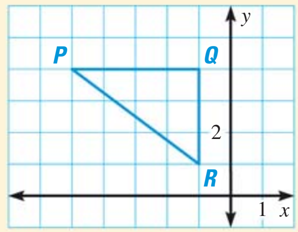
Which are the coordinates of the vertices of a triangle congruent to $\triangle PQR$?

Ⓐ $(-1, 1), (-1, 5), (-4, 5)$

Ⓑ $(-2, 4), (-7, 4), (-4, 6)$

Ⓒ $(-3, 2), (-1, 3), (-3, 1)$

Ⓓ $(-7, 7), (-7, 9), (-3, 7)$



Solution

By counting, $PQ = 4$ and $QR = 3$. Use the Distance Formula to find PR.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PR = \sqrt{(-1 - (-5))^2 + (1 - 4)^2} = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$$

By the SSS Congruence Postulate, any triangle with side lengths 3, 4, and 5 will be congruent to $\triangle PQR$. The distance from $(-1, 1)$ to $(-1, 5)$ is 4. The distance from $(-1, 5)$ to $(-4, 5)$ is 3. The distance from $(-1, 1)$ to $(-4, 5)$ is $\sqrt{(5-1)^2 + ((-4) - (-1))^2} = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$.

▶ The correct answer is A. Ⓐ Ⓑ Ⓒ Ⓓ

ELIMINATE CHOICES

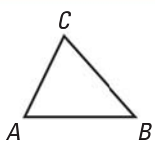
Once you know the side lengths of $\triangle PQR$, look for pairs of coordinates with the same x-coordinates or the same y-coordinates. In Choice C, $(-3, 2)$ and $(-3, 1)$ are only 1 unit apart. You can eliminate D in the same way.

✓ GUIDED PRACTICE for Example 2

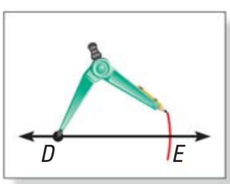
4. $\triangle JKL$ has vertices $J(-3, -2)$, $K(0, -2)$, and $L(-3, -8)$. $\triangle RST$ has vertices $R(10, 0)$, $S(10, -3)$, and $T(4, 0)$. Graph the triangles in the same coordinate plane and show that they are congruent.

ACTIVITY COPY A TRIANGLE

Follow the steps below to construct a triangle that is congruent to $\triangle ABC$.

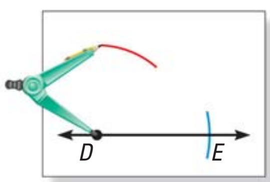


STEP 1



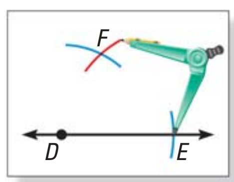
Construct $\overline{DE}$ so that it is congruent to $\overline{AB}$.

STEP 2



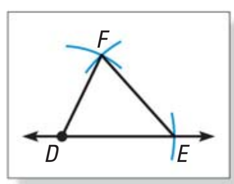
Open your compass to the length AC. Use this length to draw an arc with the compass point at D.

STEP 3



Draw an arc with radius BC and center E that intersects the arc from Step 2. Label the intersection point F.

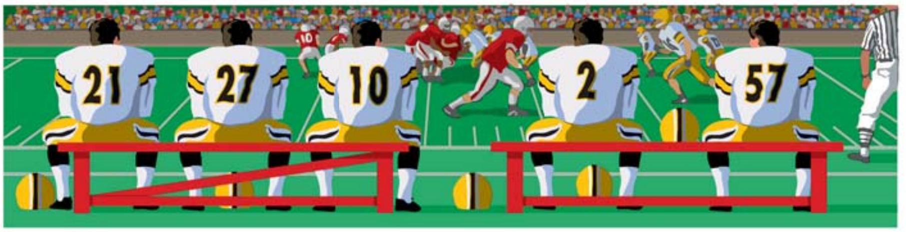
STEP 4



Draw $\triangle DEF$. By the SSS Congruence Postulate, $\triangle ABC \cong \triangle DEF$.

EXAMPLE 3 Solve a real-world problem

STRUCTURAL SUPPORT Explain why the bench with the diagonal support is stable, while the one without the support can collapse.



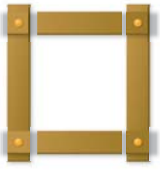
Solution

The bench with a diagonal support forms triangles with fixed side lengths. By the SSS Congruence Postulate, these triangles cannot change shape, so the bench is stable. The bench without a diagonal support is not stable because there are many possible quadrilaterals with the given side lengths.

✓ GUIDED PRACTICE for Example 3

Determine whether the figure is stable. *Explain* your reasoning.

5.



6.

7.

4.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 9, and 25

★ = **STANDARDIZED TEST PRACTICE** Exs. 16, 17, and 28

SKILL PRACTICE

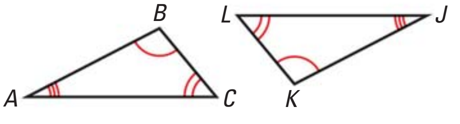
VOCABULARY **Tell whether the angles or sides are *corresponding angles*, *corresponding sides*, or *neither*.**

1. $\angle C$ and $\angle L$

2. $\overline{AC}$ and $\overline{JK}$

3. $\overline{BC}$ and $\overline{KL}$

4. $\angle B$ and $\angle L$



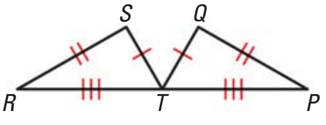
EXAMPLE 1 on p. 234 for Exs. 5–7

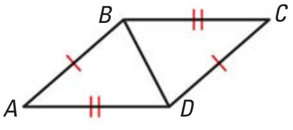
DETERMINING CONGRUENCE **Decide whether the congruence statement is true. *Explain* your reasoning.**

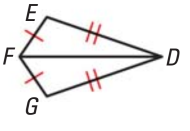
5. $\triangle RST \cong \triangle TQP$

6. $\triangle ABD \cong \triangle CDB$

7. $\triangle DEF \cong \triangle DGF$

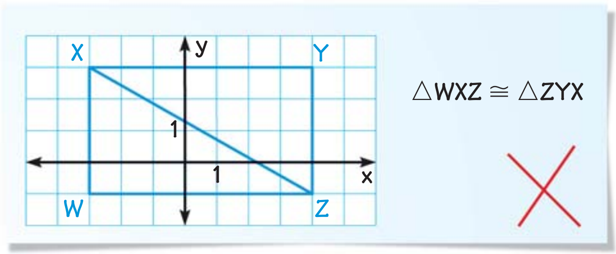






EXAMPLE 2
on p. 235
for Exs. 8–12

8. **ERROR ANALYSIS** *Describe* and correct the error in writing a congruence statement for the triangles in the coordinate plane.



ALGEBRA **Use the given coordinates to determine if $\triangle ABC \cong \triangle DEF$.**

9. $A(-2, -2)$, $B(4, -2)$, $C(4, 6)$, $D(5, 7)$, $E(5, 1)$, $F(13, 1)$

10. $A(-2, 1)$, $B(3, -3)$, $C(7, 5)$, $D(3, 6)$, $E(8, 2)$, $F(10, 11)$

11. $A(0, 0)$, $B(6, 5)$, $C(9, 0)$, $D(0, -1)$, $E(6, -6)$, $F(9, -1)$

12. $A(-5, 7)$, $B(-5, 2)$, $C(0, 2)$, $D(0, 6)$, $E(0, 1)$, $F(4, 1)$

EXAMPLE 3
on p. 236
for Exs. 13–15

USING DIAGRAMS **Decide whether the figure is stable. *Explain*.**

13.

14.

15.

16. ★ **MULTIPLE CHOICE** Let $\triangle FGH$ be an equilateral triangle with point J as the midpoint of $\overline{FG}$. Which of the statements below is *not* true?

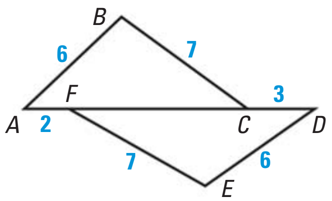
Ⓐ $\overline{FH} \cong \overline{GH}$ Ⓑ $\overline{FJ} \cong \overline{FH}$ Ⓒ $\overline{FJ} \cong \overline{GJ}$ Ⓓ $\triangle FHJ \cong \triangle GHJ$

17. ★ **MULTIPLE CHOICE** Let $ABCD$ be a rectangle separated into two triangles by $\overline{DB}$. Which of the statements below is *not* true?

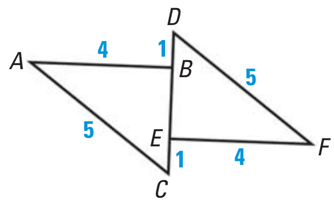
Ⓐ $\overline{AD} \cong \overline{CB}$ Ⓑ $\overline{AB} \cong \overline{AD}$ Ⓒ $\overline{AB} \cong \overline{CD}$ Ⓓ $\triangle DAB \cong \triangle BCD$

APPLYING SEGMENT ADDITION **Determine whether $\triangle ABC \cong \triangle DEF$. If they are congruent, write a congruence statement. *Explain* your reasoning.**

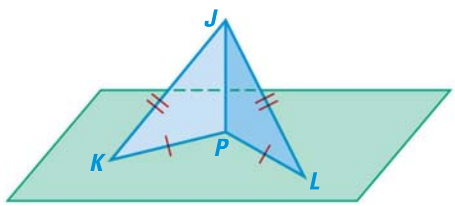
18.



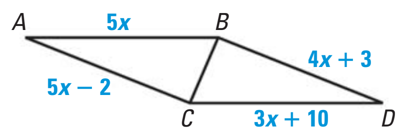
19.



20. **3-D FIGURES** In the diagram, $\overline{PK} \cong \overline{PL}$ and $\overline{JK} \cong \overline{JL}$. Show that $\triangle JPK \cong \triangle JPL$.



21. **CHALLENGE** Find all values of x that make the triangles congruent. *Explain.*



PROBLEM SOLVING

EXAMPLE 1
on p. 234
for Ex. 22

22. TILE FLOORS You notice two triangles in the tile floor of a hotel lobby. You want to determine if the triangles are congruent, but you only have a piece of string. Can you determine if the triangles are congruent? *Explain*.

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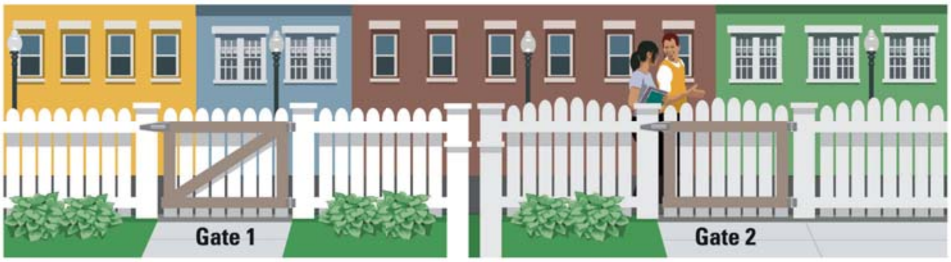
EXAMPLE 3
on p. 236
for Ex. 23

23. GATES Which gate is stable? *Explain* your reasoning.



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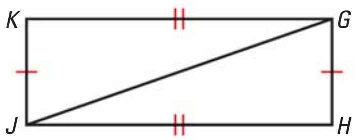
PROOF **Write a proof.**

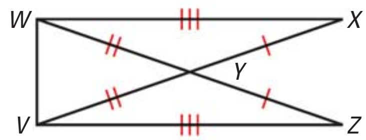
24. GIVEN ▶ $\overline{GH} \cong \overline{JK}$, $\overline{HJ} \cong \overline{KG}$

PROVE ▶ $\triangle GHJ \cong \triangle JKG$



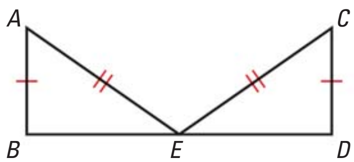
25. GIVEN ▶ $\overline{WX} \cong \overline{VZ}$, $\overline{WY} \cong \overline{VY}$, $\overline{YZ} \cong \overline{YX}$

PROVE ▶ $\triangle VWX \cong \triangle WVZ$



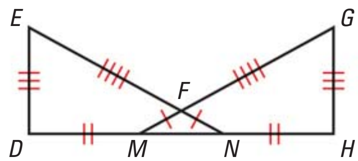
26. GIVEN ▶ $\overline{AE} \cong \overline{CE}$, $\overline{AB} \cong \overline{CD}$, E is the midpoint of $\overline{BD}$.

PROVE ▶ $\triangle EAB \cong \triangle ECD$



27. GIVEN ▶ $\overline{FM} \cong \overline{FN}$, $\overline{DM} \cong \overline{HN}$, $\overline{EF} \cong \overline{GF}$, $\overline{DE} \cong \overline{HG}$

PROVE ▶ $\triangle DEN \cong \triangle HGM$



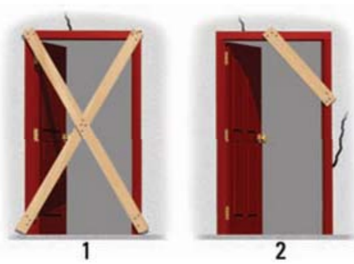
28. ★ EXTENDED RESPONSE When rescuers enter a partially collapsed building they often have to reinforce damaged doors for safety.

a. Diagonal braces are added to Door 1 as shown below. *Explain* why the door is more stable with the braces.

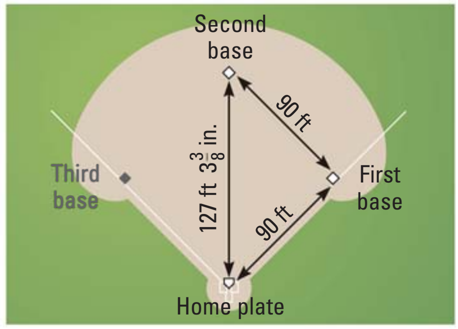
b. Would these braces be a good choice for rescuers needing to enter and exit the building through this doorway?

c. In the diagram, Door 2 has only a corner brace. Does this solve the problem from part (b)?

d. *Explain* why the corner brace makes the door more stable.



29. BASEBALL FIELD To create a baseball field, start by placing home plate. Then, place second base 127 feet $3\frac{3}{8}$ inches from home plate. Then, you can find first base using two tape measures. Stretch one from second base toward first base and the other from home plate toward first base. The point where the two tape measures cross at the 90 foot mark is first base. You can find third base in a similar manner. *Explain* how and why this process will always work.



30. CHALLENGE Draw and label the figure described below. Then, identify what is given and write a two-column proof.

In an isosceles triangle, if a segment is added from the vertex between the congruent sides to the midpoint of the third side, then two congruent triangles are formed.

MIXED REVIEW

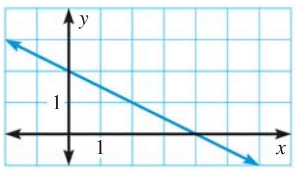
PREVIEW
Prepare for Lesson 4.4 in Exs. 31–33.

Find the slope of the line that passes through the points. *(p. 171)*

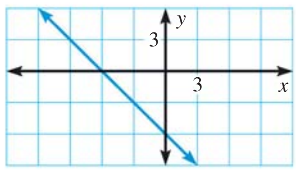
31. $A(3, 0)$, $B(7, 4)$ **32.** $F(1, 8)$, $G(-9, 2)$ **33.** $M(-4, -10)$, $N(6, 2)$

Use the *x*- and *y*-intercepts to write an equation of the line. *(p. 180)*

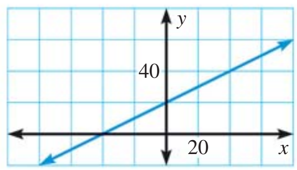
34.



35.



36.



37. Write an equation of a line that passes through $(-3, -1)$ and is parallel to $y = 3x + 2$. *(p. 180)*

QUIZ for Lessons 4.1–4.3

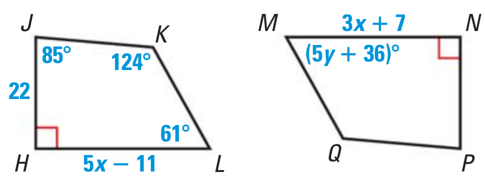
A triangle has the given vertices. Graph the triangle and classify it by its sides. Then determine if it is a right triangle. *(p. 217)*

1. $A(-3, 0)$, $B(0, 4)$, $C(3, 0)$ **2.** $A(2, -4)$, $B(5, -1)$, $C(2, -1)$ **3.** $A(-7, 0)$, $B(1, 6)$, $C(-3, 4)$

In the diagram, $HJKL \cong NPQM$. *(p. 225)*

4. Find the value of x.

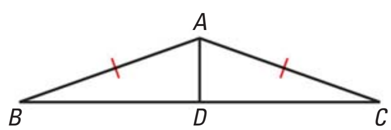
5. Find the value of y.



6. Write a proof. *(p. 234)*

GIVEN ▶ $\overline{AB} \cong \overline{AC}$, $\overline{AD}$ bisects $\overline{BC}$.

PROVE ▶ $\triangle ABD \cong \triangle ACD$



4.4 Prove Triangles Congruent by SAS and HL

Before You used the SSS Congruence Postulate.

Now You will use sides and angles to prove congruence.

Why? So you can show triangles are congruent, as in Ex. 33.

Key Vocabulary
- **leg of a right triangle**
- **hypotenuse**

Consider a relationship involving two sides and the angle they form, their *included* angle. To picture the relationship, form an angle using two pencils.

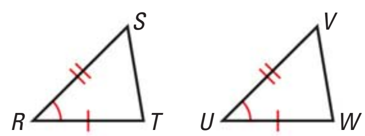
Any time you form an angle of the same measure with the pencils, the side formed by connecting the pencil points will have the same length. In fact, any two triangles formed in this way are congruent.

POSTULATE *For Your Notebook*

POSTULATE 20 Side-Angle-Side (SAS) Congruence Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If	**S**ide	$\overline{RS} \cong \overline{UV}$,
	Angle	$\angle R \cong \angle U$, and
	Side	$\overline{RT} \cong \overline{UW}$,
then		$\triangle RST \cong \triangle UVW$.

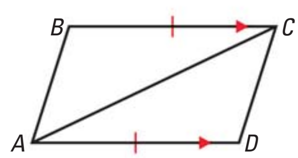


EXAMPLE 1 Use the SAS Congruence Postulate

Write a proof.

GIVEN ▶ $\overline{BC} \cong \overline{DA}$, $\overline{BC} \parallel \overline{AD}$

PROVE ▶ $\triangle ABC \cong \triangle CDA$

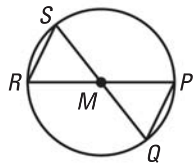


WRITE PROOFS
Make your proof easier to read by identifying the steps where you show congruent sides (S) and angles (A).

	STATEMENTS	REASONS
S	**1.** $\overline{BC} \cong \overline{DA}$	**1.** Given
	2. $\overline{BC} \parallel \overline{AD}$	**2.** Given
A	**3.** $\angle BCA \cong \angle DAC$	**3.** Alternate Interior Angles Theorem
S	**4.** $\overline{AC} \cong \overline{CA}$	**4.** Reflexive Property of Congruence
	5. $\triangle ABC \cong \triangle CDA$	**5.** SAS Congruence Postulate

EXAMPLE 2 Use SAS and properties of shapes

In the diagram, $\overline{QS}$ and $\overline{RP}$ pass through the center M of the circle. What can you conclude about $\triangle MRS$ and $\triangle MPQ$?



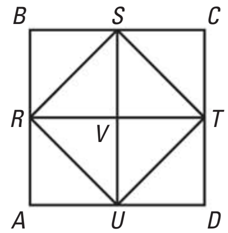
Solution

Because they are vertical angles, $\angle PMQ \cong \angle RMS$. All points on a circle are the same distance from the center, so MP, MQ, MR, and MS are all equal.

▶ $\triangle MRS$ and $\triangle MPQ$ are congruent by the SAS Congruence Postulate.

✓ GUIDED PRACTICE for Examples 1 and 2

In the diagram, *ABCD* is a square with four congruent sides and four right angles. *R*, *S*, *T*, and *U* are the midpoints of the sides of *ABCD*. Also, $\overline{RT} \perp \overline{SU}$ and $\overline{SV} \cong \overline{VU}$.

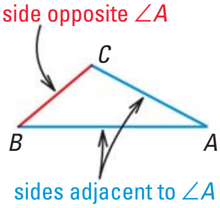


1. Prove that $\triangle SVR \cong \triangle UVR$.

2. Prove that $\triangle BSR \cong \triangle DUT$.

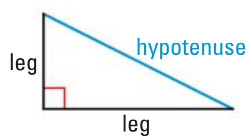
In general, if you know the lengths of two sides and the measure of an angle that is *not included* between them, you can create two different triangles.

READ VOCABULARY
The two sides of a triangle that form an angle are *adjacent* to the angle. The side not adjacent to the angle is *opposite* the angle.



Therefore, SSA is *not* a valid method for proving that triangles are congruent, although there is a special case for right triangles.

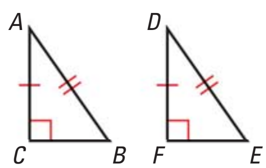
RIGHT TRIANGLES In a right triangle, the sides adjacent to the right angle are called the **legs**. The side opposite the right angle is called the **hypotenuse** of the right triangle.



THEOREM *For Your Notebook*

THEOREM 4.5 Hypotenuse-Leg (HL) Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.



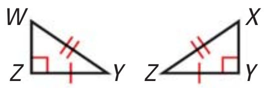
$\triangle ABC \cong \triangle DEF$

Proofs: Ex. 37, p. 439; p. 932

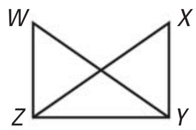
EXAMPLE 3 Use the Hypotenuse-Leg Congruence Theorem

USE DIAGRAMS

If you have trouble matching vertices to letters when you separate the overlapping triangles, leave the triangles in their original orientations.



Write a proof.

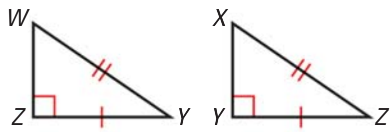


GIVEN ▶ $\overline{WY} \cong \overline{XZ}$, $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$

PROVE ▶ $\triangle WYZ \cong \triangle XZY$

Solution

Redraw the triangles so they are side by side with corresponding parts in the same position. Mark the given information in the diagram.



STATEMENTS	REASONS
H **1.** $\overline{WY} \cong \overline{XZ}$	**1.** Given
2. $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$	**2.** Given
3. $\angle Z$ and $\angle Y$ are right angles.	**3.** Definition of $\perp$ lines
4. $\triangle WYZ$ and $\triangle XZY$ are right triangles.	**4.** Definition of a right triangle
L **5.** $\overline{ZY} \cong \overline{YZ}$	**5.** Reflexive Property of Congruence
6. $\triangle WYZ \cong \triangle XZY$	**6.** HL Congruence Theorem

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EXAMPLE 4 Choose a postulate or theorem

SIGN MAKING You are making a canvas sign to hang on the triangular wall over the door to the barn shown in the picture. You think you can use two identical triangular sheets of canvas. You know that $\overline{RP} \perp \overline{QS}$ and $\overline{PQ} \cong \overline{PS}$. What postulate or theorem can you use to conclude that $\triangle PQR \cong \triangle PSR$?

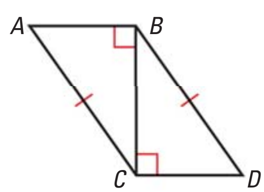
Solution

You are given that $\overline{PQ} \cong \overline{PS}$. By the Reflexive Property, $\overline{RP} \cong \overline{RP}$. By the definition of perpendicular lines, both $\angle RPQ$ and $\angle RPS$ are right angles, so they are congruent. So, two sides and their included angle are congruent.

▶ You can use the SAS Congruence Postulate to conclude that $\triangle PQR \cong \triangle PSR$.

✓ GUIDED PRACTICE for Examples 3 and 4

Use the diagram at the right.



3. Redraw $\triangle ACB$ and $\triangle DBC$ side by side with corresponding parts in the same position.

4. Use the information in the diagram to prove that $\triangle ACB \cong \triangle DBC$.

4.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 13, 19, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 23, and 39

SKILL PRACTICE

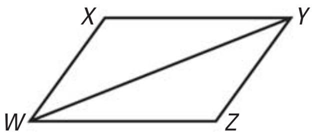
1. **VOCABULARY** Copy and complete: The angle between two sides of a triangle is called the __?__ angle.

2. ★ **WRITING** *Explain* the difference between proving triangles congruent using the SAS and SSS Congruence Postulates.

EXAMPLE 1 on p. 240 for Exs. 3–15

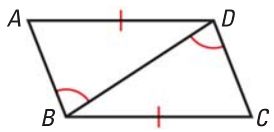
NAMING INCLUDED ANGLES Use the diagram to name the included angle between the given pair of sides.

3. $\overline{XY}$ and $\overline{YW}$
4. $\overline{WZ}$ and $\overline{ZY}$
5. $\overline{ZW}$ and $\overline{YW}$
6. $\overline{WX}$ and $\overline{YX}$
7. $\overline{XY}$ and $\overline{YZ}$
8. $\overline{WX}$ and $\overline{WZ}$

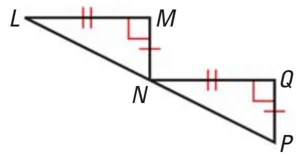


REASONING Decide whether enough information is given to prove that the triangles are congruent using the SAS Congruence Postulate.

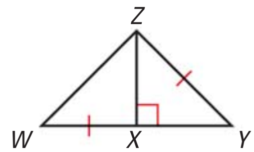
9. $\triangle ABD$, $\triangle CDB$



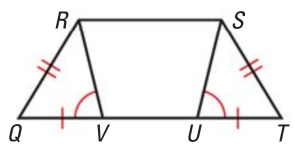
10. $\triangle LMN$, $\triangle NQP$



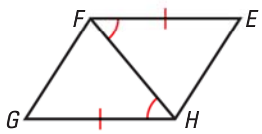
11. $\triangle YXZ$, $\triangle WXZ$



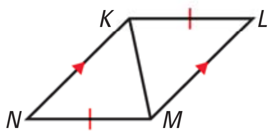
12. $\triangle QRV$, $\triangle TSU$



13. $\triangle EFH$, $\triangle GHF$



14. $\triangle KLM$, $\triangle MNK$



15. ★ **MULTIPLE CHOICE** Which of the following sets of information does not allow you to conclude that $\triangle ABC \cong \triangle DEF$?

Ⓐ $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\angle B \cong \angle E$

Ⓑ $\overline{AB} \cong \overline{DF}$, $\overline{AC} \cong \overline{DE}$, $\angle C \cong \angle E$

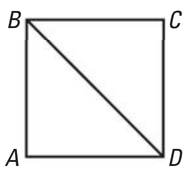
Ⓒ $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$, $\overline{BA} \cong \overline{DE}$

Ⓓ $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\angle A \cong \angle D$

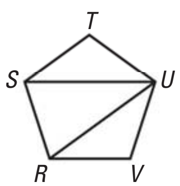
EXAMPLE 2 on p. 241 for Exs. 16–18

APPLYING SAS In Exercises 16–18, use the given information to name two triangles that are congruent. *Explain* your reasoning.

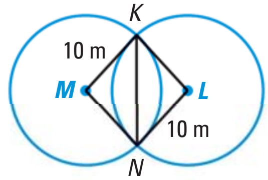
16. *ABCD* is a square with four congruent sides and four congruent angles.



17. *RSTUV* is a regular pentagon.

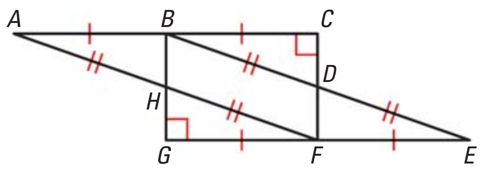


18. $\overline{MK} \perp \overline{MN}$ and $\overline{KL} \perp \overline{NL}$.



EXAMPLE 3
on p. 242
for Ex. 19

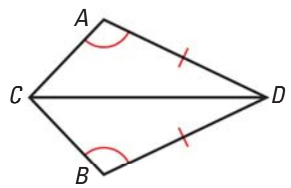
19. **OVERLAPPING TRIANGLES** Redraw $\triangle ACF$ and $\triangle EGB$ so they are side by side with corresponding parts in the same position. *Explain* how you know that $\triangle ACF \cong \triangle EGB$.



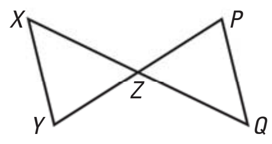
EXAMPLE 4
on p. 242
for Exs. 20–22

REASONING **Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, state the congruence postulate or theorem you would use.**

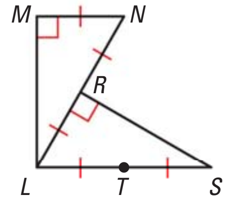
20.



21. Z is the midpoint of $\overline{PY}$ and $\overline{XQ}$.

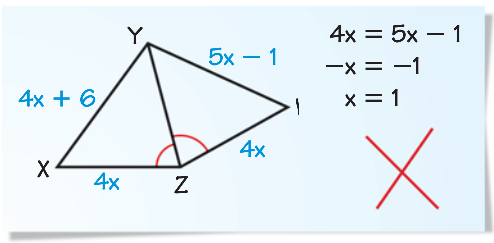


22.



23. ★ **WRITING** Suppose both pairs of corresponding legs of two right triangles are congruent. Are the triangles congruent? *Explain*.

24. **ERROR ANALYSIS** *Describe* and correct the error in finding the value of x.

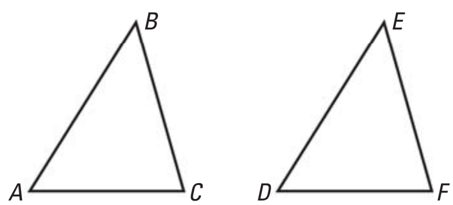


USING DIAGRAMS **In Exercises 25–27, state the third congruence that must be given to prove that $\triangle ABC \cong \triangle DEF$ using the indicated postulate.**

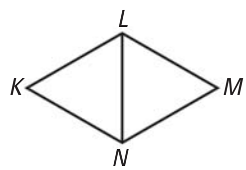
25. **GIVEN ▶** $\overline{AB} \cong \overline{DE}$, $\overline{CB} \cong \overline{FE}$, _?_ $\cong$ _?_
Use the SSS Congruence Postulate.

26. **GIVEN ▶** $\angle A \cong \angle D$, $\overline{CA} \cong \overline{FD}$, _?_ $\cong$ _?_
Use the SAS Congruence Postulate.

27. **GIVEN ▶** $\angle B \cong \angle E$, $\overline{AB} \cong \overline{DE}$, _?_ $\cong$ _?_
Use the SAS Congruence Postulate.

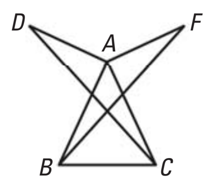


28. **USING ISOSCELES TRIANGLES** Suppose $\triangle KLN$ and $\triangle MLN$ are isosceles triangles with bases $\overline{KN}$ and $\overline{MN}$ respectively, and $\overline{NL}$ bisects $\angle KLM$. Is there enough information to prove that $\triangle KLN \cong \triangle MLN$? *Explain*.



29. **REASONING** Suppose M is the midpoint of $\overline{PQ}$ in $\triangle PQR$. If $\overline{RM} \perp \overline{PQ}$, *explain* why $\triangle RMP \cong \triangle RMQ$.

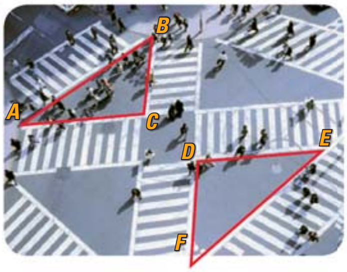
30. **CHALLENGE** Suppose $\overline{AB} \cong \overline{AC}$, $\overline{AD} \cong \overline{AF}$, $\overline{AD} \perp \overline{AB}$, and $\overline{AF} \perp \overline{AC}$. *Explain* why you can conclude that $\triangle ACD \cong \triangle ABF$.

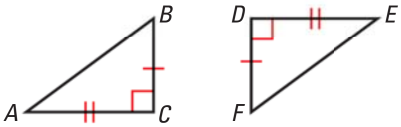


PROBLEM SOLVING

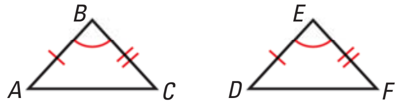
CONGRUENT TRIANGLES **In Exercises 31 and 32, identify the theorem or postulate you would use to prove the triangles congruent.**

31.





32.



33. SAILBOATS Suppose you have two sailboats. What information do you need to know to prove that the triangular sails are congruent using SAS? using HL?

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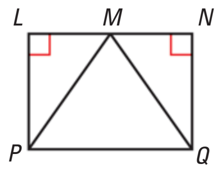
EXAMPLE 3
on p. 242
for Ex. 34

34. DEVELOPING PROOF Copy and complete the proof.



GIVEN ▶ Point M is the midpoint of $\overline{LN}$.
$\triangle PMQ$ is an isosceles triangle with base $\overline{PQ}$.
$\angle L$ and $\angle N$ are right angles.

PROVE ▶ $\triangle LMP \cong \triangle NMQ$

STATEMENTS	REASONS
1. $\angle L$ and $\angle N$ are right angles.	1. Given
2. $\triangle LMP$ and $\triangle NMQ$ are right triangles.	2. _?_
3. Point M is the midpoint of $\overline{LN}$.	3. _?_
4. _?_	4. Definition of midpoint
5. $\triangle PMQ$ is an isosceles triangle.	5. Given
6. _?_	6. Definition of isosceles triangle
7. $\triangle LMP \cong \triangle NMQ$	7. _?_

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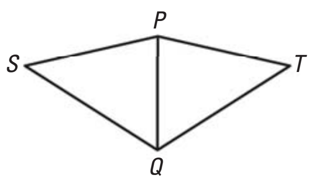
PROOF **In Exercises 35 and 36, write a proof.**

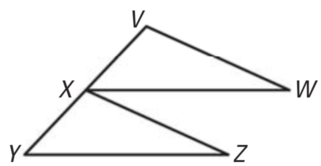
35. GIVEN ▶ $\overline{PQ}$ bisects $\angle SPT$, $\overline{SP} \cong \overline{TP}$

PROVE ▶ $\triangle SPQ \cong \triangle TPQ$



36. GIVEN ▶ $\overline{VX} \cong \overline{XY}$, $\overline{XW} \cong \overline{YZ}$, $\overline{XW} \parallel \overline{YZ}$

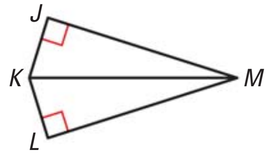
PROVE ▶ $\triangle VXW \cong \triangle XYZ$



PROOF **In Exercises 37 and 38, write a proof.**

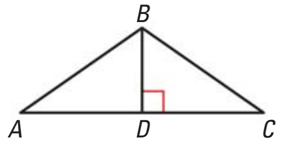
37. **GIVEN** ▶ $\overline{JM} \cong \overline{LM}$

PROVE ▶ $\triangle JKM \cong \triangle LKM$



38. **GIVEN** ▶ D is the midpoint of $\overline{AC}$.

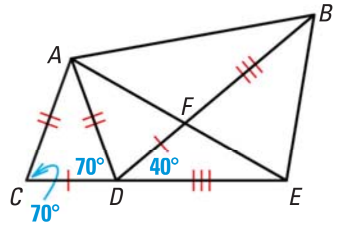
PROVE ▶ $\triangle ABD \cong \triangle CBD$



39. ★ **MULTIPLE CHOICE** Which triangle congruence can you prove, then use to prove that $\angle FED \cong \angle ABF$?

Ⓐ $\triangle ABE \cong \triangle ABF$ Ⓒ $\triangle AED \cong \triangle ABD$

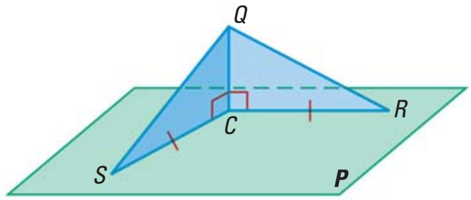
Ⓑ $\triangle ACD \cong \triangle ADF$ Ⓓ $\triangle AEC \cong \triangle ABD$



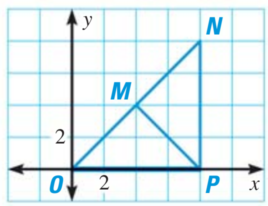
40. **PROOF** Write a two-column proof.

GIVEN ▶ $\overline{CR} \cong \overline{CS}$, $\overline{QC} \perp \overline{CR}$, $\overline{QC} \perp \overline{CS}$

PROVE ▶ $\triangle QCR \cong \triangle QCS$



41. **CHALLENGE** *Describe* how to show that $\triangle PMO \cong \triangle PMN$ using the SSS Congruence Postulate. Then show that the triangles are congruent using the SAS Congruence Postulate without measuring any angles. *Compare* the two methods.



MIXED REVIEW

Draw a figure that fits the description. *(p. 42)*

42. A pentagon that is not regular.

43. A quadrilateral that is equilateral but not equiangular.

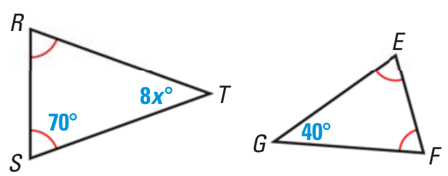
Write an equation of the line that passes through point P and is perpendicular to the line with the given equation. *(p. 180)*

44. $P(3, -1)$, $y = -x + 2$ **45.** $P(3, 3)$, $y = \frac{1}{3}x + 2$ **46.** $P(-4, -7)$, $y = -5$

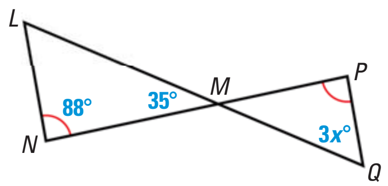
PREVIEW
Prepare for Lesson 4.5 in Exs. 47–48.

Find the value of x. *(p. 225)*

47.



48.



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Keystrokes

4.4 Investigate Triangles and Congruence

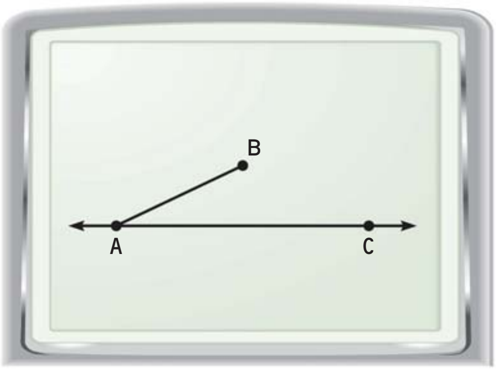
MATERIALS • graphing calculator or computer

QUESTION Can you prove triangles are congruent by SSA?

You can use geometry drawing software to show that if two sides and a nonincluded angle of one triangle are congruent to two sides and a nonincluded angle of another triangle, the triangles are not necessarily congruent.

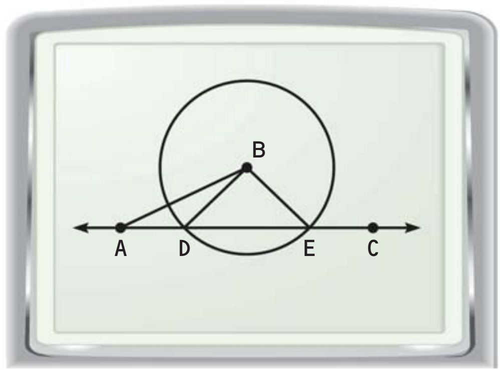
EXAMPLE Draw two triangles

STEP 1



Draw a line Draw points A and C. Draw line $\overleftrightarrow{AC}$. Then choose point B so that $\angle BAC$ is acute. Draw $\overline{AB}$.

STEP 2



Draw a circle Draw a circle with center at B so that the circle intersects $\overleftrightarrow{AC}$ at two points. Label the points D and E. Draw $\overline{BD}$ and $\overline{BE}$. Save as "EXAMPLE".

STEP 3 ***Use your drawing***

Explain why $\overline{BD} \cong \overline{BE}$. In $\triangle ABD$ and $\triangle ABE$, what other sides are congruent? What angles are congruent?

PRACTICE

1. *Explain* how your drawing shows that $\triangle ABD \ncong \triangle ABE$.
2. Change the diameter of your circle so that it intersects $\overleftrightarrow{AC}$ in only one point. Measure $\angle BDA$. *Explain* why there is exactly one triangle you can draw with the measures AB, BD, and a 90° angle at $\angle BDA$.
3. *Explain* why your results show that SSA cannot be used to show that two triangles are congruent but that HL can.

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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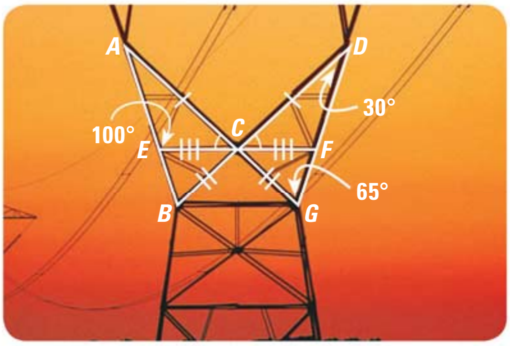
Lessons 4.1–4.4

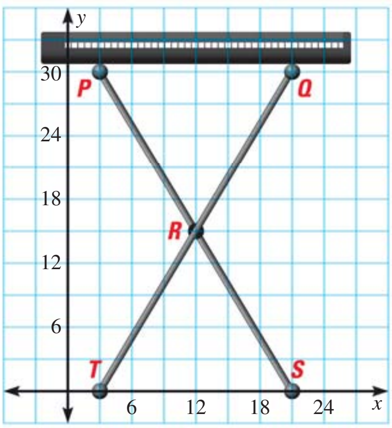
1. **MULTI-STEP PROBLEM** In the diagram, $\overline{AC} \cong \overline{CD}$, $\overline{BC} \cong \overline{CG}$, $\overline{EC} \cong \overline{CF}$, and $\angle ACE \cong \angle DCF$.



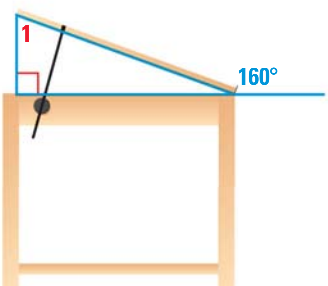
   a. Classify each triangle in the figure by angles.

   b. Classify each triangle in the figure by sides.

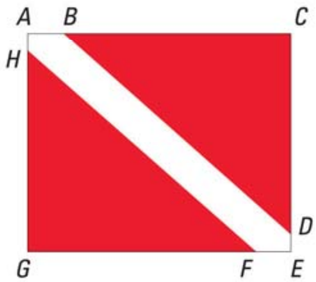
2. **OPEN-ENDED** *Explain* how you know that $\triangle PQR \cong \triangle STR$ in the keyboard stand shown.



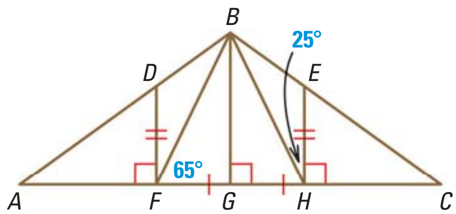
3. **GRIDDED ANSWER** In the diagram below, find the measure of $\angle 1$ in degrees.



4. **SHORT RESPONSE** A rectangular "diver down" flag is used to indicate that scuba divers are in the water. On the flag, $\overline{AB} \cong \overline{FE}$, $\overline{AH} \cong \overline{DE}$, $\overline{CE} \cong \overline{AG}$, and $\overline{EG} \cong \overline{AC}$. Also, $\angle A$, $\angle C$, $\angle E$, and $\angle G$ are right angles. Is $\triangle BCD \cong \triangle FGH$? *Explain*.



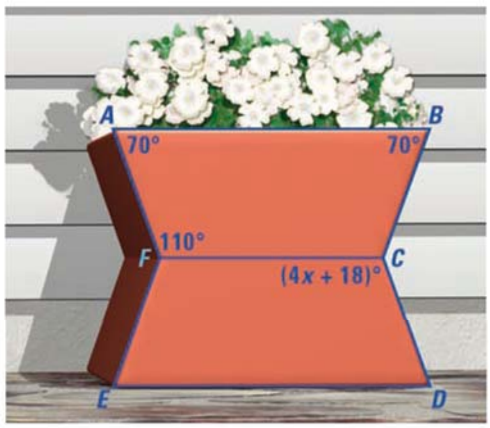
5. **EXTENDED RESPONSE** A roof truss is a network of pieces of wood that forms a stable structure to support a roof, as shown below.



   a. Prove that $\triangle FGB \cong \triangle HGB$.

   b. Is $\triangle BDF \cong \triangle BEH$? If so, prove it.

6. **GRIDDED ANSWER** In the diagram below, $BAFC \cong DEFC$. Find the value of x.



4.5 Prove Triangles Congruent by ASA and AAS

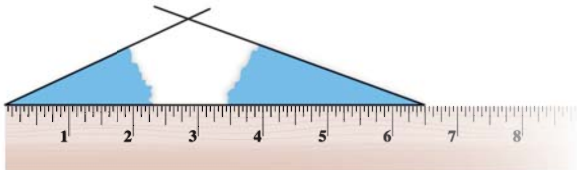
Before You used the SSS, SAS, and HL congruence methods.

Now You will use two more methods to prove congruences.

Why? So you can recognize congruent triangles in bikes, as in Exs. 23–24.

Key Vocabulary
- **flow proof**

Suppose you tear two angles out of a piece of paper and place them at a fixed distance on a ruler. Can you form more than one triangle with a given length and two given angle measures as shown below?



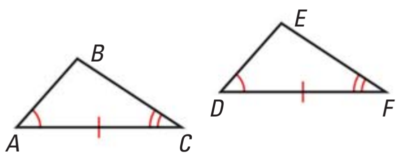
In a polygon, the side connecting the vertices of two angles is the *included* side. Given two angle measures and the length of the included side, you can make only one triangle. So, all triangles with those measurements are congruent.

THEOREMS — *For Your Notebook*

POSTULATE 21 Angle-Side-Angle (ASA) Congruence Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

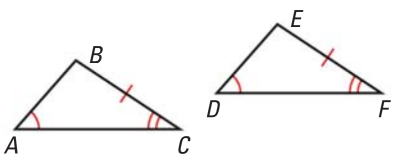
If **A**ngle $\angle A \cong \angle D$,
Side $\overline{AC} \cong \overline{DF}$, and
Angle $\angle C \cong \angle F$,
then $\triangle ABC \cong \triangle DEF$.



THEOREM 4.6 Angle-Angle-Side (AAS) Congruence Theorem

If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

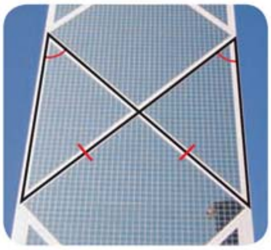
If **A**ngle $\angle A \cong \angle D$,
Angle $\angle C \cong \angle F$, and
Side $\overline{BC} \cong \overline{EF}$,
then $\triangle ABC \cong \triangle DEF$.



Proof: Example 2, p. 250

EXAMPLE 1 Identify congruent triangles

Can the triangles be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

a. 

b. 

c. 

Solution

a. The vertical angles are congruent, so two pairs of angles and a pair of non-included sides are congruent. The triangles are congruent by the AAS Congruence Theorem.

b. There is not enough information to prove the triangles are congruent, because no sides are known to be congruent.

c. Two pairs of angles and their included sides are congruent. The triangles are congruent by the ASA Congruence Postulate.

AVOID ERRORS
You need at least one pair of congruent corresponding sides to prove two triangles congruent.

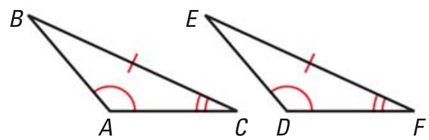
FLOW PROOFS You have written two-column proofs and paragraph proofs. A **flow proof** uses arrows to show the flow of a logical argument. Each reason is written below the statement it justifies.

EXAMPLE 2 Prove the AAS Congruence Theorem

Prove the Angle-Angle-Side Congruence Theorem.

GIVEN ▶ $\angle A \cong \angle D$, $\angle C \cong \angle F$, $\overline{BC} \cong \overline{EF}$

PROVE ▶ $\triangle ABC \cong \triangle DEF$



$\angle A \cong \angle D$ → $\angle B \cong \angle E$
Given → Third $\triangle$s Thm.

$\angle C \cong \angle F$ → $\angle B \cong \angle E$; $\angle C \cong \angle F$ → $\triangle ABC \cong \triangle DEF$
Given

$\overline{BC} \cong \overline{EF}$ → $\triangle ABC \cong \triangle DEF$
Given

$\angle B \cong \angle E$ → $\triangle ABC \cong \triangle DEF$
ASA Congruence Post.

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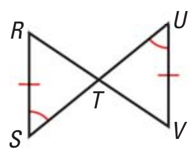
✓ GUIDED PRACTICE for Examples 1 and 2

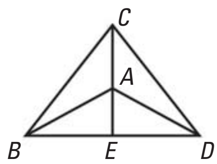
1. In the diagram at the right, what postulate or theorem can you use to prove that $\triangle RST \cong \triangle VUT$? *Explain.*

2. Rewrite the proof of the Triangle Sum Theorem on page 219 as a flow proof.



EXAMPLE 3 Write a flow proof

In the diagram, $\overline{CE} \perp \overline{BD}$ and $\angle CAB \cong \angle CAD$. Write a flow proof to show $\triangle ABE \cong \triangle ADE$.



Solution

GIVEN ▶ $\overline{CE} \perp \overline{BD}$, $\angle CAB \cong \angle CAD$

PROVE ▶ $\triangle ABE \cong \triangle ADE$

$\angle CAB \cong \angle CAD$
Given

$\angle BAE$ and $\angle CAB$ are supplements.
$\angle DAE$ and $\angle CAD$ are supplements.
Def. of supplementary angles

$\overline{CE} \perp \overline{BD}$
Given

$\angle BAE \cong \angle DAE$
Congruent Supps. Thm.

$\overline{AE} \cong \overline{AE}$
Reflexive Prop.

$m\angle AEB = m\angle AED = 90°$
Def. of $\perp$ lines

$\angle AEB \cong \angle ADE$
All right $\angle$s are $\cong$.

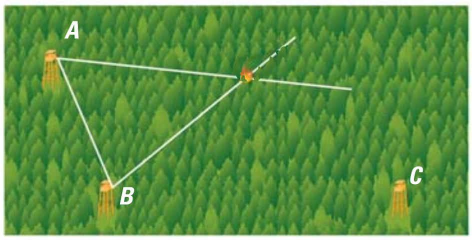
$\triangle ABE \cong \triangle ADE$
ASA Congruence Post.

★ EXAMPLE 4 Standardized Test Practice

FIRE TOWERS The forestry service uses fire tower lookouts to watch for forest fires. When the lookouts spot a fire, they measure the angle of their view and radio a dispatcher. The dispatcher then uses the angles to locate the fire. How many lookouts are needed to locate a fire?

Ⓐ 1 Ⓑ 2 Ⓒ 3 Ⓓ Not enough information

The locations of tower A, tower B, and the fire form a triangle. The dispatcher knows the distance from tower A to tower B and the measures of $\angle A$ and $\angle B$. So, he knows the measures of two angles and an included side of the triangle.



By the ASA Congruence Postulate, all triangles with these measures are congruent. So, the triangle formed is unique and the fire location is given by the third vertex. Two lookouts are needed to locate the fire.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 3 and 4

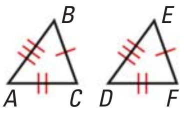
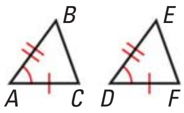
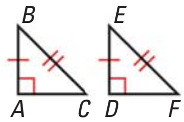
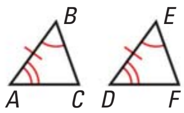
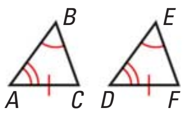
3. In Example 3, suppose $\angle ABE \cong \angle ADE$ is also given. What theorem or postulate besides ASA can you use to prove that $\triangle ABE \cong \triangle ADE$?

4. **WHAT IF?** In Example 4, suppose a fire occurs directly between tower B and tower C. Could towers B and C be used to locate the fire? *Explain.*

CONCEPT SUMMARY *For Your Notebook*

Triangle Congruence Postulates and Theorems

You have learned five methods for proving that triangles are congruent.

SSS	SAS	HL (right △ only)	ASA	AAS
				
All three sides are congruent.	Two sides and the included angle are congruent.	The hypotenuse and one of the legs are congruent.	Two angles and the included side are congruent.	Two angles and a (non-included) side are congruent.

In the Exercises, you will prove three additional theorems about the congruence of right triangles: **A**ngle-**L**eg, **L**eg-**L**eg, and **H**ypotenuse-**A**ngle.

4.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 9, and 27

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 21, and 26

SKILL PRACTICE

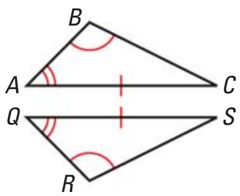
1. **VOCABULARY** Name one advantage of using a flow proof rather than a two-column proof.

2. ★ **WRITING** You know that a pair of triangles has two pairs of congruent corresponding angles. What other information do you need to show that the triangles are congruent?

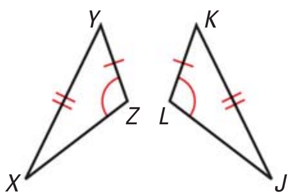
EXAMPLE 1 on p. 250 for Exs. 3–7

IDENTIFY CONGRUENT TRIANGLES **Is it possible to prove that the triangles are congruent? If so, state the postulate or theorem you would use.**

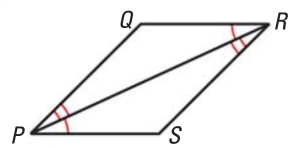
3. $\triangle ABC$, $\triangle QRS$



4. $\triangle XYZ$, $\triangle JKL$

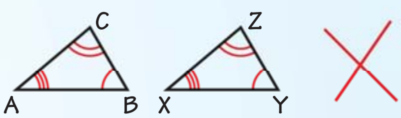


5. $\triangle PQR$, $\triangle RSP$

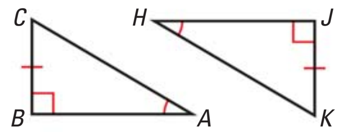


6. **ERROR ANALYSIS** *Describe* the error in concluding that $\triangle ABC \cong \triangle XYZ$.

By AAA, $\triangle ABC \cong \triangle XYZ$.



7. ★ **MULTIPLE CHOICE** Which postulate or theorem can you use to prove that $\triangle ABC \cong \triangle HJK$?

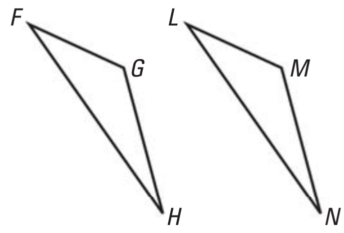


Ⓐ ASA Ⓑ AAS

Ⓒ SAS Ⓓ Not enough information

EXAMPLE 2
on p. 250
for Exs. 8–13

DEVELOPING PROOF **State the third congruence that is needed to prove that $\triangle FGH \cong \triangle LMN$ using the given postulate or theorem.**



8. **GIVEN** ▶ $\overline{GH} \cong \overline{MN}$, $\angle G \cong \angle M$, $\underline{\ ?\ } \cong \underline{\ ?\ }$
Use the AAS Congruence Theorem.

9. **GIVEN** ▶ $\overline{FG} \cong \overline{LM}$, $\angle G \cong \angle M$, $\underline{\ ?\ } \cong \underline{\ ?\ }$
Use the ASA Congruence Postulate.

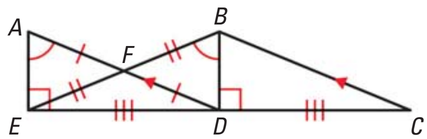
10. **GIVEN** ▶ $\overline{FH} \cong \overline{LN}$, $\angle H \cong \angle N$, $\underline{\ ?\ } \cong \underline{\ ?\ }$
Use the SAS Congruence Postulate.

OVERLAPPING TRIANGLES ***Explain*** **how you can prove that the indicated triangles are congruent using the given postulate or theorem.**

11. $\triangle AFE \cong \triangle DFB$ by SAS

12. $\triangle AED \cong \triangle BDE$ by AAS

13. $\triangle AED \cong \triangle BDC$ by ASA



DETERMINING CONGRUENCE **Tell whether you can use the given information to determine whether $\triangle ABC \cong \triangle DEF$. *Explain* your reasoning.**

14. $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$

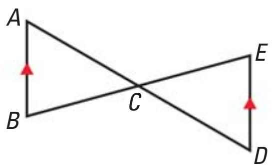
15. $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

16. $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{AC} \cong \overline{DE}$

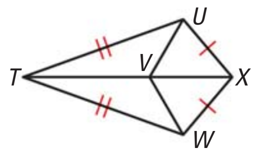
17. $\overline{AB} \cong \overline{EF}$, $\overline{BC} \cong \overline{FD}$, $\overline{AC} \cong \overline{DE}$

IDENTIFY CONGRUENT TRIANGLES **Is it possible to prove that the triangles are congruent? If so, state the postulate(s) or theorem(s) you would use.**

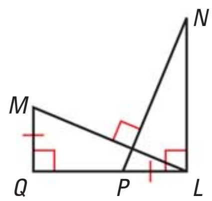
18. $\triangle ABC$, $\triangle DEC$



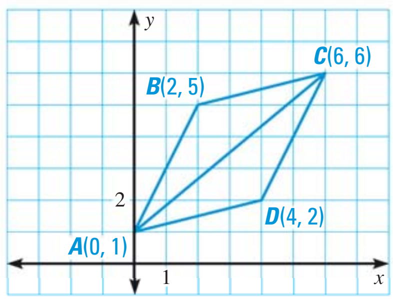
19. $\triangle TUV$, $\triangle TWV$



20. $\triangle QML$, $\triangle LPN$



21. ★ **EXTENDED RESPONSE** Use the graph at the right.



a. Show that $\angle CAD \cong \angle ACB$. *Explain* your reasoning.

b. Show that $\angle ACD \cong \angle CAB$. *Explain* your reasoning.

c. Show that $\triangle ABC \cong \triangle CDA$. *Explain* your reasoning.

22. **CHALLENGE** Use a coordinate plane.

a. Graph the lines $y = 2x + 5$, $y = 2x - 3$, and $x = 0$ in the same coordinate plane.

b. Consider the equation $y = mx + 1$. For what values of m will the graph of the equation form two triangles if added to your graph? For what values of m will those triangles be congruent? *Explain*.

PROBLEM SOLVING

CONGRUENCE IN BICYCLES *Explain* why the triangles are congruent.

23.

24.

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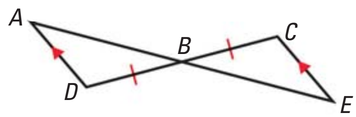
EXAMPLE 3
on p. 251
for Ex. 25

25. FLOW PROOF Copy and complete the flow proof.

GIVEN ▶ $\overline{AD} \parallel \overline{CE}$, $\overline{BD} \cong \overline{BC}$

PROVE ▶ $\triangle ABD \cong \triangle EBC$



$\underline{\ ?\ }$ (Given) → $\angle A \cong \angle E$ ($\underline{\ ?\ }$)

$\underline{\ ?\ }$ (Given) → $\angle C \cong \angle D$ ($\underline{\ ?\ }$)

$\overline{BD} \cong \overline{BC}$ ($\underline{\ ?\ }$)

$\angle A \cong \angle E$, $\angle C \cong \angle D$, $\overline{BD} \cong \overline{BC}$ → $\triangle ABD \cong \triangle EBC$ ($\underline{\ ?\ }$)

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EXAMPLE 4
on p. 251
for Ex. 26

26. ★ SHORT RESPONSE You are making a map for an orienteering race. Participants start at a large oak tree, find a boulder 250 yards due east of the oak tree, and then find a maple tree that is 50° west of north of the boulder and 35° east of north of the oak tree. Sketch a map. Can you locate the maple tree? *Explain*.

27. AIRPLANE In the airplane at the right, $\angle C$ and $\angle F$ are right angles, $\overline{BC} \cong \overline{EF}$, and $\angle A \cong \angle D$. What postulate or theorem allows you to conclude that $\triangle ABC \cong \triangle DEF$?

RIGHT TRIANGLES **In Lesson 4.4, you learned the Hypotenuse-Leg Theorem for right triangles. In Exercises 28–30, write a paragraph proof for these other theorems about right triangles.**

28. Leg-Leg (LL) Theorem If the legs of two right triangles are congruent, then the triangles are congruent.

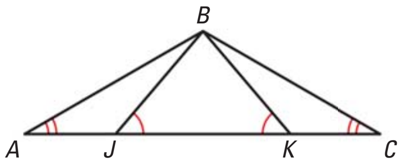
29. Angle-Leg (AL) Theorem If an angle and a leg of a right triangle are congruent to an angle and a leg of a second right triangle, then the triangles are congruent.

30. Hypotenuse-Angle (HA) Theorem If an angle and the hypotenuse of a right triangle are congruent to an angle and the hypotenuse of a second right triangle, then the triangles are congruent.

31. PROOF Write a two-column proof.

GIVEN ▶ $\overline{AK} \cong \overline{CJ}$, $\angle BJK \cong \angle BKJ$, $\angle A \cong \angle C$

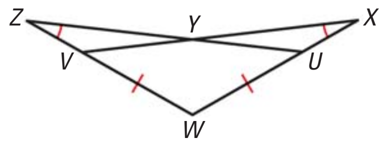
PROVE ▶ $\triangle ABK \cong \triangle CBJ$



32. PROOF Write a flow proof.

GIVEN ▶ $\overline{VW} \cong \overline{UW}$, $\angle X \cong \angle Z$

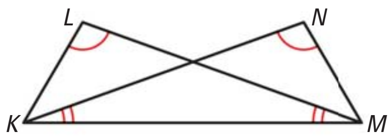
PROVE ▶ $\triangle XWV \cong \triangle ZWU$



33. PROOF Write a proof.

GIVEN ▶ $\angle NKM \cong \angle LMK$, $\angle L \cong \angle N$

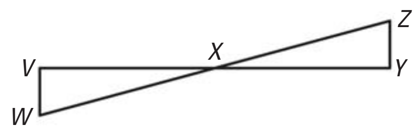
PROVE ▶ $\triangle NMK \cong \triangle LKM$



34. PROOF Write a proof.

GIVEN ▶ X is the midpoint of $\overline{VY}$ and $\overline{WZ}$.

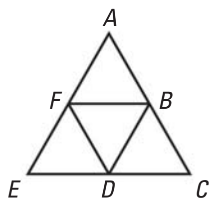
PROVE ▶ $\triangle VWX \cong \triangle YZX$



35. CHALLENGE Write a proof.

GIVEN ▶ $\triangle ABF \cong \triangle DFB$, F is the midpoint of $\overline{AE}$, B is the midpoint of $\overline{AC}$.

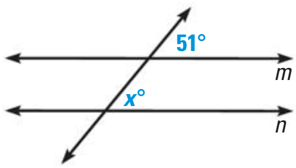
PROVE ▶ $\triangle FDE \cong \triangle BCD \cong \triangle ABF$



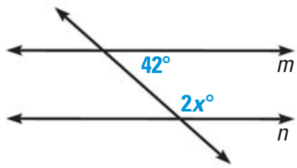
MIXED REVIEW

Find the value of x that makes $m \parallel n$. *(p. 161)*

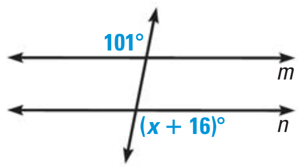
36.



37.



38.



Write an equation of the line that passes through point P and is parallel to the line with the given equation. *(p. 180)*

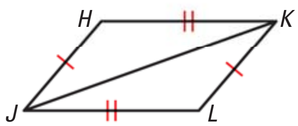
39. $P(0, 3)$, $y = x - 8$

40. $P(-2, 4)$, $y = -2x + 3$

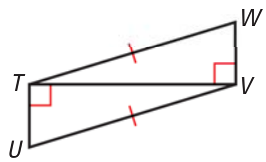
PREVIEW
Prepare for Lesson 4.6 in Exs. 41–43.

Decide which method, SSS, SAS, or HL, can be used to prove that the triangles are congruent. *(pp. 234, 240)*

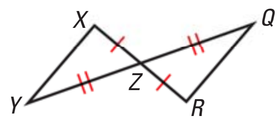
41. $\triangle HJK \cong \triangle LKJ$



42. $\triangle UTV \cong \triangle WVT$



43. $\triangle XYZ \cong \triangle RQZ$



4.6 Use Congruent Triangles

Before You used corresponding parts to prove triangles congruent.

Now You will use congruent triangles to prove corresponding parts congruent.

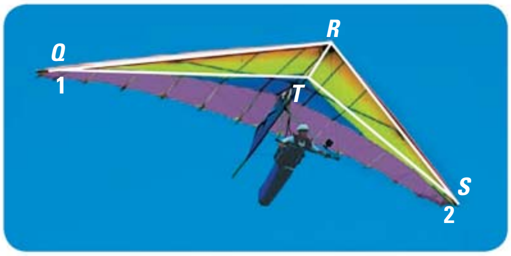
Why? So you can find the distance across a half pipe, as in Ex. 30.

Key Vocabulary
- **corresponding parts,** *p. 225*

By definition, congruent triangles have congruent corresponding parts. So, if you can prove that two triangles are congruent, you know that their corresponding parts must be congruent as well.

EXAMPLE 1 Use congruent triangles

Explain how you can use the given information to prove that the hanglider parts are congruent.



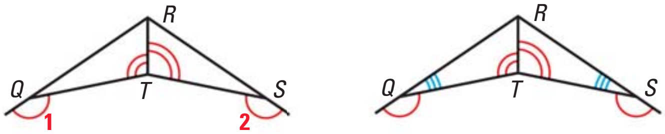
GIVEN ▶ $\angle 1 \cong \angle 2$, $\angle RTQ \cong \angle RTS$

PROVE ▶ $\overline{QT} \cong \overline{ST}$

Solution

If you can show that $\triangle QRT \cong \triangle SRT$, you will know that $\overline{QT} \cong \overline{ST}$. First, copy the diagram and mark the given information. Then add the information that you can deduce. In this case, $\angle RQT$ and $\angle RST$ are supplementary to congruent angles, so $\angle RQT \cong \angle RST$. Also, $\overline{RT} \cong \overline{RT}$.

Mark given information. **Add deduced information.**



Two angle pairs and a non-included side are congruent, so by the AAS Congruence Theorem, $\triangle QRT \cong \triangle SRT$. Because corresponding parts of congruent triangles are congruent, $\overline{QT} \cong \overline{ST}$.

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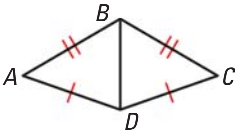
✓ GUIDED PRACTICE for Example 1

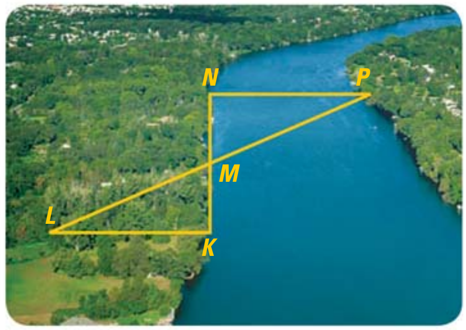
1. *Explain* how you can prove that $\angle A \cong \angle C$.



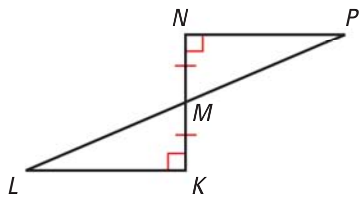
EXAMPLE 2 Use congruent triangles for measurement

INDIRECT MEASUREMENT
When you cannot easily measure a length directly, you can make conclusions about the length *indirectly*, usually by calculations based on known lengths.

SURVEYING Use the following method to find the distance across a river, from point N to point P.



- Place a stake at K on the near side so that $\overline{NK} \perp \overline{NP}$.
- Find M, the midpoint of $\overline{NK}$.
- Locate the point L so that $\overline{NK} \perp \overline{KL}$ and L, P, and M are collinear.
- Explain how this plan allows you to find the distance.

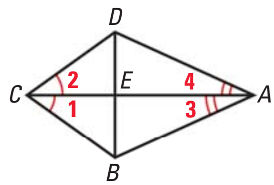


Solution

Because $\overline{NK} \perp \overline{NP}$ and $\overline{NK} \perp \overline{KL}$, $\angle N$ and $\angle K$ are congruent right angles. Because M is the midpoint of $\overline{NK}$, $\overline{NM} \cong \overline{KM}$. The vertical angles $\angle KML$ and $\angle NMP$ are congruent. So, $\triangle MLK \cong \triangle MPN$ by the ASA Congruence Postulate. Then, because corresponding parts of congruent triangles are congruent, $\overline{KL} \cong \overline{NP}$. So, you can find the distance NP across the river by measuring $\overline{KL}$.

EXAMPLE 3 Plan a proof involving pairs of triangles

Use the given information to write a plan for proof.



GIVEN ▶ $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

PROVE ▶ $\triangle BCE \cong \triangle DCE$

Solution

In $\triangle BCE$ and $\triangle DCE$, you know $\angle 1 \cong \angle 2$ and $\overline{CE} \cong \overline{CE}$. If you can show that $\overline{CB} \cong \overline{CD}$, you can use the SAS Congruence Postulate.

To prove that $\overline{CB} \cong \overline{CD}$, you can first prove that $\triangle CBA \cong \triangle CDA$. You are given $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. $\overline{CA} \cong \overline{CA}$ by the Reflexive Property. You can use the ASA Congruence Postulate to prove that $\triangle CBA \cong \triangle CDA$.

▶ **Plan for Proof** Use the ASA Congruence Postulate to prove that $\triangle CBA \cong \triangle CDA$. Then state that $\overline{CB} \cong \overline{CD}$. Use the SAS Congruence Postulate to prove that $\triangle BCE \cong \triangle DCE$.

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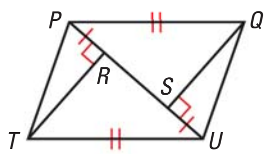
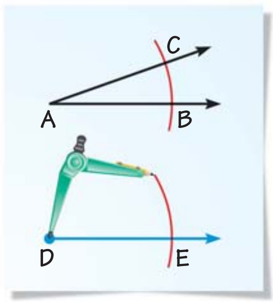
✓ GUIDED PRACTICE for Examples 2 and 3



2. In Example 2, does it matter how far from point N you place a stake at point K? *Explain.*
3. Using the information in the diagram at the right, write a plan to prove that $\triangle PTU \cong \triangle UQP$.

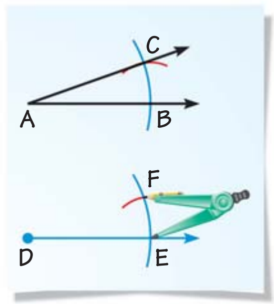
PROVING CONSTRUCTIONS On page 34, you learned how to use a compass and a straightedge to copy an angle. The construction is shown below. You can use congruent triangles to prove that this construction is valid.

STEP 1



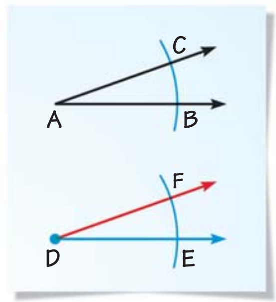
To copy $\angle A$, draw a segment with initial point D. Draw an arc with center A. Using the same radius, draw an arc with center D. Label points B, C, and E.

STEP 2



Draw an arc with radius BC and center E. Label the intersection F.

STEP 3



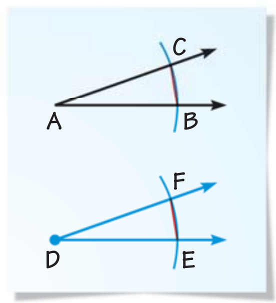
Draw $\overrightarrow{DF}$. In Example 4, you will prove that $\angle D \cong \angle A$.

EXAMPLE 4 Prove a construction

Write a proof to verify that the construction for copying an angle is valid.

Solution

Add $\overline{BC}$ and $\overline{EF}$ to the diagram. In the construction, $\overline{AB}$, $\overline{DE}$, $\overline{AC}$, and $\overline{DF}$ are all determined by the same compass setting, as are $\overline{BC}$ and $\overline{EF}$. So, you can assume the following as given statements.



GIVEN ▶ $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$

PROVE ▶ $\angle D \cong \angle A$

Plan for Proof Show that $\triangle CAB \cong \triangle FDE$, so you can conclude that the corresponding parts $\angle A$ and $\angle D$ are congruent.

Plan in Action

STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$	**1.** Given
2. $\triangle FDE \cong \triangle CAB$	**2.** SSS Congruence Postulate
3. $\angle D \cong \angle A$	**3.** Corresp. parts of $\cong$ △s are $\cong$.

✓ GUIDED PRACTICE for Example 4

4. Look back at the construction of an angle bisector in Explore 4 on page 34. What segments can you assume are congruent?

4.6 EXERCISES

HOMEWORK KEY
◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 19, 23, and 31
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 14, 31, and 36

SKILL PRACTICE

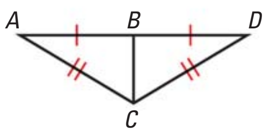
1. VOCABULARY Copy and complete: Corresponding parts of congruent triangles are _?_.

2. ★ WRITING *Explain* why you might choose to use congruent triangles to measure the distance across a river. Give another example where it may be easier to measure with congruent triangles rather than directly.

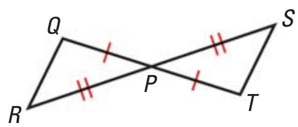
EXAMPLES 1 and 2 on p. 256–257 for Exs. 3–11

CONGRUENT TRIANGLES Tell which triangles you can show are congruent in order to prove the statement. What postulate or theorem would you use?

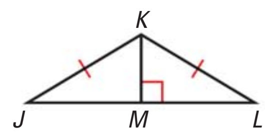
3. $\angle A \cong \angle D$



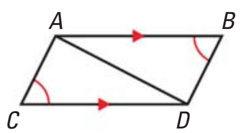
4. $\angle Q \cong \angle T$



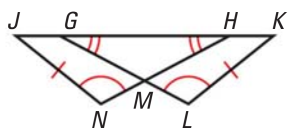
5. $\overline{JM} \cong \overline{LM}$



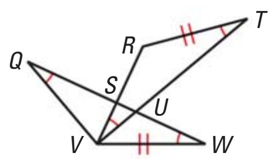
6. $\overline{AC} \cong \overline{BD}$



7. $\overline{GK} \cong \overline{HJ}$

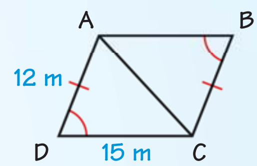


8. $\overline{QW} \cong \overline{TV}$



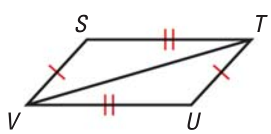
9. ERROR ANALYSIS *Describe* the error in the statement.

△ABC ≅ △CDA by SAS.
So, AB = 15 meters.

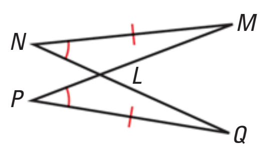


PLANNING FOR PROOF Use the diagram to write a plan for proof.

10. PROVE ▶ $\angle S \cong \angle U$



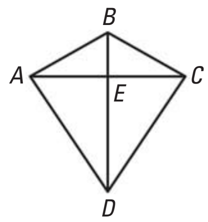
11. PROVE ▶ $\overline{LM} \cong \overline{LQ}$



12. PENTAGONS *Explain* why segments connecting any pair of corresponding vertices of congruent pentagons are congruent. Make a sketch to support your answer.

13. xy ALGEBRA Given that $\triangle ABC \cong \triangle DEF$, $m\angle A = 70°$, $m\angle B = 60°$, $m\angle C = 50°$, $m\angle D = (3x + 10)°$, $m\angle E = \left(\frac{y}{3} + 20\right)°$, and $m\angle F = (z^2 + 14)°$, find the values of x, y, and z.

14. ★ **MULTIPLE CHOICE** Which set of given information does *not* allow you to conclude that $\overline{AD} \cong \overline{CD}$?



Ⓐ $\overline{AE} \cong \overline{CE}$, $m\angle BEA = 90°$

Ⓑ $\overline{BA} \cong \overline{BC}$, $\angle BDC \cong \angle BDA$

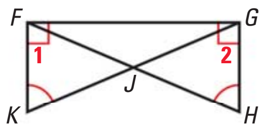
Ⓒ $\overline{AB} \cong \overline{CB}$, $\angle ABE \cong \angle CBE$

Ⓓ $\overline{AE} \cong \overline{CE}$, $\overline{AB} \cong \overline{CB}$

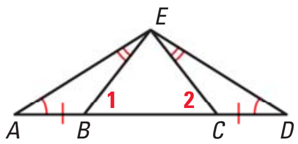
EXAMPLE 3
on p. 257
for Exs. 15–20

PLANNING FOR PROOF **Use the information given in the diagram to write a plan for proving that $\angle 1 \cong \angle 2$.**

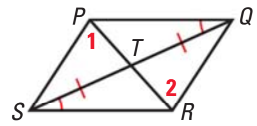
15.



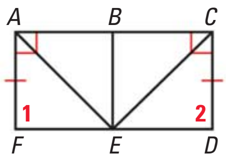
16.



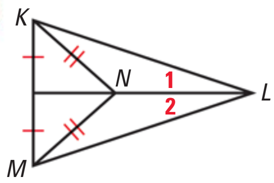
17.



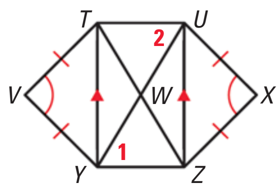
18.



19.



20.



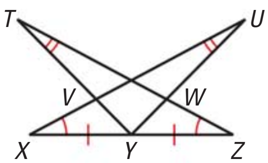
USING COORDINATES **Use the vertices of $\triangle ABC$ and $\triangle DEF$ to show that $\angle A \cong \angle D$. *Explain* your reasoning.**

21. $A(3, 7)$, $B(6, 11)$, $C(11, 13)$, $D(2, -4)$, $E(5, -8)$, $F(10, -10)$

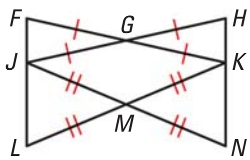
22. $A(3, 8)$, $B(3, 2)$, $C(11, 2)$, $D(-1, 5)$, $E(5, 5)$, $F(5, 13)$

PROOF **Use the information given in the diagram to write a proof.**

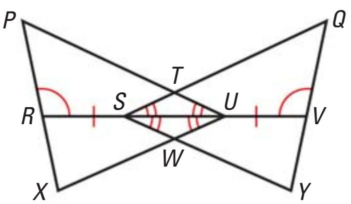
23. **PROVE** ▶ $\angle VYX \cong \angle WYZ$



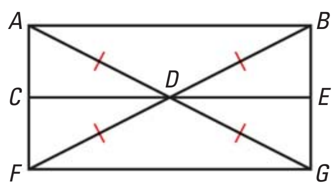
24. **PROVE** ▶ $\overline{FL} \cong \overline{HN}$



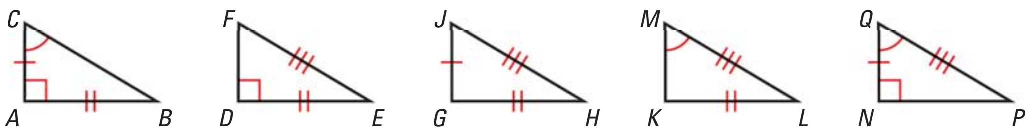
25. **PROVE** ▶ $\triangle PUX \cong \triangle QSY$



26. **PROVE** ▶ $\overline{AC} \cong \overline{GE}$



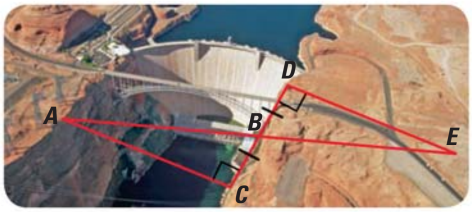
27. **CHALLENGE** Which of the triangles below are congruent?



PROBLEM SOLVING

EXAMPLE 2 on p. 257 for Ex. 28

28. CANYON *Explain* how you can find the distance across the canyon.



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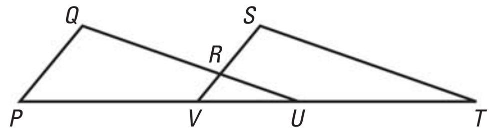
29. PROOF Use the given information and the diagram to write a two-column proof.



GIVEN ▶ $\overline{PQ} \parallel \overline{VS}$, $\overline{QU} \parallel \overline{ST}$, $\overline{PQ} \cong \overline{VS}$

PROVE ▶ $\angle Q \cong \angle S$

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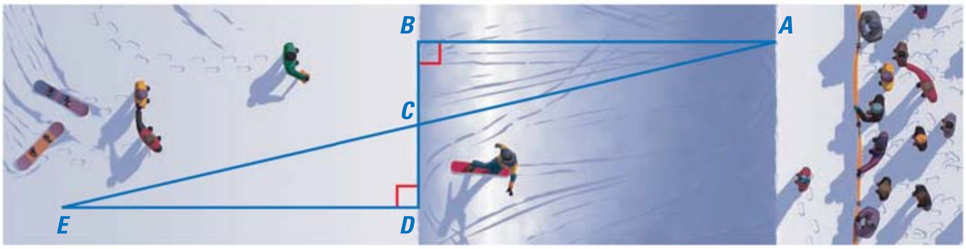
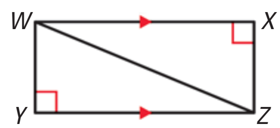
30. SNOWBOARDING In the diagram of the half pipe below, C is the midpoint of $\overline{BD}$. If $EC \approx 11.5$ m, and $CD \approx 2.5$ m, find the approximate distance across the half pipe. *Explain* your reasoning.



31. ★ **MULTIPLE CHOICE** Using the information in the diagram, you can prove that $\overline{WY} \cong \overline{ZX}$. Which reason would *not* appear in the proof?



Ⓐ SAS Congruence Postulate

Ⓑ AAS Congruence Theorem

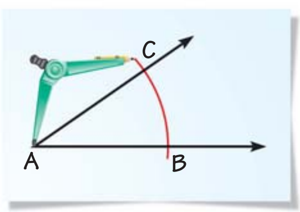
Ⓒ Alternate Interior Angles Theorem

Ⓓ Right Angle Congruence Theorem

EXAMPLE 4 on p. 258 for Ex. 32

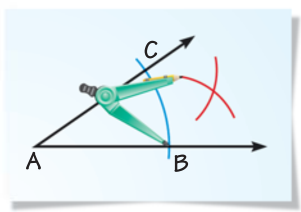
32. PROVING A CONSTRUCTION The diagrams below show the construction on page 34 used to bisect $\angle A$. By construction, you can assume that $\overline{AB} \cong \overline{AC}$ and $\overline{BG} \cong \overline{CG}$. Write a proof to verify that $\overrightarrow{AG}$ bisects $\angle A$.

STEP 1



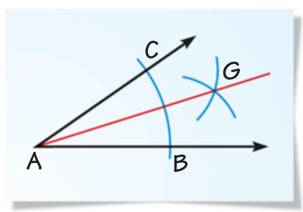
First draw an arc with center A. Label the points where the arc intersects the sides of the angle points B and C.

STEP 2



Draw an arc with center C. Using the same radius, draw an arc with center B. Label the intersection point G.

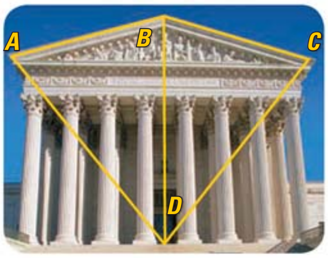
STEP 3



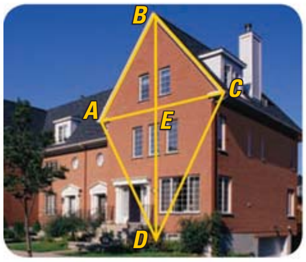
Draw $\overrightarrow{AG}$. It follows that $\angle BAG \cong \angle CAG$.

ARCHITECTURE **Can you use the given information to determine that $\overline{AB} \cong \overline{BC}$? *Justify* your answer.**

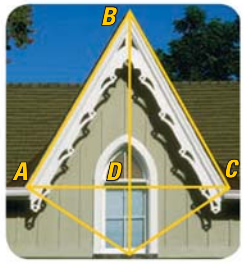
33. $\angle ABD \cong \angle CBD$, $AD = CD$



34. $\overline{AC} \perp \overline{BD}$, $\triangle ADE \cong \triangle CDE$

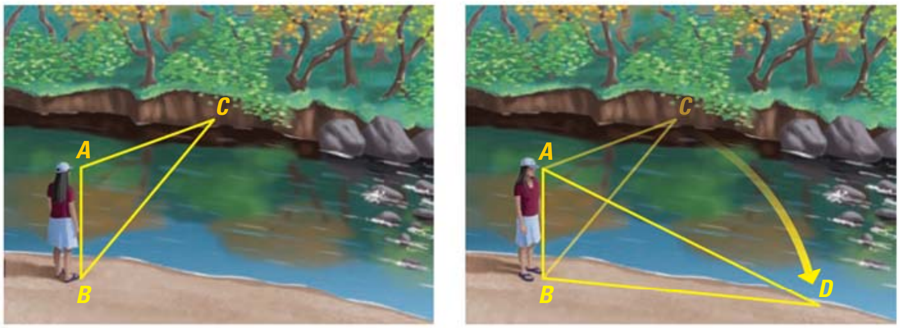


35. $\overline{BD}$ bisects $\overline{AC}$, $\overline{AD} \perp \overline{BD}$



36. ★ **EXTENDED RESPONSE** You can use the method described below to find the distance across a river. You will need a cap with a visor.

- Stand on one side of the river and look straight across to a point on the other side. Align the visor of your cap with that point.
- Without changing the inclination of your neck and head, turn sideways until the visor is in line with a point on your side of the stream.
- Measure the distance *BD* between your feet and that point.

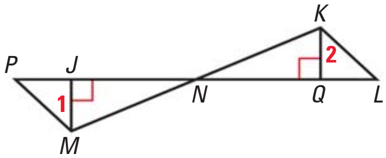


a. What corresponding parts of the two triangles can you assume are congruent? What postulate or theorem can you use to show that the two triangles are congruent?

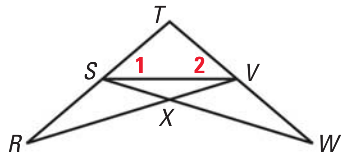
b. *Explain* why *BD* is also the distance across the stream.

PROOF **Use the given information and the diagram to prove that $\angle 1 \cong \angle 2$.**

37. **GIVEN** ▶ $\overline{MN} \cong \overline{KN}$, $\angle PMN \cong \angle NKL$



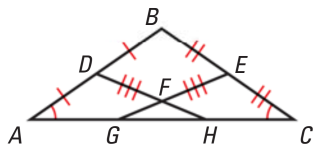
38. **GIVEN** ▶ $\overline{TS} \cong \overline{TV}$, $\overline{SR} \cong \overline{VW}$



39. **PROOF** Write a proof.

GIVEN ▶ $\overline{BA} \cong \overline{BC}$, *D* and *E* are midpoints, $\angle A \cong \angle C$, $\overline{DF} \cong \overline{EF}$

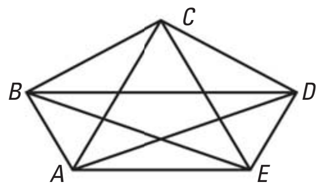
PROVE ▶ $\overline{FG} \cong \overline{FH}$



◯ = **WORKED-OUT SOLUTIONS** on p. WS1

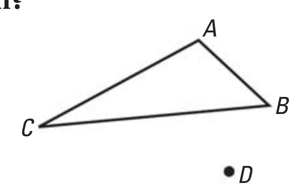
★ = **STANDARDIZED TEST PRACTICE**

40. **CHALLENGE** In the diagram of pentagon $ABCDE$, $\overline{AB} \parallel \overline{EC}$, $\overline{AC} \parallel \overline{ED}$, $\overline{AB} \cong \overline{ED}$, and $\overline{AC} \cong \overline{EC}$. Write a proof that shows $\overline{AD} \cong \overline{EB}$.



MIXED REVIEW

How many lines can be drawn that fit each description? Copy the diagram and sketch all the lines. *(p. 147)*



41. Line(s) through B and parallel to $\overleftrightarrow{AC}$

42. Line(s) through A and perpendicular to $\overleftrightarrow{BC}$

43. Line(s) through D and C

PREVIEW
Prepare for Lesson 4.7 in Exs. 44–46.

The variable expressions represent the angle measures of a triangle. Find the measure of each angle. Then classify the triangle by its angles. *(p. 217)*

44. $m\angle A = x°$
$m\angle B = (4x)°$
$m\angle C = (5x)°$

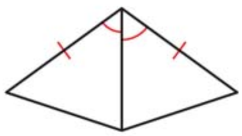
45. $m\angle A = x°$
$m\angle B = (5x)°$
$m\angle C = (x + 19)°$

46. $m\angle A = (x - 22)°$
$m\angle B = (x + 16)°$
$m\angle C = (2x - 14)°$

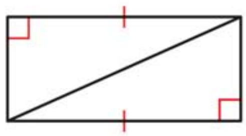
QUIZ *for Lessons 4.4–4.6*

Decide which method, SAS, ASA, AAS, or HL, can be used to prove that the triangles are congruent. *(pp. 240, 249)*

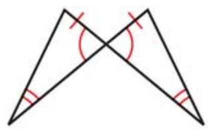
1.



2.

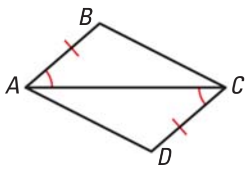


3.

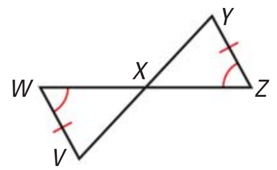


Use the given information to write a proof.

4. **GIVEN** ▶ $\angle BAC \cong \angle DCA$, $\overline{AB} \cong \overline{CD}$
PROVE ▶ $\triangle ABC \cong \triangle CDA$ *(p. 240)*

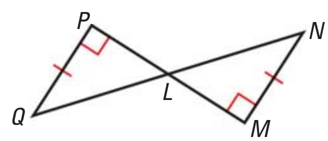


5. **GIVEN** ▶ $\angle W \cong \angle Z$, $\overline{VW} \cong \overline{YZ}$
PROVE ▶ $\triangle VWX \cong \triangle YZX$ *(p. 249)*



6. Write a plan for a proof. *(p. 256)*

GIVEN ▶ $\overline{PQ} \cong \overline{MN}$, $m\angle P = m\angle M = 90°$
PROVE ▶ $\overline{QL} \cong \overline{NL}$



4.7 Use Isosceles and Equilateral Triangles

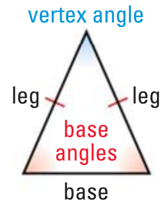
Before You learned about isosceles and equilateral triangles.

Now You will use theorems about isosceles and equilateral triangles.

Why? So you can solve a problem about architecture, as in Ex. 40.

Key Vocabulary
- **legs**
- **vertex angle**
- **base**
- **base angles**

In Lesson 4.1, you learned that a triangle is isosceles if it has at least two congruent sides. When an isosceles triangle has exactly two congruent sides, these two sides are the **legs**. The angle formed by the legs is the **vertex angle**. The third side is the **base** of the isosceles triangle. The two angles adjacent to the base are called **base angles**.



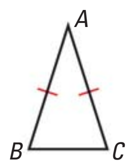
THEOREMS — For Your Notebook

THEOREM 4.7 Base Angles Theorem

If two sides of a triangle are congruent, then the angles opposite them are congruent.

If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.

Proof: p. 265

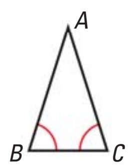


THEOREM 4.8 Converse of Base Angles Theorem

If two angles of a triangle are congruent, then the sides opposite them are congruent.

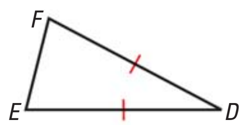
If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.

Proof: Ex. 45, p. 269



EXAMPLE 1 Apply the Base Angles Theorem

In $\triangle DEF$, $\overline{DE} \cong \overline{DF}$. Name two congruent angles.

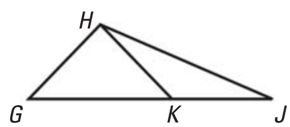


Solution

▶ $\overline{DE} \cong \overline{DF}$, so by the Base Angles Theorem, $\angle E \cong \angle F$.

✓ GUIDED PRACTICE for Example 1

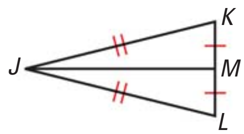
Copy and complete the statement.



1. If $\overline{HG} \cong \overline{HK}$, then $\angle$ _?_ $\cong \angle$ _?_.

2. If $\angle KHJ \cong \angle KJH$, then _?_ $\cong$ _?_.

PROOF Base Angles Theorem



GIVEN ▶ $\overline{JK} \cong \overline{JL}$

PROVE ▶ $\angle K \cong \angle L$

Plan for Proof

a. Draw $\overline{JM}$ so that it bisects $\overline{KL}$.

b. Use SSS to show that $\triangle JMK \cong \triangle JML$.

c. Use properties of congruent triangles to show that $\angle K \cong \angle L$.

Plan in Action

	STATEMENTS	REASONS
	1. M is the midpoint of $\overline{KL}$.	**1.** Definition of midpoint
a.	**2.** Draw $\overline{JM}$.	**2.** Two points determine a line.
	3. $\overline{MK} \cong \overline{ML}$	**3.** Definition of midpoint
	4. $\overline{JK} \cong \overline{JL}$	**4.** Given
	5. $\overline{JM} \cong \overline{JM}$	**5.** Reflexive Property of Congruence
b.	**6.** $\triangle JMK \cong \triangle JML$	**6.** SSS Congruence Postulate
c.	**7.** $\angle K \cong \angle L$	**7.** Corresp. parts of $\cong$ △s are $\cong$.

Recall that an equilateral triangle has three congruent sides.

WRITE A BICONDITIONAL

The corollaries state that a triangle is *equilateral* if and only if it is *equiangular.*

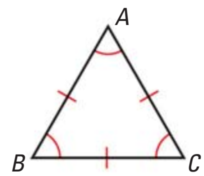
COROLLARIES *For Your Notebook*

Corollary to the Base Angles Theorem

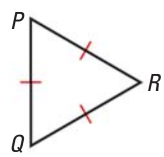
If a triangle is equilateral, then it is equiangular.

Corollary to the Converse of Base Angles Theorem

If a triangle is equiangular, then it is equilateral.



EXAMPLE 2 Find measures in a triangle



Find the measures of $\angle P$, $\angle Q$, and $\angle R$.

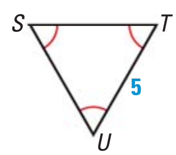
The diagram shows that $\triangle PQR$ is equilateral. Therefore, by the Corollary to the Base Angles Theorem, $\triangle PQR$ is equiangular. So, $m\angle P = m\angle Q = m\angle R$.

$3(m\angle P) = 180°$ **Triangle Sum Theorem**

$m\angle P = 60°$ **Divide each side by 3.**

▶ The measures of $\angle P$, $\angle Q$, and $\angle R$ are all 60°.

✓ GUIDED PRACTICE for Example 2

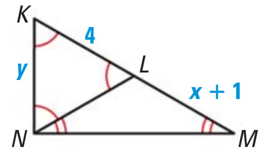


3. Find ST in the triangle at the right.

4. Is it possible for an equilateral triangle to have an angle measure other than 60°? *Explain.*

EXAMPLE 3 Use isosceles and equilateral triangles

xy ALGEBRA **Find the values of x and y in the diagram.**



Solution

STEP 1 **Find** the value of y. Because $\triangle KLN$ is equiangular, it is also equilateral and $\overline{KN} \cong \overline{KL}$. Therefore, $y = 4$.

AVOID ERRORS
You cannot use $\angle N$ to refer to $\angle LNM$ because three angles have N as their vertex.

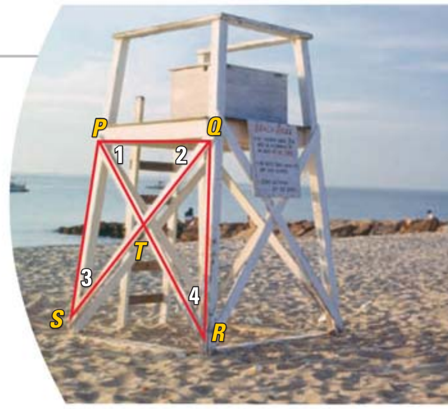
STEP 2 **Find** the value of x. Because $\angle LNM \cong \angle LMN$, $\overline{LN} \cong \overline{LM}$ and $\triangle LMN$ is isosceles. You also know that $LN = 4$ because $\triangle KLN$ is equilateral.

$LN = LM$ **Definition of congruent segments**

$4 = x + 1$ **Substitute 4 for LN and $x + 1$ for LM.**

$3 = x$ **Subtract 1 from each side.**

EXAMPLE 4 Solve a multi-step problem



LIFEGUARD TOWER **In the lifeguard tower, $\overline{PS} \cong \overline{QR}$ and $\angle QPS \cong \angle PQR$.**

a. What congruence postulate can you use to prove that $\triangle QPS \cong \triangle PQR$?

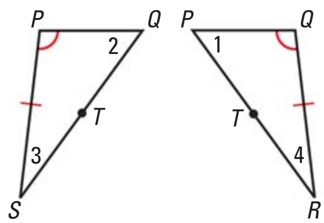
b. Explain why $\triangle PQT$ is isosceles.

c. Show that $\triangle PTS \cong \triangle QTR$.

Solution

AVOID ERRORS
When you redraw the triangles so that they do not overlap, be careful to copy all given information and labels correctly.

a. Draw and label $\triangle QPS$ and $\triangle PQR$ so that they do not overlap. You can see that $\overline{PQ} \cong \overline{QP}$, $\overline{PS} \cong \overline{QR}$, and $\angle QPS \cong \angle PQR$. So, by the SAS Congruence Postulate, $\triangle QPS \cong \triangle PQR$.

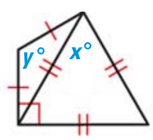


b. From part (a), you know that $\angle 1 \cong \angle 2$ because corresp. parts of $\cong$ $\triangle$s are $\cong$. By the Converse of the Base Angles Theorem, $\overline{PT} \cong \overline{QT}$, and $\triangle PQT$ is isosceles.

c. You know that $\overline{PS} \cong \overline{QR}$, and $\angle 3 \cong \angle 4$ because corresp. parts of $\cong$ $\triangle$s are $\cong$. Also, $\angle PTS \cong \angle QTR$ by the Vertical Angles Congruence Theorem. So, $\triangle PTS \cong \triangle QTR$ by the AAS Congruence Theorem.

✓ GUIDED PRACTICE for Examples 3 and 4

5. Find the values of x and y in the diagram.



6. **REASONING** Use parts (b) and (c) in Example 4 and the SSS Congruence Postulate to give a different proof that $\triangle QPS \cong \triangle PQR$.

4.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 17, and 41

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, 19, 30, 31, 42, and 46

SKILL PRACTICE

1. VOCABULARY Define the *vertex angle* of an isosceles triangle.

2. ★ **WRITING** What is the relationship between the base angles of an isosceles triangle? *Explain*.

EXAMPLE 1 on p. 264 for Exs. 3–6

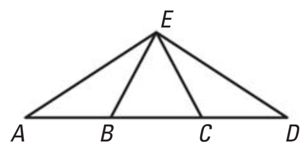
USING DIAGRAMS In Exercises 3–6, use the diagram. Copy and complete the statement. Tell what theorem you used.

3. If $\overline{AE} \cong \overline{DE}$, then $\angle$ _?_ $\cong \angle$ _?_ .

4. If $\overline{AB} \cong \overline{EB}$, then $\angle$ _?_ $\cong \angle$ _?_ .

5. If $\angle D \cong \angle CED$, then _?_ $\cong$ _?_ .

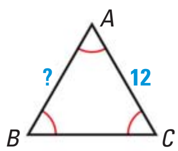
6. If $\angle EBC \cong \angle ECB$, then _?_ $\cong$ _?_ .



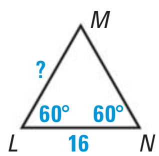
EXAMPLE 2 on p. 265 for Exs. 7–14

REASONING Find the unknown measure.

7.



8.



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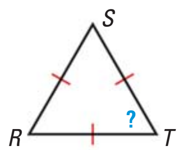
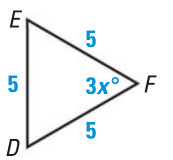


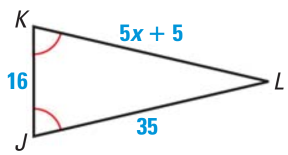
10. DRAWING DIAGRAMS A base angle in an isosceles triangle measures 37°. Draw and label the triangle. What is the measure of the vertex angle?

ALGEBRA Find the value of *x*.

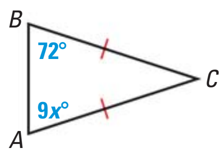
11.



12.

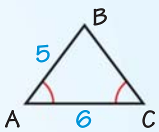


13.



14. ERROR ANALYSIS *Describe* and correct the error made in finding *BC* in the diagram shown.

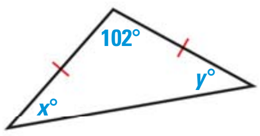
$\angle A \cong \angle C$, therefore $\overline{AC} \cong \overline{BC}$. So, $BC = 6$



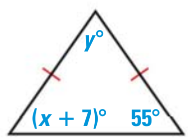
EXAMPLE 3 on p. 266 for Exs. 15–17

ALGEBRA Find the values of *x* and *y*.

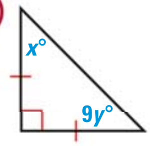
15.



16.



17.

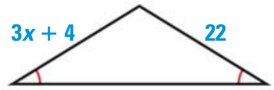


18. ★ **SHORT RESPONSE** Are isosceles triangles always acute triangles? *Explain* your reasoning.

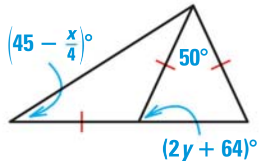
19. ★ **MULTIPLE CHOICE** What is the value of x in the diagram?

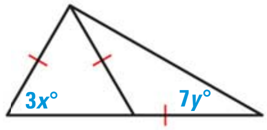
Ⓐ 5   Ⓑ 6

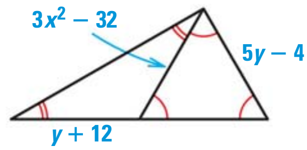
Ⓒ 7   Ⓓ 9



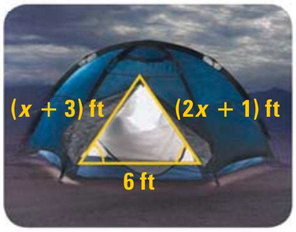
xy ALGEBRA **Find the values of x and y, if possible. *Explain* your reasoning.**

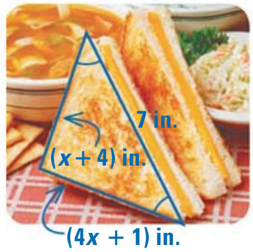
20.


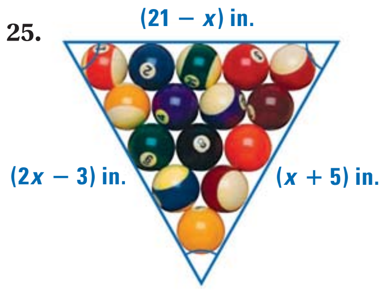
21.


22.


xy ALGEBRA **Find the perimeter of the triangle.**

23.


24.


25.


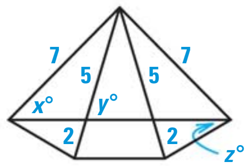
REASONING **In Exercises 26–29, use the diagram. State whether the given values for x, y, and z are possible or not. If not, *explain*.**

26. $x = 90, y = 68, z = 42$

27. $x = 40, y = 72, z = 36$

28. $x = 25, y = 25, z = 15$

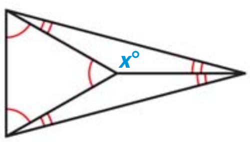
29. $x = 42, y = 72, z = 33$

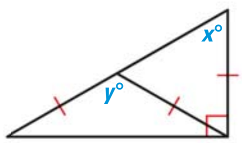


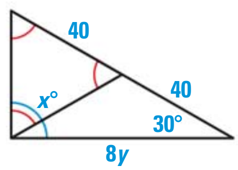
30. ★ **SHORT RESPONSE** In $\triangle DEF$, $m\angle D = (4x + 2)°$, $m\angle E = (6x - 30)°$, and $m\angle F = 3x°$. What type of triangle is $\triangle DEF$? *Explain* your reasoning.

31. ★ **SHORT RESPONSE** In $\triangle ABC$, D is the midpoint of $\overline{AC}$, and $\overline{BD}$ is perpendicular to $\overline{AC}$. *Explain* why $\triangle ABC$ is isosceles.

xy ALGEBRA **Find the value(s) of the variable(s). *Explain* your reasoning.**

32.


33.


34.


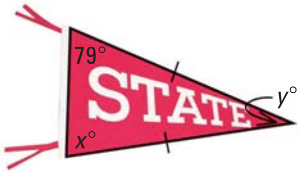
35. **REASONING** The measure of an exterior angle of an isosceles triangle is 130°. What are the possible angle measures of the triangle? *Explain*.

36. **PROOF** Let $\triangle ABC$ be isosceles with vertex angle $\angle A$. Suppose $\angle A$, $\angle B$, and $\angle C$ have integer measures. Prove that $m\angle A$ must be even.

37. **CHALLENGE** The measure of an exterior angle of an isosceles triangle is $x°$. What are the possible angle measures of the triangle in terms of x? *Describe* all the possible values of x.

PROBLEM SOLVING

38. SPORTS The dimensions of a sports pennant are given in the diagram. Find the values of x and y.



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39. ADVERTISING A logo in an advertisement is an equilateral triangle with a side length of 5 centimeters. Sketch the logo and give the measure of each side and angle.

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40. ARCHITECTURE The Transamerica Pyramid building shown in the photograph has four faces shaped like isosceles triangles. The measure of a base angle of one of these triangles is about 85°. What is the approximate measure of the vertex angle of the triangle?

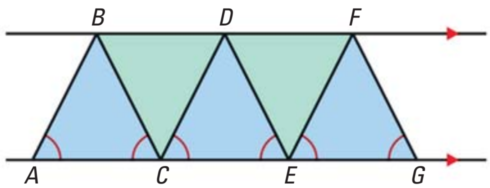
EXAMPLE 4
on p. 266
for Exs. 41–42

41. MULTI-STEP PROBLEM To make a zig-zag pattern, a graphic designer sketches two parallel line segments. Then the designer draws blue and green triangles as shown below.

a. Prove that $\triangle ABC \cong \triangle BCD$.

b. Name all the isosceles triangles in the diagram.

c. Name four angles that are congruent to $\angle ABC$.



42. ★ VISUAL REASONING In the pattern below, each small triangle is an equilateral triangle with an area of 1 square unit.

Triangle				
Area	1 square unit	?	?	?

a. Reasoning *Explain* how you know that any triangle made out of equilateral triangles will be an equilateral triangle.

b. Area Find the areas of the first four triangles in the pattern.

c. Make a Conjecture *Describe* any patterns in the areas. Predict the area of the seventh triangle in the pattern. *Explain* your reasoning.

43. REASONING Let $\triangle PQR$ be an isosceles right triangle with hypotenuse $\overline{QR}$. Find $m\angle P$, $m\angle Q$, and $m\angle R$.

44. REASONING *Explain* how the Corollary to the Base Angles Theorem follows from the Base Angles Theorem.

45. PROVING THEOREM 4.8 Write a proof of the Converse of the Base Angles Theorem.

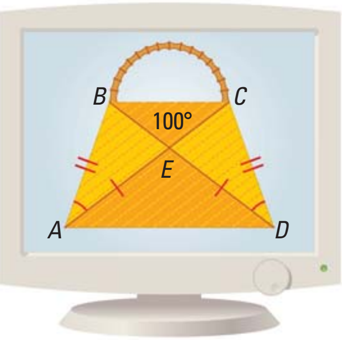
46. ★ **EXTENDED RESPONSE** Sue is designing fabric purses that she will sell at the school fair. Use the diagram of one of her purses.

a. Prove that $\triangle ABE \cong \triangle DCE$.

b. Name the isosceles triangles in the purse.

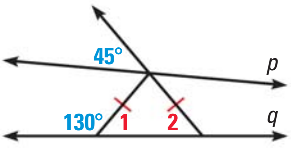
c. Name three angles that are congruent to $\angle EAD$.

d. **What If?** If the measure of $\angle BEC$ changes, does your answer to part (c) change? *Explain*.

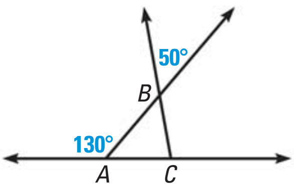


REASONING FROM DIAGRAMS **Use the information in the diagram to answer the question. *Explain* your reasoning.**

47. Is $p \parallel q$?



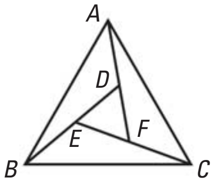
48. Is $\triangle ABC$ isosceles?



49. **PROOF** Write a proof.

GIVEN ▶ $\triangle ABC$ is equilateral, $\angle CAD \cong \angle ABE \cong \angle BCF$.

PROVE ▶ $\triangle DEF$ is equilateral.



50. **COORDINATE GEOMETRY** The coordinates of two vertices of $\triangle TUV$ are $T(0, 4)$ and $U(4, 0)$. *Explain* why the triangle will always be an isosceles triangle if V is any point on the line $y = x$ except $(2, 2)$.

51. **CHALLENGE** The lengths of the sides of a triangle are $3t$, $5t - 12$, and $t + 20$. Find the values of t that make the triangle isosceles. *Explain*.

MIXED REVIEW

What quadrant contains the point? *(p. 878)*

52. $(-1, -3)$ **53.** $(-2, 4)$ **54.** $(5, -2)$

Copy and complete the given function table. *(p. 884)*

55.

x	-7	0	5
$y = x - 4$	?	?	?

56.

?	-2	0	1
?	-6	0	3

PREVIEW
Prepare for Lesson 4.8 in Exs. 57–60.

Use the Distance Formula to decide whether $\overline{AB} \cong \overline{AC}$. *(p. 15)*

57. $A(0, 0)$, $B(-5, -6)$, $C(6, 5)$ **58.** $A(3, -3)$, $B(0, 1)$, $C(-1, 0)$

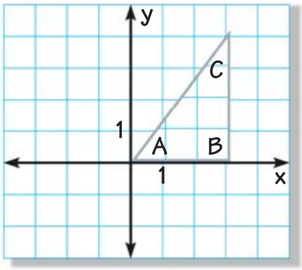
59. $A(0, 1)$, $B(4, 7)$, $C(-6, 3)$ **60.** $A(-3, 0)$, $B(2, 2)$, $C(2, -2)$

4.8 Investigate Slides and Flips

MATERIALS • graph paper • pencil

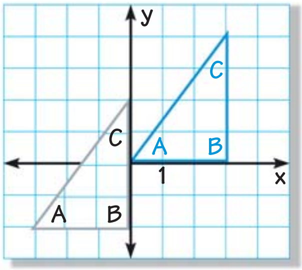
QUESTION What happens when you slide or flip a triangle?

EXPLORE 1 Slide a triangle



STEP 1 ***Draw a triangle*** Draw a scalene right triangle with legs of length 3 units and 4 units on a piece of graph paper. Cut out the triangle.

STEP 2 ***Draw coordinate plane*** Draw axes on the graph paper. Place the cut-out triangle so that the coordinates of the vertices are integers. Trace around the triangle and label the vertices.

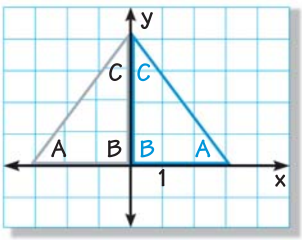


STEP 3 ***Slide triangle*** Slide the cut-out triangle so it moves left and down. Write a description of the *transformation* and record ordered pairs in a table like the one shown. Repeat this step three times, sliding the triangle left or right *and* up or down to various places in the coordinate plane.

Slide 2 units left and 3 units down.		
Vertex	**Original position**	**New position**
A	(0, 0)	(−3, −2)
B	(3, 0)	(0, −2)
C	(3, 4)	(0, 2)

EXPLORE 2 Flip a triangle

STEP 1 ***Draw a coordinate plane*** Draw and label a second coordinate plane. Place the cut-out triangle so that one vertex is at the origin and one side is along the *y*-axis, as shown.



STEP 2 ***Flip triangle*** Flip the cut-out triangle over the *y*-axis. Record a description of the *transformation* and record the ordered pairs in a table. Repeat this step, flipping the triangle over the *x*-axis.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. How are the coordinates of the original position of the triangle related to the new position in a slide? in a flip?

2. Is the original triangle congruent to the new triangle in a slide? in a flip? *Explain* your reasoning.

4.8 Perform Congruence Transformations

Before You determined whether two triangles are congruent.

Now You will create an image congruent to a given triangle.

Why So you can describe chess moves, as in Ex. 41.

Key Vocabulary
- **transformation**
- **image**
- **translation**
- **reflection**
- **rotation**
- **congruence transformation**

A **transformation** is an operation that moves or changes a geometric figure in some way to produce a new figure. The new figure is called the **image.** A transformation can be shown using an arrow.

The order of the vertices in the transformation statement tells you that P is the image of A, Q is the image of B, and R is the image of C.

$$\triangle ABC \rightarrow \triangle PQR$$

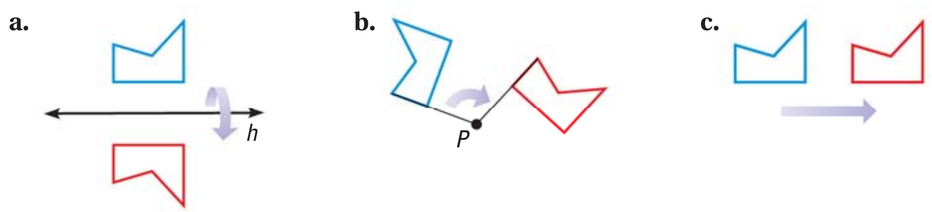
Original figure Image

There are three main types of transformations. A **translation** moves every point of a figure the same distance in the same direction. A **reflection** uses a *line of reflection* to create a mirror image of the original figure. A **rotation** turns a figure about a fixed point, called the *center of rotation*.

EXAMPLE 1 Identify transformations

TRANSFORMATIONS You will learn more about transformations in Lesson 6.7 and in Chapter 9.

Name the type of transformation demonstrated in each picture.



Reflection in a horizontal line

Rotation about a point

Translation in a straight path

✓ GUIDED PRACTICE for Example 1

1. Name the type of transformation shown.

CONGRUENCE Translations, reflections, and rotations are three types of *congruence transformations*. A **congruence transformation** changes the position of the figure without changing its size or shape.

TRANSLATIONS In a coordinate plane, a translation moves an object a given distance right or left and up or down. You can use coordinate notation to describe a translation.

READ DIAGRAMS

In this book, the original figure is blue and the transformation of the figure is red, unless otherwise stated.

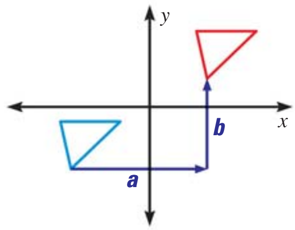
KEY CONCEPT *For Your Notebook*

Coordinate Notation for a Translation

You can describe a translation by the notation

$(x, y) \rightarrow (x + a, y + b)$

which shows that each point (x, y) of the blue figure is translated horizontally a units and vertically b units.



EXAMPLE 2 Translate a figure in the coordinate plane

Figure $ABCD$ has the vertices $A(-4, 3)$, $B(-2, 4)$, $C(-1, 1)$, and $D(-3, 1)$. Sketch $ABCD$ and its image after the translation $(x, y) \rightarrow (x + 5, y - 2)$.

Solution

First draw $ABCD$. Find the translation of each vertex by adding 5 to its x-coordinate and subtracting 2 from its y-coordinate. Then draw $ABCD$ and its image.

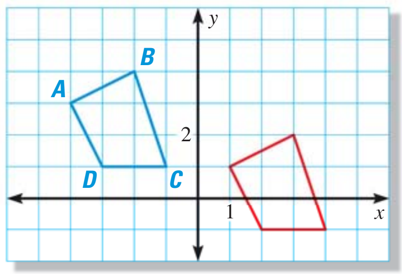
$(x, y) \rightarrow (x + 5, y - 2)$

$A(-4, 3) \rightarrow (1, 1)$

$B(-2, 4) \rightarrow (3, 2)$

$C(-1, 1) \rightarrow (4, -1)$

$D(-3, 1) \rightarrow (2, -1)$

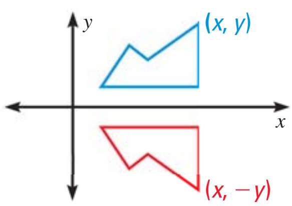


REFLECTIONS In this lesson, when a reflection is shown in a coordinate plane, the line of reflection is always the x-axis or the y-axis.

KEY CONCEPT *For Your Notebook*

Coordinate Notation for a Reflection

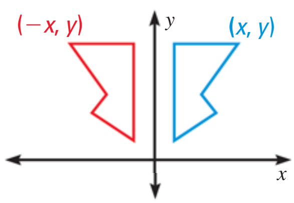
Reflection in the x-axis



Multiply the y-coordinate by -1.

$(x, y) \rightarrow (x, -y)$

Reflection in the y-axis

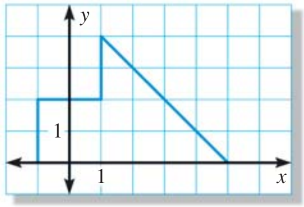


Multiply the x-coordinate by -1.

$(x, y) \rightarrow (-x, y)$

EXAMPLE 3 Reflect a figure in the *y*-axis

WOODWORK You are drawing a pattern for a wooden sign. Use a reflection in the *x*-axis to draw the other half of the pattern.



Solution

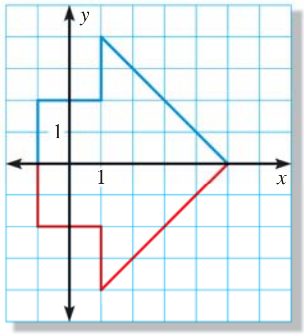
Multiply the *y*-coordinate of each vertex by −1 to find the corresponding vertex in the image.

$$(x, y) \rightarrow (x, -y)$$

$(-1, 0) \rightarrow (-1, 0)$ $\quad$ $(-1, 2) \rightarrow (-1, -2)$

$(1, 2) \rightarrow (1, -2)$ $\quad$ $(1, 4) \rightarrow (1, -4)$

$(5, 0) \rightarrow (5, 0)$



Use the vertices to draw the image. You can check your results by looking to see if each original point and its image are the same distance from the *x*-axis.

Animated Geometry at classzone.com

✓ GUIDED PRACTICE for Examples 2 and 3

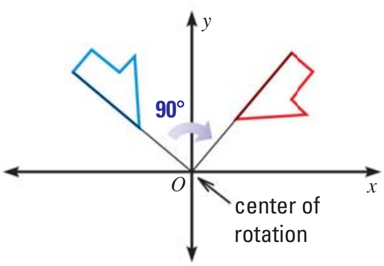
2. The vertices of $\triangle ABC$ are $A(1, 2)$, $B(0, 0)$, and $C(4, 0)$. A translation of $\triangle ABC$ results in the image $\triangle DEF$ with vertices $D(2, 1)$, $E(1, -1)$, and $F(5, -1)$. *Describe* the translation in words and in coordinate notation.

3. The endpoints of $\overline{RS}$ are $R(4, 5)$ and $S(1, -3)$. A reflection of $\overline{RS}$ results in the image $\overline{TU}$, with coordinates $T(4, -5)$ and $U(1, 3)$. Tell which axis $\overline{RS}$ was reflected in and write the coordinate rule for the reflection.

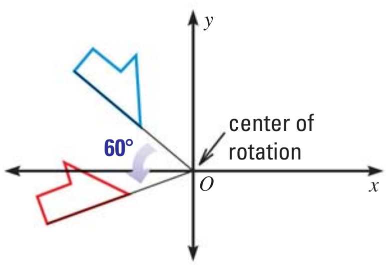
ROTATIONS In this lesson, if a rotation is shown in a coordinate plane, the center of rotation is the origin.

The direction of rotation can be either *clockwise* or *counterclockwise*. The *angle of rotation* is formed by rays drawn from the center of rotation through corresponding points on the original figure and its image.

90° clockwise rotation



60° counterclockwise rotation



Notice that rotations preserve distances from the center of rotation. So, segments drawn from the center of rotation to corresponding points on the figures are congruent.

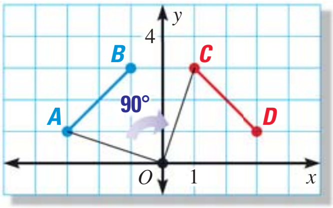
EXAMPLE 4 Identify a rotation

Graph $\overline{AB}$ and $\overline{CD}$. Tell whether $\overline{CD}$ is a rotation of $\overline{AB}$ about the origin. If so, give the angle and direction of rotation.

a. $A(-3, 1)$, $B(-1, 3)$, $C(1, 3)$, $D(3, 1)$ **b.** $A(0, 1)$, $B(1, 3)$, $C(-1, 1)$, $D(-3, 2)$

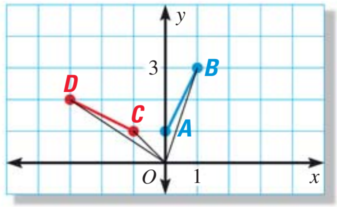
Solution

a.



$m\angle AOC = m\angle BOD = 90°$
This is a 90° clockwise rotation.

b.

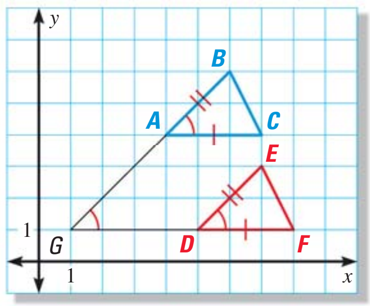


$m\angle AOC < m\angle BOD$
This is not a rotation.

EXAMPLE 5 Verify congruence

The vertices of $\triangle ABC$ are $A(4, 4)$, $B(6, 6)$, and $C(7, 4)$. The notation $(x, y) \to (x + 1, y - 3)$ describes the translation of $\triangle ABC$ to $\triangle DEF$. Show that $\triangle ABC \cong \triangle DEF$ to verify that the translation is a congruence transformation.

Solution



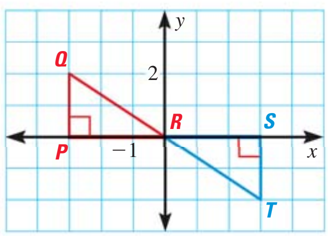
S You can see that $AC = DF = 3$, so $\overline{AC} \cong \overline{DF}$.

A Using the slopes, $\overline{AB} \parallel \overline{DE}$ and $\overline{AC} \parallel \overline{DF}$. If you extend $\overline{AB}$ and $\overline{DF}$ to form $\angle G$, the Corresponding Angles Postulate gives you $\angle BAC \cong \angle G$ and $\angle G \cong \angle EDF$. Then, $\angle BAC \cong \angle EDF$ by the Transitive Property of Congruence.

S Using the Distance Formula, $AB = DE = 2\sqrt{2}$ so $\overline{AB} \cong \overline{DE}$. So, $\triangle ABC \cong \triangle DEF$ by the SAS Congruence Postulate.

▶ Because $\triangle ABC \cong \triangle DEF$, the translation is a congruence transformation.

✓ GUIDED PRACTICE for Examples 4 and 5



4. Tell whether $\triangle PQR$ is a rotation of $\triangle STR$. If so, give the angle and direction of rotation.

5. Show that $\triangle PQR \cong \triangle STR$ to verify that the transformation is a congruence transformation.

4.8 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 23, and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 25, 40, 41, and 43

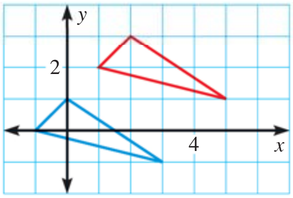
SKILL PRACTICE

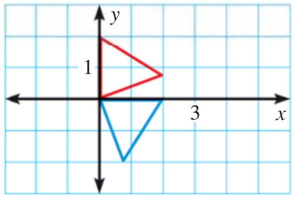
1. **VOCABULARY** *Describe* the translation $(x, y) \to (x - 1, y + 4)$ in words.

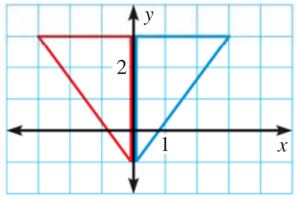
2. ★ **WRITING** *Explain* why the term *congruence transformation* is used in describing translations, reflections, and rotations.

EXAMPLE 1 on p. 272 for Exs. 3–8

IDENTIFYING TRANSFORMATIONS **Name the type of transformation shown.**

3.


4.


5.


WINDOWS **Decide whether the moving part of the window is a translation.**

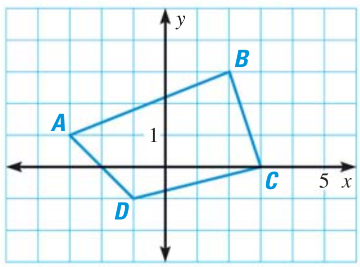
6. Double hung

7. Casement

8. Sliding

EXAMPLE 2 on p. 273 for Exs. 9–16

DRAWING A TRANSLATION **Copy figure *ABCD* and draw its image after the translation.**



9. $(x, y) \to (x + 2, y - 3)$

10. $(x, y) \to (x - 1, y - 5)$

11. $(x, y) \to (x + 4, y + 1)$

12. $(x, y) \to (x - 2, y + 3)$

COORDINATE NOTATION **Use coordinate notation to *describe* the translation.**

13. 4 units to the left, 2 units down

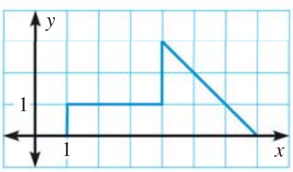
14. 6 units to the right, 3 units up

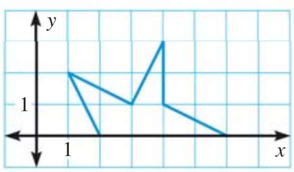
15. 2 units to the right, 1 unit down

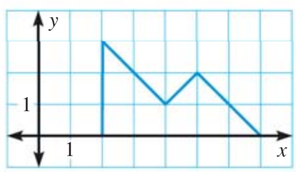
16. 7 units to the left, 9 units up

EXAMPLE 3 on p. 274 for Exs. 17–19

DRAWING **Use a reflection in the x-axis to draw the other half of the figure.**

17.


18.


19.


EXAMPLE 4
on p. 275
for Exs. 20–23

ROTATIONS **Use the coordinates to graph $\overline{AB}$ and $\overline{CD}$. Tell whether $\overline{CD}$ is a rotation of $\overline{AB}$ about the origin. If so, give the angle and direction of rotation.**

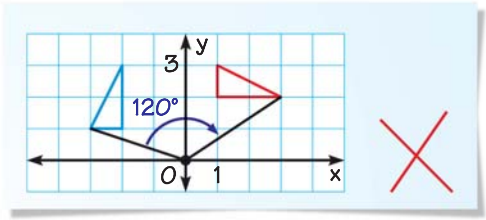
20. $A(1, 2)$, $B(3, 4)$, $C(2, -1)$, $D(4, -3)$

21. $A(-2, -4)$, $B(-1, -2)$, $C(4, 3)$, $D(2, 1)$

22. $A(-4, 0)$, $B(4, -4)$, $C(4, 4)$, $D(0, 4)$

23. $A(1, 2)$, $B(3, 0)$, $C(2, -1)$, $D(2, -3)$

24. **ERROR ANALYSIS** A student says that the red triangle is a 120° clockwise rotation of the blue triangle about the origin. *Describe* and correct the error.



25. ★ **WRITING** Can a point or a line segment be its own image under a transformation? *Explain* and illustrate your answer.

APPLYING TRANSLATIONS **Complete the statement using the description of the translation. In the description, points (0, 3) and (2, 5) are two vertices of a hexagon.**

26. If (0, 3) translates to (0, 0), then (2, 5) translates to __?__.

27. If (0, 3) translates to (1, 2), then (2, 5) translates to __?__.

28. If (0, 3) translates to (−3, −2), then (2, 5) translates to __?__.

ALGEBRA **A point on an image and the translation are given. Find the corresponding point on the original figure.**

29. Point on image: (4, 0); translation: $(x, y) \rightarrow (x + 2, y - 3)$

30. Point on image: (−3, 5); translation: $(x, y) \rightarrow (-x, y)$

31. Point on image: (6, −9); translation: $(x, y) \rightarrow (x - 7, y - 4)$

32. **CONGRUENCE** Show that the transformation in Exercise 3 is a congruence transformation.

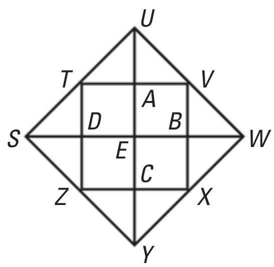
DESCRIBING AN IMAGE **State the segment or triangle that represents the image. You can use tracing paper to help you see the rotation.**

33. 90° clockwise rotation of $\overline{ST}$ about E

34. 90° counterclockwise rotation of $\overline{BX}$ about E

35. 180° rotation of $\triangle BWX$ about E

36. 180° rotation of $\triangle TUA$ about E



37. **CHALLENGE** Solve for the variables in the transformation of $\overline{AB}$ to $\overline{CD}$ and then to $\overline{EF}$.

$A(2, 3)$, $B(4, 2a)$ → Translation: $(x, y) \rightarrow (x - 2, y + 1)$ → $C(m - 3, 4)$, $D(n - 9, 5)$ → Reflection: in x-axis → $E(0, g - 6)$, $F(8h, -5)$

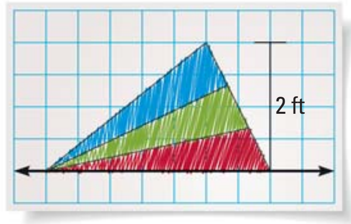
PROBLEM SOLVING

EXAMPLE 3
on p. 274
for Ex. 38

38. KITES The design for a kite shows the layout and dimensions for only half of the kite.

a. What type of transformation can a designer use to create plans for the entire kite?

b. What is the maximum width of the entire kite?



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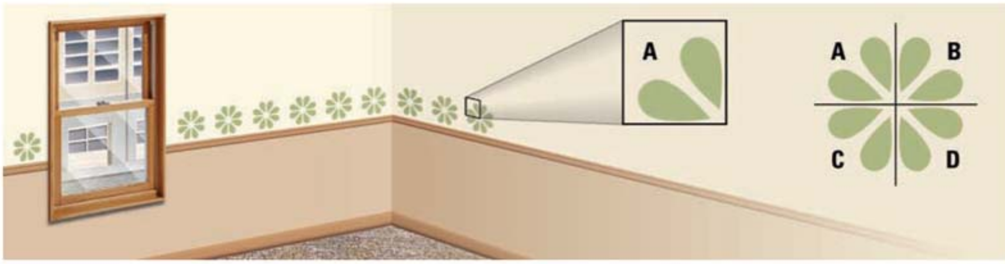
39. STENCILING You are stenciling a room in your home. You want to use the stencil pattern below on the left to create the design shown. Give the angles and directions of rotation you will use to move the stencil from *A* to *B* and from *A* to *C*.



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40. ★ **OPEN-ENDED MATH** Some words reflect onto themselves through a vertical line of reflection. An example is shown.

a. Find two other words with vertical lines of reflection. Draw the line of reflection for each word.

b. Find two words with horizontal lines of reflection. Draw the line of reflection for each word.

41. ★ **SHORT RESPONSE** In chess, six different kinds of pieces are moved according to individual rules. The Knight (shaped like a horse) moves in an "L" shape. It moves two squares horizontally or vertically and then one additional square perpendicular to its original direction. When a knight lands on a square with another piece, it *captures* that piece.

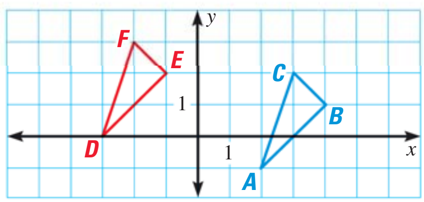
a. *Describe* the translation used by the Black Knight to capture the White Pawn.

b. *Describe* the translation used by the White Knight to capture the Black Pawn.

c. After both pawns are captured, can the Black Knight capture the White Knight? *Explain*.

EXAMPLE 5
on p. 275
for Ex. 42

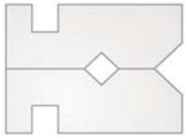
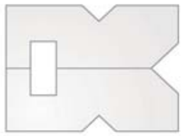
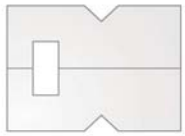
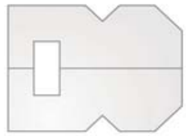
42. VERIFYING CONGRUENCE Show that $\triangle ABC$ and $\triangle DEF$ are right triangles and use the HL Congruence Theorem to verify that $\triangle DEF$ is a congruence transformation of $\triangle ABC$.



○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

43. ★ **MULTIPLE CHOICE** A piece of paper is folded in half and some cuts are made, as shown. Which figure represents the unfolded piece of paper?

Ⓐ  Ⓑ  Ⓒ  Ⓓ 

44. **CHALLENGE** A triangle is rotated 90° counterclockwise and then translated three units up. The vertices of the final image are $A(-4, 4)$, $B(-1, 6)$, and $C(-1, 4)$. Find the vertices of the original triangle. Would the final image be the same if the original triangle was translated 3 units up and then rotated 90° counterclockwise? *Explain* your reasoning.

MIXED REVIEW

PREVIEW
Prepare for Lesson 5.1 in Exs. 45–50.

Simplify the expression. Variables *a* and *b* are positive.

45. $\dfrac{-a-0}{0-(-b)}$ *(p. 870)* **46.** $|(a+b)-a|$ *(p. 870)* **47.** $\dfrac{2a+2b}{2}$ *(p. 139)*

Simplify the expression. Variables *a* and *b* are positive. *(p. 139)*

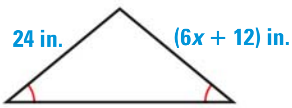
48. $\sqrt{(-b)^2}$ **49.** $\sqrt{(2a)^2}$ **50.** $\sqrt{(2a-a)^2+(0-b)^2}$

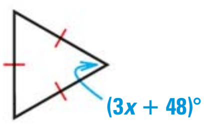
51. Use the SSS Congruence Postulate to show $\triangle RST \cong \triangle UVW$. *(p. 234)*

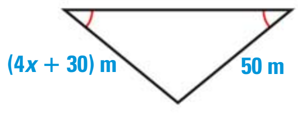
$R(1, -4)$, $S(1, -1)$, $T(6, -1)$ $U(1, 4)$, $V(1, 1)$, $W(6, 1)$

QUIZ for Lessons 4.7–4.8

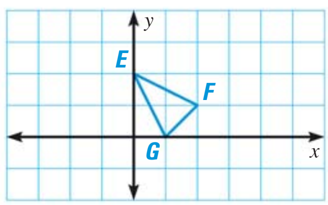
Find the value of *x*. *(p. 264)*

1. 

2. 

3. 

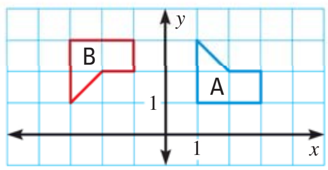
Copy $\triangle EFG$ and draw its image after the transformation. Identify the type of transformation. *(p. 272)*



4. $(x, y) \to (x+4, y-1)$ **5.** $(x, y) \to (-x, y)$

6. $(x, y) \to (x, -y)$ **7.** $(x, y) \to (x-3, y+2)$

8. Is Figure B a rotation of Figure A about the origin? If so, give the angle and direction of rotation. *(p. 272)*



MIXED REVIEW of Problem Solving

STATE TEST PRACTICE
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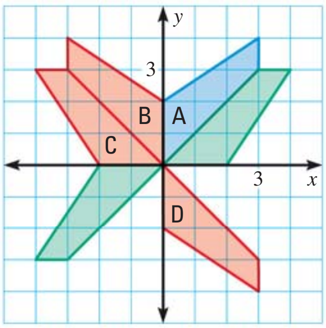
Lessons 4.5–4.8

1. **MULTI-STEP PROBLEM** Use the quilt pattern shown below.



   a. Figure B is the image of Figure A. Name and *describe* the transformation.

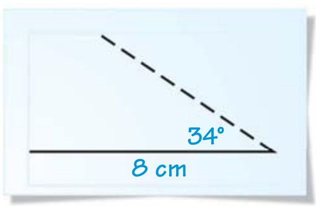
   b. Figure C is the image of Figure A. Name and *describe* the transformation.

   c. Figure D is the image of Figure A. Name and *describe* the transformation.

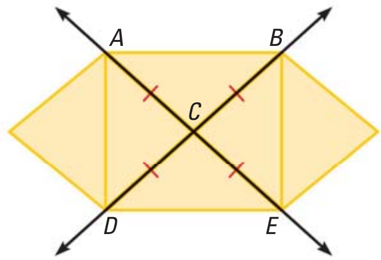
   d. *Explain* how you could complete the quilt pattern using transformations of Figure A.

2. **SHORT RESPONSE** You are told that a triangle has sides that are 5 centimeters and 3 centimeters long. You are also told that the side that is 5 centimeters long forms an angle with the third side that measures 28°. Is there only one triangle that has these given dimensions? *Explain* why or why not.

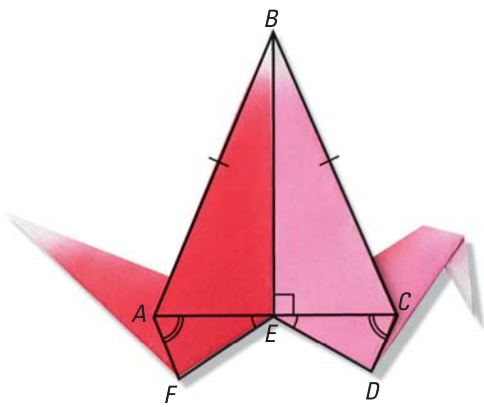
3. **OPEN-ENDED** A friend has drawn a triangle on a piece of paper and she is describing the triangle so that you can draw one that is congruent to hers. So far, she has told you that the length of one side is 8 centimeters and one of the angles formed with this side is 34°. *Describe* three pieces of additional information you could use to construct the triangle.



4. **SHORT RESPONSE** Can the triangles ACD and BCE be proven congruent using the information given in the diagram? Can you show that $\overline{AD} \cong \overline{BE}$? *Explain*.



5. **EXTENDED RESPONSE** Use the information given in the diagram to prove the statements below.



   a. Prove that $\angle BCE \cong \angle BAE$.

   b. Prove that $\overline{AF} \cong \overline{CD}$.

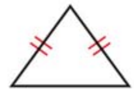
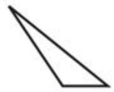
6. **GRIDDED ANSWER** Find the value of x in the diagram.

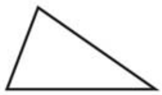
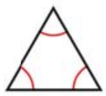
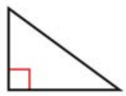
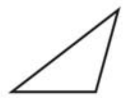
4 CHAPTER SUMMARY

BIG IDEAS — *For Your Notebook*

Big Idea 1

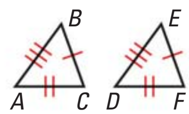
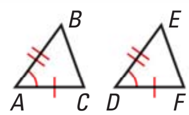
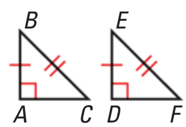
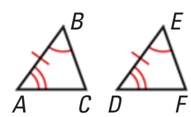
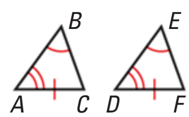
Classifying Triangles by Sides and Angles

	Equilateral	Isosceles	Scalene
Sides			
	3 congruent sides	2 or 3 congruent sides	No congruent sides

	Acute	Equiangular	Right	Obtuse
Angles				
	3 angles $< 90°$	3 angles $= 60°$	1 angle $= 90°$	1 angle $> 90°$

Big Idea 2

Proving That Triangles Are Congruent

SSS	All three sides are congruent.	$\triangle ABC \cong \triangle DEF$	
SAS	Two sides and the included angle are congruent.	$\triangle ABC \cong \triangle DEF$	
HL	The hypotenuse and one of the legs are congruent. (Right triangles only)	$\triangle ABC \cong \triangle DEF$	
ASA	Two angles and the included side are congruent.	$\triangle ABC \cong \triangle DEF$	
AAS	Two angles and a (non-included) side are congruent.	$\triangle ABC \cong \triangle DEF$	

Big Idea 3

Using Coordinate Geometry to Investigate Triangle Relationships

You can use the Distance and Midpoint Formulas to apply postulates and theorems to triangles in the coordinate plane.

4 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- triangle, *p. 217*
  scalene, isosceles, equilateral, acute, right, obtuse, equiangular
- interior angles, *p. 218*
- exterior angles, *p. 218*
- corollary to a theorem, *p. 220*
- congruent figures, *p. 225*
- corresponding parts, *p. 225*
- right triangle, *p. 241*
  legs, hypotenuse
- flow proof, *p. 250*
- isosceles triangle, *p. 264*
  legs, vertex angle, base, base angles
- transformation, *p. 272*
- image, *p. 272*
- congruence transformation, *p. 272*
  translation, reflection, rotation

VOCABULARY EXERCISES

1. Copy and complete: A triangle with three congruent angles is called __?__.
2. **WRITING** *Compare* vertex angles and base angles.
3. **WRITING** *Describe* the difference between isosceles and scalene triangles.
4. Sketch an acute scalene triangle. Label its interior angles 1, 2, and 3. Then draw and shade its exterior angles.
5. If $\triangle PQR \cong \triangle LMN$, which angles are corresponding angles? Which sides are corresponding sides?

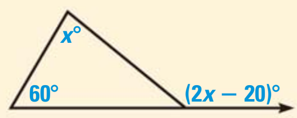
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 4.

4.1 Apply Triangle Sum Properties *pp. 217–224*

EXAMPLE

Find the measure of the exterior angle shown.



Use the Exterior Angle Theorem to write and solve an equation to find the value of x.

$(2x - 20)° = 60° + x°$ **Apply the Exterior Angle Theorem.**

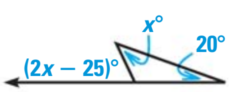
$x = 80$ **Solve for x.**

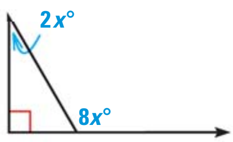
The measure of the exterior angle is $(2 \cdot 80 - 20)°$, or $140°$.

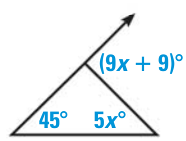
EXERCISES

EXAMPLE 3 on p. 219 for Exs. 6–8

Find the measure of the exterior angle shown.

6. 

7. 

8. 

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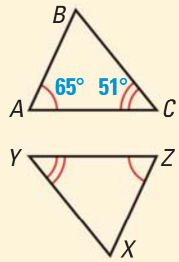
Chapter Review Practice

4.2 Apply Congruence and Triangles

pp. 225–231

EXAMPLE

Use the Third Angles Theorem to find $m\angle X$.



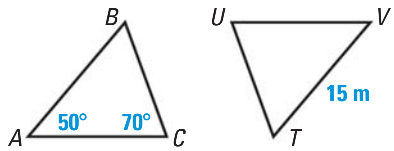
In the diagram, $\angle A \cong \angle Z$ and $\angle C \cong \angle Y$. By the Third Angles Theorem, $\angle B \cong \angle X$. Then by the Triangle Sum Theorem, $m\angle B = 180° - 65° - 51° = 64°$.

So, $m\angle X = m\angle B = 64°$ by the definition of congruent angles.

EXERCISES

EXAMPLES 2 and 4
on pp. 226–227
for Exs. 9–14

In the diagram, $\triangle ABC \cong \triangle VTU$. Find the indicated measure.



9. $m\angle B$

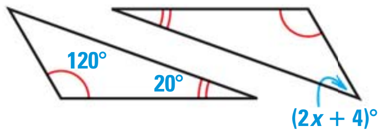
10. AB

11. $m\angle T$

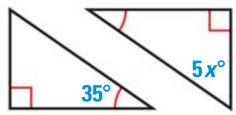
12. $m\angle V$

Find the value of x.

13.



14.

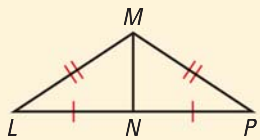


4.3 Prove Triangles Congruent by SSS

pp. 234–239

EXAMPLE

Prove that $\triangle LMN \cong \triangle PMN$.



The marks on the diagram show that $\overline{LM} \cong \overline{PM}$ and $\overline{LN} \cong \overline{PN}$. By the Reflexive Property, $\overline{MN} \cong \overline{MN}$.

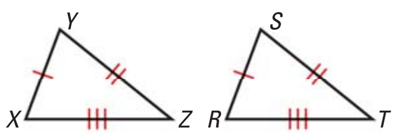
So, by the SSS Congruence Postulate, $\triangle LMN \cong \triangle PMN$.

EXERCISES

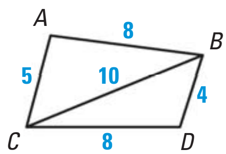
EXAMPLE 1
on p. 234
for Exs. 15–16

Decide whether the congruence statement is true. *Explain* your reasoning.

15. $\triangle XYZ \cong \triangle RST$



16. $\triangle ABC \cong \triangle DCB$



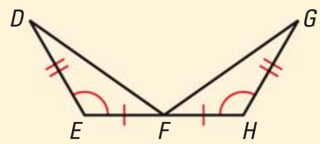
4 CHAPTER REVIEW

4.4 Prove Triangles Congruent by SAS and HL

pp. 240–246

EXAMPLE

Prove that $\triangle DEF \cong \triangle GHF$.



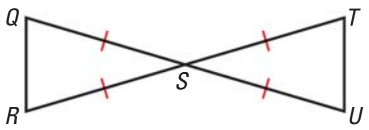
From the diagram, $\overline{DE} \cong \overline{GH}$, $\angle E \cong \angle H$, and $\overline{EF} \cong \overline{HF}$. By the SAS Congruence Postulate, $\triangle DEF \cong \triangle GHF$.

EXERCISES

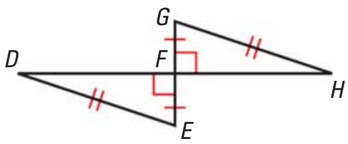
EXAMPLES 1 and 3 on pp. 240, 242 for Exs. 17–18

Decide whether the congruence statement is true. *Explain* your reasoning.

17. $\triangle QRS \cong \triangle TUS$



18. $\triangle DEF \cong \triangle GHF$

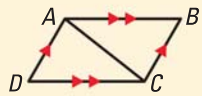


4.5 Prove Triangles Congruent by ASA and AAS

pp. 249–255

EXAMPLE

Prove that $\triangle DAC \cong \triangle BCA$.

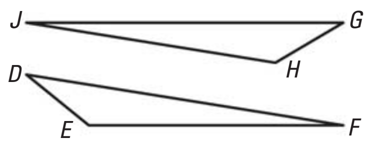


By the Reflexive Property, $\overline{AC} \cong \overline{AC}$. Because $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{DC}$, $\angle DAC \cong \angle BCA$ and $\angle DCA \cong \angle BAC$ by the Alternate Interior Angles Theorem. So, by the ASA Congruence Postulate, $\triangle ADC \cong \triangle ABC$.

EXERCISES

EXAMPLES 1 and 2 on p. 250 for Exs. 19–20

State the third congruence that is needed to prove that $\triangle DEF \cong \triangle GHJ$ using the given postulate or theorem.



19. **GIVEN ▶** $\overline{DE} \cong \overline{GH}$, $\angle D \cong \angle G$, _?_ $\cong$ _?_

Use the AAS Congruence Theorem.

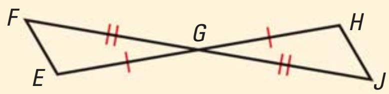
20. **GIVEN ▶** $\overline{DF} \cong \overline{GJ}$, $\angle F \cong \angle J$, _?_ $\cong$ _?_

Use the ASA Congruence Postulate.

4.6 Use Congruent Triangles

pp. 256–263

EXAMPLE



GIVEN ▶ $\overline{FG} \cong \overline{JG}$, $\overline{EG} \cong \overline{HG}$

PROVE ▶ $\overline{EF} \cong \overline{HJ}$

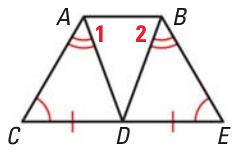
You are given that $\overline{FG} \cong \overline{JG}$ and $\overline{EG} \cong \overline{HG}$. By the Vertical Angles Theorem, $\angle FGE \cong \angle JGH$. So, $\triangle FGE \cong \triangle JGH$ by the SAS Congruence Postulate. Corres. parts of $\cong$ ⧍ are $\cong$, so $\overline{EF} \cong \overline{HJ}$.

EXAMPLE 3
on p. 257
for Exs. 21–23

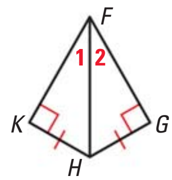
EXERCISES

Write a plan for proving that $\angle 1 \cong \angle 2$.

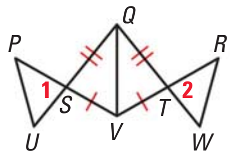
21.



22.



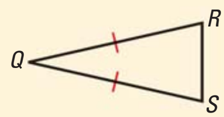
23.



4.7 Use Isosceles and Equilateral Triangles *pp. 264–270*

EXAMPLE

$\triangle QRS$ is isosceles. Name two congruent angles.



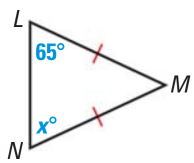
$\overline{QR} \cong \overline{QS}$, so by the Base Angles Theorem, $\angle R \cong \angle S$.

EXAMPLE 3
on p. 266
for Exs. 24–26

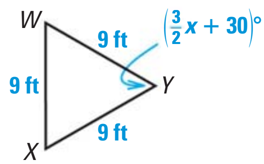
EXERCISES

Find the value of x.

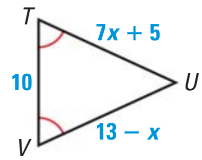
24.



25.



26.



4.8 Perform Congruence Transformations *pp. 272–279*

EXAMPLE

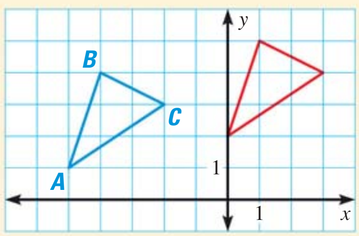
Triangle ABC has vertices $A(-5, 1)$, $B(-4, 4)$, and $C(-2, 3)$. Sketch $\triangle ABC$ and its image after the translation $(x, y) \to (x + 5, y + 1)$.

$(x, y) \to (x + 5, y + 1)$

$A(-5, 1) \to (0, 2)$

$B(-4, 4) \to (1, 5)$

$C(-2, 3) \to (3, 4)$



EXAMPLES 2 and 3
on pp. 273–274
for Exs. 27–29

EXERCISES

Triangle QRS has vertices $Q(2, -1)$, $R(5, -2)$, and $S(2, -3)$. Sketch $\triangle QRS$ and its image after the transformation.

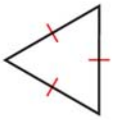
27. $(x, y) \to (x - 1, y + 5)$

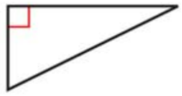
28. $(x, y) \to (x, -y)$

29. $(x, y) \to (-x, -y)$

4 CHAPTER TEST

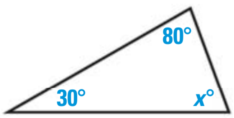
Classify the triangle by its sides and by its angles.

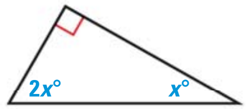
1.


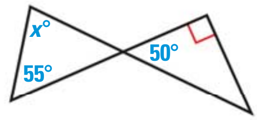
2.


3.

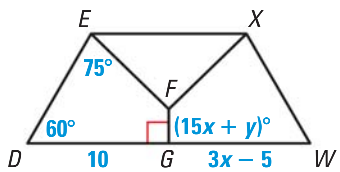

In Exercises 4–6, find the value of *x*.

4.


5.


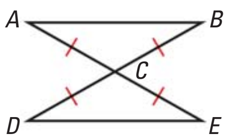
6.


7. In the diagram, $DEFG \cong WXFG$. Find the values of x and y.

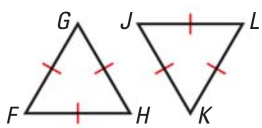


In Exercises 8–10, decide whether the triangles can be proven congruent by the given postulate.

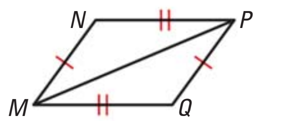
8. $\triangle ABC \cong \triangle EDC$ by SAS



9. $\triangle FGH \cong \triangle JKL$ by ASA



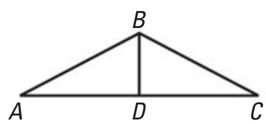
10. $\triangle MNP \cong \triangle PQM$ by SSS



11. Write a proof.

GIVEN ▶ $\triangle ABC$ is isosceles, $\overline{BD}$ bisects $\angle B$.

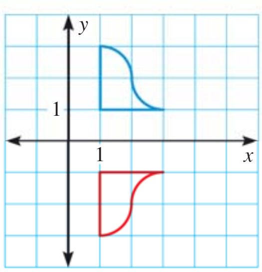
PROVE ▶ $\triangle ABD \cong \triangle CBD$



12. What is the third congruence needed to prove that $\triangle PQR \cong \triangle STU$ using the indicated theorem?

a. HL **b.** AAS

Decide whether the transformation is a *translation, reflection*, or *rotation*.

13.


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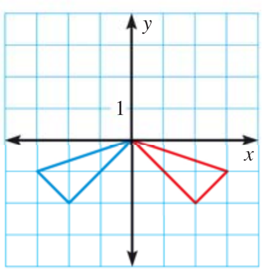

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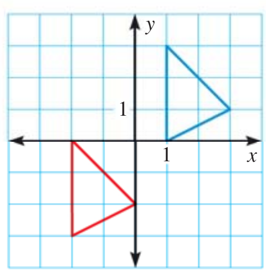

4 ALGEBRA REVIEW

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SOLVE INEQUALITIES AND ABSOLUTE VALUE EQUATIONS

EXAMPLE 1 Solve inequalities

Solve $-3x + 7 \leq 28$. Then graph the solution.

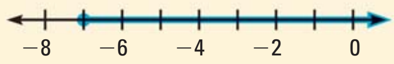
When you multiply or divide each side of an inequality by a *negative* number, you must reverse the inequality symbol to obtain an equivalent inequality.

$-3x + 7 \leq 28$ **Write original inequality.**

$-3x \leq 21$ **Subtract 7 from both sides.**

$x \geq -7$ **Divide each side by -3. Reverse the inequality symbol.**

▶ The solutions are all real numbers greater than or equal to -7. The graph is shown at the right.



EXAMPLE 2 Solve absolute value equations

Solve $|2x + 1| = 5$.

The expression inside the absolute value bars can represent 5 or -5.

STEP 1 Assume $2x + 1$ represents 5.

$2x + 1 = 5$

$2x = 4$

$x = 2$

STEP 2 Assume $2x + 1$ represents -5.

$2x + 1 = -5$

$2x = -6$

$x = -3$

▶ The solutions are 2 and -3.

EXERCISES

EXAMPLE 1 for Exs. 1–12

Solve the inequality. Then graph the solution.

1. $x - 6 > -4$
2. $7 - c \leq -1$
3. $-54 \geq 6x$
4. $\frac{5}{2}t + 8 \leq 33$
5. $3(y + 2) < 3$
6. $\frac{1}{4}z < 2$
7. $5k + 1 \geq -11$
8. $13.6 > -0.8 - 7.2r$
9. $6x + 7 < 2x - 3$
10. $-v + 12 \leq 9 - 2v$
11. $4(n + 5) \geq 5 - n$
12. $5y + 3 \geq 2(y - 9)$

EXAMPLE 2 for Exs. 13–27

Solve the equation.

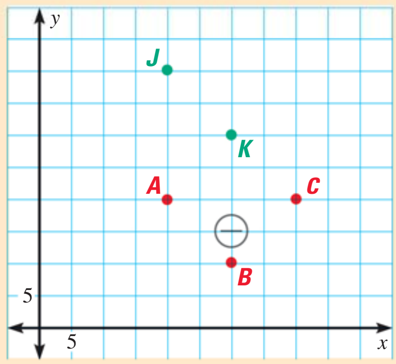
13. $|x - 5| = 3$
14. $|x + 6| = 2$
15. $|4 - x| = 4$
16. $|2 - x| = 0.5$
17. $|3x - 1| = 8$
18. $|4x + 5| = 7$
19. $|x - 1.3| = 2.1$
20. $|3x - 15| = 0$
21. $|6x - 2| = 4$
22. $|8x + 1| = 17$
23. $|9 - 2x| = 19$
24. $|0.5x - 4| = 2$
25. $|5x - 2| = 8$
26. $|7x + 4| = 11$
27. $|3x - 11| = 4$

CONTEXT-BASED MULTIPLE CHOICE QUESTIONS

Some of the information you need to solve a context-based multiple choice question may appear in a table, a diagram, or a graph.

PROBLEM 1

Five of six players on a lacrosse team are set up in a 2-3-1 formation. In this formation, the players form two congruent triangles. Three **attackmen** form one triangle. Three **midfielders** form the second triangle. In the diagram, where should player L stand so that $\triangle ABC \cong \triangle JKL$?



Ⓐ (8, 8) Ⓑ (20, 60)

Ⓒ (40, 40) Ⓓ (30, 15)

Plan

INTERPRET THE GRAPH Use the graph to determine the coordinates of each player. Use the Distance Formula to check the coordinates in the choices.

Solution

STEP 1 Find the coordinates of each vertex.

For $\triangle ABC$, the coordinates are $A(20, 20)$, $B(30, 10)$, and $C(40, 20)$. For $\triangle JKL$, the coordinates are $J(20, 40)$, $K(30, 30)$, and $L(\underline{\ ?\ }, \underline{\ ?\ })$.

STEP 2 Calculate EH and GE.

Because $\triangle ABC \cong \triangle JKL$, $BC = KL$ and $CA = LJ$. Find BC and CA.

By the Distance Formula, $BC = \sqrt{(40-30)^2 + (20-10)^2} = \sqrt{200} = 10\sqrt{2}$ yards.

Also, $CA = \sqrt{(20-40)^2 + (20-20)^2} = \sqrt{400} = 20$ yards.

STEP 3 Check the choices to find the coordinates that produce the congruent.

Check the coordinates given in the choices to see whether $LJ = CA = 20$ yards and $KL = BC = 10\sqrt{2}$ yards. As soon as one set of coordinates does not work for the first side length, you can move to the next set.

Choice A: $L(8, 8)$, so $LJ = \sqrt{(20-8)^2 + (40-8)^2} = 4\sqrt{73} \neq 20$ ✗

Choice B: $L(20, 60)$, so $LJ = \sqrt{(20-20)^2 + (40-60)^2} = \sqrt{400} = 20$ ✓

and $KL = \sqrt{(20-30)^2 + (60-30)^2} = \sqrt{1000} \neq 10\sqrt{2}$ ✗

Choice C: $L(40, 40)$, so $LJ = \sqrt{(20-40)^2 + (40-40)^2} = \sqrt{400} = 20$ ✓

and $KL = \sqrt{(40-30)^2 + (40-30)^2} = \sqrt{200} = 10\sqrt{2}$ ✓

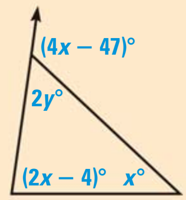
Player L should stand at (40, 40). The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

PROBLEM 2

Use the diagram to find the value of y.

Ⓐ 15.5 Ⓑ 27.5

Ⓒ 43 Ⓓ 82



Plan

INTERPRET THE DIAGRAM All of the angle measures in the diagram are labeled with algebraic expressions. Use what you know about the angles in a triangle to find the value of y.

Solution

STEP 1
Find the value of x.

Use the Exterior Angle Theorem to find the value of x.

$(4x - 47)^\circ = (2x - 4)^\circ + x^\circ$	**Exterior Angle Theorem**
$4x - 47 = 3x - 4$	**Combine like terms.**
$x = 43$	**Solve for x.**

STEP 2
Find the value of y.

Use the Linear Pair Postulate to find the value of y.

$(4x - 47)^\circ + 2y^\circ = 180^\circ$	**Linear Pair Postulate**
$[4(43) - 47] + 2y = 180$	**Substitute 43 for x.**
$125 + 2y = 180$	**Simplify.**
$y = 27.5$	**Solve for y.**

The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

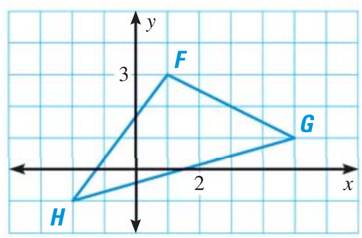
PRACTICE

1. In Problem 2, what are the measures of the interior angles of the triangle?

Ⓐ 27.5°, 43°, 109.5° Ⓑ 27.5°, 51°, 86°

Ⓒ 40°, 60°, 80° Ⓓ 43°, 55°, 82°

2. What are the coordinates of the vertices of the image of $\triangle FGH$ after the translation $(x, y) \rightarrow (x - 2, y + 3)$?



Ⓐ (3, 4), (−4, 4), (−1, 6)

Ⓑ (−2, −1), (1, 3), (5, 1)

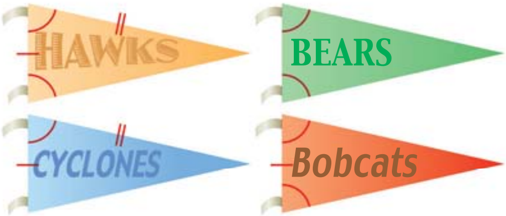
Ⓒ (4, 1), (7, −1), (1, −3)

Ⓓ (−4, 2), (−1, 6), (3, 4)

★ Standardized TEST PRACTICE

MULTIPLE CHOICE

1. A teacher has the pennants shown below. Which pennants can you prove are congruent?



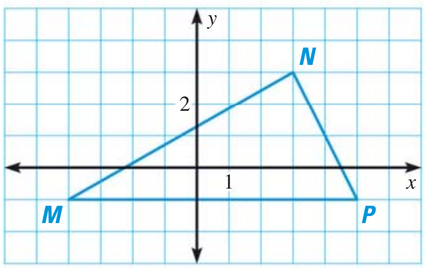
Ⓐ All of the pennants can be proven congruent.

Ⓑ The Hawks, Cyclones, and Bobcats pennants can be proven congruent.

Ⓒ The Bobcats and Bears pennants can be proven congruent.

Ⓓ None of the pennants can be proven congruent.

In Exercises 2 and 3, use the graph below.



2. What type of triangle is $\triangle MNP$?

Ⓐ Scalene

Ⓑ Isosceles

Ⓒ Right

Ⓓ Not enough information

3. Which are the coordinates of point Q such that $\triangle MNP \cong \triangle QPN$?

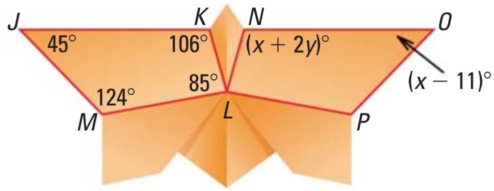
Ⓐ $(0, -3)$

Ⓑ $(-6, 3)$

Ⓒ $(12, 3)$

Ⓓ $(3, -5)$

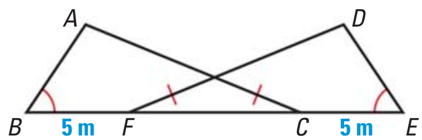
4. The diagram shows the final step in folding an origami butterfly. Use the congruent quadrilaterals, outlined in red, to find the value of $x + y$.



Ⓐ 25 Ⓑ 56

Ⓒ 81 Ⓓ 106

5. Which reason cannot be used to prove that $\angle A \cong \angle D$?



Ⓐ Base Angles Theorem

Ⓑ Segment Addition Postulate

Ⓒ SSS Congruence Postulate

Ⓓ Corresponding parts of congruent triangles are congruent.

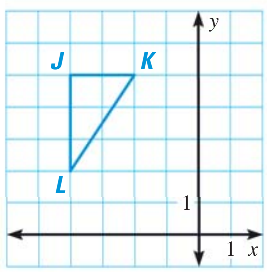
6. Which coordinates are the vertices of a triangle congruent to $\triangle JKL$?

Ⓐ $(-5, 0), (-5, 6), (-1, 6)$

Ⓑ $(-1, -5), (-1, -1), (1, -5)$

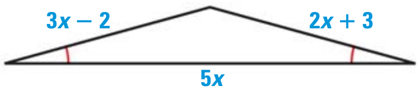
Ⓒ $(2, 1), (2, 3), (5, 1)$

Ⓓ $(4, 6), (6, 6), (6, 4)$



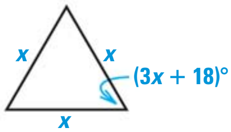
GRIDDED ANSWER

7. What is the perimeter of the triangle?



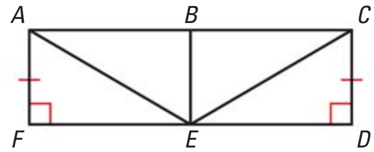
8. Figure $ABCD$ has vertices $A(0, 2)$, $B(-2, -4)$, $C(2, 7)$, and $D(5, 0)$. What is the y-coordinate of the image of vertex B after the translation $(x, y) \rightarrow (x + 8, y - 0.5)$?

9. What is the value of x?



SHORT RESPONSE

10. If $\triangle ABE \cong \triangle EDC$, show that $\triangle EFA \cong \triangle CBE$.



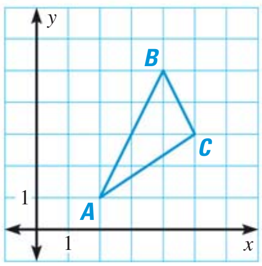
11. Two triangles have the same base and height. Are the triangles congruent? *Justify* your answer using an example.

12. If two people construct wooden frames for a triangular weaving loom using the instructions below, will the frames be congruent triangles? *Explain* your reasoning.

> Construct the frame so that the loom has a 90° angle at the bottom and 45° angles at the two upper corners. The piece of wood at the top should measure 72 inches.

EXTENDED RESPONSE

13. Use the diagram at the right.

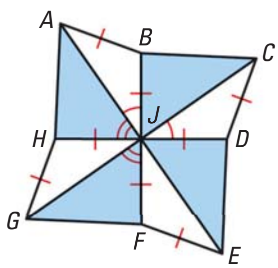


   a. Copy the diagram onto a piece of graph paper. Reflect $\triangle ABC$ in the x-axis.

   b. Copy and complete the table. *Describe* what you notice about the coordinates of the image compared to the coordinates of $\triangle ABC$.

	A	B	C
Coordinates of $\triangle ABC$	?	?	?
Coordinates of image	?	?	?

14. Kylie is designing a quilting pattern using two different fabrics. The diagram shows her progress so far.



   a. Use the markings on the diagram to prove that all of the white triangles are congruent.

   b. Prove that all of the blue triangles are congruent.

   c. Can you prove that the blue triangles are right triangles? *Explain*.