5 Relationships within Triangles

5.1 Midsegment Theorem and Coordinate Proof

5.2 Use Perpendicular Bisectors

5.3 Use Angle Bisectors of Triangles

5.4 Use Medians and Altitudes

5.5 Use Inequalities in a Triangle

5.6 Inequalities in Two Triangles and Indirect Proof

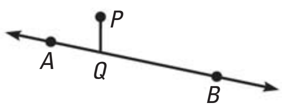
Before

In previous courses and in Chapters 1–4, you learned the following skills, which you'll use in Chapter 5: simplifying expressions, finding distances and slopes, using properties of triangles, and solving equations and inequalities.

Prerequisite Skills

VOCABULARY CHECK

1. Is the *distance from point P to line AB* equal to the length of $\overline{PQ}$? *Explain* why or why not.



SKILLS AND ALGEBRA CHECK

Simplify the expression. All variables are positive. *(Review pp. 139, 870 for 5.1.)*

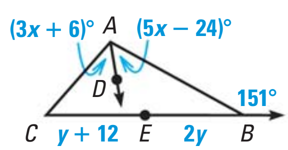
2. $\sqrt{(0-h)^2}$ **3.** $\frac{2m+2n}{2}$ **4.** $|(x+a)-a|$ **5.** $\sqrt{r^2+r^2}$

$\triangle PQR$ has the given vertices. Graph the triangle and classify it by its sides. Then determine if it is a right triangle. *(Review p. 217 for 5.1, 5.4.)*

6. $P(2, 0)$, $Q(6, 6)$, and $R(12, 2)$ **7.** $P(2, 3)$, $Q(4, 7)$, and $R(11, 3)$

Ray AD bisects $\angle BAC$ and point E bisects $\overline{CB}$. Find the measurement. *(Review pp. 15, 24, 217 for 5.2, 5.3, 5.5.)*

8. CE **9.** $m\angle BAC$ **10.** $m\angle ACB$



Solve. *(Review pp. 287, 882 for 5.3, 5.5.)*

11. $x^2 + 24^2 = 26^2$ **12.** $48 + x^2 = 60$ **13.** $43 > x + 35$

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 5, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 343. You will also use the key vocabulary listed below.

Big Ideas

1. **Using properties of special segments in triangles**
2. **Using triangle inequalities to determine what triangles are possible**
3. **Extending methods for justifying and proving relationships**

KEY VOCABULARY

- midsegment of a triangle, *p. 295*
- coordinate proof, *p. 296*
- perpendicular bisector, *p. 303*
- equidistant, *p. 303*
- point of concurrency, *p. 305*
- circumcenter, *p. 306*
- incenter, *p. 312*
- median of a triangle, *p. 319*
- centroid, *p. 319*
- altitude of a triangle, *p. 320*
- orthocenter, *p. 321*
- indirect proof, *p. 337*

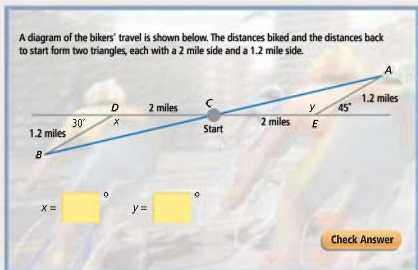
Why?

You can use triangle relationships to find and compare angle measures and distances. For example, if two sides of a triangle represent travel along two roads, then the third side represents the distance back to the starting point.

Animated Geometry

The animation illustrated below for Example 2 on page 336 helps you answer this question: After taking different routes, which group of bikers is farther from the camp?

Two groups of bikers head out from the same point and use different routes.



Enter values for *x* and *y*. Predict which bikers are farther from the start.

Geometry at classzone.com

Animated Geometry at classzone.com

Other animations for Chapter 5: pages 296, 304, 312, 321, and 330

5.1 Investigate Segments in Triangles

MATERIALS • graph paper • ruler • pencil

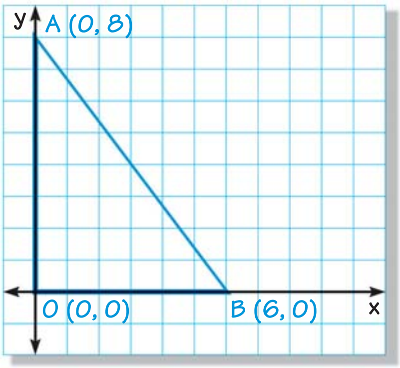
QUESTION **How are the midsegments of a triangle related to the sides of the triangle?**

A *midsegment* of a triangle connects the midpoints of two sides of a triangle.

EXPLORE **Draw and find a midsegment**

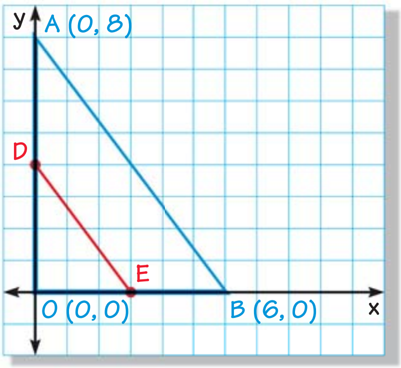
STEP 1 ***Draw a right triangle***

Draw a right triangle with legs on the x-axis and the y-axis. Use vertices $A(0, 8)$, $B(6, 0)$, and $O(0, 0)$ as Case 1.



STEP 2 ***Draw the midsegment***

Find the midpoints of $\overline{OA}$ and $\overline{OB}$. Plot the midpoints and label them D and E. Connect them to create the midsegment $\overline{DE}$.



STEP 3 ***Make a table***

Draw the Case 2 triangle below. Copy and complete the table.

	Case 1	Case 2
O	(0, 0)	(0, 0)
A	(0, 8)	(0, 11)
B	(6, 0)	(5, 0)
D	?	?
E	?	?
Slope of $\overline{AB}$	?	?
Slope of $\overline{DE}$	?	?
Length of $\overline{AB}$	?	?
Length of $\overline{DE}$	?	?

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Choose two other right triangles with legs on the axes. Add these triangles as Cases 3 and 4 to your table.
2. Expand your table in Step 3 for Case 5 with $A(0, n)$, $B(k, 0)$, and $O(0, 0)$.
3. Expand your table in Step 3 for Case 6 with $A(0, 2n)$, $B(2k, 0)$, and $O(0, 0)$.
4. What do you notice about the slopes of $\overline{AB}$ and $\overline{DE}$? What do you notice about the lengths of $\overline{AB}$ and $\overline{DE}$?
5. In each case, is the midsegment $\overline{DE}$ parallel to $\overline{AB}$? *Explain.*
6. Are your observations true for the midsegment created by connecting the midpoints of $\overline{OA}$ and $\overline{AB}$? What about the midsegment connecting the midpoints of $\overline{AB}$ and $\overline{OB}$?
7. Make a conjecture about the relationship between a midsegment and a side of the triangle. Test your conjecture using an acute triangle.

5.1 Midsegment Theorem and Coordinate Proof

Before You used coordinates to show properties of figures.

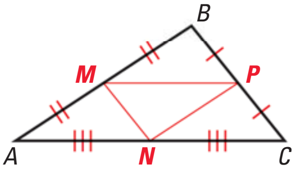
Now You will use properties of midsegments and write coordinate proofs.

Why? So you can use indirect measure to find a height, as in Ex. 35.

Key Vocabulary
- **midsegment of a triangle**
- **coordinate proof**

A **midsegment of a triangle** is a segment that connects the midpoints of two sides of the triangle. Every triangle has three midsegments.

The midsegments of $\triangle ABC$ at the right are $\overline{MP}$, $\overline{MN}$, and $\overline{NP}$.

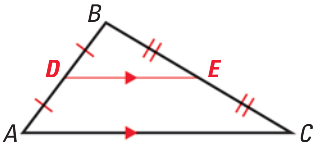


THEOREM — *For Your Notebook*

THEOREM 5.1 Midsegment Theorem

The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long as that side.

Proof: Example 5, p. 297; Ex. 41, p. 300

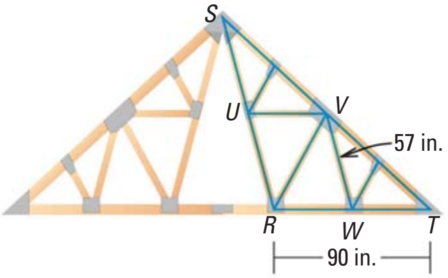


$\overline{DE} \parallel \overline{AC}$ and $DE = \frac{1}{2}AC$

EXAMPLE 1 Use the Midsegment Theorem to find lengths

READ DIAGRAMS
In the diagram for Example 1, midsegment $\overline{UV}$ can be called "the midsegment opposite $\overline{RT}$."

CONSTRUCTION Triangles are used for strength in roof trusses. In the diagram, $\overline{UV}$ and $\overline{VW}$ are midsegments of $\triangle RST$. Find UV and RS.



Solution

$$UV = \frac{1}{2} \cdot RT = \frac{1}{2}(90 \text{ in.}) = 45 \text{ in.}$$

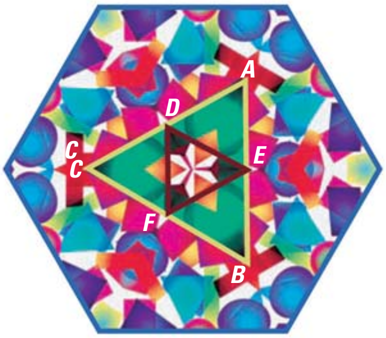
$$RS = 2 \cdot VW = 2(57 \text{ in.}) = 114 \text{ in.}$$

✓ GUIDED PRACTICE for Example 1

1. Copy the diagram in Example 1. Draw and name the third midsegment.
2. In Example 1, suppose the distance UW is 81 inches. Find VS.

EXAMPLE 2 Use the Midsegment Theorem

In the kaleidoscope image, $\overline{AE} \cong \overline{BE}$ and $\overline{AD} \cong \overline{CD}$. Show that $\overline{CB} \parallel \overline{DE}$.



Solution

Because $\overline{AE} \cong \overline{BE}$ and $\overline{AD} \cong \overline{CD}$, E is the midpoint of $\overline{AB}$ and D is the midpoint of $\overline{AC}$ by definition. Then $\overline{DE}$ is a midsegment of $\triangle ABC$ by definition and $\overline{CB} \parallel \overline{DE}$ by the Midsegment Theorem.

COORDINATE PROOF A **coordinate proof** involves placing geometric figures in a coordinate plane. When you use variables to represent the coordinates of a figure in a coordinate proof, the results are true for all figures of that type.

EXAMPLE 3 Place a figure in a coordinate plane

Place each figure in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex.

a. A rectangle

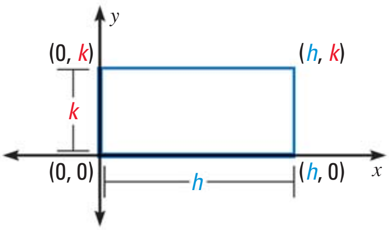
b. A scalene triangle

Solution

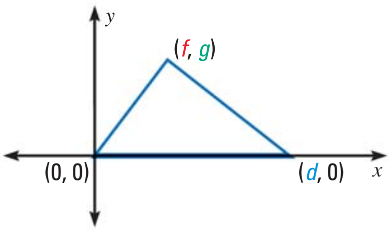
It is easy to find lengths of horizontal and vertical segments and distances from (0, 0), so place one vertex at the origin and one or more sides on an axis.

USE VARIABLES
The rectangle shown represents a general rectangle because the choice of coordinates is based only on the definition of a rectangle. If you use this rectangle to prove a result, the result will be true for all rectangles.

a. Let h represent the length and k represent the width.



b. Notice that you need to use three different variables.



AnimatedGeometry at classzone.com

✓ GUIDED PRACTICE for Examples 2 and 3

3. In Example 2, if F is the midpoint of $\overline{CB}$, what do you know about $\overline{DF}$?

4. Show another way to place the rectangle in part (a) of Example 3 that is convenient for finding side lengths. Assign new coordinates.

5. Is it possible to find any of the side lengths in part (b) of Example 3 without using the Distance Formula? *Explain.*

6. A square has vertices (0, 0), $(m, 0)$, and $(0, m)$. Find the fourth vertex.

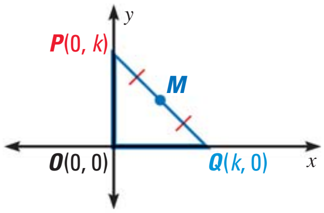
EXAMPLE 4 Apply variable coordinates

Place an isosceles right triangle in a coordinate plane. Then find the length of the hypotenuse and the coordinates of its midpoint *M*.

ANOTHER WAY
For an alternative method for solving the problem in Example 4, turn to page 302 for the **Problem Solving Workshop**.

Solution

Place $\triangle PQO$ with the right angle at the origin. Let the length of the legs be k. Then the vertices are located at $P(0, k)$, $Q(k, 0)$, and $O(0, 0)$.



Use the Distance Formula to find PQ.

$$PQ = \sqrt{(k - 0)^2 + (0 - k)^2} = \sqrt{k^2 + (-k)^2} = \sqrt{k^2 + k^2} = \sqrt{2k^2} = k\sqrt{2}$$

Use the Midpoint Formula to find the midpoint M of the hypotenuse.

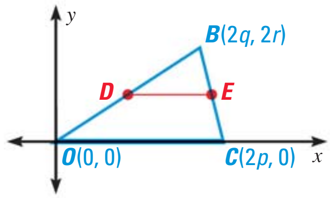
$$M\left(\frac{0 + k}{2}, \frac{k + 0}{2}\right) = M\left(\frac{k}{2}, \frac{k}{2}\right)$$

EXAMPLE 5 Prove the Midsegment Theorem

Write a coordinate proof of the Midsegment Theorem for one midsegment.

GIVEN ▶ $\overline{DE}$ is a midsegment of $\triangle OBC$.

PROVE ▶ $\overline{DE} \parallel \overline{OC}$ and $DE = \frac{1}{2}OC$



Solution

WRITE PROOFS
You can often assign coordinates in several ways, so choose a way that makes computation easier. In Example 5, you can avoid fractions by using $2p$, $2q$, and $2r$.

STEP 1 **Place** $\triangle OBC$ and assign coordinates. Because you are finding midpoints, use $2p$, $2q$, and $2r$. Then find the coordinates of D and E.

$$D\left(\frac{2q + 0}{2}, \frac{2r + 0}{2}\right) = D(q, r) \qquad E\left(\frac{2q + 2p}{2}, \frac{2r + 0}{2}\right) = E(q + p, r)$$

STEP 2 **Prove** $\overline{DE} \parallel \overline{OC}$. The y-coordinates of D and E are the same, so $\overline{DE}$ has a slope of 0. $\overline{OC}$ is on the x-axis, so its slope is 0.

▶ Because their slopes are the same, $\overline{DE} \parallel \overline{OC}$.

STEP 3 **Prove** $DE = \frac{1}{2}OC$. Use the Ruler Postulate to find $\overline{DE}$ and $\overline{OC}$.

$$DE = |(q + p) - q| = p \qquad OC = |2p - 0| = 2p$$

▶ So, the length of $\overline{DE}$ is half the length of $\overline{OC}$.

✓ GUIDED PRACTICE for Examples 4 and 5

7. In Example 5, find the coordinates of F, the midpoint of $\overline{OC}$. Then show that $\overline{EF} \parallel \overline{OB}$.

8. Graph the points $O(0, 0)$, $H(m, n)$, and $J(m, 0)$. Is $\triangle OHJ$ a right triangle? Find the side lengths and the coordinates of the midpoint of each side.

5.1 EXERCISES

HOMEWORK KEY

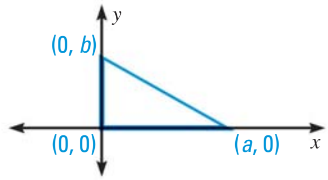
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 21, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 31, and 39

SKILL PRACTICE

1. VOCABULARY Copy and complete: In $\triangle ABC$, D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$. $\overline{DE}$ is a _?_ of $\triangle ABC$.

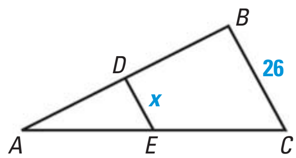
2. ★ WRITING *Explain* why it is convenient to place a right triangle on the grid as shown when writing a coordinate proof. How might you want to relabel the coordinates of the vertices if the proof involves midpoints?



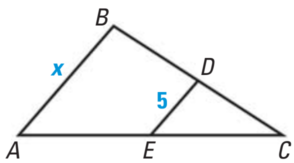
EXAMPLES 1 and 2 on pp. 295–296 for Exs. 3–11

FINDING LENGTHS $\overline{DE}$ **is a midsegment of** $\triangle ABC$**. Find the value of** x**.**

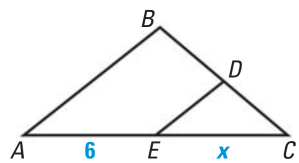
3.



4.



5.



USING THE MIDSEGMENT THEOREM In $\triangle XYZ$, $\overline{XJ} \cong \overline{JY}$, $\overline{YL} \cong \overline{LZ}$, **and** $\overline{XK} \cong \overline{KZ}$**. Copy and complete the statement.**

6. $\overline{JK} \parallel$ _?_

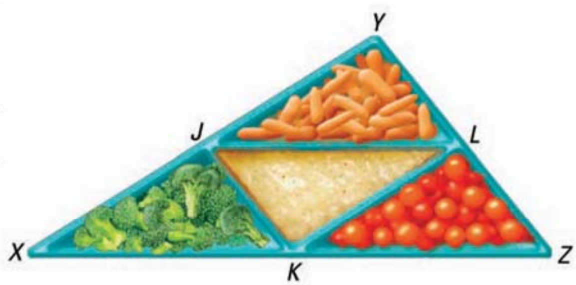
7. $\overline{JL} \parallel$ _?_

8. $\overline{XY} \parallel$ _?_

9. $\overline{YJ} \cong$ _?_ $\cong$ _?_

10. $\overline{JL} \cong$ _?_ $\cong$ _?_

11. $\overline{JK} \cong$ _?_ $\cong$ _?_



EXAMPLE 3 on p. 296 for Exs. 12–19

PLACING FIGURES Place the figure in a coordinate plane in a convenient way. Assign coordinates to each vertex.

12. Right triangle: leg lengths are 3 units and 2 units

13. Isosceles right triangle: leg length is 7 units

14. Square: side length is 3 units

15. Scalene triangle: one side length is $2m$

16. Rectangle: length is a and width is b

17. Square: side length is s

18. Isosceles right triangle: leg length is p

19. Right triangle: leg lengths are r and s

EXAMPLES 4 and 5 on p. 297 for Exs. 20–23

20. COMPARING METHODS Find the length of the hypotenuse in Exercise 19. Then place the triangle another way and use the new coordinates to find the length of the hypotenuse. Do you get the same result?

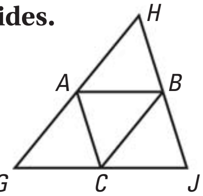
APPLYING VARIABLE COORDINATES Sketch $\triangle ABC$**. Find the length and the slope of each side. Then find the coordinates of each midpoint. Is** $\triangle ABC$ **a right triangle? Is it isosceles?** ***Explain.*** **(Assume all variables are positive,** $p \neq q$**, and** $m \neq n$**.)**

21. $A(0, 0)$, $B(p, q)$, $C(2p, 0)$

22. $A(0, 0)$, $B(h, h)$, $C(2h, 0)$

23. $A(0, n)$, $B(m, n)$, $C(m, 0)$

ALGEBRA **Use $\triangle GHJ$, where A, B, and C are midpoints of the sides.**

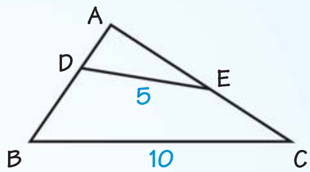


24. If $AB = 3x + 8$ and $GJ = 2x + 24$, what is AB?

25. If $AC = 3y - 5$ and $HJ = 4y + 2$, what is HB?

26. If $GH = 7z - 1$ and $BC = 4z - 3$, what is GH?

27. **ERROR ANALYSIS** *Explain* why the conclusion is incorrect.

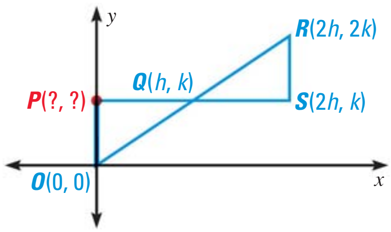


$DE = \frac{1}{2}BC$, so by the Midsegment Theorem $\overline{AD} \cong \overline{DB}$ and $\overline{AE} \cong \overline{EC}$.

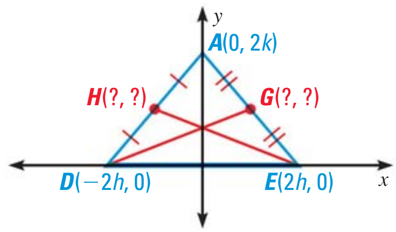
28. **FINDING PERIMETER** The midpoints of the three sides of a triangle are $P(2, 0)$, $Q(7, 12)$, and $R(16, 0)$. Find the length of each midsegment and the perimeter of $\triangle PQR$. Then find the perimeter of the original triangle.

APPLYING VARIABLE COORDINATES **Find the coordinates of the red point(s) in the figure. Then show that the given statement is true.**

29. $\triangle OPQ \cong \triangle RSQ$



30. slope of $\overline{HE} = -$(slope of $\overline{DG}$)



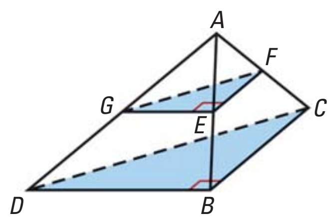
31. ★ **MULTIPLE CHOICE** A rectangle with side lengths $3h$ and k has a vertex at $(-h, k)$. Which point *cannot* be a vertex of the rectangle?

Ⓐ (h, k) Ⓑ $(-h, 0)$ Ⓒ $(2h, 0)$ Ⓓ $(2h, k)$

32. **RECONSTRUCTING A TRIANGLE** The points $T(2, 1)$, $U(4, 5)$, and $V(7, 4)$ are the midpoints of the sides of a triangle. Graph the three midsegments. Then show how to use your graph and the properties of midsegments to draw the original triangle. Give the coordinates of each vertex.

33. **3-D FIGURES** Points A, B, C, and D are the vertices of a *tetrahedron* (a solid bounded by four triangles). $\overline{EF}$ is a midsegment of $\triangle ABC$, $\overline{GE}$ is a midsegment of $\triangle ABD$, and $\overline{FG}$ is a midsegment of $\triangle ACD$.

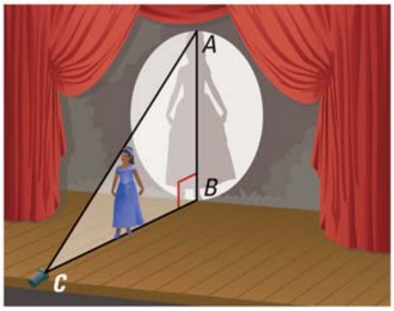
Show that Area of $\triangle EFG = \frac{1}{4} \cdot$ Area of $\triangle BCD$.



34. **CHALLENGE** In $\triangle PQR$, the midpoint of $\overline{PQ}$ is $K(4, 12)$, the midpoint of $\overline{QR}$ is $L(5, 15)$, and the midpoint of $\overline{PR}$ is $M(6.4, 10.8)$. Show how to find the vertices of $\triangle PQR$. *Compare* your work for this exercise with your work for Exercise 32. How were your methods different?

PROBLEM SOLVING

35. FLOODLIGHTS A floodlight on the edge of the stage shines upward onto the curtain as shown. Constance is 5 feet tall. She stands halfway between the light and the curtain, and the top of her head is at the midpoint of $\overline{AC}$. The edge of the light just reaches the top of her head. How tall is her shadow?



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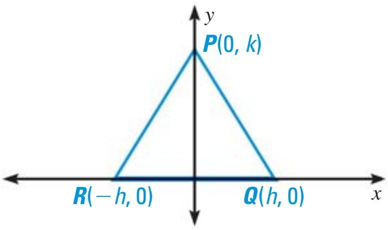
EXAMPLE 5 on p. 297 for Exs. 36–37

COORDINATE PROOF Write a coordinate proof.

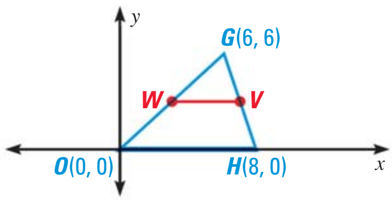
36. GIVEN ▶ $P(0, k)$, $Q(h, 0)$, $R(-h, 0)$

PROVE ▶ $\triangle PQR$ is isosceles.



37. GIVEN ▶ $O(0, 0)$, $G(6, 6)$, $H(8, 0)$, $\overline{WV}$ is a midsegment.

PROVE ▶ $\overline{WV} \parallel \overline{OH}$ and $WV = \frac{1}{2}OH$



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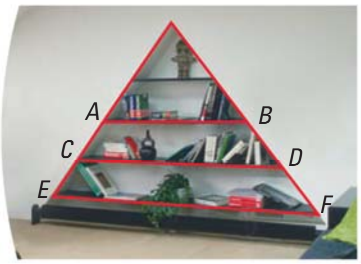
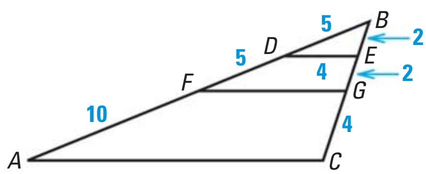
38. CARPENTRY In the set of shelves shown, the third shelf, labeled $\overline{CD}$, is closer to the bottom shelf, $\overline{EF}$, than midsegment $\overline{AB}$ is. If $\overline{EF}$ is 8 feet long, is it possible for $\overline{CD}$ to be 3 feet long? 4 feet long? 6 feet long? 8 feet long? *Explain.*



39. ★ SHORT RESPONSE Use the information in the diagram at the right. What is the length of side $\overline{AC}$ of $\triangle ABC$? *Explain* your reasoning.

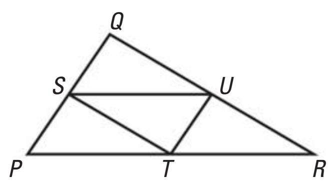


40. PLANNING FOR PROOF Copy and complete the plan for proof.

GIVEN ▶ $\overline{ST}$, $\overline{TU}$, and $\overline{SU}$ are midsegments of $\triangle PQR$.

PROVE ▶ $\triangle PST \cong \triangle SQU$

Use __?__ to show that $\overline{PS} \cong \overline{SQ}$. Use __?__ to show that $\angle QSU \cong \angle SPT$. Use __?__ to show that $\angle$ __?__ $\cong \angle$ __?__. Use __?__ to show that $\triangle PST \cong \triangle SQU$.



41. PROVING THEOREM 5.1 Use the figure in Example 5. Draw the midpoint F of $\overline{OC}$. Prove that $\overline{DF}$ is parallel to $\overline{BC}$ and $DF = \frac{1}{2}BC$.

○ = WORKED-OUT SOLUTIONS on p. WS1

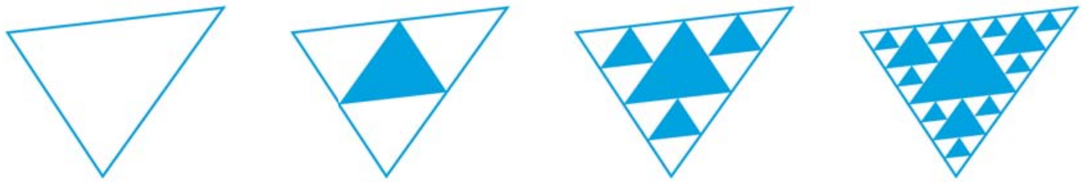
★ = STANDARDIZED TEST PRACTICE

42. COORDINATE PROOF Write a coordinate proof.

GIVEN ▶ $\triangle ABD$ is a right triangle, with the right angle at vertex A. Point C is the midpoint of hypotenuse BD.

PROVE ▶ Point C is the same distance from each vertex of $\triangle ABD$.

43. MULTI-STEP PROBLEM To create the design below, shade the triangle formed by the three midsegments of a triangle. Then repeat the process for each unshaded triangle. Let the perimeter of the original triangle be 1.



Stage 0 **Stage 1** **Stage 2** **Stage 3**

a. What is the perimeter of the triangle that is shaded in Stage 1?

b. What is the total perimeter of all the shaded triangles in Stage 2?

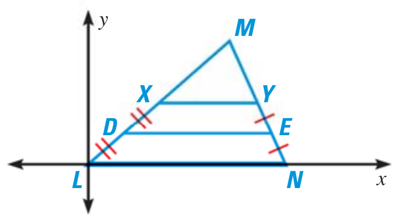
c. What is the total perimeter of all the shaded triangles in Stage 3?

RIGHT ISOSCELES TRIANGLES In Exercises 44 and 45, write a coordinate proof.

44. Any right isosceles triangle can be subdivided into a pair of congruent right isosceles triangles. (*Hint:* Draw the segment from the right angle to the midpoint of the hypotenuse.)

45. Any two congruent right isosceles triangles can be combined to form a single right isosceles triangle.

46. CHALLENGE XY is a midsegment of $\triangle LMN$. Suppose $\overline{DE}$ is called a "quarter-segment" of $\triangle LMN$. What do you think an "eighth-segment" would be? Make a conjecture about the properties of a quarter-segment and of an eighth-segment. Use variable coordinates to verify your conjectures.

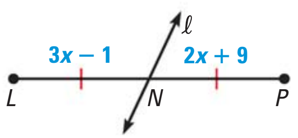


MIXED REVIEW

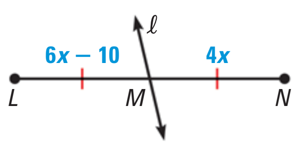
PREVIEW
Prepare for Lesson 5.2 in Exs. 47–49.

Line ℓ bisects the segment. Find LN. *(p. 15)*

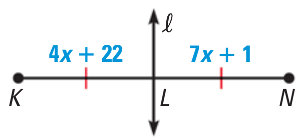
47.



48.

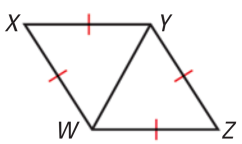


49.

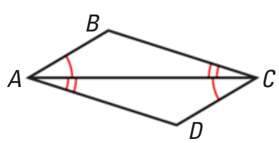


State which postulate or theorem you can use to prove that the triangles are congruent. Then write a congruence statement. *(pp. 225, 249)*

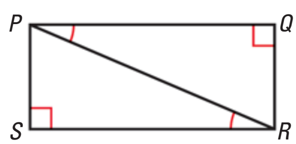
50.



51.



52.



PROBLEM SOLVING WORKSHOP
LESSON 5.1

Using ALTERNATIVE METHODS

Another Way to Solve Example 4, page 297

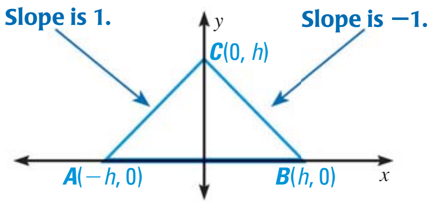
MULTIPLE REPRESENTATIONS When you write a coordinate proof, you often have several options for how to place the figure in the coordinate plane and how to assign variables.

PROBLEM

Place an isosceles right triangle in a coordinate plane. Then find the length of the hypotenuse and the coordinates of its midpoint M.

METHOD

Placing Hypotenuse on an Axis Place the triangle with point C at $(0, h)$ on the y-axis and the hypotenuse $\overline{AB}$ on the x-axis. To make $\angle ACB$ be a right angle, position A and B so that legs $\overline{CA}$ and $\overline{CB}$ have slopes of 1 and -1.



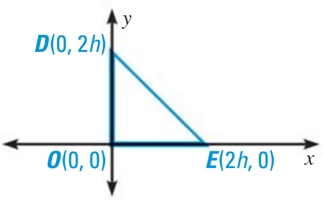
Length of hypotenuse $= 2h$

$$M = \left(\frac{-h+h}{2}, \frac{0+0}{2}\right) = (0, 0)$$

PRACTICE

1. **VERIFYING TRIANGLE PROPERTIES** Verify that $\angle C$ above is a right angle. Verify that $\triangle ABC$ is isosceles by showing $AC = BC$.

2. **MULTIPLES OF 2** Find the midpoint and length of each side using the placement below. What is the advantage of using $2h$ instead of h for the leg lengths?



3. **OTHER ALTERNATIVES** Graph $\triangle JKL$ and verify that it is an isosceles right triangle. Then find the length and midpoint of $\overline{JK}$.

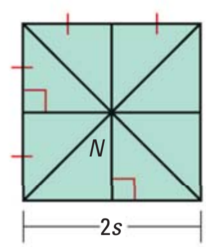
   a. $J(0, 0)$, $K(h, h)$, $L(h, 0)$

   b. $J(-2h, 0)$, $K(2h, 0)$, $L(0, 2h)$

4. **CHOOSE** Suppose you need to place a right isosceles triangle on a coordinate grid and assign variable coordinates. You know you will need to find all three side lengths and all three midpoints. How would you place the triangle? *Explain* your reasoning.

5. **RECTANGLES** Place rectangle $PQRS$ with length m and width n in the coordinate plane. Draw $\overline{PR}$ and $\overline{QS}$ connecting opposite corners of the rectangle. Then use coordinates to show that $\overline{PR} \cong \overline{QS}$.

6. **PARK** A square park has paths as shown. Use coordinates to determine whether a snack cart at point N is the same distance from each corner.



5.2 Use Perpendicular Bisectors

Before You used segment bisectors and perpendicular lines.

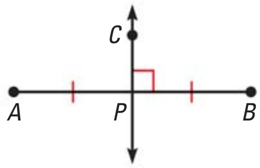
Now You will use perpendicular bisectors to solve problems.

Why? So you can solve a problem in archaeology, as in Ex. 28.

Key Vocabulary
- perpendicular bisector
- equidistant
- concurrent
- point of concurrency
- circumcenter

In Lesson 1.3, you learned that a segment bisector intersects a segment at its midpoint. A segment, ray, line, or plane that is perpendicular to a segment at its midpoint is called a **perpendicular bisector**.

A point is **equidistant** from two figures if the point is the *same distance* from each figure. Points on the perpendicular bisector of a segment are equidistant from the segment's endpoints.



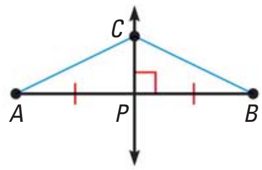
$\overleftrightarrow{CP}$ is a $\perp$ bisector of $\overline{AB}$.

THEOREMS — *For Your Notebook*

THEOREM 5.2 Perpendicular Bisector Theorem

In a plane, if a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

If $\overleftrightarrow{CP}$ is the $\perp$ bisector of $\overline{AB}$, then $CA = CB$.

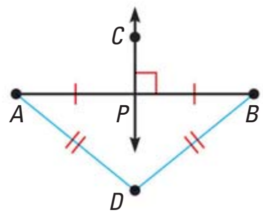


Proof: Ex. 26, p. 308

THEOREM 5.3 Converse of the Perpendicular Bisector Theorem

In a plane, if a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

If $DA = DB$, then D lies on the $\perp$ bisector of $\overline{AB}$.

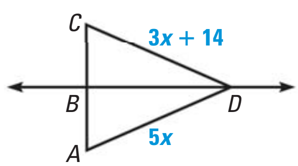


Proof: Ex. 27, p. 308

EXAMPLE 1 Use the Perpendicular Bisector Theorem

xy ALGEBRA $\overleftrightarrow{BD}$ **is the perpendicular bisector of** $\overline{AC}$**. Find** AD**.**

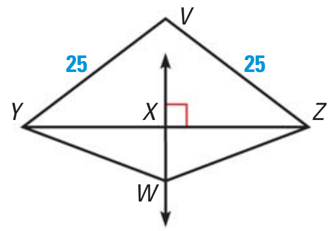
$AD = CD$	Perpendicular Bisector Theorem
$5x = 3x + 14$	Substitute.
$x = 7$	Solve for x.



▶ $AD = 5x = 5(7) = 35.$

EXAMPLE 2 Use perpendicular bisectors

In the diagram, $\overleftrightarrow{WX}$ is the perpendicular bisector of $\overline{YZ}$.



a. What segment lengths in the diagram are equal?

b. Is V on $\overleftrightarrow{WX}$?

Solution

a. $\overleftrightarrow{WX}$ bisects $\overline{YZ}$, so $XY = XZ$. Because W is on the perpendicular bisector of $\overline{YZ}$, $WY = WZ$ by Theorem 5.2. The diagram shows that $VY = VZ = 25$.

b. Because $VY = VZ$, V is equidistant from Y and Z. So, by the Converse of the Perpendicular Bisector Theorem, V is on the perpendicular bisector of $\overline{YZ}$, which is $\overleftrightarrow{WX}$.

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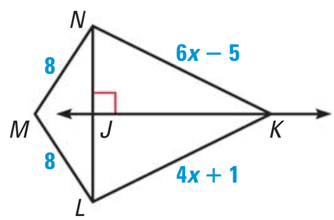
✓ GUIDED PRACTICE for Examples 1 and 2

In the diagram, $\overleftrightarrow{JK}$ is the perpendicular bisector of $\overline{NL}$.



1. What segment lengths are equal? *Explain* your reasoning.
2. Find NK.
3. *Explain* why M is on $\overleftrightarrow{JK}$.

ACTIVITY FOLD THE PERPENDICULAR BISECTORS OF A TRIANGLE

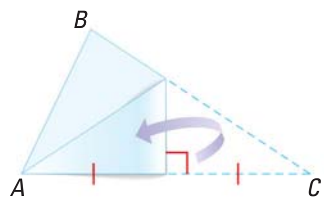
QUESTION Where do the perpendicular bisectors of a triangle meet?

Materials:
- paper
- scissors
- ruler

Follow the steps below and answer the questions about perpendicular bisectors of triangles.

STEP 1 Cut four large acute scalene triangles out of paper. Make each one different.

STEP 2 Choose one triangle. Fold it to form the perpendicular bisectors of the sides. Do the three bisectors intersect at the same point?



STEP 3 Repeat the process for the other three triangles. Make a conjecture about the perpendicular bisectors of a triangle.

STEP 4 Choose one triangle. Label the vertices A, B, and C. Label the point of intersection of the perpendicular bisectors as P. Measure $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$. What do you observe?

CONCURRENCY When three or more lines, rays, or segments intersect in the same point, they are called **concurrent** lines, rays, or segments. The point of intersection of the lines, rays, or segments is called the **point of concurrency**.

READ VOCABULARY
The perpendicular bisector of a side of a triangle can be referred to as a *perpendicular bisector of the triangle.*

As you saw in the Activity on page 304, the three perpendicular bisectors of a triangle are concurrent and the point of concurrency has a special property.

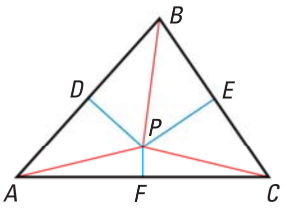
THEOREM *For Your Notebook*

THEOREM 5.4 Concurrency of Perpendicular Bisectors of a Triangle

The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.

If $\overline{PD}$, $\overline{PE}$, and $\overline{PF}$ are perpendicular bisectors, then $PA = PB = PC$.

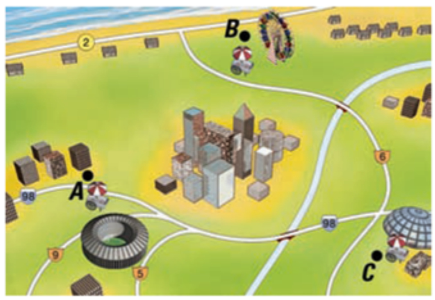
Proof: p. 933



EXAMPLE 3 Use the concurrency of perpendicular bisectors

FROZEN YOGURT Three snack carts sell frozen yogurt from points A, B, and C outside a city. Each of the three carts is the same distance from the frozen yogurt distributor.

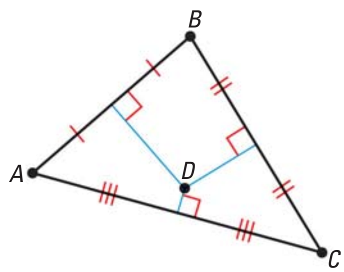
Find a location for the distributor that is equidistant from the three carts.



Solution

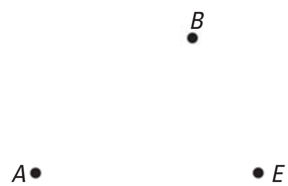
Theorem 5.4 shows you that you can find a point equidistant from three points by using the perpendicular bisectors of the triangle formed by those points.

Copy the positions of points A, B, and C and connect those points to draw $\triangle ABC$. Then use a ruler and protractor to draw the three perpendicular bisectors of $\triangle ABC$. The point of concurrency D is the location of the distributor.



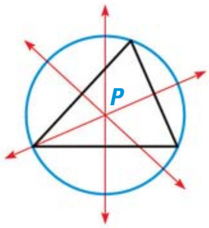
✓ GUIDED PRACTICE for Example 3

4. **WHAT IF?** Hot pretzels are sold from points A and B and also from a cart at point E. Where could the pretzel distributor be located if it is equidistant from those three points? Sketch the triangle and show the location.

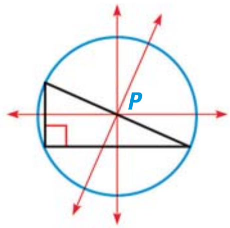


READ VOCABULARY
The prefix *circum-* means "around" or "about" as in *circumference* (distance around a circle).

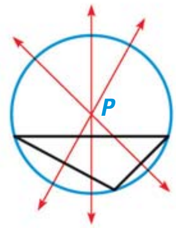
CIRCUMCENTER The point of concurrency of the three perpendicular bisectors of a triangle is called the **circumcenter** of the triangle. The circumcenter P is equidistant from the three vertices, so P is the center of a circle that passes through all three vertices.



Acute triangle
***P* is inside triangle.**



Right triangle
***P* is on triangle.**



Obtuse triangle
***P* is outside triangle.**

As shown above, the location of P depends on the type of triangle. The circle with the center P is said to be *circumscribed* about the triangle.

5.2 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 15, 17, and 25
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 25, and 28

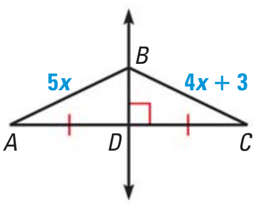
SKILL PRACTICE

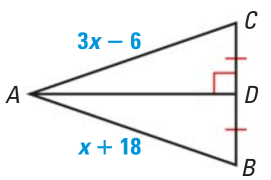
1. **VOCABULARY** Suppose you draw a circle with a compass. You choose three points on the circle to use as the vertices of a triangle. Copy and complete: The center of the circle is also the __?__ of the triangle.

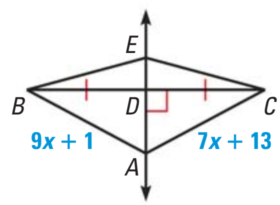
2. ★ **WRITING** Consider $\overline{AB}$. How can you *describe* the set of all points in a plane that are equidistant from A and B?

EXAMPLES 1 and 2
on pp. 303–304 for Exs. 3–15

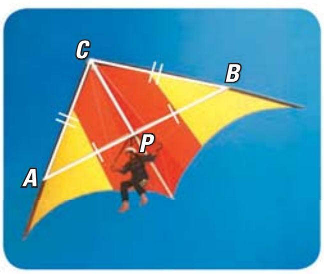
ALGEBRA **Find the length of $\overline{AB}$.**

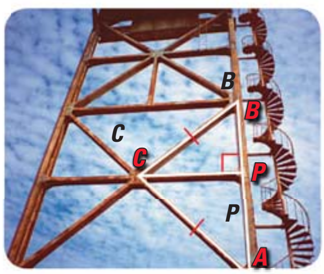
3.


4.


5.


REASONING **Tell whether the information in the diagram allows you to conclude that C is on the perpendicular bisector of $\overline{AB}$.**

6.


7.


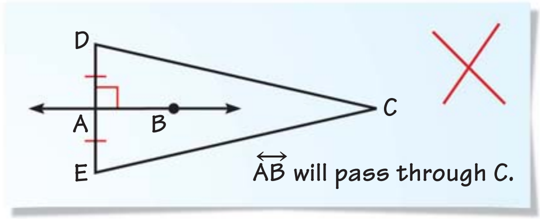
8.


9. ★ **MULTIPLE CHOICE** Point P is inside $\triangle ABC$ and is equidistant from points A and B. On which of the following segments must P be located?

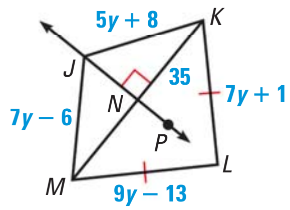
Ⓐ $\overline{AB}$ Ⓑ The perpendicular bisector of $\overline{AB}$

Ⓒ The midsegment opposite $\overline{AB}$ Ⓓ The perpendicular bisector of $\overline{AC}$

10. **ERROR ANALYSIS** *Explain* why the conclusion is not correct given the information in the diagram.



PERPENDICULAR BISECTORS **In Exercises 11–15, use the diagram. $\overleftrightarrow{JN}$ is the perpendicular bisector of $\overline{MK}$.**



11. Find NM. 12. Find JK.

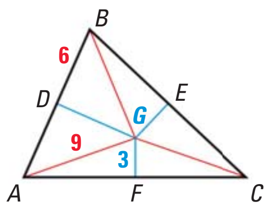
13. Find KL. 14. Find ML.

15. Is L on $\overleftrightarrow{JP}$? *Explain* your reasoning.

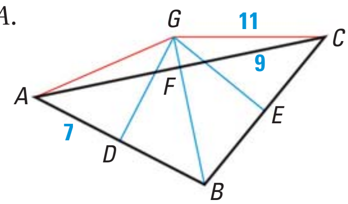
EXAMPLE 3
on p. 305
for Exs. 16–17

USING CONCURRENCY **In the diagram, the perpendicular bisectors of $\triangle ABC$ meet at point G and are shown in blue. Find the indicated measure.**

16. Find BG.



17. Find GA.



18. **CONSTRUCTING PERPENDICULAR BISECTORS** Use the construction shown on page 33 to construct the bisector of a segment. *Explain* why the bisector you constructed is actually the perpendicular bisector.

19. **CONSTRUCTION** Draw a right triangle. Use a compass and straightedge to find its circumcenter. Use a compass to draw the circumscribed circle.

ANALYZING STATEMENTS **Copy and complete the statement with *always*, *sometimes*, or *never*. *Justify* your answer.**

20. The circumcenter of a scalene triangle is _?_ inside the triangle.

21. If the perpendicular bisector of one side of a triangle goes through the opposite vertex, then the triangle is _?_ isosceles.

22. The perpendicular bisectors of a triangle intersect at a point that is _?_ equidistant from the midpoints of the sides of the triangle.

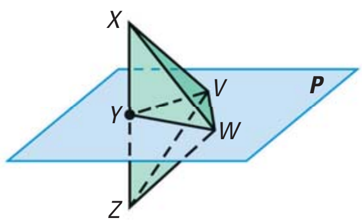
23. **CHALLENGE** Prove the statements in parts (a) – (c).

GIVEN ▶ Plane P is a perpendicular bisector of $\overline{XZ}$ at Y.

PROVE ▶ **a.** $\overline{XW} \cong \overline{ZW}$

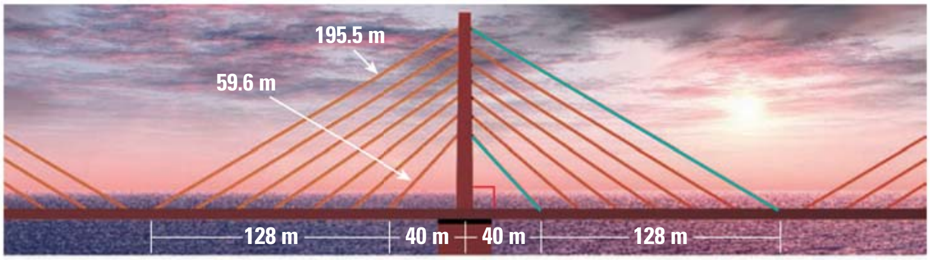
b. $\overline{XV} \cong \overline{ZV}$

c. $\angle VXW \cong \angle VZW$



PROBLEM SOLVING

24. **BRIDGE** A cable-stayed bridge is shown below. Two cable lengths are given. Find the lengths of the blue cables. *Justify* your answer.



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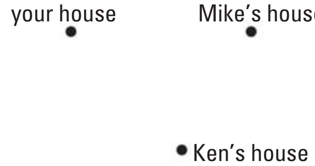
EXAMPLE 3 on p. 305 for Exs. 25, 28

25. ★ **SHORT RESPONSE** You and two friends plan to walk your dogs together. You want your meeting place to be the same distance from each person's house. *Explain* how you can use the diagram to locate the meeting place.



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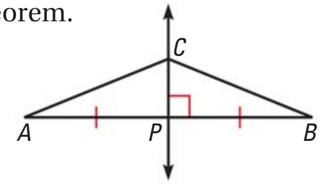
26. **PROVING THEOREM 5.2** Prove the Perpendicular Bisector Theorem.

GIVEN ▶ $\overleftrightarrow{CP}$ is the perpendicular bisector of $\overline{AB}$.

PROVE ▶ $CA = CB$

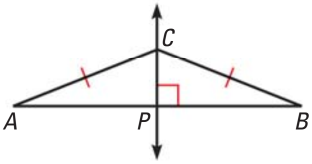
Plan for Proof Show that right triangles $\triangle APC$ and $\triangle BPC$ are congruent. Then show that $\overline{CA} \cong \overline{CB}$.



27. **PROVING THEOREM 5.3** Prove the converse of Theorem 5.2. (*Hint:* Construct a line through C perpendicular to $\overline{AB}$.)

GIVEN ▶ $CA = CB$

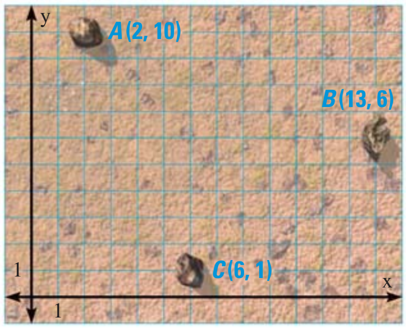
PROVE ▶ C is on the perpendicular bisector of $\overline{AB}$.



28. ★ **EXTENDED RESPONSE** Archaeologists find three stones. They believe that the stones were once part of a circle of stones with a community firepit at its center. They mark the locations of Stones A, B, and C on a graph where distances are measured in feet.

a. *Explain* how the archaeologists can use a sketch to estimate the center of the circle of stones.

b. Copy the diagram and find the approximate coordinates of the point at which the archaeologists should look for the firepit.



29. **TECHNOLOGY** Use geometry drawing software to construct $\overline{AB}$. Find the midpoint C. Draw the perpendicular bisector of $\overline{AB}$ through C. Construct a point D along the perpendicular bisector and measure $\overline{DA}$ and $\overline{DB}$. Move D along the perpendicular bisector. What theorem does this construction demonstrate?

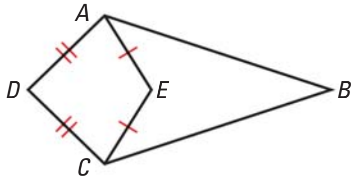
○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

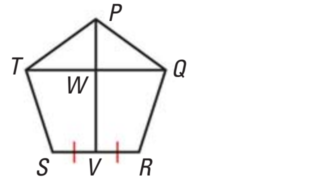
30. **COORDINATE PROOF** Where is the circumcenter located in any right triangle? Write a coordinate proof of this result.

PROOF Use the information in the diagram to prove the given statement.

31. $\overline{AB} \cong \overline{BC}$ if and only if D, E, and B are collinear.



32. $\overline{PV}$ is the perpendicular bisector of $\overline{TQ}$ for regular polygon $PQRST$.



33. **CHALLENGE** The four towns on the map are building a common high school. They have agreed that the school should be an equal distance from each of the four towns. Is there a single point where they could agree to build the school? If so, find it. If not, *explain* why not. Use a diagram to *explain* your answer.

MIXED REVIEW

Solve the equation. Write your answer in simplest radical form. *(p. 882)*

34. $5^2 + x^2 = 13^2$

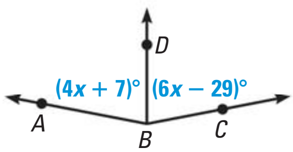
35. $x^2 + 15^2 = 17^2$

36. $x^2 + 10 = 38$

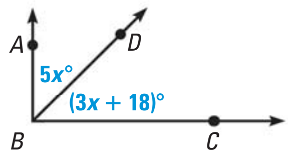
PREVIEW
Prepare for Lesson 5.3 in Exs. 37–38.

Ray $\overrightarrow{BD}$ bisects $\angle ABC$. Find the value of x. Then find $m\angle ABC$. *(p. 24)*

37.



38.



Describe **the pattern in the numbers. Write the next number.** *(p. 72)*

39. 21, 16, 11, 6, . . .

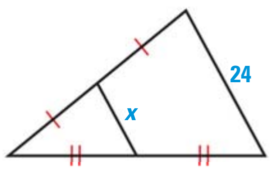
40. 2, 6, 18, 54, . . .

41. 3, 3, 4, 6, . . .

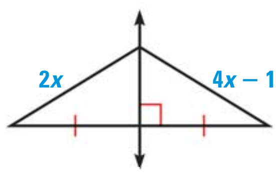
QUIZ *for Lessons 5.1–5.2*

Find the value of x. Identify the theorem used to find the answer. *(pp. 295, 303)*

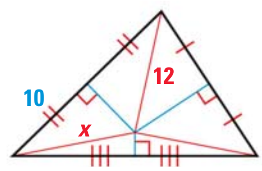
1.



2.



3.



4. Graph the triangle $R(2a, 0)$, $S(0, 2b)$, $T(2a, 2b)$, where a and b are positive. Find RT and ST. Then find the slope of $\overline{SR}$ and the coordinates of the midpoint of $\overline{SR}$. *(p. 295)*

5.3 Use Angle Bisectors of Triangles

Before You used angle bisectors to find angle relationships.

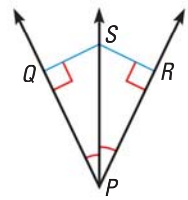
Now You will use angle bisectors to find distance relationships.

Why? So you can apply geometry in sports, as in Example 2.

Key Vocabulary
- **incenter**
- **angle bisector,** *p. 28*
- **distance from a point to a line,** *p. 192*

Remember that an *angle bisector* is a ray that divides an angle into two congruent adjacent angles. Remember also that the *distance from a point to a line* is the length of the perpendicular segment from the point to the line.

So, in the diagram, $\overrightarrow{PS}$ is the bisector of $\angle QPR$ and the distance from S to $\overrightarrow{PQ}$ is SQ, where $\overline{SQ} \perp \overrightarrow{PQ}$.

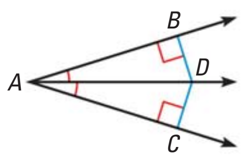


THEOREMS — *For Your Notebook*

THEOREM 5.5 Angle Bisector Theorem

If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.

If $\overrightarrow{AD}$ bisects $\angle BAC$ and $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$, then $DB = DC$.

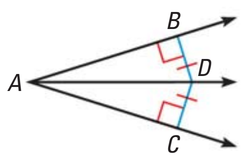


Proof: Ex. 34, p. 315

THEOREM 5.6 Converse of the Angle Bisector Theorem

If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.

If $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$ and $DB = DC$, then $\overrightarrow{AD}$ bisects $\angle BAC$.

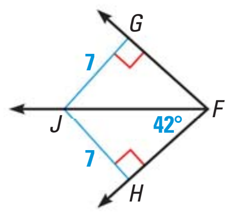


Proof: Ex. 35, p. 315

REVIEW DISTANCE
In Geometry, *distance* means the *shortest* length between two objects.

EXAMPLE 1 Use the Angle Bisector Theorems

Find the measure of $\angle GFJ$.

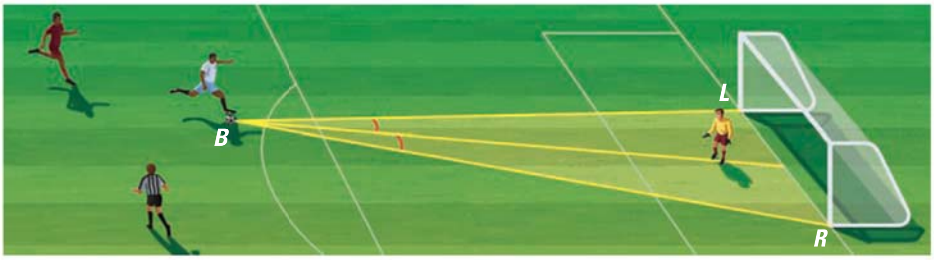


Solution

Because $\overline{JG} \perp \overrightarrow{FG}$ and $\overline{JH} \perp \overrightarrow{FH}$ and $JG = JH = 7$, $\overrightarrow{FJ}$ bisects $\angle GFH$ by the Converse of the Angle Bisector Theorem. So, $m\angle GFJ = m\angle HFJ = 42°$.

EXAMPLE 2 Solve a real-world problem

SOCCER A soccer goalie's position relative to the ball and goalposts forms congruent angles, as shown. Will the goalie have to move farther to block a shot toward the right goalpost R or the left goalpost L?



Solution

The congruent angles tell you that the goalie is on the bisector of $\angle LBR$. By the Angle Bisector Theorem, the goalie is equidistant from $\overrightarrow{BR}$ and $\overrightarrow{BL}$.

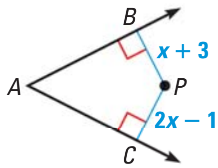
▶ So, the goalie must move the same distance to block either shot.

EXAMPLE 3 Use algebra to solve a problem

xy ALGEBRA For what value of x does P lie on the bisector of $\angle A$?

Solution

From the Converse of the Angle Bisector Theorem, you know that P lies on the bisector of $\angle A$ if P is equidistant from the sides of $\angle A$, so when $BP = CP$.



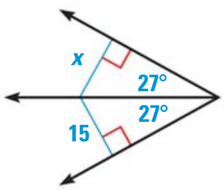
$BP = CP$	**Set segment lengths equal.**
$x + 3 = 2x - 1$	**Substitute expressions for segment lengths.**
$4 = x$	**Solve for x.**

▶ Point P lies on the bisector of $\angle A$ when $x = 4$.

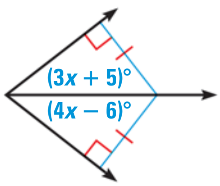
✓ GUIDED PRACTICE for Examples 1, 2, and 3

In Exercises 1–3, find the value of x.

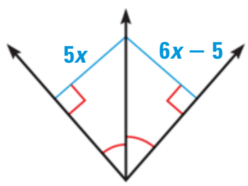
1.



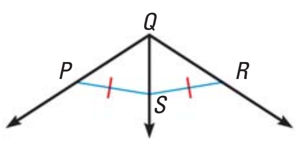
2.



3.



4. Do you have enough information to conclude that $\overrightarrow{QS}$ bisects $\angle PQR$? *Explain.*



READ VOCABULARY

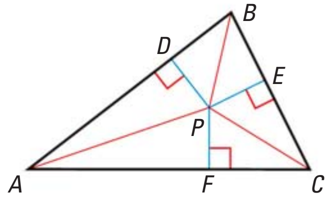
An *angle bisector of a triangle* is the bisector of an interior angle of the triangle.

THEOREM — For Your Notebook

THEOREM 5.7 Concurrency of Angle Bisectors of a Triangle

The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

If $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are angle bisectors of $\triangle ABC$, then $PD = PE = PF$.



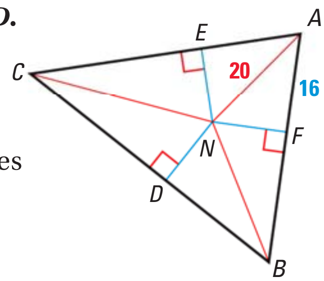
Proof: Ex. 36, p. 316

The point of concurrency of the three angle bisectors of a triangle is called the **incenter** of the triangle. The incenter always lies inside the triangle.

Because the incenter P is equidistant from the three sides of the triangle, a circle drawn using P as the center and the distance to one side as the radius will just touch the other two sides. The circle is said to be *inscribed* within the triangle.

EXAMPLE 4 Use the concurrency of angle bisectors

In the diagram, N is the incenter of $\triangle ABC$. Find ND.



Solution

By the Concurrency of Angle Bisectors of a Triangle Theorem, the incenter N is equidistant from the sides of $\triangle ABC$. So, to find ND, you can find NF in $\triangle NAF$. Use the Pythagorean Theorem stated on page 18.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$20^2 = NF^2 + 16^2$	Substitute known values.
$400 = NF^2 + 256$	Multiply.
$144 = NF^2$	Subtract 256 from each side.
$12 = NF$	Take the positive square root of each side.

▶ Because $NF = ND$, $ND = 12$.

REVIEW QUADRATIC EQUATIONS

For help with solving a quadratic equation by taking square roots, see page 882. Use only the positive square root when finding a distance, as in Example 4.

AnimatedGeometry at classzone.com

✓ **GUIDED PRACTICE** for Example 4

5. **WHAT IF?** In Example 4, suppose you are not given AF or AN, but you are given that $BF = 12$ and $BN = 13$. Find ND.

5.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 15, and 29

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, 23, 30, and 31

SKILL PRACTICE

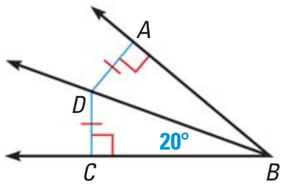
1. **VOCABULARY** Copy and complete: Point C is in the interior of $\angle ABD$. If $\angle ABC$ and $\angle DBC$ are congruent, then $\overrightarrow{BC}$ is the __?__ of $\angle ABD$.

2. ★ **WRITING** How are perpendicular bisectors and angle bisectors of a triangle different? How are they alike?

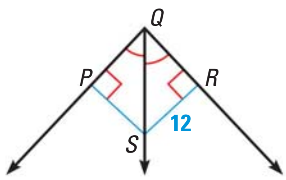
EXAMPLE 1 on p. 310 for Exs. 3–5

FINDING MEASURES **Use the information in the diagram to find the measure.**

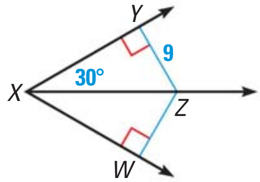
3. Find $m\angle ABD$.



4. Find PS.



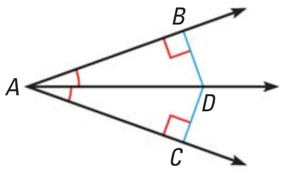
5. $m\angle YXW = 60°$. Find WZ.



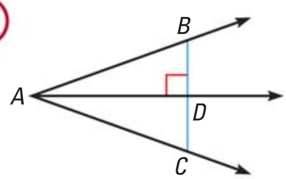
EXAMPLE 2 on p. 311 for Exs. 6–11

ANGLE BISECTOR THEOREM **Is $DB = DC$? *Explain.***

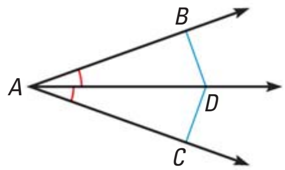
6.



7.

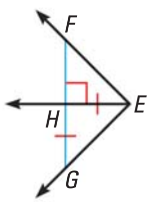


8.

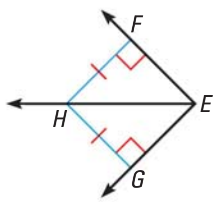


REASONING **Can you conclude that $\overrightarrow{EH}$ bisects $\angle FEG$? *Explain.***

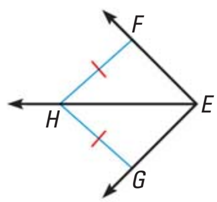
9.



10.



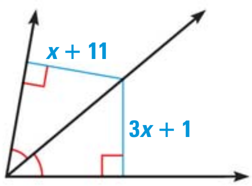
11.



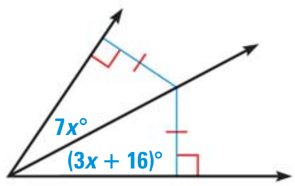
EXAMPLE 3 on p. 311 for Exs. 12–18

xy ALGEBRA **Find the value of x.**

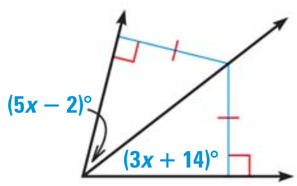
12.



13.

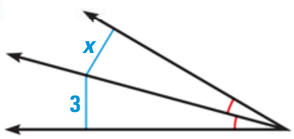


14.

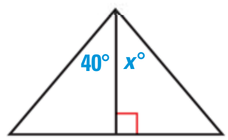


RECOGNIZING MISSING INFORMATION **Can you find the value of x? *Explain.***

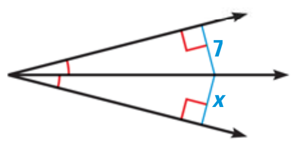
15.



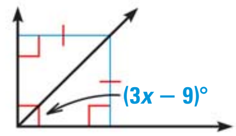
16.



17.



18. ★ **MULTIPLE CHOICE** What is the value of x in the diagram?



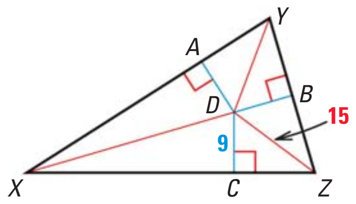
Ⓐ 13 Ⓑ 18

Ⓒ 33 Ⓓ Not enough information

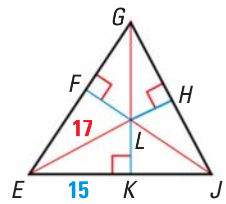
EXAMPLE 4 on p. 312 for Exs. 19–22

USING INCENTERS **Find the indicated measure.**

19. Point D is the incenter of $\triangle XYZ$. Find DB.

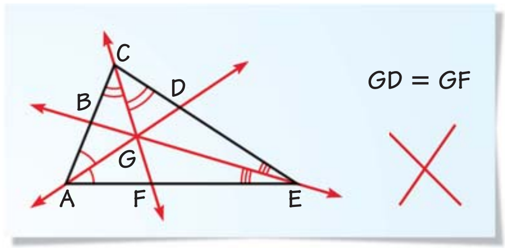


20. Point L is the incenter of $\triangle EGJ$. Find HL.

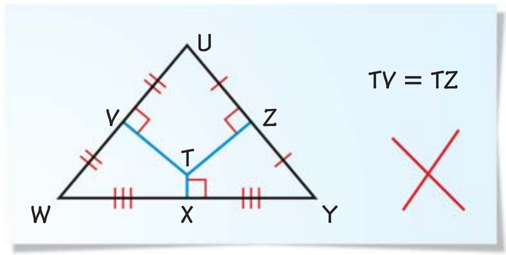


ERROR ANALYSIS ***Describe*** **the error in reasoning. Then state a correct conclusion about distances that can be deduced from the diagram.**

21.



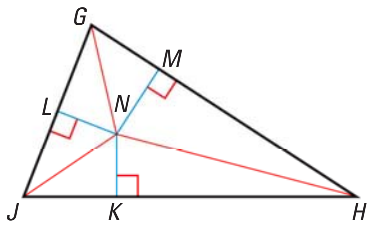
22.



23. ★ **MULTIPLE CHOICE** In the diagram, N is the incenter of $\triangle GHJ$. Which statement cannot be deduced from the given information?

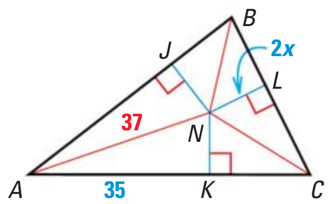
Ⓐ $\overline{NM} \cong \overline{NK}$ Ⓑ $\overline{NL} \cong \overline{NM}$

Ⓒ $\overline{NG} \cong \overline{NJ}$ Ⓓ $\overline{HK} \cong \overline{HM}$

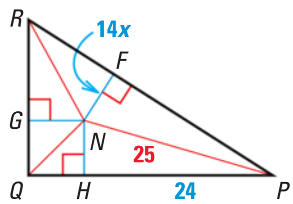


ALGEBRA **Find the value of x that makes N the incenter of the triangle.**

24.

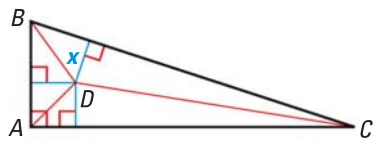


25.



26. **CONSTRUCTION** Use a compass and a straightedge to draw $\triangle ABC$ with incenter D. Label the angle bisectors and the perpendicular segments from D to each of the sides of $\triangle ABC$. Measure each segment. What do you notice? What theorem have you verified for your $\triangle ABC$?

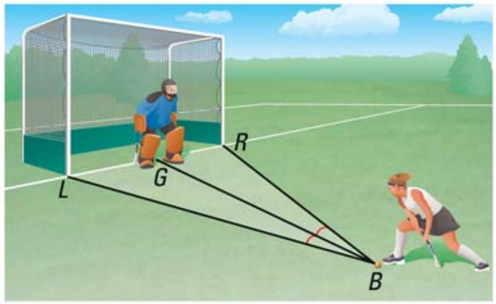
27. **CHALLENGE** Point D is the incenter of $\triangle ABC$. Write an expression for the length x in terms of the three side lengths AB, AC, and BC.



PROBLEM SOLVING

EXAMPLE 2 on p. 311 for Ex. 28

28. FIELD HOCKEY In a field hockey game, the goalkeeper is at point G and a player from the opposing team hits the ball from point B. The goal extends from left goalpost L to right goalpost R. Will the goalkeeper have to move farther to keep the ball from hitting L or R? *Explain*.



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29. KOI POND You are constructing a fountain in a triangular koi pond. You want the fountain to be the same distance from each edge of the pond. Where should you build the fountain? *Explain* your reasoning. Use a sketch to support your answer.

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30. ★ SHORT RESPONSE What congruence postulate or theorem would you use to prove the Angle Bisector Theorem? to prove the Converse of the Angle Bisector Theorem? Use diagrams to show your reasoning.

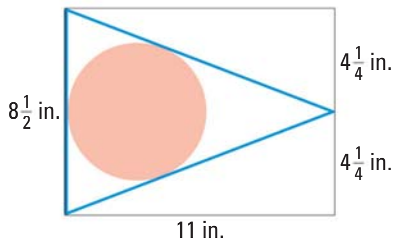
31. ★ EXTENDED RESPONSE Suppose you are given a triangle and are asked to draw all of its perpendicular bisectors and angle bisectors.

a. For what type of triangle would you need the fewest segments? What is the minimum number of segments you would need? *Explain*.

b. For what type of triangle would you need the most segments? What is the maximum number of segments you would need? *Explain*.

CHOOSING A METHOD In Exercises 32 and 33, tell whether you would use *perpendicular bisectors* or *angle bisectors*. Then solve the problem.

32. BANNER To make a banner, you will cut a triangle from an $8\frac{1}{2}$ inch by 11 inch sheet of white paper and paste a red circle onto it as shown. The circle should just touch each side of the triangle. Use a model to decide whether the circle's radius should be *more* or *less* than $2\frac{1}{2}$ inches. Can you cut the circle from a 5 inch by 5 inch red square? *Explain*.



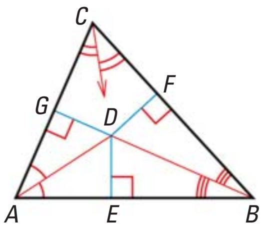
33. CAMP A map of a camp shows a pool at (10, 20), a nature center at (16, 2), and a tennis court at (2, 4). A new circular walking path will connect the three locations. Graph the points and find the approximate center of the circle. Estimate the radius of the circle if each unit on the grid represents 10 yards. Then use the formula $C = 2\pi r$ to estimate the length of the path.

PROVING THEOREMS 5.5 AND 5.6 Use Exercise 30 to prove the theorem.

34. Angle Bisector Theorem

35. Converse of the Angle Bisector Theorem

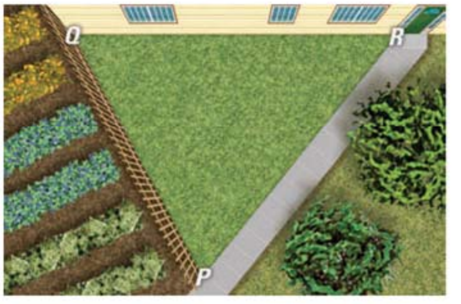
36. **PROVING THEOREM 5.7** Write a proof of the Concurrency of Angle Bisectors of a Triangle Theorem.



GIVEN ▶ $\triangle ABC$, $\overline{AD}$ bisects $\angle CAB$, $\overline{BD}$ bisects $\angle CBA$, $\overline{DE} \perp \overline{AB}$, $\overline{DF} \perp \overline{BC}$, $\overline{DG} \perp \overline{CA}$

PROVE ▶ The angle bisectors intersect at D, which is equidistant from $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.

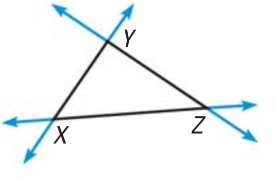
37. **CELEBRATION** You are planning a graduation party in the triangular courtyard shown. You want to fit as large a circular tent as possible on the site without extending into the walkway.



a. Copy the triangle and show how to place the tent so that it just touches each edge. Then *explain* how you can be sure that there is no place you could fit a larger tent on the site. Use sketches to support your answer.

b. Suppose you want to fit as large a tent as possible while leaving at least one foot of space around the tent. Would you put the center of the tent in the same place as you did in part (a)? *Justify* your answer.

38. **CHALLENGE** You have seen that there is a point inside any triangle that is equidistant from the three sides of the triangle. Prove that if you extend the sides of the triangle to form lines, you can find three points outside the triangle, each of which is equidistant from those three lines.



MIXED REVIEW

PREVIEW
Prepare for Lesson 5.4 in Exs. 39–41.

Find the length of $\overline{AB}$ and the coordinates of the midpoint of $\overline{AB}$. *(p. 15)*

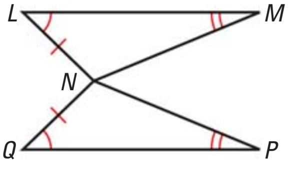
39. $A(-2, 2)$, $B(-10, 2)$

40. $A(0, 6)$, $B(5, 8)$

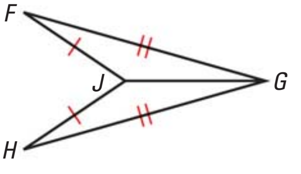
41. $A(-1, -3)$, $B(7, -5)$

Explain **how to prove the given statement.** *(p. 256)*

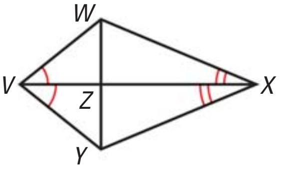
42. $\angle QNP \cong \angle LNM$



43. $\overline{JG}$ bisects $\angle FGH$.

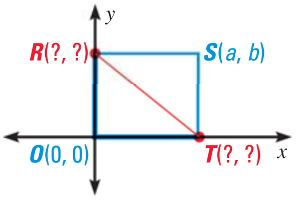


44. $\triangle ZWX \cong \triangle ZYX$

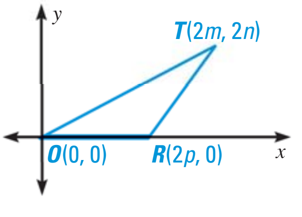


Find the coordinates of the red points in the figure if necessary. Then find OR and the coordinates of the midpoint M of $\overline{RT}$. *(p. 295)*

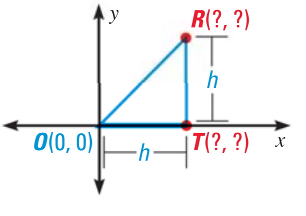
45.



46.



47.



MIXED REVIEW of Problem Solving

STATE TEST PRACTICE
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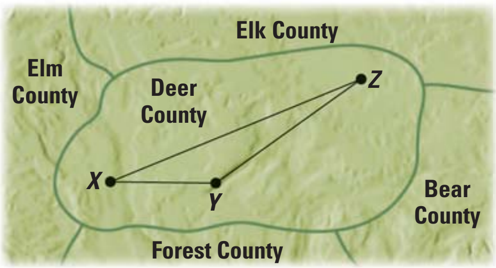
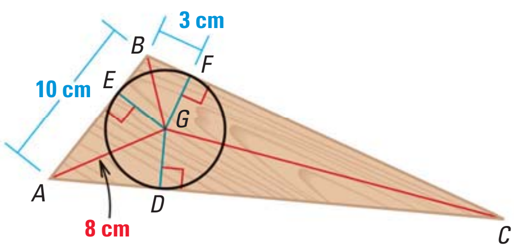
Lessons 5.1–5.3

1. SHORT RESPONSE A committee has decided to build a park in Deer County. The committee agreed that the park should be equidistant from the three largest cities in the county, which are labeled X, Y, and Z in the diagram. *Explain* why this may not be the best place to build the park. Use a sketch to support your answer.



2. EXTENDED RESPONSE A woodworker is trying to cut as large a wheel as possible from a triangular scrap of wood. The wheel just touches each side of the triangle as shown below.



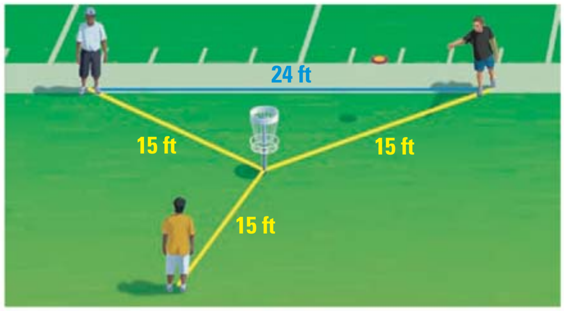
a. Which point of concurrency is the woodworker using for the center of the circle? What type of special segment are $\overline{BG}$, $\overline{CG}$, and $\overline{AG}$?

b. Which postulate or theorem can you use to prove that $\triangle BGF \cong \triangle BGE$?

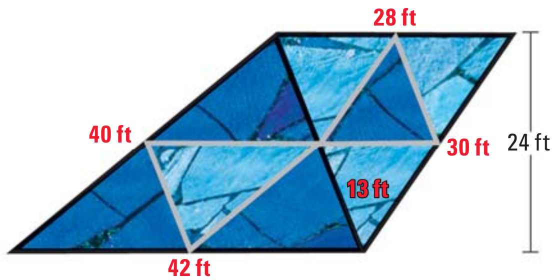
c. Find the radius of the wheel to the nearest tenth of a centimeter. *Explain* your reasoning.

3. SHORT RESPONSE Graph $\triangle GHJ$ with vertices $G(2, 2)$, $H(6, 8)$, and $J(10, 4)$ and draw its midsegments. Each midsegment is contained in a line. Which of those lines has the greatest y-intercept? Write the equation of that line. *Justify* your answer.

4. GRIDDED ANSWER Three friends are practicing disc golf, in which a flying disk is thrown into a set of targets. Each player is 15 feet from the target. Two players are 24 feet from each other along one edge of the nearby football field. How far is the target from that edge of the football field?



5. MULTI-STEP PROBLEM An artist created a large floor mosaic consisting of eight triangular sections. The grey segments are the midsegments of the two black triangles.

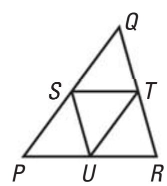


a. The gray and black edging was created using special narrow tiles. What is the total length of all the edging used?

b. What is the total area of the mosaic?

6. OPEN-ENDED If possible, draw a triangle whose incenter and circumcenter are the same point. *Describe* this triangle as specifically as possible.

7. SHORT RESPONSE Points S, T, and U are the midpoints of the sides of $\triangle PQR$. Which angles are congruent to $\angle QST$? *Justify* your answer.



Investigating Geometry ACTIVITY *Use before Lesson 5.4*

5.4 Intersecting Medians

MATERIALS • cardboard • straightedge • scissors • metric ruler

QUESTION What is the relationship between segments formed by the medians of a triangle?

EXPLORE 1 Find the balance point of a triangle

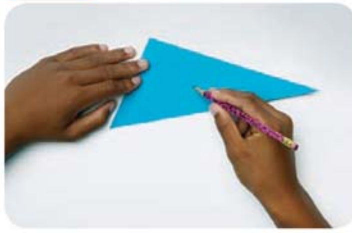
STEP 1

Cut out triangle Draw a triangle on a piece of cardboard. Then cut it out.

STEP 2

Balance the triangle Balance the triangle on the eraser end of a pencil.

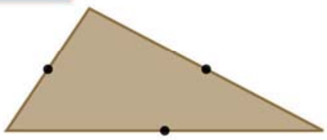
STEP 3



Mark the balance point Mark the point on the triangle where it balanced on the pencil.

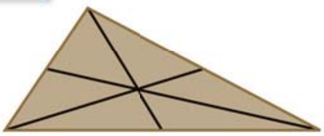
EXPLORE 2 Construct the medians of a triangle

STEP 1



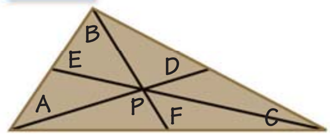
Find the midpoint Use a ruler to find the midpoint of each side of the triangle.

STEP 2



Draw medians Draw a segment, or *median*, from each midpoint to the vertex of the opposite angle.

STEP 3



Label points Label your triangle as shown. What do you notice about point P and the balance point in Explore 1?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Copy and complete the table. Measure in millimeters.

Length of segment from vertex to midpoint of opposite side	$AD = ?$	$BF = ?$	$CE = ?$
Length of segment from vertex to P	$AP = ?$	$BP = ?$	$CP = ?$
Length of segment from P to midpoint	$PD = ?$	$PF = ?$	$PE = ?$

2. How does the length of the segment from a vertex to P compare with the length of the segment from P to the midpoint of the opposite side?

3. How does the length of the segment from a vertex to P compare with the length of the segment from the vertex to the midpoint of the opposite side?

5.4 Use Medians and Altitudes

Before You used perpendicular bisectors and angle bisectors of triangles.

Now You will use medians and altitudes of triangles.

Why? So you can find the balancing point of a triangle, as in Ex. 37.

Key Vocabulary
- median of a triangle
- centroid
- altitude of a triangle
- orthocenter

As shown by the Activity on page 318, a triangle will balance at a particular point. This point is the intersection of the *medians* of the triangle.

A **median of a triangle** is a segment from a vertex to the midpoint of the opposite side. The three medians of a triangle are concurrent. The point of concurrency, called the **centroid**, is inside the triangle.

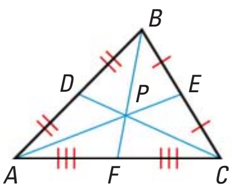
Three medians meet at the centroid.

THEOREM *For Your Notebook*

THEOREM 5.8 Concurrency of Medians of a Triangle

The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

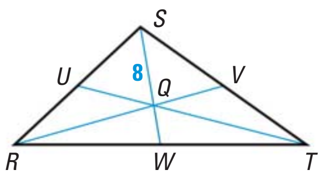
The medians of $\triangle ABC$ meet at P and $AP = \frac{2}{3}AE$, $BP = \frac{2}{3}BF$, and $CP = \frac{2}{3}CD$.



Proof: Ex. 32, p. 323; p. 934

EXAMPLE 1 Use the centroid of a triangle

In $\triangle RST$, Q is the centroid and $SQ = 8$. Find QW and SW.



Solution

$SQ = \frac{2}{3}SW$ — Concurrency of Medians of a Triangle Theorem

$8 = \frac{2}{3}SW$ — Substitute 8 for SQ.

$12 = SW$ — Multiply each side by the reciprocal, $\frac{3}{2}$.

Then $QW = SW - SQ = 12 - 8 = 4$.

▶ So, $QW = 4$ and $SW = 12$.

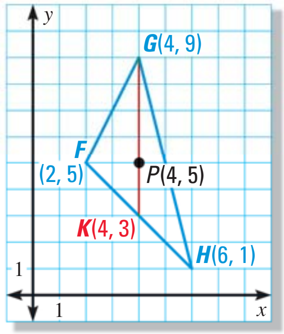
EXAMPLE 2 Standardized Test Practice

The vertices of $\triangle FGH$ are $F(2, 5)$, $G(4, 9)$, and $H(6, 1)$. Which ordered pair gives the coordinates of the centroid P of $\triangle FGH$?

Ⓐ (3, 5)　Ⓑ (4, 5)　Ⓒ (4, 7)　Ⓓ (5, 3)

Solution

Sketch $\triangle FGH$. Then use the Midpoint Formula to find the midpoint K of $\overline{FH}$ and sketch median $\overline{GK}$.



$$K\left(\frac{2+6}{2}, \frac{5+1}{2}\right) = K(4, 3).$$

The centroid is two thirds of the distance from each vertex to the midpoint of the opposite side.

The distance from vertex $G(4, 9)$ to $K(4, 3)$ is $9 - 3 = 6$ units. So, the centroid is $\frac{2}{3}(6) = 4$ units down from G on $\overline{GK}$.

The coordinates of the centroid P are $(4, 9 - 4)$, or $(4, 5)$.

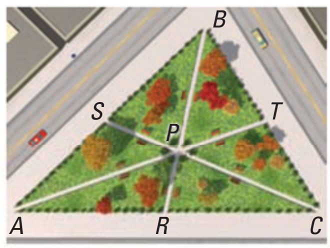
▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

CHECK ANSWERS

Median $\overline{GK}$ was used in Example 2 because it is easy to find distances on a vertical segment. It is a good idea to check by finding the centroid using a different median.

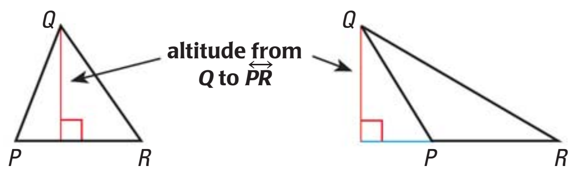
✓ GUIDED PRACTICE for Examples 1 and 2

There are three paths through a triangular park. Each path goes from the midpoint of one edge to the opposite corner. The paths meet at point *P*.



1. If $SC = 2100$ feet, find PS and PC.
2. If $BT = 1000$ feet, find TC and BC.
3. If $PT = 800$ feet, find PA and TA.

ALTITUDES An **altitude of a triangle** is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side.



MEASURES OF TRIANGLES

In the area formula for a triangle, $A = \frac{1}{2}bh$, you can use the length of any side for the base b. The height h is the length of the altitude to that side from the opposite vertex.

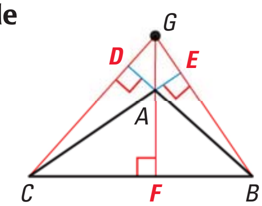
THEOREM — For Your Notebook

THEOREM 5.9 Concurrency of Altitudes of a Triangle

The lines containing the altitudes of a triangle are concurrent.

The lines containing $\overline{AF}$, $\overline{BE}$, and $\overline{CD}$ meet at G.

Proof: Exs. 29–31, p. 323; p. 936



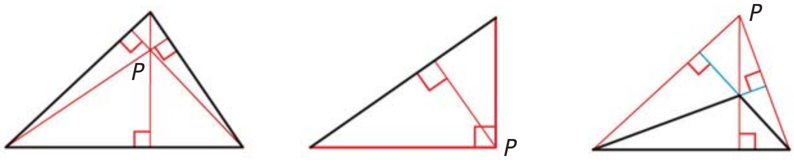
CONCURRENCY OF ALTITUDES The point at which the lines containing the three altitudes of a triangle intersect is called the **orthocenter** of the triangle.

EXAMPLE 3 Find the orthocenter

Find the orthocenter P in an acute, a right, and an obtuse triangle.

READ DIAGRAMS
The altitudes are shown in red. Notice that in the right triangle the legs are also altitudes. The altitudes of the obtuse triangle are extended to find the orthocenter.

Solution



Acute triangle *P* **is inside triangle.** **Right triangle** *P* **is on triangle.** **Obtuse triangle** *P* **is outside triangle.**

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ISOSCELES TRIANGLES In an isosceles triangle, the perpendicular bisector, angle bisector, median, and altitude from the vertex angle to the base are all the same segment. In an equilateral triangle, this is true for the special segment from any vertex.

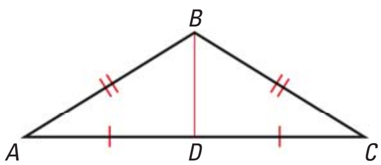
EXAMPLE 4 Prove a property of isosceles triangles

Prove that the median to the base of an isosceles triangle is an altitude.

Solution

GIVEN ▶ $\triangle ABC$ is isosceles, with base $\overline{AC}$.
$\overline{BD}$ is the median to base $\overline{AC}$.

PROVE ▶ $\overline{BD}$ is an altitude of $\triangle ABC$.



Proof Legs $\overline{AB}$ and $\overline{BC}$ of isosceles $\triangle ABC$ are congruent. $\overline{CD} \cong \overline{AD}$ because $\overline{BD}$ is the median to $\overline{AC}$. Also, $\overline{BD} \cong \overline{BD}$. Therefore, $\triangle ABD \cong \triangle CBD$ by the SSS Congruence Postulate.

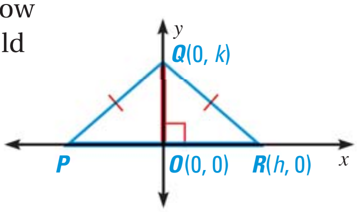
$\angle ADB \cong \angle CDB$ because corresponding parts of $\cong$ ⚠ are $\cong$. Also, $\angle ADB$ and $\angle CDB$ are a linear pair. $\overline{BD}$ and $\overline{AC}$ intersect to form a linear pair of congruent angles, so $\overline{BD} \perp \overline{AC}$ and $\overline{BD}$ is an altitude of $\triangle ABC$.

✓ GUIDED PRACTICE for Examples 3 and 4

4. Copy the triangle in Example 4 and find its orthocenter.

5. **WHAT IF?** In Example 4, suppose you wanted to show that median $\overline{BD}$ is also an angle bisector. How would your proof be different?

6. Triangle PQR is an isosceles triangle and segment $\overline{OQ}$ is an altitude. What else do you know about $\overline{OQ}$? What are the coordinates of P?



5.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 21, and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 11, 12, 28, 40, and 44

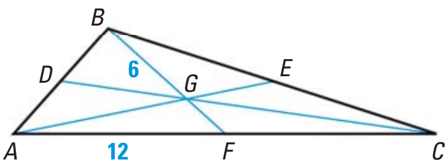
SKILL PRACTICE

1. **VOCABULARY** Name the four types of points of concurrency introduced in Lessons 5.2–5.4. When is each type inside the triangle? on the triangle? outside the triangle?

2. ★ **WRITING** *Compare* a perpendicular bisector and an altitude of a triangle. *Compare* a perpendicular bisector and a median of a triangle.

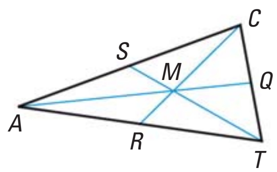
EXAMPLE 1 on p. 319 for Exs. 3–7

FINDING LENGTHS **G is the centroid of $\triangle ABC$, $BG = 6$, $AF = 12$, and $AE = 15$. Find the length of the segment.**



3. $\overline{FC}$
4. $\overline{BF}$
5. $\overline{AG}$
6. $\overline{GE}$

7. ★ **MULTIPLE CHOICE** In the diagram, M is the centroid of $\triangle ACT$, $CM = 36$, $MQ = 30$, and $TS = 56$. What is AM?

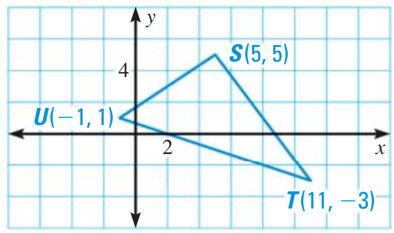


Ⓐ 15 Ⓑ 30

Ⓒ 36 Ⓓ 60

EXAMPLE 2 on p. 320 for Exs. 8–11

8. **FINDING A CENTROID** Use the graph shown.
   a. Find the coordinates of P, the midpoint of $\overline{ST}$. Use the median $\overline{UP}$ to find the coordinates of the centroid Q.
   b. Find the coordinates of R, the midpoint of $\overline{TU}$. Verify that $SQ = \frac{2}{3}SR$.



GRAPHING CENTROIDS **Find the coordinates of the centroid P of $\triangle ABC$.**

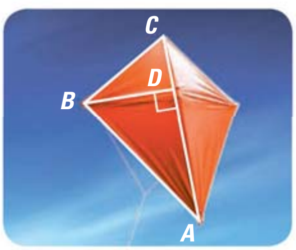
9. $A(-1, 2)$, $B(5, 6)$, $C(5, -2)$
10. $A(0, 4)$, $B(3, 10)$, $C(6, -2)$

11. ★ **OPEN-ENDED MATH** Draw a large right triangle and find its centroid.

EXAMPLE 3 on p. 321 for Exs. 12–16

12. ★ **OPEN-ENDED MATH** Draw a large obtuse, scalene triangle and find its orthocenter.

IDENTIFYING SEGMENTS **Is $\overline{BD}$ a *perpendicular bisector* of $\triangle ABC$? Is $\overline{BD}$ a *median*? an *altitude*?**

13.


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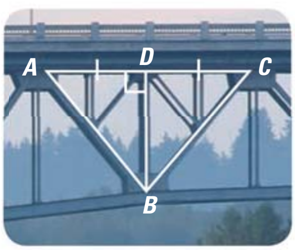

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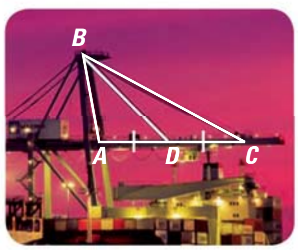
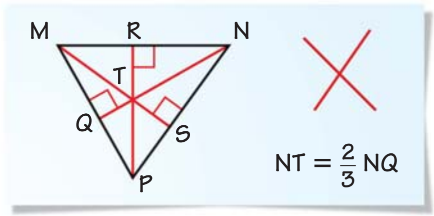
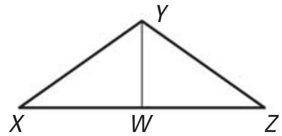


16. **ERROR ANALYSIS** A student uses the fact that T is a point of concurrency to conclude that $NT = \frac{2}{3}NQ$. *Explain* what is wrong with this reasoning.



EXAMPLE 4
on p. 321
for Exs. 17–22

REASONING **Use the diagram shown and the given information to decide whether $\overline{YW}$ is a *perpendicular bisector*, an *angle bisector*, a *median*, or an *altitude* of $\triangle XYZ$. There may be more than one right answer.**



17. $\overline{YW} \perp \overline{XZ}$

18. $\angle XYW \cong \angle ZYW$

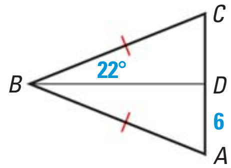
19. $\overline{XW} \cong \overline{ZW}$

20. $\overline{YW} \perp \overline{XZ}$ and $\overline{XW} \cong \overline{ZW}$

21. $\triangle XYW \cong \triangle ZYW$

22. $\overline{YW} \perp \overline{XZ}$ and $\overline{XY} \cong \overline{ZY}$

ISOSCELES TRIANGLES **Find the measurements. *Explain* your reasoning.**



23. Given that $\overline{DB} \perp \overline{AC}$, find DC and $m\angle ABD$.

24. Given that $AD = DC$, find $m\angle ADB$ and $m\angle ABD$.

RELATING LENGTHS **Copy and complete the statement for $\triangle DEF$ with medians $\overline{DH}$, $\overline{EJ}$, and $\overline{FG}$, and centroid K.**

25. $EJ = \underline{\ ?\ }\ KJ$

26. $DK = \underline{\ ?\ }\ KH$

27. $FG = \underline{\ ?\ }\ KF$

28. ★ **SHORT RESPONSE** Any isosceles triangle can be placed in the coordinate plane with its base on the x-axis and the opposite vertex on the y-axis as in Guided Practice Exercise 6 on page 321. *Explain* why.

CONSTRUCTION **Verify the Concurrency of Altitudes of a Triangle by drawing a triangle of the given type and constructing its altitudes.** (*Hint:* To construct an altitude, use the construction in Exercise 25 on page 195.)

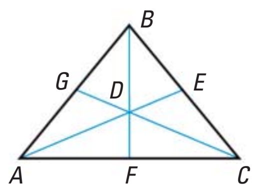
29. Equilateral triangle

30. Right scalene triangle

31. Obtuse isosceles triangle

32. **VERIFYING THEOREM 5.8** Use Example 2 on page 320. Verify that Theorem 5.8, the Concurrency of Medians of a Triangle, holds for the median from vertex F and for the median from vertex H.

xy **ALGEBRA** **Point D is the centroid of $\triangle ABC$. Use the given information to find the value of x.**

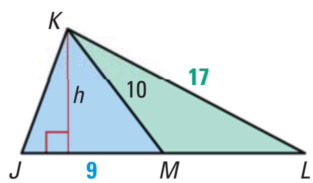


33. $BD = 4x + 5$ and $BF = 9x$

34. $GD = 2x - 8$ and $GC = 3x + 3$

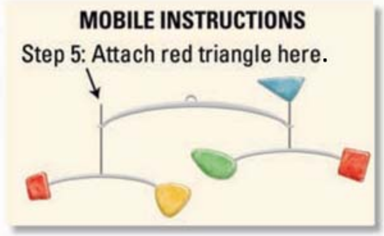
35. $AD = 5x$ and $DE = 3x - 2$

36. **CHALLENGE** $\overline{KM}$ is a median of $\triangle JKL$. Find the areas of $\triangle JKM$ and $\triangle LKM$. Compare the areas. Do you think that the two areas will always compare in this way, regardless of the shape of the triangle? *Explain*.



PROBLEM SOLVING

37. MOBILES To complete the mobile, you need to balance the red triangle on the tip of a metal rod. Copy the triangle and decide if you should place the rod at A or B. *Explain.*



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38. DEVELOPING PROOF Show two different ways that you can place an isosceles triangle with base $2n$ and height h on the coordinate plane. Label the coordinates for each vertex.

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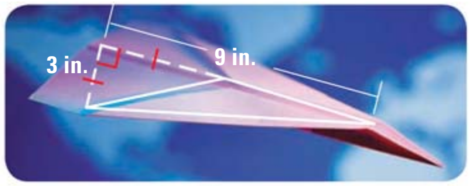
39. PAPER AIRPLANE Find the area of the triangular part of the paper airplane wing that is outlined in red. Which special segment of the triangle did you use?



40. ★ SHORT RESPONSE In what type(s) of triangle can a vertex of the triangle be one of the points of concurrency of the triangle? *Explain.*

41. COORDINATE GEOMETRY Graph the lines on the same coordinate plane and find the centroid of the triangle formed by their intersections.

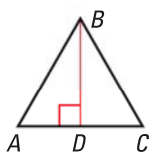
$y_1 = 3x - 4$ $\qquad$ $y_2 = \frac{3}{4}x + 5$ $\qquad$ $y_3 = -\frac{3}{2}x - 4$

EXAMPLE 4
on p. 321
for Ex. 42

42. PROOF Write proofs using different methods.

GIVEN ▶ $\triangle ABC$ is equilateral.
$\overline{BD}$ is an altitude of $\triangle ABC$.

PROVE ▶ $\overline{BD}$ is also a perpendicular bisector of $\overline{AC}$.



a. Write a proof using congruent triangles.

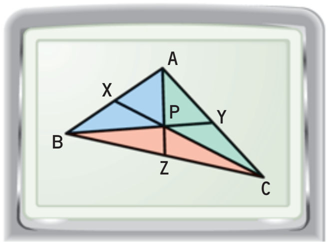
b. Write a proof using the Perpendicular Postulate on page 148.

43. TECHNOLOGY Use geometry drawing software.

a. Construct a triangle and its medians. Measure the areas of the blue, green, and red triangles.

b. What do you notice about the triangles?

c. If a triangle is of uniform thickness, what can you conclude about the weight of the three interior triangles? How does this support the idea that a triangle will balance on its centroid?



44. ★ EXTENDED RESPONSE Use $P(0, 0)$, $Q(8, 12)$, and $R(14, 0)$.

a. What is the slope of the altitude from R to $\overline{PQ}$?

b. Write an equation for each altitude of $\triangle PQR$. Find the orthocenter by finding the ordered pair that is a solution of the three equations.

c. How would your steps change if you were finding the circumcenter?

○ = WORKED-OUT SOLUTIONS on p. WS1 $\qquad$ ★ = STANDARDIZED TEST PRACTICE

45. CHALLENGE Prove the results in parts (a) – (c).

GIVEN ▶ $\overline{LP}$ and $\overline{MQ}$ are medians of scalene $\triangle LMN$. Point R is on $\overrightarrow{LP}$ such that $\overline{LP} \cong \overline{PR}$. Point S is on $\overrightarrow{MQ}$ such that $\overline{MQ} \cong \overline{QS}$.

PROVE ▶ **a.** $\overline{NS} \cong \overline{NR}$

b. $\overline{NS}$ and $\overline{NR}$ are both parallel to $\overline{LM}$.

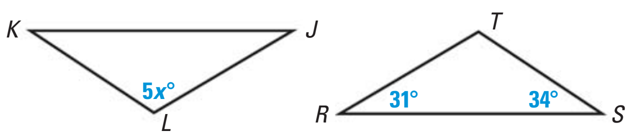
c. R, N, and S are collinear.

MIXED REVIEW

In Exercises 46–48, write an equation of the line that passes through points *A* and *B*. *(p. 180)*

46. $A(0, 7)$, $B(1, 10)$ **47.** $A(4, -8)$, $B(-2, -5)$ **48.** $A(5, -21)$, $B(0, 4)$

49. In the diagram, $\triangle JKL \cong \triangle RST$. Find the value of x. *(p. 225)*



PREVIEW
Prepare for Lesson 5.5 in Exs. 50–52.

Solve the inequality. *(p. 287)*

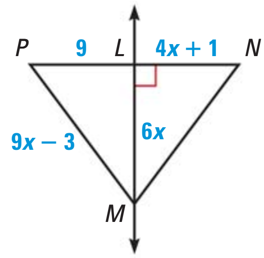
50. $2x + 13 < 35$ **51.** $12 > -3x - 6$ **52.** $6x < x + 20$

In the diagram, $\overline{LM}$ is the perpendicular bisector of $\overline{PN}$. *(p. 303)*

53. What segment lengths are equal?

54. What is the value of x?

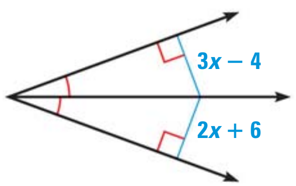
55. Find MN.



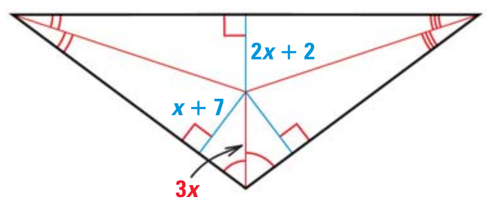
QUIZ for Lessons 5.3–5.4

Find the value of x. Identify the theorem used to find the answer. *(p. 310)*

1.



2.

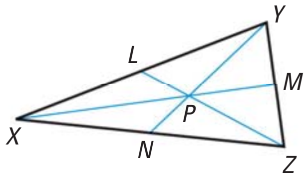


In the figure, P is the centroid of $\triangle XYZ$, $YP = 12$, $LX = 15$, and $LZ = 18$. *(p. 319)*

3. Find the length of $\overline{LY}$.

4. Find the length of $\overline{YN}$.

5. Find the length of $\overline{LP}$.



Technology **ACTIVITY** *Use after Lesson 5.4*

5.4 Investigate Points of Concurrency

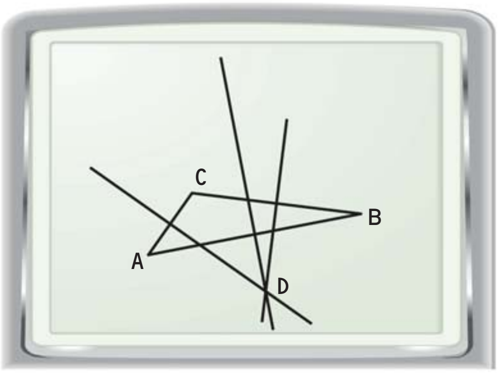
MATERIALS • graphing calculator or computer

QUESTION How are the points of concurrency in a triangle related?

You can use geometry drawing software to investigate concurrency.

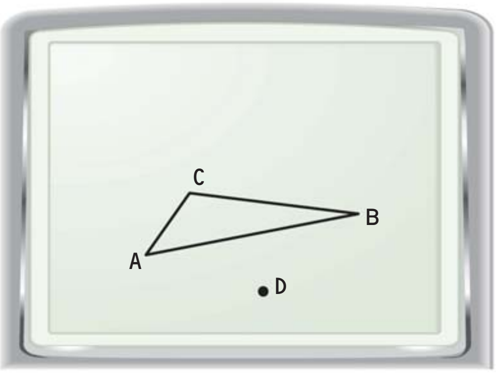
EXAMPLE 1 Draw the perpendicular bisectors of a triangle

STEP 1



Draw perpendicular bisectors Draw a line perpendicular to each side of a $\triangle ABC$ at the midpoint. Label the point of concurrency D.

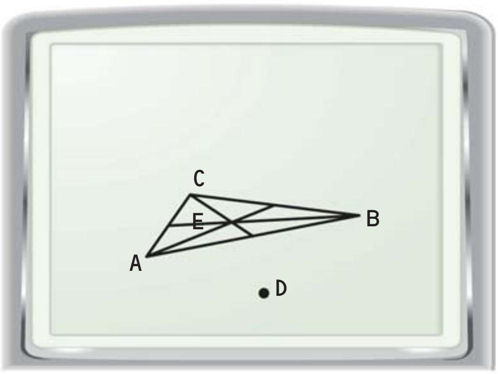
STEP 2



Hide the lines Use the *HIDE* feature to hide the perpendicular bisectors. Save as "EXAMPLE1."

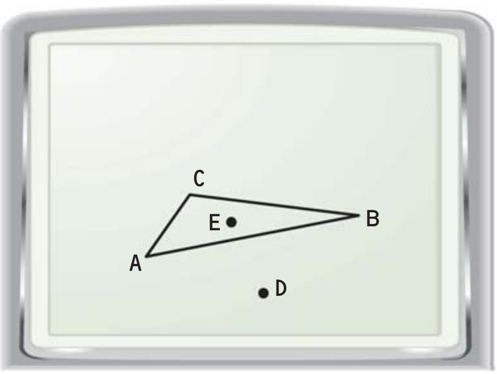
EXAMPLE 2 Draw the medians of the triangle

STEP 1



Draw medians Start with the figure you saved as "EXAMPLE1." Draw the medians of $\triangle ABC$. Label the point of concurrency E.

STEP 2

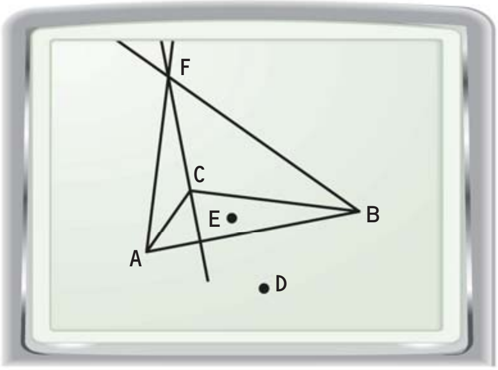


Hide the lines Use the *HIDE* feature to hide the medians. Save as "EXAMPLE2."

@HomeTutor classzone.com Keystrokes

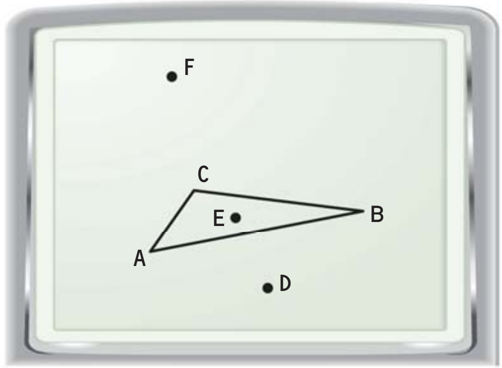
EXAMPLE 3 **Draw the altitudes of the triangle**

STEP 1



Draw altitudes Start with the figure you saved as "EXAMPLE2." Draw the altitudes of $\triangle ABC$. Label the point of concurrency F.

STEP 2



Hide the lines Use the *HIDE* feature to hide the altitudes. Save as "EXAMPLE3."

PRACTICE

1. Try to draw a line through points D, E, and F. Are the points collinear?
2. Try dragging point A. Do points D, E, and F remain collinear?

In Exercises 3–5, use the triangle you saved as "EXAMPLE3."

3. Draw the angle bisectors. Label the point of concurrency as point G.
4. How does point G relate to points D, E, and F?
5. Try dragging point A. What do you notice about points D, E, F, and G?

DRAW CONCLUSIONS

In 1765, Leonhard Euler (pronounced "oi′-ler") proved that the circumcenter, the centroid, and the orthocenter are all collinear. The line containing these three points is called *Euler's line*. Save the triangle from Exercise 5 as "EULER" and use that for Exercises 6–8.

6. Try moving the triangle's vertices. Can you verify that the same three points lie on Euler's line whatever the shape of the triangle? *Explain.*
7. Notice that some of the four points can be outside of the triangle. Which points lie outside the triangle? Why? What happens when you change the shape of the triangle? Are there any points that never lie outside the triangle? Why?
8. Draw the three midsegments of the triangle. Which, if any, of the points seem contained in the triangle formed by the midsegments? Do those points stay there when the shape of the large triangle is changed?

5.5 Use Inequalities in a Triangle

Before You found what combinations of angles are possible in a triangle.

Now You will find possible side lengths of a triangle.

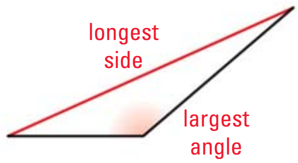
Why? So you can find possible distances, as in Ex. 39.

Key Vocabulary
- **side opposite,** *p. 241*
- **inequality,** *p. 876*

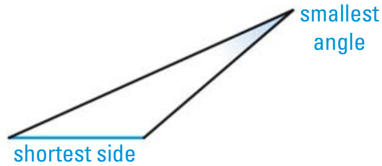
EXAMPLE 1 Relate side length and angle measure

Draw an obtuse scalene triangle. Find the largest angle and longest side and mark them in red. Find the smallest angle and shortest side and mark them in blue. What do you notice?

Solution



The longest side and largest angle are opposite each other.



The shortest side and smallest angle are opposite each other.

The relationships in Example 1 are true for all triangles as stated in the two theorems below. These relationships can help you to decide whether a particular arrangement of side lengths and angle measures in a triangle may be possible.

AVOID ERRORS

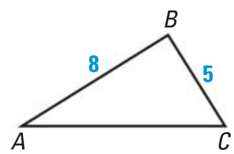
Be careful not to confuse the symbol ∠ meaning *angle* with the symbol < meaning *is less than*. Notice that the bottom edge of the angle symbol is horizontal.

THEOREMS *For Your Notebook*

THEOREM 5.10

If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

Proof: p. 329

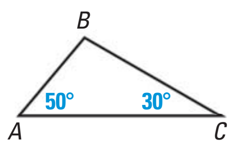


$AB > BC$, so $m\angle C > m\angle A$.

THEOREM 5.11

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

Proof: Ex. 24, p. 340



$m\angle A > m\angle C$, so $BC > AB$.

EXAMPLE 2 Standardized Test Practice

STAGE PROP You are constructing a stage prop that shows a large triangular mountain. The bottom edge of the mountain is about 27 feet long, the left slope is about 24 feet long, and the right slope is about 20 feet long. You are told that one of the angles is about 46° and one is about 59°. What is the angle measure of the peak of the mountain?

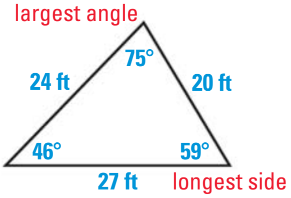
Ⓐ 46° Ⓑ 59° Ⓒ 75° Ⓓ 85°

ELIMINATE CHOICES

You can eliminate choice D because a triangle with a 46° angle and a 59° angle cannot have an 85° angle. The sum of the three angles in a triangle must be 180°, but the sum of 46, 59, and 85 is 190, not 180.

Solution

Draw a diagram and label the side lengths. The peak angle is opposite the longest side so, by Theorem 5.10, the peak angle is the largest angle.

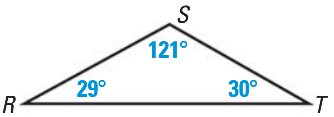


The angle measures sum to 180°, so the third angle measure is $180° - (46° + 59°) = 75°$. You can now label the angle measures in your diagram.

▶ The greatest angle measure is 75°, so the correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 1 and 2

1. List the sides of $\triangle RST$ in order from shortest to longest.

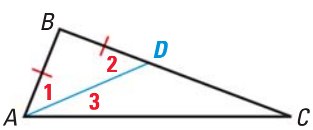


2. Another stage prop is a right triangle with sides that are 6, 8, and 10 feet long and angles of 90°, about 37°, and about 53°. Sketch and label a diagram with the shortest side on the bottom and the right angle at the left.

PROOF Theorem 5.10

GIVEN ▶ $BC > AB$

PROVE ▶ $m\angle BAC > m\angle C$



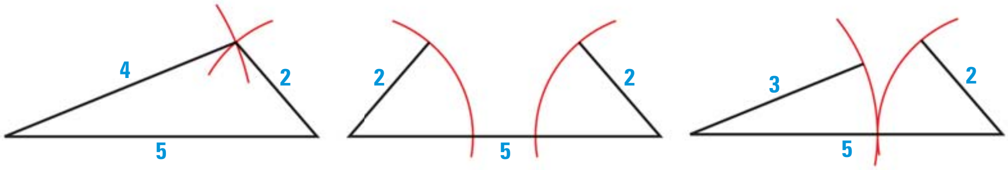
Locate a point D on $\overline{BC}$ such that $DB = BA$. Then draw $\overline{AD}$. In the isosceles triangle $\triangle ABD$, $\angle 1 \cong \angle 2$.

Because $m\angle BAC = m\angle 1 + m\angle 3$, it follows that $m\angle BAC > m\angle 1$. Substituting $m\angle 2$ for $m\angle 1$ produces $m\angle BAC > m\angle 2$.

By the Exterior Angle Theorem, $m\angle 2 = m\angle 3 + m\angle C$, so it follows that $m\angle 2 > m\angle C$ (see Exercise 27, page 332). Finally, because $m\angle BAC > m\angle 2$ and $m\angle 2 > m\angle C$, you can conclude that $m\angle BAC > m\angle C$.

THE TRIANGLE INEQUALITY Not every group of three segments can be used to form a triangle. The lengths of the segments must fit a certain relationship.

For example, three attempted triangle constructions for sides with given lengths are shown below. Only the first set of side lengths forms a triangle.



If you start with the longest side and attach the other two sides at its endpoints, you can see that the other two sides are not long enough to form a triangle in the second and third figures. This leads to the *Triangle Inequality Theorem*.

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THEOREM — *For Your Notebook*

THEOREM 5.12 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$AB + BC > AC$ $AC + BC > AB$ $AB + AC > BC$

Proof: Ex. 47, p. 334

EXAMPLE 3 Find possible side lengths

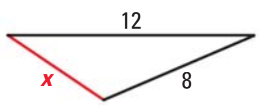
ALGEBRA A triangle has one side of length 12 and another of length 8. Describe the possible lengths of the third side.

Solution

Let x represent the length of the third side. Draw diagrams to help visualize the small and large values of x. Then use the Triangle Inequality Theorem to write and solve inequalities.

USE SYMBOLS
You can combine the two inequalities, $x > 4$ and $x < 20$, to write the compound inequality $4 < x < 20$. This can be read as *x is between 4 and 20.*

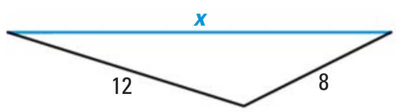
Small values of x



$x + 8 > 12$

$x > 4$

Large values of x



$8 + 12 > x$

$20 > x$, or $x < 20$

▶ The length of the third side must be greater than 4 and less than 20.

✓ GUIDED PRACTICE for Example 3

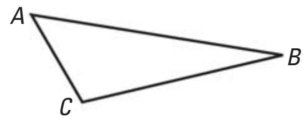
3. A triangle has one side of 11 inches and another of 15 inches. *Describe* the possible lengths of the third side.

5.5 EXERCISES

HOMEWORK KEY
◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 17, and 39
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 12, 20, 30, 39, and 45

SKILL PRACTICE

1. **VOCABULARY** Use the diagram at the right. For each angle, name the side that is *opposite* that angle.



2. ★ **WRITING** How can you tell from the angle measures of a triangle which side of the triangle is the longest? the shortest?

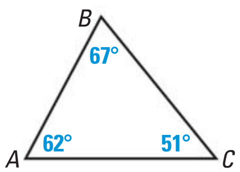
EXAMPLE 1 on p. 328 for Exs. 3–5

MEASURING Use a ruler and protractor to draw the given type of triangle. Mark the largest angle and longest side in red and the smallest angle and shortest side in blue. What do you notice?

3. Acute scalene
4. Right scalene
5. Obtuse isosceles

EXAMPLE 2 on p. 329 for Exs. 6–15

WRITING MEASUREMENTS IN ORDER List the sides and the angles in order from smallest to largest.

6.


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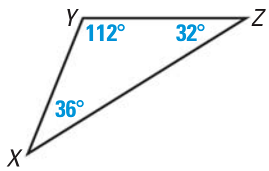

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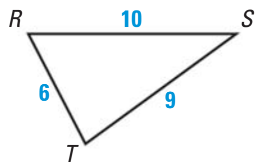

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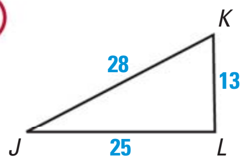

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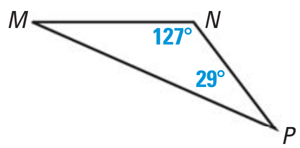

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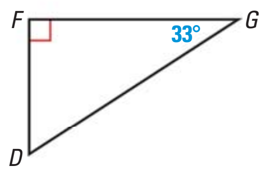
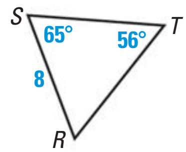


12. ★ **MULTIPLE CHOICE** In $\triangle RST$, which is a possible side length for ST?

Ⓐ 7 Ⓑ 8

Ⓒ 9 Ⓓ Cannot be determined



DRAWING TRIANGLES Sketch and label the triangle described.

13. Side lengths: about 3 m, 7 m, and 9 m, with longest side on the bottom
Angle measures: 16°, 41°, and 123°, with smallest angle at the left

14. Side lengths: 37 ft, 35 ft, and 12 ft, with shortest side at the right
Angle measures: about 71°, about 19°, and 90°, with right angle at the top

15. Side lengths: 11 in., 13 in., and 14 in., with middle-length side at the left
Two angle measures: about 48° and 71°, with largest angle at the top

EXAMPLE 3 on p. 330 for Exs. 16–26

IDENTIFYING POSSIBLE TRIANGLES Is it possible to construct a triangle with the given side lengths? If not, *explain* why not.

16. 6, 7, 11
17. 3, 6, 9
18. 28, 34, 39
19. 35, 120, 125

20. ★ **MULTIPLE CHOICE** Which group of side lengths can be used to construct a triangle?

(A) 3 yd, 4 ft, 5 yd (B) 3 yd, 5 ft, 8 ft

(C) 11 in., 16 in., 27 in. (D) 2 ft, 11 in., 12 in.

POSSIBLE SIDE LENGTHS ***Describe*** **the possible lengths of the third side of the triangle given the lengths of the other two sides.**

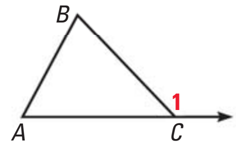
21. 5 inches, 12 inches **22.** 3 meters, 4 meters **23.** 12 feet, 18 feet

24. 10 yards, 23 yards **25.** 2 feet, 40 inches **26.** 25 meters, 25 meters

27. **EXTERIOR ANGLE INEQUALITY** Another triangle inequality relationship is given by the Exterior Inequality Theorem. It states:

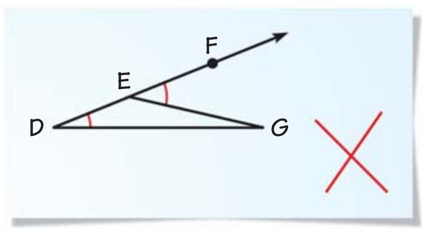
The measure of an exterior angle of a triangle is greater than the measure of either of the nonadjacent interior angles.

Use a relationship from Chapter 4 to *explain* how you know that $m\angle 1 > m\angle A$ and $m\angle 1 > m\angle B$ in $\triangle ABC$ with exterior angle $\angle 1$.



ERROR ANALYSIS **Use Theorems 5.10–5.12 and the theorem in Exercise 27 to** ***explain*** **why the diagram must be incorrect.**

28.



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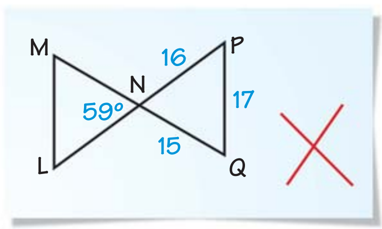


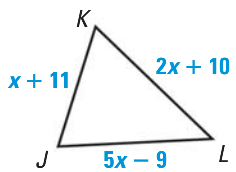
30. ★ **SHORT RESPONSE** *Explain* why the hypotenuse of a right triangle must always be longer than either leg.

ORDERING MEASURES **Is it possible to build a triangle using the given side lengths? If so, order the angles measures of the triangle from least to greatest.**

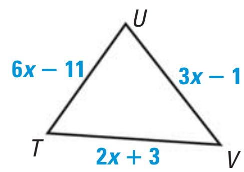
31. $PQ = \sqrt{58}$, $QR = 2\sqrt{13}$, $PR = 5\sqrt{2}$ **32.** $ST = \sqrt{29}$, $TU = 2\sqrt{17}$, $SU = 13.9$

ALGEBRA ***Describe*** **the possible values of** x**.**

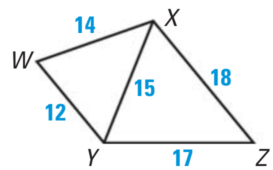
33.



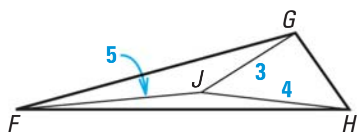
34.



35. **USING SIDE LENGTHS** Use the diagram at the right. Suppose $\overline{XY}$ bisects $\angle WYZ$. List all six angles of $\triangle XYZ$ and $\triangle WXY$ in order from smallest to largest. *Explain* your reasoning.

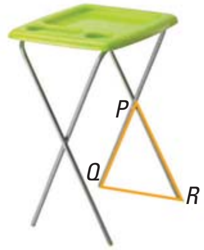


36. **CHALLENGE** The perimeter of $\triangle HGF$ must be between what two integers? *Explain* your reasoning.



PROBLEM SOLVING

37. TRAY TABLE In the tray table shown, $\overline{PQ} \cong \overline{PR}$ and $QR < PQ$. Write two inequalities about the angles in $\triangle PQR$. What other angle relationship do you know?



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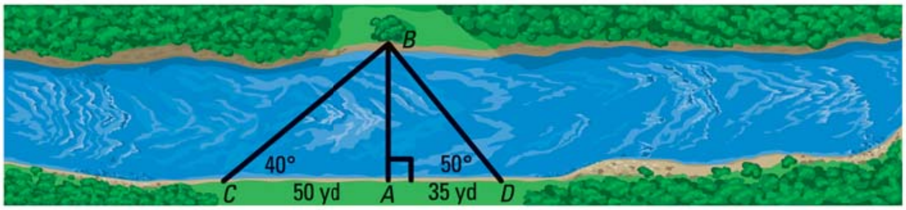
38. INDIRECT MEASUREMENT You can estimate the width of the river at point A by taking several sightings to the tree across the river at point B. The diagram shows the results for locations C and D along the riverbank. Using $\triangle BCA$ and $\triangle BDA$, what can you conclude about AB, the width of the river at point A? What could you do if you wanted a closer estimate?



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EXAMPLE 3 on p. 330 for Ex. 39

39. ★ **EXTENDED RESPONSE** You are planning a vacation to Montana. You want to visit the destinations shown in the map.

a. A brochure states that the distance between Granite Peak and Fort Peck Lake is 1080 kilometers. *Explain* how you know that this distance is a misprint.

b. Could the distance from Granite Peak to Fort Peck Lake be 40 kilometers? *Explain*.

c. Write two inequalities to represent the range of possible distances from Granite Peak to Fort Peck Lake.

d. What can you say about the distance between Granite Peak and Fort Peck Lake if you know that $m\angle 2 < m\angle 1$ and $m\angle 2 < m\angle 3$?

FORMING TRIANGLES In Exercises 40–43, you are given a 24 centimeter piece of string. You want to form a triangle out of the string so that the length of each side is a whole number. Draw figures accurately.

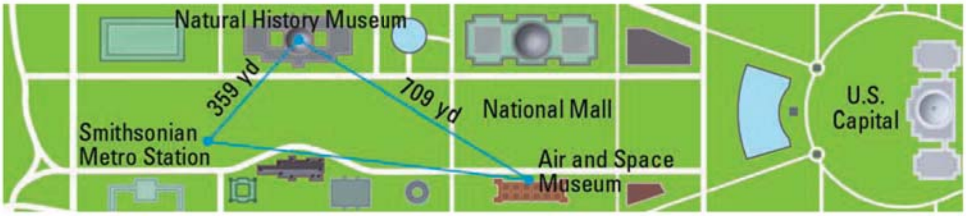
40. Can you decide if three side lengths form a triangle without checking all three inequalities shown for Theorem 5.12? If so, *describe* your shortcut.

41. Draw four possible isosceles triangles and label each side length. Tell whether each of the triangles you formed is *acute*, *right*, or *obtuse*.

42. Draw three possible scalene triangles and label each side length. Try to form at least one scalene acute triangle and one scalene obtuse triangle.

43. List three combinations of side lengths that will not produce triangles.

44. SIGHTSEEING You get off the Washington, D.C., subway system at the Smithsonian Metro station. First you visit the Museum of Natural History. Then you go to the Air and Space Museum. You record the distances you walk on your map as shown. *Describe* the range of possible distances you might have to walk to get back to the Smithsonian Metro station.



45. ★ SHORT RESPONSE Your house is 2 miles from the library. The library is $\frac{3}{4}$ mile from the grocery store. What do you know about the distance from your house to the grocery store? *Explain*. Include the special case when the three locations are all in a straight line.

46. ISOSCELES TRIANGLES For what combinations of angle measures in an isosceles triangle are the congruent sides shorter than the base of the triangle? longer than the base of the triangle?

47. PROVING THEOREM 5.12 Prove the Triangle Inequality Theorem.

GIVEN ▶ $\triangle ABC$

PROVE ▶ (1) $AB + BC > AC$
(2) $AC + BC > AB$
(3) $AB + AC > BC$

Plan for Proof One side, say BC, is longer than or at least as long as each of the other sides. Then (1) and (2) are true. To prove (3), extend $\overline{AC}$ to D so that $\overline{AB} \cong \overline{AD}$ and use Theorem 5.11 to show that $DC > BC$.

48. CHALLENGE Prove the following statements.

a. The length of any one median of a triangle is less than half the perimeter of the triangle.

b. The sum of the lengths of the three medians of a triangle is greater than half the perimeter of the triangle.

MIXED REVIEW

PREVIEW
Prepare for Lesson 5.6 in Exs. 49–50.

In Exercises 49 and 50, write the if-then form, the converse, the inverse, and the contrapositive of the given statement. *(p. 79)*

49. A redwood is a large tree.

50. $5x - 2 = 18$, because $x = 4$.

51. A triangle has vertices $A(22, 21)$, $B(0, 0)$, and $C(22, 2)$. Graph $\triangle ABC$ and classify it by its sides. Then determine if it is a right triangle. *(p. 217)*

Graph figure *LMNP* with vertices *L*(−4, 6), *M*(4, 8), *N*(2, 2), and *P*(−4, 0). Then draw its image after the transformation. *(p. 272)*

52. $(x, y) \rightarrow (x + 3, y - 4)$

53. $(x, y) \rightarrow (x, -y)$

54. $(x, y) \rightarrow (-x, y)$

5.6 Inequalities in Two Triangles and Indirect Proof

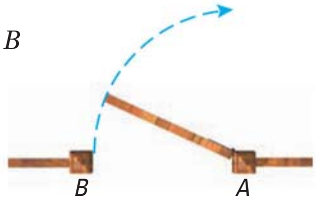
Before You used inequalities to make comparisons in one triangle.

Now You will use inequalities to make comparisons in two triangles.

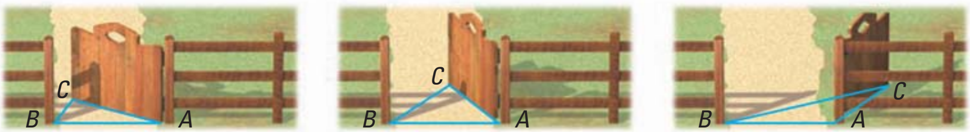
Why? So you can compare the distances hikers traveled, as in Ex. 22.

Key Vocabulary
- **indirect proof**
- **included angle,** *p. 240*

Imagine a gate between fence posts A and B that has hinges at A and swings open at B.



As the gate swings open, you can think of $\triangle ABC$, with side $\overline{AC}$ formed by the gate itself, side $\overline{AB}$ representing the distance between the fence posts, and side $\overline{BC}$ representing the opening between post B and the outer edge of the gate.

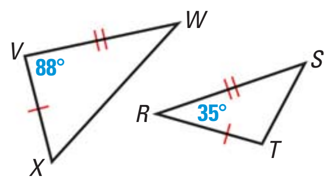


Notice that as the gate opens wider, both the measure of $\angle A$ and the distance CB increase. This suggests the *Hinge Theorem*.

THEOREMS — For Your Notebook

THEOREM 5.13 Hinge Theorem

If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second.

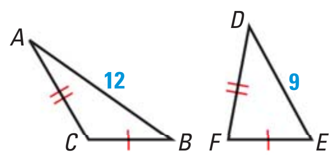


$WX > ST$

Proof: Ex. 28, p. 341

THEOREM 5.14 Converse of the Hinge Theorem

If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second.

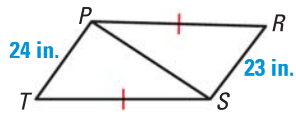


$m\angle C > m\angle F$

Proof: Example 4, p. 338

EXAMPLE 1 Use the Converse of the Hinge Theorem

Given that $\overline{ST} \cong \overline{PR}$, how does $\angle PST$ compare to $\angle SPR$?



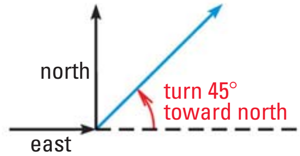
Solution

You are given that $\overline{ST} \cong \overline{PR}$ and you know that $\overline{PS} \cong \overline{PS}$ by the Reflexive Property. Because 24 inches > 23 inches, $PT > RS$. So, two sides of $\triangle STP$ are congruent to two sides of $\triangle PRS$ and the third side in $\triangle STP$ is longer.

▶ By the Converse of the Hinge Theorem, $m\angle PST > m\angle SPR$.

EXAMPLE 2 Solve a multi-step problem

BIKING Two groups of bikers leave the same camp heading in opposite directions. Each group goes 2 miles, then changes direction and goes 1.2 miles. Group A starts due east and then turns 45° toward north as shown. Group B starts due west and then turns 30° toward south.



Which group is farther from camp? Explain your reasoning.

Solution

Draw a diagram and mark the given measures. The distances biked and the distances back to camp form two triangles, with congruent 2 mile sides and congruent 1.2 mile sides. Add the third sides of the triangles to your diagram.

Next use linear pairs to find and mark the included angles of 150° and 135°.

▶ Because 150° > 135°, Group B is farther from camp by the Hinge Theorem.

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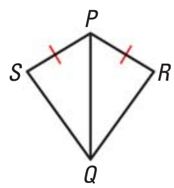
✓ GUIDED PRACTICE for Examples 1 and 2

Use the diagram at the right.



1. If $PR = PS$ and $m\angle QPR > m\angle QPS$, which is longer, $\overline{SQ}$ or $\overline{RQ}$?

2. If $PR = PS$ and $RQ < SQ$, which is larger, $\angle RPQ$ or $\angle SPQ$?

3. **WHAT IF?** In Example 2, suppose Group C leaves camp and goes 2 miles due north. Then they turn 40° toward east and continue 1.2 miles. *Compare* the distances from camp for all three groups.

INDIRECT REASONING Suppose a student looks around the cafeteria, concludes that hamburgers are not being served, and explains as follows.

> *At first I assumed that we are having hamburgers because today is Tuesday and Tuesday is usually hamburger day.*
>
> *There is always ketchup on the table when we have hamburgers, so I looked for the ketchup, but I didn't see any.*
>
> *So, my assumption that we are having hamburgers must be false.*

The student used *indirect* reasoning. So far in this book, you have reasoned *directly* from given information to prove desired conclusions.

In an **indirect proof**, you start by making the temporary assumption that the desired conclusion is false. By then showing that this assumption leads to a logical impossibility, you prove the original statement true *by contradiction*.

KEY CONCEPT *For Your Notebook*

How to Write an Indirect Proof

STEP 1 **Identify** the statement you want to prove. **Assume** temporarily that this statement is false by assuming that its opposite is true.

STEP 2 **Reason** logically until you reach a contradiction.

STEP 3 **Point out** that the desired conclusion must be true because the contradiction proves the temporary assumption false.

EXAMPLE 3 Write an indirect proof

Write an indirect proof that an odd number is not divisible by 4.

GIVEN ▶ x is an odd number.

PROVE ▶ x is not divisible by 4.

Solution

STEP 1 Assume temporarily that x is divisible by 4. This means that $\frac{x}{4} = n$ for some whole number n. So, multiplying both sides by 4 gives $x = 4n$.

STEP 2 If x is odd, then, by definition, x cannot be divided evenly by 2. However, $x = 4n$ so $\frac{x}{2} = \frac{4n}{2} = 2n$. We know that $2n$ is a whole number because n is a whole number, so x *can* be divided evenly by 2. This contradicts the given statement that x is odd.

STEP 3 Therefore, the assumption that x is divisible by 4 must be false, which proves that x is not divisible by 4.

READ VOCABULARY
You have reached a *contradiction* when you have two statements that cannot both be true at the same time.

GUIDED PRACTICE for Example 3

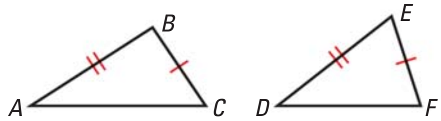
4. Suppose you wanted to prove the statement "If $x + y \neq 14$ and $y = 5$, then $x \neq 9$." What temporary assumption could you make to prove the conclusion indirectly? How does that assumption lead to a contradiction?

EXAMPLE 4 Prove the Converse of the Hinge Theorem

Write an indirect proof of Theorem 5.14.

GIVEN ▶ $\overline{AB} \cong \overline{DE}$
$\overline{BC} \cong \overline{EF}$
$AC > DF$

PROVE ▶ $m\angle B > m\angle E$



Proof Assume temporarily that $m\angle B \ngtr m\angle E$. Then, it follows that either $m\angle B = m\angle E$ or $m\angle B < m\angle E$.

Case 1 If $m\angle B = m\angle E$, then $\angle B \cong \angle E$. So, $\triangle ABC \cong \triangle DEF$ by the SAS Congruence Postulate and $AC = DF$.

Case 2 If $m\angle B < m\angle E$, then $AC < DF$ by the Hinge Theorem.

Both conclusions contradict the given statement that $AC > DF$. So, the temporary assumption that $m\angle B \ngtr m\angle E$ cannot be true. This proves that $m\angle B > m\angle E$.

✓ GUIDED PRACTICE for Example 4

5. Write a temporary assumption you could make to prove the Hinge Theorem indirectly. What two cases does that assumption lead to?

5.6 EXERCISES

HOMEWORK KEY
◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 7, and 23
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 19, and 25

SKILL PRACTICE

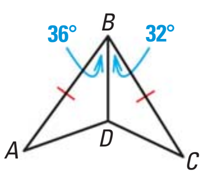
1. **VOCABULARY** Why is indirect proof also called *proof by contradiction*?

2. ★ **WRITING** *Explain* why the name "Hinge Theorem" is used for Theorem 5.13.

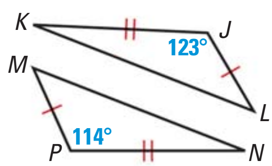
EXAMPLE 1 on p. 336 for Exs. 3–10

APPLYING THEOREMS **Copy and complete with <, >, or =. *Explain.***

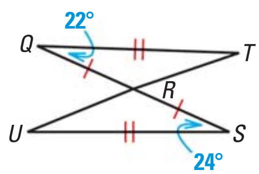
3. AD _?_ CD



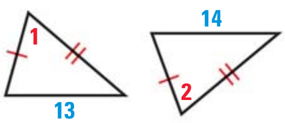
4. MN _?_ LK



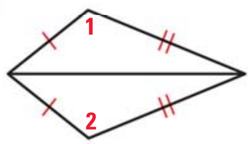
5. TR _?_ UR



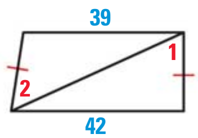
6. $m\angle 1$ _?_ $m\angle 2$



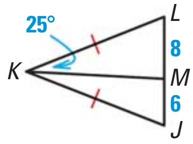
7. $m\angle 1$ _?_ $m\angle 2$



8. $m\angle 1$ _?_ $m\angle 2$



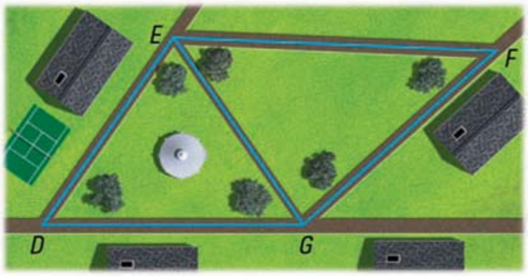
9. ★ **MULTIPLE CHOICE** Which is a possible measure for $\angle JKM$?



Ⓐ 20° Ⓑ 25°

Ⓒ 30° Ⓓ Cannot be determined

10. **USING A DIAGRAM** The path from E to F is longer than the path from E to D. The path from G to D is the same length as the path from G to F. What can you conclude about the angles of the paths? *Explain* your reasoning.



EXAMPLES 3 and 4 on p. 337–338 for Exs. 11–13

STARTING AN INDIRECT PROOF In Exercises 11 and 12, write a temporary assumption you could make to prove the conclusion indirectly.

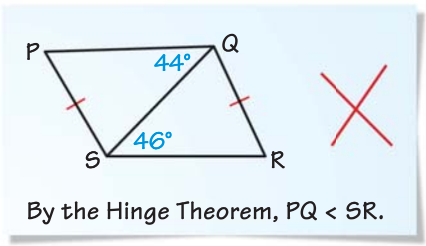
11. If x and y are odd integers, then xy is odd.

12. In $\triangle ABC$, if $m\angle A = 100°$, then $\angle B$ is not a right angle.

13. **REASONING** Your study partner is planning to write an indirect proof to show that $\angle A$ is an obtuse angle. She states "Assume temporarily that $\angle A$ is an acute angle." What has your study partner overlooked?

ERROR ANALYSIS ***Explain*** **why the student's reasoning is not correct.**

14.



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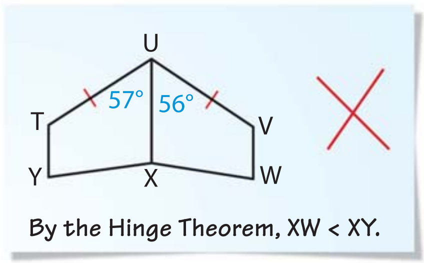
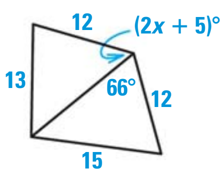


xy **ALGEBRA Use the Hinge Theorem or its converse and properties of triangles to write and solve an inequality to describe a restriction on the value of** $\boldsymbol{x}$**.**

16.



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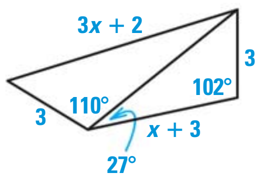

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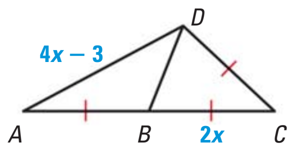

19. ★ **SHORT RESPONSE** If $\overline{NR}$ is a median of $\triangle NPQ$ and $NQ > NP$, *explain* why $\angle NRQ$ is obtuse.

20. **ANGLE BISECTORS** In $\triangle EFG$, the bisector of $\angle F$ intersects the bisector of $\angle G$ at point H. *Explain* why $\overline{FG}$ must be longer than $\overline{FH}$ or $\overline{HG}$.

21. **CHALLENGE** In $\triangle ABC$, the altitudes from B and C meet at D. What is true about $\triangle ABC$ if $m\angle BAC > m\angle BDC$? *Justify* your answer.

PROBLEM SOLVING

EXAMPLE 2
on p. 336
for Ex. 22

22. HIKING Two hikers start at the visitor center. The first hikes 4 miles due west, then turns 40° toward south and hikes 1.8 miles. The second hikes 4 miles due east, then turns 52° toward north and and hikes 1.8 miles. Which hiker is farther from camp? *Explain* how you know.

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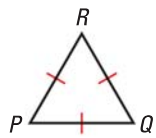
EXAMPLES 3 and 4
on pp. 337–338
for Exs. 23–24

23. INDIRECT PROOF Arrange statements A–E in order to write an indirect proof of the corollary: If $\triangle ABC$ is *equilateral*, then it is *equiangular*.



GIVEN ▶ $\triangle PQR$ is equilateral.

A. That means that for some pair of vertices, say P and Q, $m\angle P > m\angle Q$.

B. But this contradicts the given statement that $\triangle PQR$ is equilateral.

C. The contradiction shows that the temporary assumption that $\triangle PQR$ is not equiangular is false. This proves that $\triangle PQR$ is equiangular.

D. Then, by Theorem 5.11, you can conclude that $QR > PR$.

E. Temporarily assume that $\triangle PQR$ is not equiangular.

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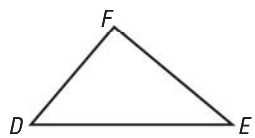
24. PROVING THEOREM 5.11 Write an indirect proof of Theorem 5.11, page 328.

GIVEN ▶ $m\angle D > m\angle E$

PROVE ▶ $EF > DF$



Plan for Proof In Case 1, assume that $EF < DF$. In Case 2, assume that $EF = DF$.

25. ★ EXTENDED RESPONSE A scissors lift can be used to adjust the height of a platform.

a. Interpret As the mechanism expands, $\overline{KL}$ gets longer. As KL increases, what happens to $m\angle LNK$? to $m\angle KNM$?

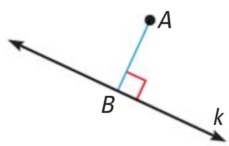
b. Apply Name a distance that decreases as $\overline{KL}$ gets longer.

c. Writing *Explain* how the adjustable mechanism illustrates the Hinge Theorem.

26. PROOF Write a proof that the shortest distance from a point to a line is the length of the perpendicular segment from the point to the line.

GIVEN ▶ Line k; point A not on k; point B on k such that $\overline{AB} \perp k$

PROVE ▶ $\overline{AB}$ is the shortest segment from A to k.



Plan for Proof Assume that there is a shorter segment from A to k and use Theorem 5.10 to show that this leads to a contradiction.

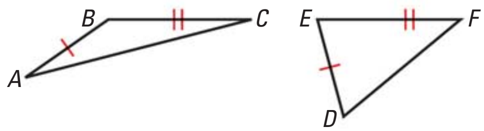
○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE

27. **USING A CONTRAPOSITIVE** Because the contrapositive of a conditional is equivalent to the original statement, you can prove the statement by proving its contrapositive. Look back at the conditional in Example 3 on page 337. Write a proof of the contrapositive that uses direct reasoning. How is your proof similar to the indirect proof of the original statement?

28. **CHALLENGE** Write a proof of Theorem 5.13, the Hinge Theorem.

GIVEN ▶ $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $m\angle ABC > m\angle DEF$

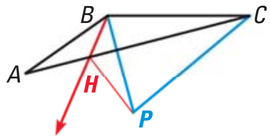
PROVE ▶ $AC > DF$



Plan for Proof

1. Because $m\angle ABC > m\angle DEF$, you can locate a point P in the interior of $\angle ABC$ so that $\angle CBP \cong \angle FED$ and $\overline{BP} \cong \overline{ED}$. Draw $\overline{BP}$ and show that $\triangle PBC \cong \triangle DEF$.
2. Locate a point H on $\overline{AC}$ so that $\overrightarrow{BH}$ bisects $\angle PBA$ and show that $\triangle ABH \cong \triangle PBH$.
3. Give reasons for each statement below to show that $AC > DF$.

$AC = AH + HC = PH + HC > PC = DF$



MIXED REVIEW

PREVIEW
Prepare for Lesson 6.1 in Exs. 29–31.

Write the conversion factor you would multiply by to change units as specified. *(p. 886)*

29. inches to feet
30. liters to kiloliters
31. pounds to ounces

Solve the equation. Write a reason for each step. *(p. 105)*

32. $1.5(x + 4) = 5(2.4)$
33. $-3(-2x + 5) = 12$
34. $2(5x) = 3(4x + 6)$

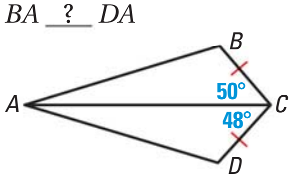
35. Simplify the expression $\frac{-6xy^2}{21x^2y}$ if possible. *(p. 139)*

QUIZ *for Lessons 5.5–5.6*

1. Is it possible to construct a triangle with side lengths 5, 6, and 12? If not, *explain* why not. *(p. 328)*
2. The lengths of two sides of a triangle are 15 yards and 27 yards. *Describe* the possible lengths of the third side of the triangle. *(p. 328)*
3. In $\triangle PQR$, $m\angle P = 48°$ and $m\angle Q = 79°$. List the sides of $\triangle PQR$ in order from shortest to longest. *(p. 328)*

Copy and complete with <, >, or =. *(p. 335)*

4. BA __?__ DA



5. $m\angle 1$ __?__ $m\angle 2$

STATE TEST PRACTICE
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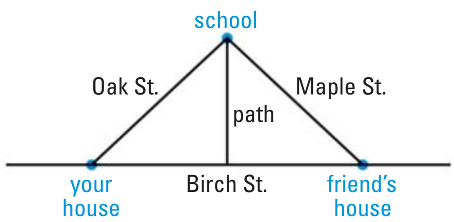
Lessons 5.4–5.6

1. **MULTI-STEP PROBLEM** In the diagram below, the entrance to the path is halfway between your house and your friend's house.

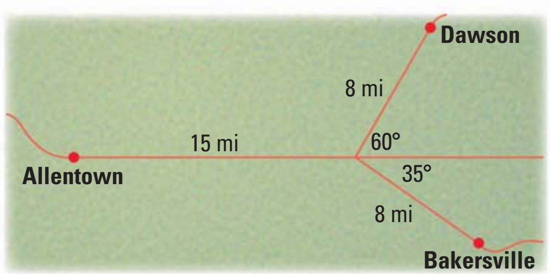


   a. Can you conclude that you and your friend live the same distance from the school if the path bisects the angle formed by Oak and Maple Streets?

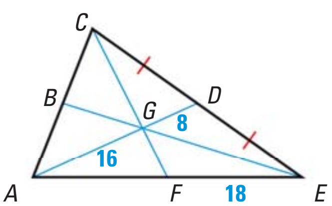
   b. Can you conclude that you and your friend live the same distance from the school if the path is perpendicular to Birch Street?

   c. Your answers to parts (a) and (b) show that a triangle must be isosceles if which two special segments are equal in length?

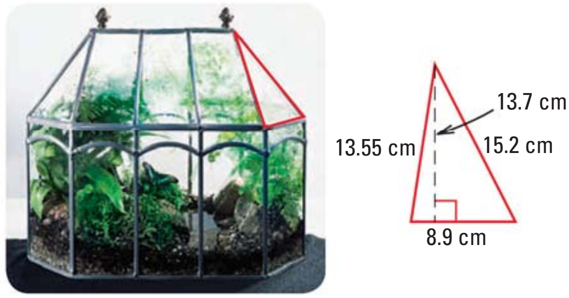
2. **SHORT RESPONSE** The map shows your driving route from Allentown to Bakersville and from Allentown to Dawson. Which city, Bakersville or Dawson, is located closer to Allentown? *Explain* your reasoning.



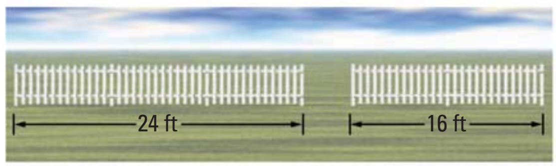
3. **GRIDDED RESPONSE** Find the length of $\overline{AF}$.



4. **SHORT RESPONSE** In the instructions for creating the terrarium shown, you are given a pattern for the pieces that form the roof. Does the diagram for the red triangle appear to be correct? *Explain* why or why not.



5. **EXTENDED RESPONSE** You want to create a triangular fenced pen for your dog. You have the two pieces of fencing shown, so you plan to move those to create two sides of the pen.

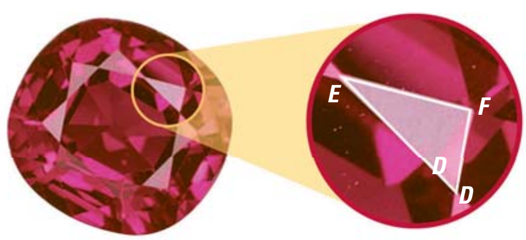


   a. *Describe* the possible lengths for the third side of the pen.

   b. The fencing is sold in 8 foot sections. If you use whole sections, what lengths of fencing are possible for the third side?

   c. You want your dog to have a run within the pen that is at least 25 feet long. Which pen(s) could you use? *Explain*.

6. **OPEN-ENDED** In the gem shown, give a possible side length of $\overline{DE}$ if $m\angle EFD > 90°$, $DF = 0.4$ mm, and $EF = 0.63$ mm.



5 CHAPTER SUMMARY

BIG IDEAS *For Your Notebook*

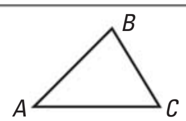
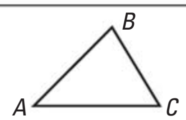
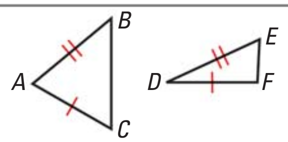
Big Idea 1

Using Properties of Special Segments in Triangles

Special segment	***Properties to remember***
Midsegment	Parallel to side opposite it and half the length of side opposite it
Perpendicular bisector	Concurrent at the circumcenter, which is: • equidistant from 3 vertices of $\triangle$ • center of *circumscribed* circle that passes through 3 vertices of $\triangle$
Angle bisector	Concurrent at the incenter, which is: • equidistant from 3 sides of $\triangle$ • center of *inscribed* circle that just touches each side of $\triangle$
Median (connects vertex to midpoint of opposite side)	Concurrent at the centroid, which is: • located two thirds of the way from vertex to midpoint of opposite side • balancing point of $\triangle$
Altitude (perpendicular to side of $\triangle$ through opposite vertex)	Concurrent at the orthocenter Used in finding area: If b is length of any side and h is length of altitude to that side, then $A = \frac{1}{2}bh$.

Big Idea 2

Using Triangle Inequalities to Determine What Triangles are Possible

Sum of lengths of any two sides of a $\triangle$ is greater than length of third side.		$AB + BC > AC$ $AB + AC > BC$ $BC + AC > AB$
In a $\triangle$, longest side is opposite largest angle and shortest side is opposite smallest angle.		If $AC > AB > BC$, then $m\angle B > m\angle C > m\angle A$. If $m\angle B > m\angle C > m\angle A$, then $AC > AB > BC$.
If two sides of a $\triangle$ are $\cong$ to two sides of another $\triangle$, then the $\triangle$ with longer third side also has larger included angle.		If $BC > EF$, then $m\angle A > m\angle D$. If $m\angle A > m\angle D$, then $BC > EF$.

Big Idea 3

Extending Methods for Justifying and Proving Relationships

Coordinate proof uses the coordinate plane and variable coordinates. *Indirect proof* involves assuming the conclusion is false and then showing that the assumption leads to a contradiction.

5 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- midsegment of a triangle, *p. 295*
- coordinate proof, *p. 296*
- perpendicular bisector, *p. 303*
- equidistant, *p. 303*
- concurrent, *p. 305*
- point of concurrency, *p. 305*
- circumcenter, *p. 306*
- incenter, *p. 312*
- median of a triangle, *p. 319*
- centroid, *p. 319*
- altitude of a triangle, *p. 320*
- orthocenter, *p. 321*
- indirect proof, *p. 337*

VOCABULARY EXERCISES

1. Copy and complete: A perpendicular bisector is a segment, ray, line, or plane that is perpendicular to a segment at its __?__.

2. **WRITING** *Explain* how to draw a circle that is circumscribed about a triangle. What is the center of the circle called? *Describe* its radius.

In Exercises 3–5, match the term with the correct definition.

3. Incenter — **A.** The point of concurrency of the medians of a triangle

4. Centroid — **B.** The point of concurrency of the angle bisectors of a triangle

5. Orthocenter — **C.** The point of concurrency of the altitudes of a triangle

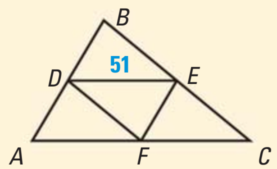
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 5.

5.1 Midsegment Theorem and Coordinate Proof — *pp. 295–301*

EXAMPLE

In the diagram, $\overline{DE}$ is a midsegment of $\triangle ABC$. Find AC.



By the Midsegment Theorem, $DE = \frac{1}{2}AC$.

So, $AC = 2DE = 2(51) = 102$.

EXERCISES

EXAMPLES 1, 4, and 5 on pp. 295, 297 for Exs. 6–8

Use the diagram above where $\overline{DF}$ and $\overline{EF}$ are midsegments of $\triangle ABC$.

6. If $AB = 72$, find EF.

7. If $DF = 45$, find EC.

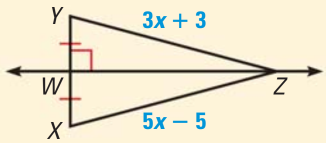
8. Graph $\triangle PQR$, with vertices $P(2a, 2b)$, $Q(2a, 0)$, and $O(0, 0)$. Find the coordinates of midpoint S of $\overline{PQ}$ and midpoint T of $\overline{QO}$. Show $\overline{ST} \parallel \overline{PO}$.

5.2 Use Perpendicular Bisectors

pp. 303–309

EXAMPLE

Use the diagram at the right to find XZ.



$\overleftrightarrow{WZ}$ is the perpendicular bisector of $\overline{XY}$.

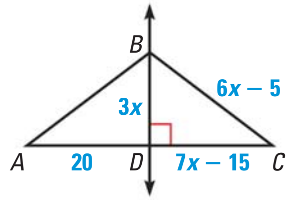
$5x - 5 = 3x + 3$	By the Perpendicular Bisector Theorem, $ZX = ZY$.
$x = 4$	Solve for x.

▶ So, $XZ = 5x - 5 = 5(4) - 5 = 15$.

EXERCISES

EXAMPLES 1 and 2
on pp. 303–304
for Exs. 9–11

In the diagram, $\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$.



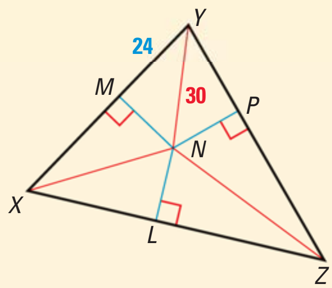
9. What segment lengths are equal?
10. What is the value of x?
11. Find AB.

5.3 Use Angle Bisectors of Triangles

pp. 310–316

EXAMPLE

In the diagram, N is the incenter of $\triangle XYZ$. Find NL.



Use the Pythagorean Theorem to find NM in $\triangle NMY$.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$30^2 = NM^2 + 24^2$	Substitute known values.
$900 = NM^2 + 576$	Multiply.
$324 = NM^2$	Subtract 576 from each side.
$18 = NM$	Take positive square root of each side.

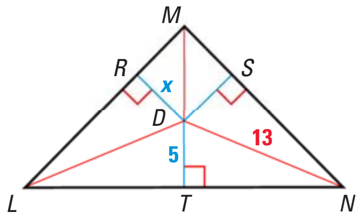
▶ By the Concurrency of Angle Bisectors of a Triangle, the incenter N of $\triangle XYZ$ is equidistant from all three sides of $\triangle XYZ$. So, because $NM = NL$, $NL = 18$.

EXERCISES

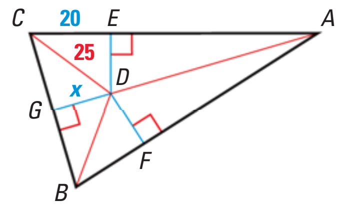
EXAMPLE 4
on p. 312
for Exs. 12–13

Point D is the incenter of the triangle. Find the value of x.

12.



13.

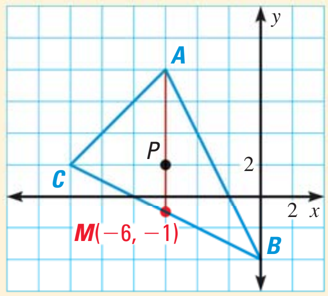


5 CHAPTER REVIEW

5.4 Use Medians and Altitudes *pp. 319–325*

EXAMPLE

The vertices of $\triangle ABC$ are $A(-6, 8)$, $B(0, -4)$, and $C(-12, 2)$. Find the coordinates of its centroid P.



Sketch $\triangle ABC$. Then find the midpoint M of $\overline{BC}$ and sketch median $\overline{AM}$.

$$M\left(\frac{-12+0}{2}, \frac{2+(-4)}{2}\right) = M(-6, -1)$$

The centroid is two thirds of the distance from a vertex to the midpoint of the opposite side.

The distance from vertex $A(-6, 8)$ to midpoint $M(-6, -1)$ is $8 - (-1) = 9$ units.

So, the centroid P is $\frac{2}{3}(9) = 6$ units down from A on $\overline{AM}$.

▶ The coordinates of the centroid P are $(-6, 8 - 6)$, or $(-6, 2)$.

EXERCISES

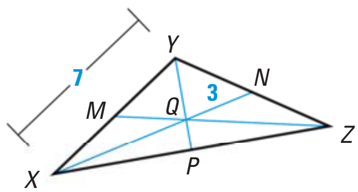
EXAMPLES 1, 2, and 3 on pp. 319–321 for Exs. 14–18

Find the coordinates of the centroid D of $\triangle RST$.

14. $R(-4, 0)$, $S(2, 2)$, $T(2, -2)$

15. $R(-6, 2)$, $S(-2, 6)$, $T(2, 4)$

Point Q is the centroid of $\triangle XYZ$.



16. Find XQ.

17. Find XM.

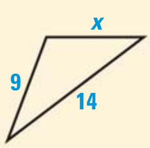
18. Draw an obtuse $\triangle ABC$. Draw its three altitudes. Then label its orthocenter D.

5.5 Use Inequalities in a Triangle *pp. 328–334*

EXAMPLE

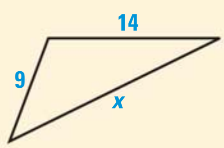
A triangle has one side of length 9 and another of length 14. Describe the possible lengths of the third side.

Let x represent the length of the third side. Draw diagrams and use the Triangle Inequality Theorem to write inequalities involving x.



$x + 9 > 14$

$x > 5$



$9 + 14 > x$

$23 > x$, or $x < 23$

▶ The length of the third side must be greater than 5 and less than 23.

EXERCISES

EXAMPLES 1, 2, and 3 on pp. 328–330 for Exs. 19–24

Describe **the possible lengths of the third side of the triangle given the lengths of the other two sides.**

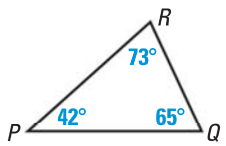
19. 4 inches, 8 inches

20. 6 meters, 9 meters

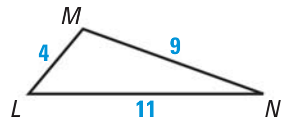
21. 12 feet, 20 feet

List the sides and the angles in order from smallest to largest.

22.



23.



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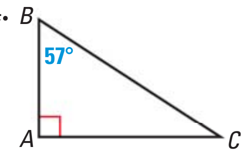
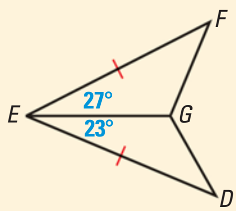


5.6 Inequalities in Two Triangles and Indirect Proof

pp. 335–341

EXAMPLE

How does the length of $\overline{DG}$ compare to the length of $\overline{FG}$?



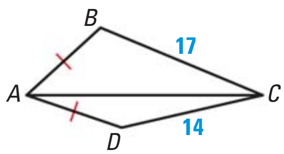
▶ Because $27° > 23°$, $m\angle GEF > m\angle GED$. You are given that $\overline{DE} \cong \overline{FE}$ and you know that $\overline{EG} \cong \overline{EG}$. Two sides of $\triangle GEF$ are congruent to two sides of $\triangle GED$ and the included angle is larger so, by the Hinge Theorem, $FG > DG$.

EXERCISES

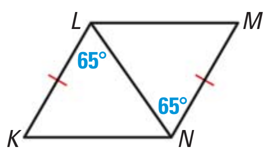
EXAMPLES 1, 3, and 4 on pp. 336–338 for Exs. 25–27

Copy and complete with <, >, or =.

25. $m\angle BAC$ _?_ $m\angle DAC$



26. LM _?_ KN



27. Arrange statements A–D in correct order to write an indirect proof of the statement: *If two lines intersect, then their intersection is exactly one point.*

GIVEN ▶ Intersecting lines m and n

PROVE ▶ The intersection of lines m and n is exactly one point.

A. But this contradicts Postulate 5, which states that through any two points there is exactly one line.

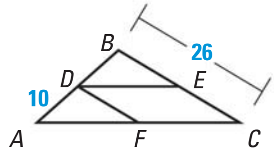
B. Then there are two lines (m and n) through points P and Q.

C. Assume that there are two points, P and Q, where m and n intersect.

D. It is false that m and n can intersect in two points, so they must intersect in exactly one point.

5 CHAPTER TEST

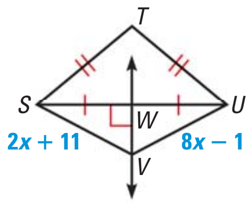
Two midsegments of $\triangle ABC$ are $\overline{DE}$ and $\overline{DF}$.



1. Find DB.
2. Find DF.
3. What can you conclude about $\overline{EF}$?

Find the value of x. *Explain* your reasoning.

4.



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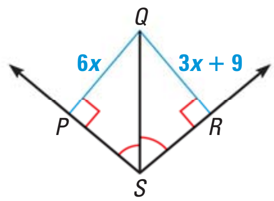

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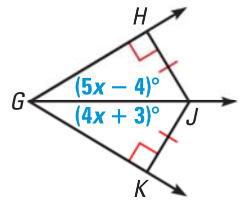
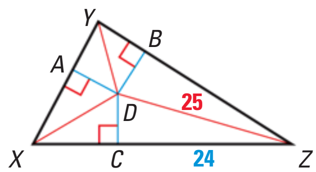
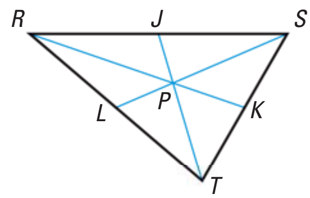


7. In Exercise 4, is point T on the perpendicular bisector of $\overline{SU}$? *Explain*.
8. In the diagram at the right, the angle bisectors of $\triangle XYZ$ meet at point D. Find DB.

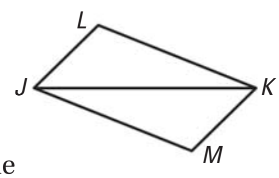


In the diagram at the right, P is the centroid of $\triangle RST$.



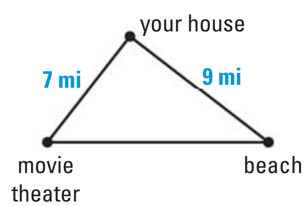
9. If $LS = 36$, find PL and PS.
10. If $TP = 20$, find TJ and PJ.
11. If $JR = 25$, find JS and RS.
12. Is it possible to construct a triangle with side lengths 9, 12, and 22? If not, *explain* why not.
13. In $\triangle ABC$, $AB = 36$, $BC = 18$, and $AC = 22$. Sketch and label the triangle. List the angles in order from smallest to largest.

In the diagram for Exercises 14 and 15, $JL = MK$.



14. If $m\angle JKM > m\angle LJK$, which is longer, $\overline{LK}$ or $\overline{MJ}$? *Explain*.
15. If $MJ < LK$, which is larger, $\angle LJK$ or $\angle JKM$? *Explain*.
16. Write a temporary assumption you could make to prove the conclusion indirectly: *If $RS + ST \neq 12$ and $ST = 5$, then $RS \neq 7$.*

Use the diagram in Exercises 17 and 18.



17. *Describe* the range of possible distances from the beach to the movie theater.
18. A market is the same distance from your house, the movie theater, and the beach. Copy the diagram and locate the market.

ALGEBRA REVIEW

Animated Algebra classzone.com

USE RATIOS AND PERCENT OF CHANGE

EXAMPLE 1 Write a ratio in simplest form

A team won 18 of its 30 games and lost the rest. Find its win-loss ratio.

The ratio of a to b, $b \neq 0$, can be written as a to b, $a:b$, and $\frac{a}{b}$.

$$\frac{\text{wins}}{\text{losses}} = \frac{18}{30 - 18}$$ To find losses, subtract wins from total.

$$= \frac{18}{12} = \frac{3}{2}$$ Simplify.

▶ The team's win-loss ratio is 3 : 2.

EXAMPLE 2 Find and interpret a percent of change

A \$50 sweater went on sale for \$28. What is the percent of change in price? The new price is what percent of the old price?

$$\text{Percent of change} = \frac{\text{Amount of increase or decrease}}{\text{Original amount}} = \frac{50 - 28}{50} = \frac{22}{50} = 0.44$$

▶ The price went down, so the change is a decrease. The percent of decrease is 44%. So, the new price is 100%−44% = 56% of the original price.

EXERCISES

EXAMPLE 1 for Exs. 1–3

1. A team won 12 games and lost 4 games. Write each ratio in simplest form.
   a. wins to losses
   b. losses out of total games
2. A scale drawing that is 2.5 feet long by 1 foot high was used to plan a mural that is 15 feet long by 6 feet high. Write each ratio in simplest form.
   a. length to height of mural
   b. length of scale drawing to length of mural
3. There are 8 males out of 18 members in the school choir. Write the ratio of females to males in simplest form.

EXAMPLE 2 for Exs. 4–13

Find the percent of change.

4. From 75 campsites to 120 campsites
5. From 150 pounds to 136.5 pounds
6. From \$480 to \$408
7. From 16 employees to 18 employees
8. From 24 houses to 60 houses
9. From 4000 ft^2 to 3990 ft^2

Write the percent comparing the new amount to the original amount. Then find the new amount.

10. 75 feet increased by 4%
11. 45 hours decreased by 16%
12. \$16,500 decreased by 85%
13. 80 people increased by 7.5%

5 ★ Standardized TEST PREPARATION

Scoring Rubric

Full Credit
- solution is complete and correct

Partial Credit
- solution is complete but has errors,
  or
- solution is without error but incomplete

No Credit
- no solution is given,
  or
- solution makes no sense

SHORT RESPONSE QUESTIONS

PROBLEM

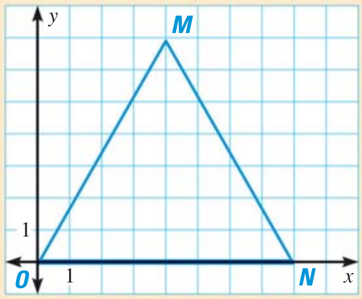
The coordinates of the vertices of a triangle are $O(0, 0)$, $M(k, k\sqrt{3})$, and $N(2k, 0)$. Classify $\triangle OMN$ by its side lengths. *Justify* your answer.

Below are sample solutions to the problem. Read each solution and the comments in blue to see why the sample represents full credit, partial credit, or no credit.

SAMPLE 1: Full credit solution

A sample triangle is graphed and an explanation is given.

Begin by graphing $\triangle OMN$ for a given value of k. I chose a value of k that makes $\triangle OMN$ easy to graph. In the diagram, $k = 4$, so the coordinates are $O(0, 0)$, $M(4, 4\sqrt{3})$, and $N(8, 0)$.



From the graph, it appears that $\triangle OMN$ is equilateral.

The Distance Formula is applied correctly.

To verify that $\triangle OMN$ is equilateral, use the Distance Formula. Show that $OM = MN = ON$ for all values of k.

$$OM = \sqrt{(k-0)^2 + (k\sqrt{3}-0)^2} = \sqrt{k^2 + 3k^2} = \sqrt{4k^2} = 2|k|$$

$$MN = \sqrt{(2k-k)^2 + \left(0 - k\sqrt{3}\right)^2} = \sqrt{k^2 + 3k^2} = \sqrt{4k^2} = 2|k|$$

$$ON = \sqrt{(2k-0)^2 + (0-0)^2} = \sqrt{4k^2} = 2|k|$$

The answer is correct.

Because all of its side lengths are equal, $\triangle OMN$ is an equilateral triangle.

SAMPLE 2: Partial credit solution

Use the Distance Formula to find the side lengths.

A calculation error is made in finding *OM* and *MN*. The value of $\left(k\sqrt{3}\right)^2$ is $k^2 \cdot \left(\sqrt{3}\right)^2$, or $3k^2$, not $9k^2$.

$$OM = \sqrt{(k-0)^2 + \left(k\sqrt{3}-0\right)^2} = \sqrt{k^2 + 9k^2} = \sqrt{10k^2} = k\sqrt{10}$$

$$MN = \sqrt{(2k-k)^2 + \left(0 - k\sqrt{3}\right)^2} = \sqrt{k^2 + 9k^2} = \sqrt{10k^2} = k\sqrt{10}$$

$$ON = \sqrt{(2k-0)^2 + (0-0)^2} = \sqrt{4k^2} = 2k$$

The answer is incorrect.

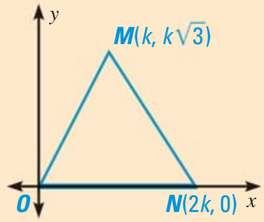
Two of the side lengths are equal, so $\triangle OMN$ is an isosceles triangle.

Sample 3: Partial credit solution

The answer is correct, but the explanation does not justify the answer.

Graph $\triangle OMN$ and compare the side lengths.

From $O(0, 0)$, move right k units and up $k\sqrt{3}$ units to $M(k, k\sqrt{3})$. Draw $\overline{OM}$. To draw $\overline{MN}$, move k units right and $k\sqrt{3}$ units down from M to $N(2k, 0)$. Then draw $\overline{ON}$, which is $2k$ units long. All side lengths appear to be equal, so $\triangle OMN$ is equilateral.



Sample 4: No credit solution

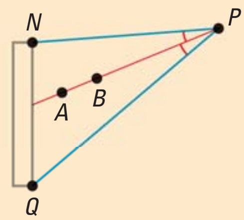
The reasoning and the answer are incorrect.

You are not given enough information to classify $\triangle OMN$ because you need to know the value of k.

PRACTICE Apply the Scoring Rubric

Use the rubric on page 350 to score the solution to the problem below as *full credit*, *partial credit*, or *no credit*. *Explain* your reasoning.

PROBLEM You are a goalie guarding the goal $\overline{NQ}$. To make a goal, Player P must send the ball across $\overline{NQ}$. Is the distance you may need to move to block the shot greater if you stand at Position A or at Position B? *Explain*.

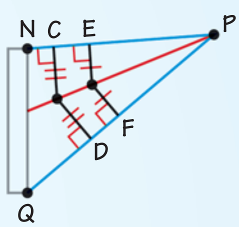


1. At either position, you are on the angle bisector of $\angle NPQ$. So, in both cases you are equidistant from the angle's sides. Therefore, the distance you need to move to block the shot from the two positions is the same.

2. Both positions lie on the angle bisector of $\angle NPQ$. So, each is equidistant from $\overline{PN}$ and $\overline{PQ}$.

The sides of an angle are farther from the angle bisector as you move away from the vertex. So, A is farther from $\overline{PN}$ and from $\overline{PQ}$ than B is.

The distance may be greater if you stand at Position A than if you stand at Position B.



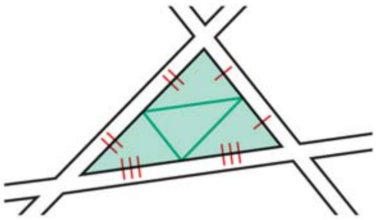
3. Because Position B is farther from the goal, you may need to move a greater distance to block the shot if you stand at Position B.

5 ★ Standardized TEST PRACTICE

SHORT RESPONSE

1. The coordinates of $\triangle OPQ$ are $O(0, 0)$, $P(a, a)$, and $Q(2a, 0)$. Classify $\triangle OPQ$ by its side lengths. Is $\triangle OPQ$ a right triangle? *Justify* your answer.

2. The local gardening club is planting flowers on a traffic triangle. They divide the triangle into four sections, as shown. The perimeter of the middle triangle is 10 feet. What is the perimeter of the traffic triangle? *Explain* your reasoning.

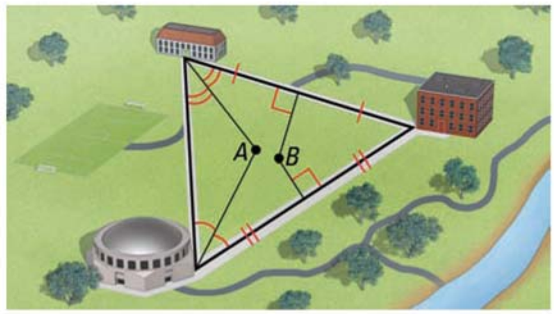


3. A wooden stepladder with a metal support is shown. The legs of the stepladder form a triangle. The support is parallel to the floor, and positioned about five inches above where the midsegment of the triangle would be. Is the length of the support from one side of the triangle to the other side of the triangle *greater than*, *less than*, or *equal to* 8 inches? *Explain* your reasoning.

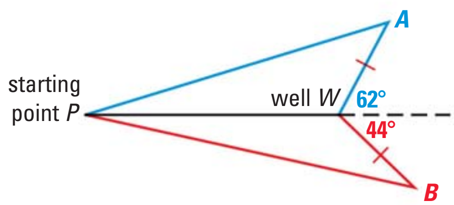
4. You are given instructions for making a triangular earring from silver wire. According to the instructions, you must first bend a wire into a triangle with side lengths of $\frac{3}{4}$ inch, $\frac{5}{8}$ inch, and $1\frac{1}{2}$ inches. *Explain* what is wrong with the first part of the instructions.

5. The centroid of $\triangle ABC$ is located at $P(-1, 2)$. The coordinates of A and B are $A(0, 6)$ and $B(-2, 4)$. What are the coordinates of vertex C? *Explain* your reasoning.

6. A college club wants to set up a booth to attract more members. They want to put the booth at a spot that is equidistant from three important buildings on campus. Without measuring, decide which spot, A or B, is the correct location for the booth. *Explain* your reasoning.



7. Contestants on a television game show must run to a well (point W), fill a bucket with water, empty it at either point A or B, and then run back to the starting point (point P). To run the shortest distance possible, which point should contestants choose, A or B? *Explain* your reasoning.



8. How is the area of the triangle formed by the midsegments of a triangle related to the area of the original triangle? Use an example to *justify* your answer.

9. You are bending an 18 inch wire to form an isosceles triangle. *Describe* the possible lengths of the base if the vertex angle is larger than 60°. *Explain* your reasoning.

MULTIPLE CHOICE

10. If $\triangle ABC$ is obtuse, which statement is always true about its circumcenter P?

Ⓐ P is equidistant from $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$.

Ⓑ P is inside $\triangle ABC$.

Ⓒ P is on $\triangle ABC$.

Ⓓ P is outside $\triangle ABC$.

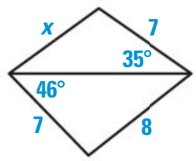
11. Which conclusion about the value of x can be made from the diagram?

Ⓐ $x < 8$

Ⓑ $x = 8$

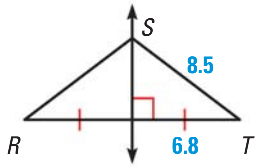
Ⓒ $x > 8$

Ⓓ No conclusion can be made.

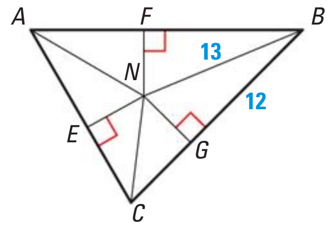


GRIDDED ANSWER

12. Find the perimeter of $\triangle RST$.



13. In the diagram, N is the incenter of $\triangle ABC$. Find NF.

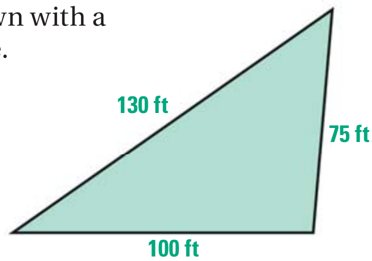


EXTENDED RESPONSE

14. A new sport is to be played on the triangular playing field shown with a basket located at a point that is equidistant from each side line.

a. Copy the diagram and show how to find the location of the basket. *Describe* your method.

b. What theorem can you use to verify that the location you chose in part (a) is correct? *Explain*.



15. A segment has endpoints $A(8, -1)$ and $B(6, 3)$.

a. Graph $\overline{AB}$. Then find the midpoint C of $\overline{AB}$ and the slope of $\overline{AB}$.

b. Use what you know about slopes of perpendicular lines to find the slope of the perpendicular bisector of $\overline{AB}$. Then sketch the perpendicular bisector of $\overline{AB}$ and write an equation of the line. *Explain* your steps.

c. Find a point D that is a solution to the equation you wrote in part (b). Find AD and BD. What do you notice? What theorem does this illustrate?

16. The coordinates of $\triangle JKL$ are $J(-2, 2)$, $K(4, 8)$, and $L(10, -4)$.

a. Find the coordinates of the centroid M. Show your steps.

b. Find the mean of the x-coordinates of the three vertices and the mean of the y-coordinates of the three vertices. *Compare* these results with the coordinates of the centroid. What do you notice?

c. Is the relationship in part (b) true for $\triangle JKP$ with $P(1, -1)$? *Explain*.

6 Similarity

6.1 Ratios, Proportions, and the Geometric Mean

6.2 Use Proportions to Solve Geometry Problems

6.3 Use Similar Polygons

6.4 Prove Triangles Similar by AA

6.5 Prove Triangles Similar by SSS and SAS

6.6 Use Proportionality Theorems

6.7 Perform Similarity Transformations

Before

In previous courses and in Chapters 1–5, you learned the following skills, which you'll use in Chapter 6: using properties of parallel lines, using properties of triangles, simplifying expressions, and finding perimeter.

Prerequisite Skills

VOCABULARY CHECK

1. The alternate interior angles formed when a transversal intersects two __?__ lines are congruent.

2. Two triangles are congruent if and only if their corresponding parts are __?__.

SKILLS AND ALGEBRA CHECK

Simplify the expression. *(Review pp. 870, 874 for 6.1.)*

3. $\frac{9 \cdot 20}{15}$ **4.** $\frac{15}{25}$ **5.** $\frac{3 + 4 + 5}{6 + 8 + 10}$ **6.** $\sqrt{5(5 \cdot 7)}$

Find the perimeter of the rectangle with the given dimensions. *(Review p. 49 for 6.1, 6.2.)*

7. $\ell = 5$ in., $w = 12$ in. **8.** $\ell = 30$ ft, $w = 10$ ft **9.** $A = 56\text{ m}^2$, $\ell = 8$ m

10. Find the slope of a line parallel to the line whose equation is $y - 4 = 7(x + 2)$. *(Review p. 171 for 6.5.)*

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 6, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 417. You will also use the key vocabulary listed below.

Big Ideas

1. Using ratios and proportions to solve geometry problems
2. Showing that triangles are similar
3. Using indirect measurement and similarity

KEY VOCABULARY

- ratio, *p. 356*
- proportion, *p. 358*
  means, extremes
- geometric mean, *p. 359*
- scale drawing, *p. 365*
- scale, *p. 365*
- similar polygons, *p. 372*
- scale factor of two similar polygons, *p. 373*
- dilation, *p. 409*
- center of dilation, *p. 409*
- scale factor of a dilation, *p. 409*
- reduction, *p. 409*
- enlargement, *p. 409*

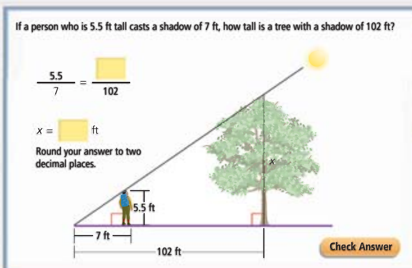
Why?

You can use similarity to measure lengths indirectly. For example, you can use similar triangles to find the height of a tree.

Animated Geometry

The animation illustrated below for Exercise 33 on page 394 helps you answer this question: What is the height of the tree?

You can use proportional reasoning to estimate the height of a tall tree.



Use similar triangles to write a proportion. Then find the value of x.

Geometry at classzone.com

Animated Geometry at classzone.com

Other animations for Chapter 6: pages 365, 375, 391, 407, and 414

Other animations for Chapter 1 appear on pages 7, 9, 14, 21, 37, and 50.

6.1 Ratios, Proportions, and the Geometric Mean

Before You solved problems by writing and solving equations.

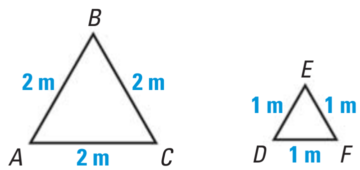
Now You will solve problems by writing and solving proportions.

Why? So you can estimate bird populations, as in Ex. 62.

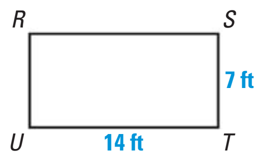
Key Vocabulary
- **ratio**
- **proportion**
  means, extremes
- **geometric mean**

If a and b are two numbers or quantities and $b \neq 0$, then the **ratio of *a* to *b*** is $\frac{a}{b}$. The ratio of a to b can also be written as $a : b$.

For example, the ratio of a side length in $\triangle ABC$ to a side length in $\triangle DEF$ can be written as $\frac{2}{1}$ or $2 : 1$.



Ratios are usually expressed in simplest form. Two ratios that have the same simplified form are called *equivalent ratios*. The ratios $7 : 14$ and $1 : 2$ in the example below are *equivalent*.



$$\frac{\text{width of } RSTU}{\text{length of } RSTU} = \frac{7 \text{ ft}}{14 \text{ ft}} = \frac{1}{2}$$

EXAMPLE 1 Simplify ratios

Simplify the ratio.

a. $64 \text{ m} : 6 \text{ m}$

b. $\frac{5 \text{ ft}}{20 \text{ in.}}$

Solution

a. Write $64 \text{ m} : 6 \text{ m}$ as $\frac{64 \text{ m}}{6 \text{ m}}$. Then divide out the units and simplify.

$$\frac{64 \cancel{\text{m}}}{6 \cancel{\text{m}}} = \frac{32}{3} = 32 : 3$$

b. To simplify a ratio with unlike units, multiply by a conversion factor.

$$\frac{5 \text{ ft}}{20 \text{ in.}} = \frac{5 \cancel{\text{ft}}}{20 \cancel{\text{in.}}} \cdot \frac{12 \cancel{\text{in.}}}{1 \cancel{\text{ft}}} = \frac{60}{20} = \frac{3}{1}$$

REVIEW UNIT ANALYSIS

For help with measures and conversion factors, see p. 886 and the Table of Measures on p. 921.

✓ GUIDED PRACTICE for Example 1

Simplify the ratio.

1. 24 yards to 3 yards

2. $150 \text{ cm} : 6 \text{ m}$

EXAMPLE 2 Use a ratio to find a dimension

PAINTING You are planning to paint a mural on a rectangular wall. You know that the perimeter of the wall is 484 feet and that the ratio of its length to its width is 9 : 2. Find the area of the wall.

Solution

WRITE EXPRESSIONS

Because the ratio in Example 2 is 9 : 2, you can write an equivalent ratio to find expressions for the length and width.

$$\frac{\text{length}}{\text{width}} = \frac{9}{2} = \frac{9}{2} \cdot \frac{x}{x} = \frac{9x}{2x}$$

STEP 1 **Write** expressions for the length and width. Because the ratio of length to width is 9 : 2, you can represent the length by $9x$ and the width by $2x$.

STEP 2 **Solve** an equation to find x.

$2\ell + 2w = P$	Formula for perimeter of rectangle
$2(9x) + 2(2x) = 484$	Substitute for ℓ, w, and P.
$22x = 484$	Multiply and combine like terms.
$x = 22$	Divide each side by 22.

STEP 3 **Evaluate** the expressions for the length and width. Substitute the value of x into each expression.

Length $= 9x = 9(22) = 198$ Width $= 2x = 2(22) = 44$

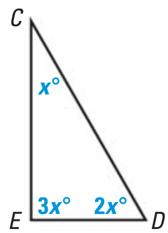
▶ The wall is 198 feet long and 44 feet wide, so its area is $198 \text{ ft} \cdot 44 \text{ ft} = 8712 \text{ ft}^2$.

EXAMPLE 3 Use extended ratios

ALGEBRA The measures of the angles in $\triangle CDE$ are in the *extended ratio* of 1 : 2 : 3. Find the measures of the angles.

Solution

Begin by sketching the triangle. Then use the extended ratio of 1 : 2 : 3 to label the measures as $x°$, $2x°$, and $3x°$.



$x° + 2x° + 3x° = 180°$	Triangle Sum Theorem
$6x = 180$	Combine like terms.
$x = 30$	Divide each side by 6.

▶ The angle measures are $30°$, $2(30°) = 60°$, and $3(30°) = 90°$.

✓ GUIDED PRACTICE for Examples 2 and 3

3. The perimeter of a room is 48 feet and the ratio of its length to its width is 7 : 5. Find the length and width of the room.

4. A triangle's angle measures are in the extended ratio of 1 : 3 : 5. Find the measures of the angles.

PROPORTIONS An equation that states that two ratios are equal is called a **proportion**.

$$\text{extreme} \rightarrow a \quad\quad c \leftarrow \text{mean}$$
$$\frac{a}{b} = \frac{c}{d}$$
$$\text{mean} \rightarrow b \quad\quad d \leftarrow \text{extreme}$$

The numbers b and c are the **means** of the proportion. The numbers a and d are the **extremes** of the proportion.

The property below can be used to solve proportions. To *solve a proportion*, you find the value of any variable in the proportion.

PROPORTIONS

You will learn more properties of proportions on p. 364.

KEY CONCEPT — *For Your Notebook*

A Property of Proportions

1. **Cross Products Property** In a proportion, the product of the extremes equals the product of the means.

If $\frac{a}{b} = \frac{c}{d}$ where $b \neq 0$ and $d \neq 0$, then $ad = bc$.

$$\frac{2}{3} = \frac{4}{6} \qquad \begin{aligned} 3 \cdot 4 &= 12 \\ 2 \cdot 6 &= 12 \end{aligned}$$

EXAMPLE 4 Solve proportions

ALGEBRA **Solve the proportion.**

a. $\frac{5}{10} = \frac{x}{16}$ **b.** $\frac{1}{y+1} = \frac{2}{3y}$

Solution

a.

$\frac{5}{10} = \frac{x}{16}$	Write original proportion.
$5 \cdot 16 = 10 \cdot x$	Cross Products Property
$80 = 10x$	Multiply.
$8 = x$	Divide each side by 10.

b.

$\frac{1}{y+1} = \frac{2}{3y}$	Write original proportion.
$1 \cdot 3y = 2(y + 1)$	Cross Products Property
$3y = 2y + 2$	Distributive Property
$y = 2$	Subtract $2y$ from each side.

ANOTHER WAY

In part (a), you could multiply each side by the denominator, 16. Then $16 \cdot \frac{5}{10} = 16 \cdot \frac{x}{16}$, so $8 = x$.

✓ GUIDED PRACTICE for Example 4

Solve the proportion.

5. $\frac{2}{x} = \frac{5}{8}$ **6.** $\frac{1}{x-3} = \frac{4}{3x}$ **7.** $\frac{y-3}{7} = \frac{y}{14}$

EXAMPLE 5 Solve a real-world problem

SCIENCE As part of an environmental study, you need to estimate the number of trees in a 150 acre area. You count 270 trees in a 2 acre area and you notice that the trees seem to be evenly distributed. Estimate the total number of trees.

Solution

Write and solve a proportion involving two ratios that compare the number of trees with the area of the land.

$\frac{270}{2} = \frac{n}{150}$ ← number of trees ← area in acres — **Write proportion.**

$270 \cdot 150 = 2 \cdot n$ — **Cross Products Property**

$20{,}250 = n$ — **Simplify.**

▶ There are about 20,250 trees in the 150 acre area.

KEY CONCEPT — *For Your Notebook*

Geometric Mean

The **geometric mean** of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x} = \frac{x}{b}$. So, $x^2 = ab$ and $x = \sqrt{ab}$.

EXAMPLE 6 Find a geometric mean

Find the geometric mean of 24 and 48.

Solution

$x = \sqrt{ab}$ — **Definition of geometric mean**

$= \sqrt{24 \cdot 48}$ — **Substitute 24 for *a* and 48 for *b*.**

$= \sqrt{24 \cdot 24 \cdot 2}$ — **Factor.**

$= 24\sqrt{2}$ — **Simplify.**

▶ The geometric mean of 24 and 48 is $24\sqrt{2} \approx 33.9$.

✓ GUIDED PRACTICE for Examples 5 and 6

8. **WHAT IF?** In Example 5, suppose you count 390 trees in a 3 acre area of the 150 acre area. Make a new estimate of the total number of trees.

Find the geometric mean of the two numbers.

9. 12 and 27 **10.** 18 and 54 **11.** 16 and 18

6.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 27, and 59

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 47, 48, 52, and 63

◆ = **MULTIPLE REPRESENTATIONS** Ex. 66

SKILL PRACTICE

1. **VOCABULARY** Copy the proportion $\frac{m}{n} = \frac{p}{q}$. Identify the means of the proportion and the extremes of the proportion.

2. ★ **WRITING** Write three ratios that are equivalent to the ratio 3 : 4. *Explain* how you found the ratios.

EXAMPLE 1 on p. 356 for Exs. 3–17

SIMPLIFYING RATIOS **Simplify the ratio.**

3. $20 : $5

4. $\frac{15\text{ cm}^2}{12\text{ cm}^2}$

5. 6 L : 10 mL

6. $\frac{1\text{ mi}}{20\text{ ft}}$

7. $\frac{7\text{ ft}}{12\text{ in.}}$

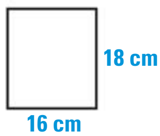
8. $\frac{80\text{ cm}}{2\text{ m}}$

9. $\frac{3\text{ lb}}{10\text{ oz}}$

10. $\frac{2\text{ gallons}}{18\text{ quarts}}$

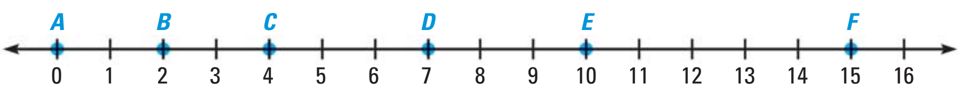
WRITING RATIOS **Find the ratio of the width to the length of the rectangle. Then simplify the ratio.**

11. 

12. 

13. 

FINDING RATIOS **Use the number line to find the ratio of the distances.**



14. $\frac{AD}{CF}$

15. $\frac{BD}{AB}$

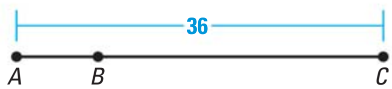
16. $\frac{CE}{EF}$

17. $\frac{BE}{CE}$

EXAMPLE 2 on p. 357 for Exs. 18–19

18. **PERIMETER** The perimeter of a rectangle is 154 feet. The ratio of the length to the width is 10 : 1. Find the length and the width.

19. **SEGMENT LENGTHS** In the diagram, $AB : BC$ is 2 : 7 and $AC = 36$. Find AB and BC.



EXAMPLE 3 on p. 357 for Exs. 20–22

USING EXTENDED RATIOS **The measures of the angles of a triangle are in the extended ratio given. Find the measures of the angles of the triangle.**

20. 3 : 5 : 10

21. 2 : 7 : 9

22. 11 : 12 : 13

EXAMPLE 4 on p. 358 for Exs. 23–30

xy **ALGEBRA** **Solve the proportion.**

23. $\frac{6}{x} = \frac{3}{2}$

24. $\frac{y}{20} = \frac{3}{10}$

25. $\frac{2}{7} = \frac{12}{z}$

26. $\frac{j+1}{5} = \frac{4}{10}$

27. $\frac{1}{c+5} = \frac{3}{24}$

28. $\frac{4}{a-3} = \frac{2}{5}$

29. $\frac{1+3b}{4} = \frac{5}{2}$

30. $\frac{3}{2p+5} = \frac{1}{9p}$

EXAMPLE 6
on p. 359
for Exs. 31–36

GEOMETRIC MEAN **Find the geometric mean of the two numbers.**

31. 2 and 18 **32.** 4 and 25 **33.** 32 and 8

34. 4 and 16 **35.** 2 and 25 **36.** 6 and 20

37. **ERROR ANALYSIS** A student incorrectly simplified the ratio. *Describe* and correct the student's error.

$$\frac{8\text{ in.}}{3\text{ ft}} = \frac{8\text{ in.}}{3\text{ ft}} \cdot \frac{12\text{ in.}}{1\text{ ft}} = \frac{96\text{ in.}}{3\text{ ft}} = \frac{32\text{ in.}}{1\text{ ft}}$$

WRITING RATIOS **Let $x = 10$, $y = 3$, and $z = 8$. Write the ratio in simplest form.**

38. $x:z$ **39.** $\frac{8y}{x}$ **40.** $\frac{4}{2x + 2z}$ **41.** $\frac{2x - z}{3y}$

ALGEBRA **Solve the proportion.**

42. $\frac{2x+5}{3} = \frac{x-5}{4}$ **43.** $\frac{2-s}{3} = \frac{2s+1}{5}$ **44.** $\frac{15}{m} = \frac{m}{5}$ **45.** $\frac{7}{q+1} = \frac{q-1}{5}$

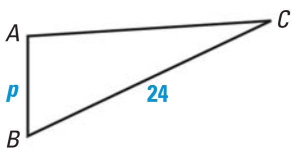
46. **ANGLE MEASURES** The ratio of the measures of two supplementary angles is 5 : 3. Find the measures of the angles.

47. ★ **SHORT RESPONSE** The ratio of the measure of an exterior angle of a triangle to the measure of the adjacent interior angle is 1 : 4. Is the triangle *acute* or *obtuse*? *Explain* how you found your answer.

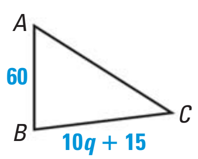
48. ★ **SHORT RESPONSE** Without knowing its side lengths, can you determine the ratio of the perimeter of a square to the length of one of its sides? *Explain*.

ALGEBRA **In Exercises 49–51, the ratio of two side lengths for the triangle is given. Solve for the variable.**

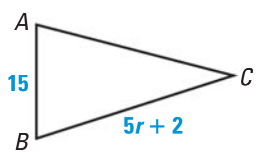
49. $AB:BC$ is 3 : 8.



50. $AB:BC$ is 3 : 4.



51. $AB:BC$ is 5 : 9.



52. ★ **MULTIPLE CHOICE** What is a value of x that makes $\frac{x}{3} = \frac{4x}{x+3}$ true?

Ⓐ 3 Ⓑ 4 Ⓒ 9 Ⓓ 12

53. **AREA** The area of a rectangle is 4320 square inches. The ratio of the width to the length is 5 : 6. Find the length and the width.

54. **COORDINATE GEOMETRY** The points $(-3, 2)$, $(1, 1)$, and $(x, 0)$ are collinear. Use slopes to write a proportion to find the value of x.

55. **ALGEBRA** Use the proportions $\frac{a+b}{2a-b} = \frac{5}{4}$ and $\frac{b}{a+9} = \frac{5}{9}$ to find a and b.

56. **CHALLENGE** Find the ratio of x to y given that $\frac{5}{y} + \frac{7}{x} = 24$ and $\frac{12}{y} + \frac{2}{x} = 24$.

PROBLEM SOLVING

EXAMPLE 2 on p. 357 for Ex. 57

57. **TILING** The perimeter of a room is 66 feet. The ratio of its length to its width is 6 : 5. You want to tile the floor with 12 inch square tiles. Find the length and width of the room, and the area of the floor. How many tiles will you need? The tiles cost \$1.98 each. What is the total cost to tile the floor?

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58. **GEARS** The *gear ratio* of two gears is the ratio of the number of teeth of the larger gear to the number of teeth of the smaller gear. In a set of three gears, the ratio of Gear A to Gear B is equal to the ratio of Gear B to Gear C. Gear A has 36 teeth and Gear C has 16 teeth. How many teeth does Gear B have?

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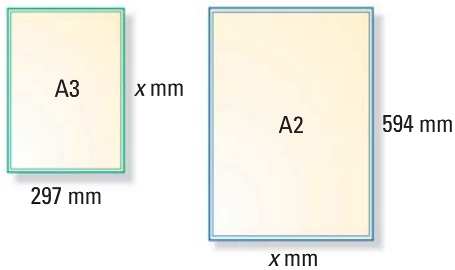
59. **TRAIL MIX** You need to make 36 one-half cup bags of trail mix for a class trip. The recipe calls for peanuts, chocolate chips, and raisins in the extended ratio 5 : 1 : 4. How many cups of each item do you need?

60. **PAPER SIZES** International standard paper sizes are commonly used all over the world. The various sizes all have the same width-to-length ratios. Two sizes of paper are shown, called A3 and A2. The distance labeled x is the geometric mean of 297 mm and 594 mm. Find the value of x.



61. **BATTING AVERAGE** The batting average of a baseball player is the ratio of the number of hits to the number of official at-bats. In 2004, Johnny Damon of the Boston Red Sox had 621 official at-bats and a batting average of .304. Use the proportion to find the number of hits made by Johnny Damon.

$$\frac{\text{Number of hits}}{\text{Number of at-bats}} = \frac{\text{Batting average}}{1.000}$$

EXAMPLE 5 on p. 359 for Ex. 62

62. **MULTI-STEP PROBLEM** The population of Red-tailed hawks is increasing in many areas of the United States. One long-term survey of bird populations suggests that the Red-tailed hawk population is increasing nationally by 2.7% each year.

    a. Write 2.7% as a ratio of hawks in year n to hawks in year $(n - 1)$.

    b. In 2004, observers in Corpus Christi, TX, spotted 180 migrating Red-tailed hawks. Assuming this population follows the national trend, about how many Red-tailed hawks can they expect to see in 2005?

    c. Observers in Lipan Point, AZ, spotted 951 migrating Red-tailed hawks in 2004. Assuming this population follows the national trend, about how many Red-tailed hawks can they expect to see in 2006?

63. ★ **SHORT RESPONSE** Some common computer screen resolutions are 1024 : 768, 800 : 600, and 640 : 480. *Explain* why these ratios are equivalent.

64. **BIOLOGY** The larvae of the Mother-of-Pearl moth is the fastest moving caterpillar. It can run at a speed of 15 inches per second. When threatened, it can curl itself up and roll away 40 times faster than it can run. How fast can it run in miles per hour? How fast can it roll?

65. **CURRENCY EXCHANGE** Emily took 500 U.S. dollars to the bank to exchange for Canadian dollars. The exchange rate on that day was 1.2 Canadian dollars per U.S. dollar. How many Canadian dollars did she get in exchange for the 500 U.S. dollars?

66. **MULTIPLE REPRESENTATIONS** Let x and y be two positive numbers whose geometric mean is 6.

   a. **Making a Table** Make a table of ordered pairs (x, y) such that $\sqrt{xy} = 6$.

   b. **Drawing a Graph** Use the ordered pairs to make a scatter plot. Connect the points with a smooth curve.

   c. **Analyzing Data** Is the data linear? Why or why not?

67. *xy* **ALGEBRA** Use algebra to verify Property 1, the Cross Products Property.

68. *xy* **ALGEBRA** Show that the geometric mean of two numbers is equal to the arithmetic mean (or average) of the two numbers only when the numbers are equal. (*Hint*: Solve $\sqrt{xy} = \frac{x + y}{2}$ with $x, y \geq 0$.)

CHALLENGE In Exercises 69–71, use the given information to find the value(s) of x. Assume that the given quantities are nonnegative.

69. The geometric mean of the quantities $(\sqrt{x})$ and $(3\sqrt{x})$ is $(x - 6)$.

70. The geometric mean of the quantities $(x + 1)$ and $(2x + 3)$ is $(x + 3)$.

71. The geometric mean of the quantities $(2x + 1)$ and $(6x + 1)$ is $(4x - 1)$.

MIXED REVIEW

PREVIEW
Prepare for Lesson 6.2 in Exs. 72–75.

Find the reciprocal. *(p. 869)*

72. -6 73. $\frac{1}{13}$ 74. $\frac{-36}{3}$ 75. -0.2

Solve the quadratic equation. *(p. 882)*

76. $5x^2 = 35$ 77. $x^2 - 20 = 29$ 78. $(x - 3)(x + 3) = 27$

Write the equation of the line with the given description. *(p. 180)*

79. Parallel to $y = 3x - 7$, passing through (1, 2)

80. Perpendicular to $y = \frac{1}{4}x + 5$, passing through (0, 24)

6.2 Use Proportions to Solve Geometry Problems

Before You wrote and solved proportions.

Now You will use proportions to solve geometry problems.

Why? So you can calculate building dimensions, as in Ex. 22.

Key Vocabulary
- scale drawing
- scale

In Lesson 6.1, you learned to use the Cross Products Property to write equations that are equivalent to a given proportion. Three more ways to do this are given by the properties below.

REVIEW RECIPROCALS
For help with reciprocals, see p. 869.

KEY CONCEPT — *For Your Notebook*

Additional Properties of Proportions

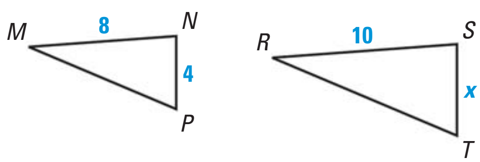
2. **Reciprocal Property** If two ratios are equal, then their reciprocals are also equal. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$.

3. If you interchange the means of a proportion, then you form another true proportion. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.

4. In a proportion, if you add the value of each ratio's denominator to its numerator, then you form another true proportion. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

EXAMPLE 1 Use properties of proportions

In the diagram, $\frac{MN}{RS} = \frac{NP}{ST}$. Write four true proportions.



Solution

Because $\frac{MN}{RS} = \frac{NP}{ST}$, then $\frac{8}{10} = \frac{4}{x}$.

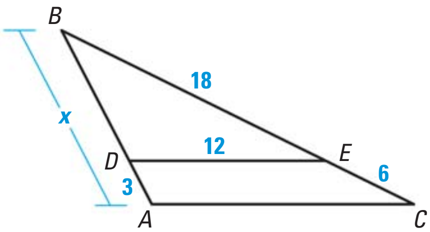
By the Reciprocal Property, the reciprocals are equal, so $\frac{10}{8} = \frac{x}{4}$.

By Property 3, you can interchange the means, so $\frac{8}{4} = \frac{10}{x}$.

By Property 4, you can add the denominators to the numerators, so $\frac{8+10}{10} = \frac{4+x}{x}$, or $\frac{18}{10} = \frac{4+x}{x}$.

EXAMPLE 2 Use proportions with geometric figures

ALGEBRA In the diagram, $\frac{BD}{DA} = \frac{BE}{EC}$. Find BA and BD.



Solution

$$\frac{BD}{DA} = \frac{BE}{EC}$$ Given

$$\frac{BD + DA}{DA} = \frac{BE + EC}{EC}$$ Property of Proportions (Property 4)

$$\frac{x}{3} = \frac{18 + 6}{6}$$ Substitution Property of Equality

$$6x = 3(18 + 6)$$ Cross Products Property

$$x = 12$$ Solve for x.

▶ So, $BA = 12$ and $BD = 12 - 3 = 9$.

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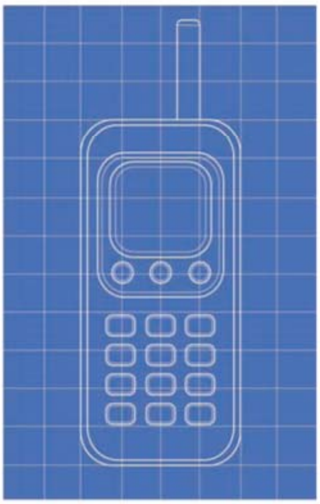
SCALE DRAWING A **scale drawing** is a drawing that is the same shape as the object it represents. The **scale** is a ratio that describes how the dimensions in the drawing are related to the actual dimensions of the object.

EXAMPLE 3 Find the scale of a drawing

BLUEPRINTS The blueprint shows a scale drawing of a cell phone. The length of the antenna on the blueprint is 5 centimeters. The actual length of the antenna is 2 centimeters. What is the scale of the blueprint?



Solution

To find the scale, write the ratio of a length in the drawing to an actual length, then rewrite the ratio so that the denominator is 1.

$$\frac{\text{length on blueprint}}{\text{length of antenna}} = \frac{5 \text{ cm}}{2 \text{ cm}} = \frac{5 \div 2}{2 \div 2} = \frac{2.5}{1}$$

▶ The scale of the blueprint is 2.5 cm : 1 cm.

✓ GUIDED PRACTICE for Examples 1, 2, and 3

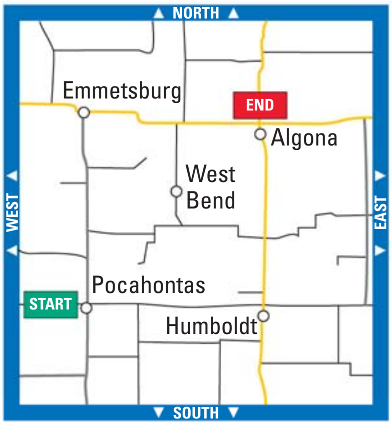
1. In Example 1, find the value of x.

2. In Example 2, $\frac{DE}{AC} = \frac{BE}{BC}$. Find AC.

3. **WHAT IF?** In Example 3, suppose the length of the antenna on the blueprint is 10 centimeters. Find the new scale of the blueprint.

EXAMPLE 4 Use a scale drawing

MAPS The scale of the map at the right is 1 inch : 26 miles. Find the actual distance from Pocahontas to Algona.



Solution

Use a ruler. The distance from Pocahontas to Algona on the map is about 1.25 inches. Let x be the actual distance in miles.

$$\frac{1.25 \text{ in.}}{x \text{ mi}} = \frac{1 \text{ in.}}{26 \text{ mi}}$$ ← **distance on map** ← **actual distance**

$x = 1.25(26)$ **Cross Products Property**

$x = 32.5$ **Simplify.**

▶ The actual distance from Pocahontas to Algona is about 32.5 miles.

EXAMPLE 5 Solve a multi-step problem

SCALE MODEL You buy a 3-D scale model of the Reunion Tower in Dallas, TX. The actual building is 560 feet tall. Your model is 10 inches tall, and the diameter of the dome on your scale model is about 2.1 inches.

a. What is the diameter of the actual dome?

b. About how many times as tall as your model is the actual building?

Solution

a. $$\frac{10 \text{ in.}}{560 \text{ ft}} = \frac{2.1 \text{ in.}}{x \text{ ft}}$$ ← **measurement on model** ← **measurement on actual building**

$10x = 1176$ **Cross Products Property**

$x = 117.6$ **Solve for *x*.**

▶ The diameter of the actual dome is about 118 feet.

b. To simplify a ratio with unlike units, multiply by a conversion factor.

$$\frac{560 \text{ ft}}{10 \text{ in.}} = \frac{560 \cancel{\text{ft}}}{10 \cancel{\text{in.}}} \cdot \frac{12 \cancel{\text{in.}}}{1 \cancel{\text{ft}}} = 672$$

▶ The actual building is 672 times as tall as the model.

✓ GUIDED PRACTICE for Examples 4 and 5

4. Two cities are 96 miles from each other. The cities are 4 inches apart on a map. Find the scale of the map.

5. **WHAT IF?** Your friend has a model of the Reunion Tower that is 14 inches tall. What is the diameter of the dome on your friend's model?

6.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 13, and 25

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, and 24

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A __?__ is a drawing that has the same shape as the object it represents.

2. ★ **WRITING** Suppose the scale of a model of the Eiffel Tower is 1 inch : 20 feet. *Explain* how to determine how many times taller the actual tower is than the model.

EXAMPLE 1 on p. 364 for Exs. 3–10

REASONING **Copy and complete the statement.**

3. If $\frac{8}{x} = \frac{3}{y}$, then $\frac{8}{3} = \frac{?}{?}$.

4. If $\frac{x}{9} = \frac{y}{20}$, then $\frac{x}{y} = \frac{?}{?}$.

5. If $\frac{x}{6} = \frac{y}{15}$, then $\frac{x+6}{6} = \frac{?}{?}$.

6. If $\frac{14}{3} = \frac{x}{y}$, then $\frac{17}{3} = \frac{?}{?}$.

REASONING **Decide whether the statement is *true* or *false*.**

7. If $\frac{8}{m} = \frac{n}{9}$, then $\frac{8+m}{m} = \frac{n+9}{9}$.

8. If $\frac{5}{7} = \frac{a}{b}$, then $\frac{7}{5} = \frac{a}{b}$.

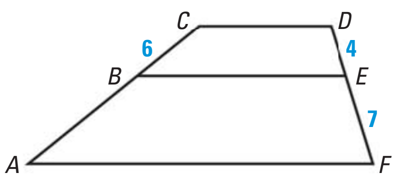
9. If $\frac{d}{2} = \frac{g+10}{11}$, then $\frac{d}{g+10} = \frac{2}{11}$.

10. If $\frac{4+x}{4} = \frac{3+y}{y}$, then $\frac{x}{4} = \frac{3}{y}$.

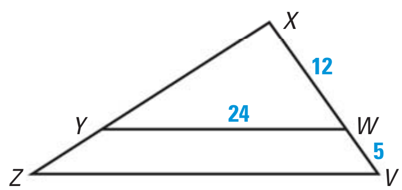
EXAMPLE 2 on p. 365 for Exs. 11–12

PROPERTIES OF PROPORTIONS **Use the diagram and the given information to find the unknown length.**

11. Given $\frac{CB}{BA} = \frac{DE}{EF}$, find BA.

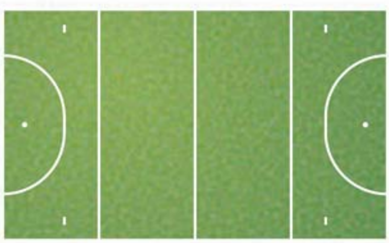


12. Given $\frac{XW}{XV} = \frac{YW}{ZV}$, find ZV.



EXAMPLES 3 and 4 on pp. 365–366 for Exs. 13–14

SCALE DIAGRAMS **In Exercises 13 and 14, use the diagram of the field hockey field in which 1 inch = 50 yards. Use a ruler to approximate the dimension.**



13. Find the actual length of the field.

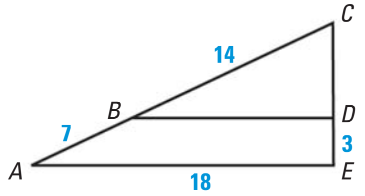
14. Find the actual width of the field.

15. **ERROR ANALYSIS** *Describe* and correct the error made in the reasoning.

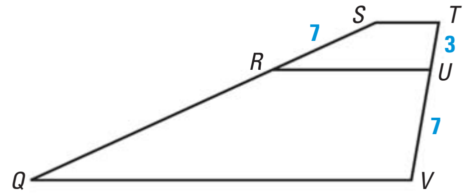
If $\frac{a}{3} = \frac{c}{4}$, then $\frac{a+3}{3} = \frac{c+3}{4}$.

PROPERTIES OF PROPORTIONS **Use the diagram and the given information to find the unknown length.**

16. Given $\frac{CA}{CB} = \frac{AE}{BD}$, find BD.



17. Given $\frac{SQ}{SR} = \frac{TV}{TU}$, find RQ.



18. ★ **MULTIPLE CHOICE** If x, y, z, and q are four different numbers, and the proportion $\frac{x}{y} = \frac{z}{q}$ is true, which of the following is false?

Ⓐ $\frac{y}{x} = \frac{q}{z}$ Ⓑ $\frac{x}{z} = \frac{y}{q}$ Ⓒ $\frac{y}{x} = \frac{z}{q}$ Ⓓ $\frac{x+y}{y} = \frac{z+q}{q}$

CHALLENGE **Two number patterns are *proportional* if there is a nonzero number k such that $(a_1, b_1, c_1, \ldots) = k(a_2, b_2, c_2, \ldots) = ka_2, kb_2, kc_2, \ldots$.**

19. Given the relationship $(8, 16, 20) = k(2, 4, 5)$, find k.

20. Given that $a_1 = ka_2$, $b_1 = kb_2$, and $c_1 = kc_2$, show that $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

21. Given that $a_1 = ka_2$, $b_1 = kb_2$, and $c_1 = kc_2$, show that $\frac{a_1 + b_1 + c_1}{a_2 + b_2 + c_2} = k$.

PROBLEM SOLVING

EXAMPLE 5 on p. 366 for Ex. 22

22. **ARCHITECTURE** A basket manufacturer has headquarters in an office building that has the same shape as a basket they sell.

a. The bottom of the basket is a rectangle with length 15 inches and width 10 inches. The base of the building is a rectangle with length 192 feet. What is the width of the base of the building?

b. About how many times as long as the bottom of the basket is the base of the building?

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Longaberger Company Home Office Newark, Ohio

23. **MAP SCALE** A street on a map is 3 inches long. The actual street is 1 mile long. Find the scale of the map.

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24. ★ **MULTIPLE CHOICE** A model train engine is 12 centimeters long. The actual engine is 18 meters long. What is the scale of the model?

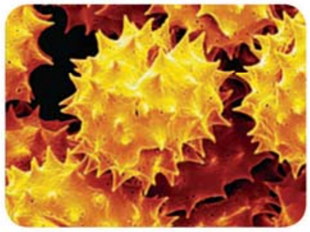
Ⓐ 3 cm : 2 m Ⓑ 1 cm : 1.5 m Ⓒ 1 cm : 3 m Ⓓ 200 cm : 3 m

MAP READING **The map of a hiking trail has a scale of 1 inch : 3.2 miles. Use a ruler to approximate the actual distance between the two shelters.**

25. Meadow View and Whispering Pines

26. Whispering Pines and Blueberry Hill

27. **POLLEN** The photograph shows a particle of goldenrod pollen that has been magnified under a microscope. The scale of the photograph is 900 : 1. Use a ruler to estimate the width in millimeters of the particle.



RAMP DESIGN **Assume that the wheelchair ramps described each have a slope of $\frac{1}{12}$, which is the maximum slope recommended for a wheelchair ramp.**

28. A wheelchair ramp has a 21 foot run. What is its rise?

29. A wheelchair ramp rises 4 feet. What is its run?

30. **STATISTICS** Researchers asked 4887 people to pick a number between 1 and 10. The results are shown in the table below.

Answer	1	2	3	4	5
Percent	4.2%	5.1%	11.4%	10.5%	10.7%
Answer	6	7	8	9	10
Percent	10.0%	27.2%	8.8%	6.0%	6.1%

a. Estimate the number of people who picked the number 3.

b. You ask a participant what number she picked. Is the participant more likely to answer 6 or 7? *Explain*.

c. Conduct this experiment with your classmates. Make a table in which you compare the new percentages with the ones given in the original survey. Why might they be different?

xy **ALGEBRA** **Use algebra to verify the property of proportions.**

31. Property 2

32. Property 3

33. Property 4

REASONING **Use algebra to *explain* why the property of proportions is true.**

34. If $\frac{a-b}{a+b} = \frac{c-d}{c+d}$, then $\frac{a}{b} = \frac{c}{d}$.

35. If $\frac{a+c}{b+d} = \frac{a-c}{b-d}$, then $\frac{a}{b} = \frac{c}{d}$.

36. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then $\frac{a+c+e}{b+d+f} = \frac{a}{b}$. (*Hint:* Let $\frac{a}{b} = r$.)

37. **CHALLENGE** When fruit is dehydrated, water is removed from the fruit. The water content in fresh apricots is about 86%. In dehydrated apricots, the water content is about 75%. Suppose 5 kilograms of raw apricots are dehydrated. How many kilograms of water are removed from the fruit? What is the approximate weight of the dehydrated apricots?

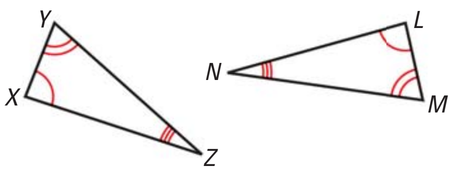
MIXED REVIEW

38. Over the weekend, Claudia drove a total of 405 miles, driving twice as far on Saturday as on Sunday. How far did Claudia travel each day? *(p. 65)*

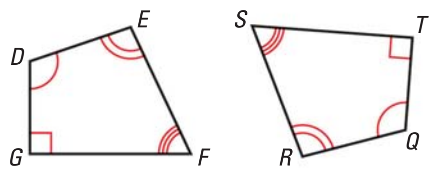
PREVIEW
Prepare for Lesson 6.3 in Exs. 39–40.

Identify all pairs of congruent corresponding parts. Then write another congruence statement for the figures. *(p. 225)*

39. $\triangle XYZ \cong \triangle LMN$



40. $DEFG \cong QRST$



QUIZ *for Lessons 6.1–6.2*

Solve the proportion. *(p. 356)*

1. $\frac{10}{y} = \frac{5}{2}$

2. $\frac{x}{6} = \frac{9}{3}$

3. $\frac{1}{a+3} = \frac{4}{16}$

4. $\frac{6}{d-6} = \frac{4}{8}$

Copy and complete the statement. *(p. 364)*

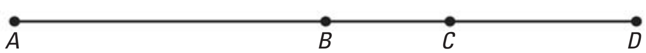
5. If $\frac{9}{x} = \frac{5}{2}$, then $\frac{9}{5} = \frac{?}{?}$.

6. If $\frac{x}{15} = \frac{y}{21}$, then $\frac{x}{y} = \frac{?}{?}$.

7. If $\frac{x}{8} = \frac{y}{12}$, then $\frac{x+8}{8} = \frac{?}{?}$.

8. If $\frac{32}{5} = \frac{x}{y}$, then $\frac{37}{5} = \frac{?}{?}$.

9. In the diagram, $AD = 10$, B is the midpoint of $\overline{AD}$, and AC is the geometric mean of AB and AD. Find AC. *(p. 364)*



6.3 Similar Polygons

MATERIALS • metric ruler • protractor

QUESTION When a figure is reduced, how are the corresponding angles related? How are the corresponding lengths related?

EXPLORE Compare measures of lengths and angles in two photos

Photo 1

STEP 1 ***Measure segments*** Photo 2 is a reduction of Photo 1. In each photo, find AB to the nearest millimeter. Write the ratio of the length of $\overline{AB}$ in Photo 1 to the length of $\overline{AB}$ in Photo 2.

STEP 2 ***Measure angles*** Use a protractor to find the measure of $\angle 1$ in each photo. Write the ratio of $m\angle 1$ in Photo 1 to $m\angle 1$ in Photo 2.

STEP 3 ***Find measurements*** Copy and complete the table. Use the same units for each measurement. Record your results in a table.

Measurement	Photo 1	Photo 2	$\frac{\text{Photo 1}}{\text{Photo 2}}$
AB	?	?	?
AC	?	?	?
DE	?	?	?
$m\angle 1$	?	?	?
$m\angle 2$	?	?	?

Photo 2

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Make a conjecture about the relationship between corresponding lengths when a figure is reduced.
2. Make a conjecture about the relationship between corresponding angles when a figure is reduced.
3. Suppose the measure of an angle in Photo 2 is 35°. What is the measure of the corresponding angle in Photo 1?
4. Suppose a segment in Photo 2 is 1 centimeters long. What is the measure of the corresponding segment in Photo 1?
5. Suppose a segment in Photo 1 is 5 centimeters long. What is the measure of the corresponding segment in Photo 2?

6.3 Use Similar Polygons

Before You used proportions to solve geometry problems.

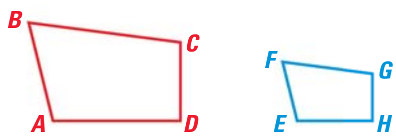
Now You will use proportions to identify similar polygons.

Why? So you can solve science problems, as in Ex. 34.

Key Vocabulary
- similar polygons
- scale factor

Two polygons are **similar polygons** if corresponding angles are congruent and corresponding side lengths are proportional.

In the diagram below, *ABCD* is similar to *EFGH*. You can write "*ABCD* is similar to *EFGH*" as $ABCD \sim EFGH$. Notice in the similarity statement that the corresponding vertices are listed in the same order.



$ABCD \sim EFGH$

Corresponding angles

$\angle A \cong \angle E$, $\angle B \cong \angle F$, $\angle C \cong \angle G$, and $\angle D \cong \angle H$

Ratios of corresponding sides

$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$$

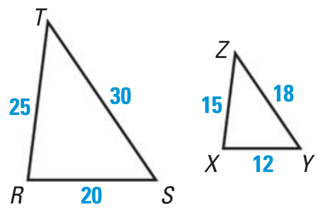
EXAMPLE 1 Use similarity statements

In the diagram, $\triangle RST \sim \triangle XYZ$.

a. List all pairs of congruent angles.

b. Check that the ratios of corresponding side lengths are equal.

c. Write the ratios of the corresponding side lengths in a *statement of proportionality*.



READ VOCABULARY
In a *statement of proportionality*, any pair of ratios forms a true proportion.

Solution

a. $\angle R \cong \angle X$, $\angle S \cong \angle Y$, and $\angle T \cong \angle Z$.

b. $\frac{RS}{XY} = \frac{20}{12} = \frac{5}{3}$ $\quad \frac{ST}{YZ} = \frac{30}{18} = \frac{5}{3}$ $\quad \frac{TR}{ZX} = \frac{25}{15} = \frac{5}{3}$

c. Because the ratios in part (b) are equal, $\frac{RS}{XY} = \frac{ST}{YZ} = \frac{TR}{ZX}$.

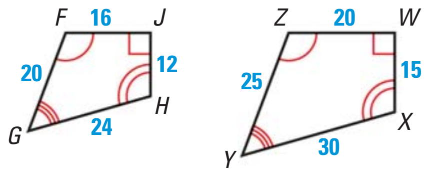
✓ GUIDED PRACTICE for Example 1

1. Given $\triangle JKL \sim \triangle PQR$, list all pairs of congruent angles. Write the ratios of the corresponding side lengths in a statement of proportionality.

SCALE FACTOR If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the **scale factor**. In Example 1, the common ratio of $\frac{5}{3}$ is the scale factor of $\triangle RST$ to $\triangle XYZ$.

EXAMPLE 2 Find the scale factor

Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor of *ZYXW* to *FGHJ*.



Solution

STEP 1 **Identify** pairs of congruent angles. From the diagram, you can see that $\angle Z \cong \angle F$, $\angle Y \cong \angle G$, and $\angle X \cong \angle H$. Angles W and J are right angles, so $\angle W \cong \angle J$. So, the corresponding angles are congruent.

STEP 2 **Show** that corresponding side lengths are proportional.

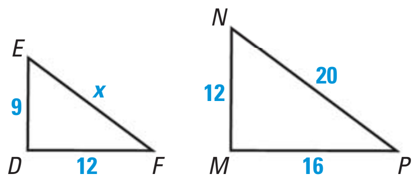
$$\frac{ZY}{FG} = \frac{25}{20} = \frac{5}{4} \qquad \frac{YX}{GH} = \frac{30}{24} = \frac{5}{4} \qquad \frac{XW}{HJ} = \frac{15}{12} = \frac{5}{4} \qquad \frac{WZ}{JF} = \frac{20}{16} = \frac{5}{4}$$

The ratios are equal, so the corresponding side lengths are proportional.

▶ So $ZYXW \sim FGHJ$. The scale factor of $ZYXW$ to $FGHJ$ is $\frac{5}{4}$.

EXAMPLE 3 Use similar polygons

ALGEBRA In the diagram, $\triangle DEF \sim \triangle MNP$. Find the value of x.



Solution

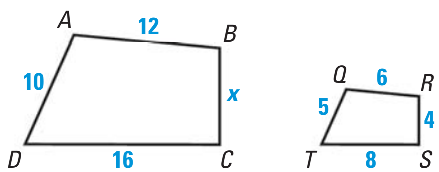
The triangles are similar, so the corresponding side lengths are proportional.

$\frac{MN}{DE} = \frac{NP}{EF}$	Write proportion.
$\frac{12}{9} = \frac{20}{x}$	Substitute.
$12x = 180$	Cross Products Property
$x = 15$	Solve for x.

ANOTHER WAY
There are several ways to write the proportion. For example, you could write $\frac{DF}{MP} = \frac{EF}{NP}$.

✓ GUIDED PRACTICE for Examples 2 and 3

In the diagram, *ABCD* ~ *QRST*.



2. What is the scale factor of $QRST$ to $ABCD$?

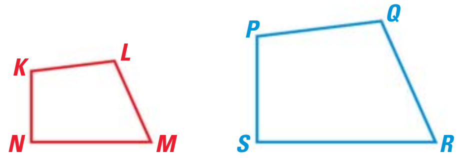
3. Find the value of x.

PERIMETERS The ratios of lengths in similar polygons is the same as the scale factor. Theorem 6.1 shows this is true for the perimeters of the polygons.

THEOREM *For Your Notebook*

THEOREM 6.1 Perimeters of Similar Polygons

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.

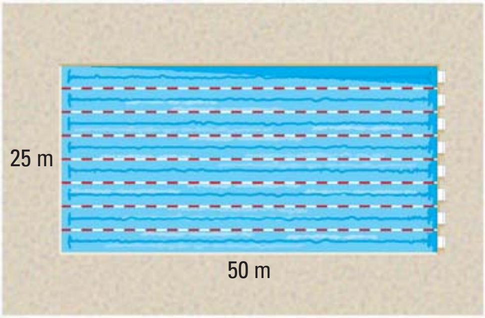


If $KLMN \sim PQRS$, then $\frac{KL + LM + MN + NK}{PQ + QR + RS + SP} = \frac{KL}{PQ} = \frac{LM}{QR} = \frac{MN}{RS} = \frac{NK}{SP}$.

Proof: Ex. 38, p. 379

EXAMPLE 4 Find perimeters of similar figures

SWIMMING A town is building a new swimming pool. An Olympic pool is rectangular with length 50 meters and width 25 meters. The new pool will be similar in shape, but only 40 meters long.



a. Find the scale factor of the new pool to an Olympic pool.

b. Find the perimeter of an Olympic pool and the new pool.

Solution

ANOTHER WAY
Another way to solve Example 4 is to write the scale factor as the decimal 0.8. Then, multiply the perimeter of the Olympic pool by the scale factor to get the perimeter of the new pool:
0.8(150) = 120.

a. Because the new pool will be similar to an Olympic pool, the scale factor is the ratio of the lengths, $\frac{40}{50} = \frac{4}{5}$.

b. The perimeter of an Olympic pool is $2(50) + 2(25) = 150$ meters. You can use Theorem 6.1 to find the perimeter x of the new pool.

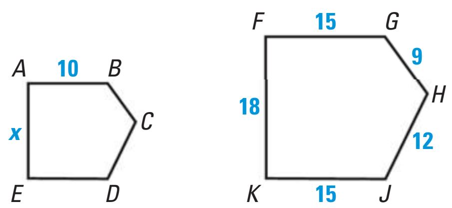
$\frac{x}{150} = \frac{4}{5}$ Use Theorem 6.1 to write a proportion.

$x = 120$ Multiply each side by 150 and simplify.

▶ The perimeter of the new pool is 120 meters.

✓ GUIDED PRACTICE for Example 4

In the diagram, *ABCDE* ~ *FGHJK*.

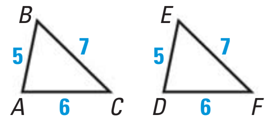


4. Find the scale factor of *FGHJK* to *ABCDE*.

5. Find the value of x.

6. Find the perimeter of *ABCDE*.

SIMILARITY AND CONGRUENCE Notice that any two congruent figures are also similar. Their scale factor is 1 : 1. In $\triangle ABC$ and $\triangle DEF$, the scale factor is $\frac{5}{5} = 1$. You can write $\triangle ABC \sim \triangle DEF$ and $\triangle ABC \cong \triangle DEF$.



READ VOCABULARY
For example, *corresponding lengths* in similar triangles include side lengths, altitudes, medians, midsegments, and so on.

CORRESPONDING LENGTHS You know that perimeters of similar polygons are in the same ratio as corresponding side lengths. You can extend this concept to other segments in polygons.

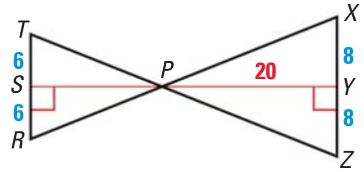
KEY CONCEPT ***For Your Notebook***

Corresponding Lengths in Similar Polygons

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

EXAMPLE 5 Use a scale factor

In the diagram, $\triangle TPR \sim \triangle XPZ$. Find the length of the altitude $\overline{PS}$.



Solution

First, find the scale factor of $\triangle TPR$ to $\triangle XPZ$.

$$\frac{TR}{XZ} = \frac{6+6}{8+8} = \frac{12}{16} = \frac{3}{4}$$

Because the ratio of the lengths of the altitudes in similar triangles is equal to the scale factor, you can write the following proportion.

$\frac{PS}{PY} = \frac{3}{4}$ **Write proportion.**

$\frac{PS}{20} = \frac{3}{4}$ **Substitute 20 for *PY*.**

$PS = 15$ **Multiply each side by 20 and simplify.**

▶ The length of the altitude $\overline{PS}$ is 15.

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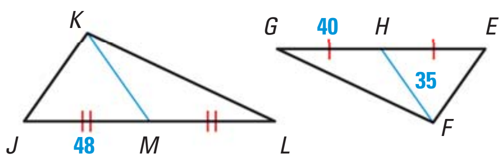
✓ GUIDED PRACTICE for Example 5

7. In the diagram, $\triangle JKL \sim \triangle EFG$. Find the length of the median $\overline{KM}$.



6.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 7, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 18, 27, 28, 35, 36, and 37

◆ = **MULTIPLE REPRESENTATIONS** Ex. 33

SKILL PRACTICE

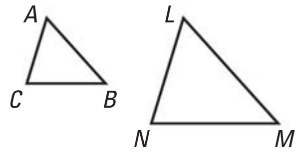
1. **VOCABULARY** Copy and complete: Two polygons are similar if corresponding angles are _?_ and corresponding side lengths are _?_.

2. ★ **WRITING** If two polygons are congruent, must they be similar? If two polygons are similar, must they be congruent? *Explain.*

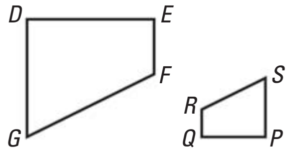
EXAMPLE 1 on p. 372 for Exs. 3–6

USING SIMILARITY List all pairs of congruent angles for the figures. Then write the ratios of the corresponding sides in a statement of proportionality.

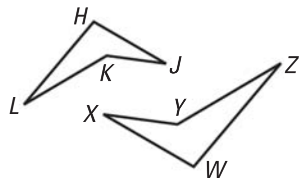
3. $\triangle ABC \sim \triangle LMN$



4. $DEFG \sim PQRS$



5. $HJKL \sim WXYZ$



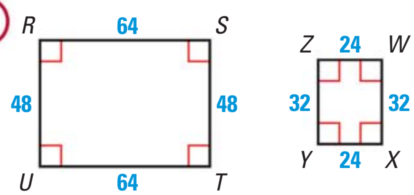
6. ★ **MULTIPLE CHOICE** Triangles *ABC* and *DEF* are similar. Which statement is *not* correct?

Ⓐ $\frac{BC}{EF} = \frac{BC}{EF}$ Ⓑ $\frac{AB}{DE} = \frac{CA}{FD}$ Ⓒ $\frac{CA}{FD} = \frac{BC}{EF}$ Ⓓ $\frac{AB}{EF} = \frac{BC}{DE}$

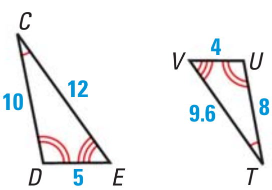
EXAMPLES 2 and 3 on p. 373 for Exs. 7–10

DETERMINING SIMILARITY Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor.

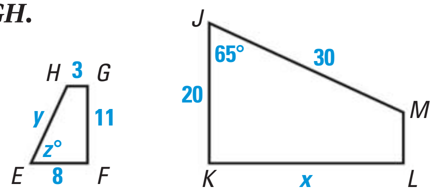
7.



8.



USING SIMILAR POLYGONS In the diagram, $JKLM \sim EFGH$.



9. Find the scale factor of *JKLM* to *EFGH*.

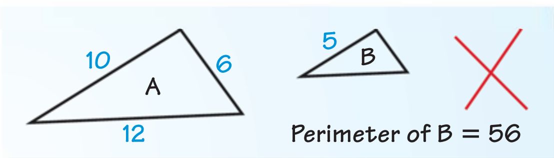
10. Find the values of *x*, *y*, and *z*.

EXAMPLE 4 on p. 374 for Exs. 11–13

11. Find the perimeter of each polygon.

12. **PERIMETER** Two similar FOR SALE signs have a scale factor of 5 : 3. The large sign's perimeter is 60 inches. Find the small sign's perimeter.

13. **ERROR ANALYSIS** The triangles are similar. *Describe* and correct the error in finding the perimeter of Triangle B.



REASONING **Are the polygons *always*, *sometimes*, or *never* similar?**

14. Two isosceles triangles

15. Two equilateral triangles

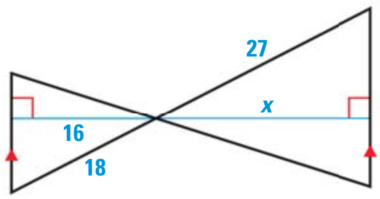
16. A right triangle and an isosceles triangle

17. A scalene triangle and an isosceles triangle

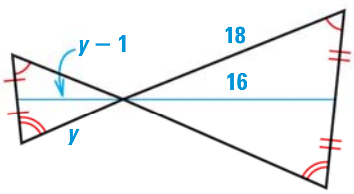
18. ★ **SHORT RESPONSE** The scale factor of Figure A to Figure B is $1:x$. What is the scale factor of Figure B to Figure A? *Explain* your reasoning.

SIMILAR TRIANGLES **Identify the type of special segment shown in blue, and find the value of the variable.**

19.



20.



EXAMPLE 5
on p. 375
for Exs. 21–22

USING SCALE FACTOR **Triangles *NPQ* and *RST* are similar. The side lengths of $\triangle NPQ$ are 6 inches, 8 inches, and 10 inches, and the length of an altitude is 4.8 inches. The shortest side of $\triangle RST$ is 8 inches long.**

21. Find the lengths of the other two sides of $\triangle RST$.

22. Find the length of the corresponding altitude in $\triangle RST$.

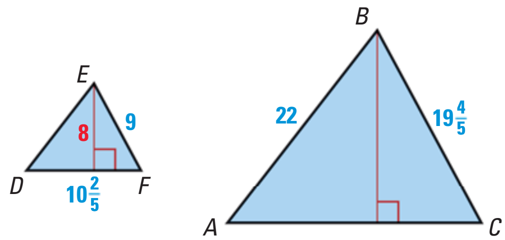
USING SIMILAR TRIANGLES **In the diagram, $\triangle ABC \sim \triangle DEF$.**

23. Find the scale factor of $\triangle ABC$ to $\triangle DEF$.

24. Find the unknown side lengths in both triangles.

25. Find the length of the altitude shown in $\triangle ABC$.

26. Find and compare the areas of both triangles.

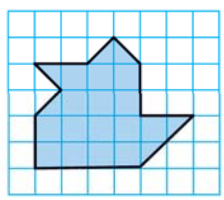


27. ★ **SHORT RESPONSE** Suppose you are told that $\triangle PQR \sim \triangle XYZ$ and that the extended ratio of the angle measures in $\triangle PQR$ is $x : x + 30 : 3x$. Do you need to know anything about $\triangle XYZ$ to be able to write its extended ratio of angle measures? *Explain* your reasoning.

28. ★ **MULTIPLE CHOICE** The lengths of the legs of right triangle ABC are 3 feet and 4 feet. The shortest side of $\triangle UVW$ is 4.5 feet and $\triangle UVW \sim \triangle ABC$. How long is the hypotenuse of $\triangle UVW$?

Ⓐ 1.5 ft Ⓑ 5 ft Ⓒ 6 ft Ⓓ 7.5 ft

29. **CHALLENGE** Copy the figure at the right and divide it into two similar figures.

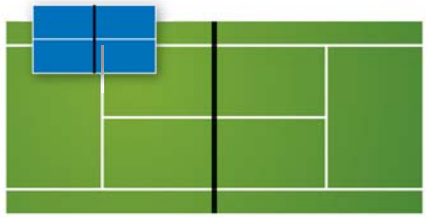


30. **REASONING** Is similarity reflexive? symmetric? transitive? Give examples to support your answers.

PROBLEM SOLVING

EXAMPLE 2 on p. 373 for Exs. 31–32

31. **TENNIS** In table tennis, the table is a rectangle 9 feet long and 5 feet wide. A tennis court is a rectangle 78 feet long and 36 feet wide. Are the two surfaces similar? *Explain*. If so, find the scale factor of the tennis court to the table.



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32. **DIGITAL PROJECTOR** You are preparing a computer presentation to be digitally projected onto the wall of your classroom. Your computer screen is 13.25 inches wide and 10.6 inches high. The projected image on the wall is 53 inches wide and 42.4 inches high. Are the two shapes similar? If so, find the scale factor of the computer screen to the projected image.

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33. **MULTIPLE REPRESENTATIONS** Use the similar figures shown. The scale factor of Figure 1 to Figure 2 is 7 : 10.

a. **Making a Table** Copy and complete the table.

	AB	BC	CD	DE	EA
Figure 1	3.5	?	?	?	?
Figure 2	5.0	4.0	6.0	8.0	3.0

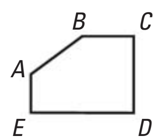


Figure 1

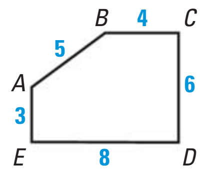
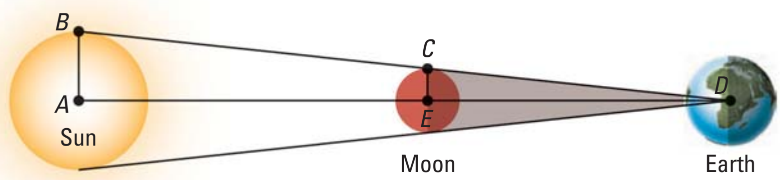


Figure 2

b. **Drawing a Graph** Graph the data in the table. Let x represent the length of a side in Figure 1 and let y represent the length of the corresponding side in Figure 2. Is the relationship linear?

c. **Writing an Equation** Write an equation that relates x and y. What is its slope? How is the slope related to the scale factor?

34. **MULTI-STEP PROBLEM** During a total eclipse of the sun, the moon is directly in line with the sun and blocks the sun's rays. The distance ED between Earth and the moon is 240,000 miles, the distance DA between Earth and the sun is 93,000,000 miles, and the radius AB of the sun is 432,500 miles.



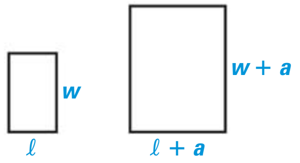
Not drawn to scale

a. Copy the diagram and label the known distances.

b. In the diagram, $\triangle BDA \sim \triangle CDE$. Use this fact to explain a total eclipse of the sun.

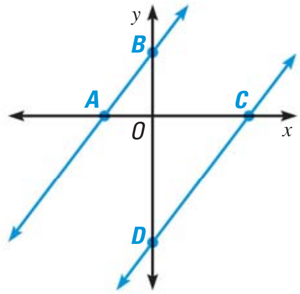
c. Estimate the radius CE of the moon.

35. ★ **SHORT RESPONSE** A rectangular image is enlarged on each side by the same amount. The angles remain unchanged. Can the larger image be similar to the original? *Explain* your reasoning, and give an example to support your answer.



36. ★ **SHORT RESPONSE** How are the areas of similar rectangles related to the scale factor? Use examples to *justify* your reasoning.

37. ★ **EXTENDED RESPONSE** The equations of two lines in the coordinate plane are $y = \frac{4}{3}x + 4$ and $y = \frac{4}{3}x - 8$.



a. *Explain* why the two lines are parallel.

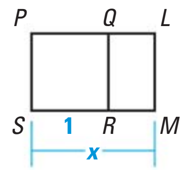
b. Show that $\angle BOA \cong \angle DOC$, $\angle OBA \cong \angle ODC$, and $\angle BAO \cong \angle DCO$.

c. Find the coordinates of points A, B, C, and D. Find the lengths of the sides of $\triangle AOB$ and $\triangle COD$.

d. Show that $\triangle AOB \sim \triangle COD$.

38. **PROVING THEOREM 6.1** Prove the Perimeters of Similar Polygons Theorem for similar rectangles. Include a diagram in your proof.

39. **CHALLENGE** In the diagram, *PQRS* is a square, and *PLMS* ~ *LMRQ*. Find the exact value of x. This value is called the *golden ratio*. Golden rectangles have their length and width in this ratio. Show that the similar rectangles in the diagram are golden rectangles.



MIXED REVIEW

PREVIEW
Prepare for Lesson 6.4 in Exs. 40–42.

Given $A(1, 1)$, $B(3, 2)$, $C(2, 4)$, and $D\left(1, \frac{7}{2}\right)$, determine whether the following lines are *parallel*, *perpendicular*, or *neither*. *(p. 171)*

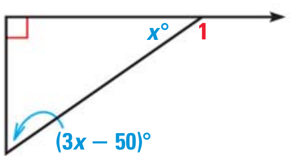
40. $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$

41. $\overleftrightarrow{CD}$ and $\overleftrightarrow{AD}$

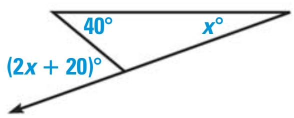
42. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$

Find the measure of the exterior angle shown. *(p. 217)*

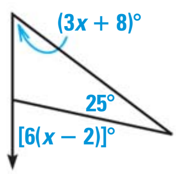
43.



44.

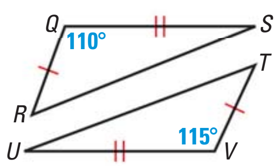


45.

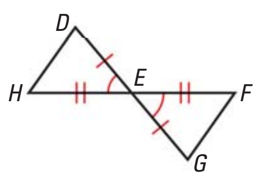


Copy and complete the statement with <, >, or =. *(p. 335)*

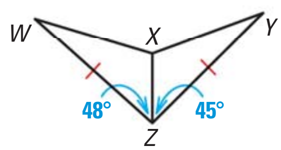
46. RS __?__ TU



47. FG __?__ HD



48. WX __?__ YX



MIXED REVIEW of Problem Solving

STATE TEST PRACTICE
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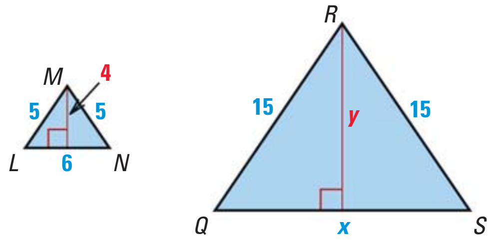
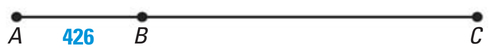
Lessons 6.1–6.3

1. **MULTI-STEP PROBLEM** In the diagram, $\triangle LMN \sim \triangle QRS$.



   a. Find the scale factor of $\triangle LMN$ to $\triangle QRS$. Then find the values of x and y.
   b. Find the perimeters of $\triangle LMN$ and $\triangle QRS$.
   c. Find the areas of $\triangle LMN$ and $\triangle QRS$.
   d. *Compare* the ratio of the perimeters to the ratio of the areas of $\triangle LMN$ to $\triangle QRS$. What do you notice?

2. **GRIDDED ANSWER** In the diagram, $AB : BC$ is $3 : 8$. Find AC.



3. **OPEN-ENDED** $\triangle UVW$ is a right triangle with side lengths of 3 cm, 4 cm, and 5 cm. Draw and label $\triangle UVW$. Then draw a triangle similar to $\triangle UVW$ and label its side lengths. What scale factor did you use?

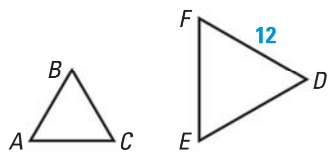
4. **MULTI-STEP PROBLEM** Kelly is going on a trip to England. She takes 600 U.S. dollars with her.

   a. In England, she exchanges her U.S. dollars for British pounds. During her stay, Kelly spends 150 pounds. How many British pounds does she have left?
   b. When she returns home, she exchanges her money back to U.S. dollars. How many U.S. dollars does she have at the end of her trip?

5. **SHORT RESPONSE** Kelly bought a 3-D scale model of the Tower Bridge in London, England. The towers of the model are 9 inches tall. The towers of the actual bridge are 206 feet tall, and there are two walkways that are 140 feet high.

   a. Approximate the height of the walkways on the model.
   b. About how many times as tall as the model is the actual structure?

6. **GRIDDED ANSWER** In the diagram, $\triangle ABC \sim \triangle DEF$. The scale factor of $\triangle ABC$ to $\triangle DEF$ is $3 : 5$. Find AC.



7. **EXTENDED RESPONSE** In the United States, 4634 million pounds of apples were consumed in 2002. The population of the United States in that year was 290 million.

   a. Divide the total number of apples consumed by the population to find the per capita consumption.
   b. About how many pounds of apples would a family of four have consumed in one year? in one month?
   c. A medium apple weighs about 5 ounces. Estimate how many apples a family of four would have consumed in one month.
   d. Is it reasonable to assume that a family of four would have eaten that many apples? What other factors could affect the per capita consumption? *Explain*.

6.4 Prove Triangles Similar by AA

Before You used the AAS Congruence Theorem.

Now You will use the AA Similarity Postulate.

Why? So you can use similar triangles to understand aerial photography, as in Ex. 34.

Key Vocabulary
- **similar polygons,** *p. 372*

ACTIVITY ANGLES AND SIMILAR TRIANGLES

QUESTION What can you conclude about two triangles if you know two pairs of corresponding angles are congruent?

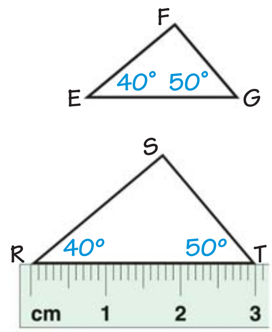
Materials:
- protractor
- metric ruler

STEP 1 **Draw** $\triangle EFG$ so that $m\angle E = 40°$ and $m\angle G = 50°$.

STEP 2 **Draw** $\triangle RST$ so that $m\angle R = 40°$ and $m\angle T = 50°$, and $\triangle RST$ is not congruent to $\triangle EFG$.

STEP 3 **Calculate** $m\angle F$ and $m\angle S$ using the Triangle Sum Theorem. Use a protractor to check that your results are true.

STEP 4 **Measure** and record the side lengths of both triangles. Use a metric ruler.



DRAW CONCLUSIONS

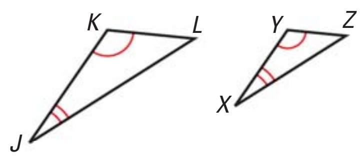
1. Are the triangles similar? Explain your reasoning.
2. Repeat the steps above using different angle measures. Make a conjecture about two triangles with two pairs of congruent corresponding angles.

TRIANGLE SIMILARITY The Activity suggests that two triangles are similar if two pairs of corresponding angles are congruent. In other words, you do not need to know the measures of the sides or the third pair of angles.

POSTULATE *For Your Notebook*

POSTULATE 22 Angle-Angle (AA) Similarity Postulate

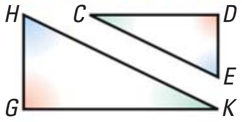
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.



$\triangle JKL \sim \triangle XYZ$

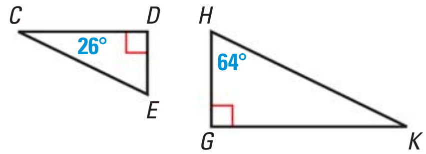
EXAMPLE 1 Use the AA Similarity Postulate

DRAW DIAGRAMS



Use colored pencils to show congruent angles. This will help you write similarity statements.

Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.



Solution

Because they are both right angles, $\angle D$ and $\angle G$ are congruent.

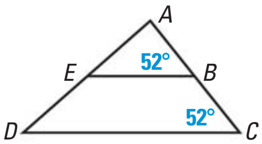
By the Triangle Sum Theorem, $26° + 90° + m\angle E = 180°$, so $m\angle E = 64°$. Therefore, $\angle E$ and $\angle H$ are congruent.

▶ So, $\triangle CDE \sim \triangle KGH$ by the AA Similarity Postulate.

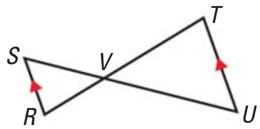
EXAMPLE 2 Show that triangles are similar

Show that the two triangles are similar.

a. $\triangle ABE$ and $\triangle ACD$



b. $\triangle SVR$ and $\triangle UVT$



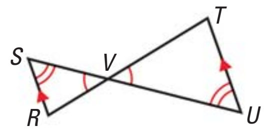
Solution

a. You may find it helpful to redraw the triangles separately.

Because $m\angle ABE$ and $m\angle C$ both equal 52°, $\angle ABE \cong \angle C$. By the Reflexive Property, $\angle A \cong \angle A$.

▶ So, $\triangle ABE \sim \triangle ACD$ by the AA Similarity Postulate.

b. You know $\angle SVR \cong \angle UVT$ by the Vertical Angles Congruence Theorem. The diagram shows $\overline{RS} \parallel \overline{UT}$ so $\angle S \cong \angle U$ by the Alternate Interior Angles Theorem.

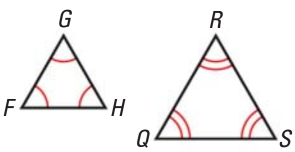


▶ So, $\triangle SVR \sim \triangle UVT$ by the AA Similarity Postulate.

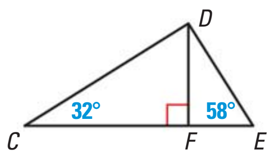
✓ GUIDED PRACTICE for Examples 1 and 2

Show that the triangles are similar. Write a similarity statement.

1. $\triangle FGH$ and $\triangle RQS$



2. $\triangle CDF$ and $\triangle DEF$



3. **REASONING** Suppose in Example 2, part (b), $\overline{SR} \nparallel \overline{TU}$. Could the triangles still be similar? *Explain.*

INDIRECT MEASUREMENT In Lesson 4.6, you learned a way to use congruent triangles to find measurements indirectly. Another useful way to find measurements indirectly is by using similar triangles.

EXAMPLE 3 Standardized Test Practice

A flagpole casts a shadow that is 50 feet long. At the same time, a woman standing nearby who is five feet four inches tall casts a shadow that is 40 inches long. How tall is the flagpole to the nearest foot?

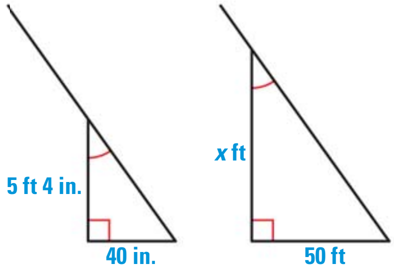
Ⓐ 12 feet Ⓑ 40 feet

Ⓒ 80 feet Ⓓ 140 feet

ELIMINATE CHOICES
Notice that the woman's height is greater than her shadow's length. So the flagpole must be taller than its shadow's length. Eliminate choices A and B.

Solution

The flagpole and the woman form sides of two right triangles with the ground, as shown below. The sun's rays hit the flagpole and the woman at the same angle. You have two pairs of congruent angles, so the triangles are similar by the AA Similarity Postulate.



You can use a proportion to find the height x. Write 5 feet 4 inches as 64 inches so that you can form two ratios of feet to inches.

$\frac{x \text{ ft}}{64 \text{ in.}} = \frac{50 \text{ ft}}{40 \text{ in.}}$ **Write proportion of side lengths.**

$40x = 64(50)$ **Cross Products Property**

$x = 80$ **Solve for *x*.**

▶ The flagpole is 80 feet tall. The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Example 3

4. **WHAT IF?** A child who is 58 inches tall is standing next to the woman in Example 3. How long is the child's shadow?

5. You are standing in your backyard, and you measure the lengths of the shadows cast by both you and a tree. Write a proportion showing how you could find the height of the tree.

6.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 13, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 18, 19, 20, 33, and 38

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are _?_.

2. ★ **WRITING** Can you assume that corresponding sides and corresponding angles of any two similar triangles are congruent? *Explain.*

EXAMPLE 1 on p. 382 for Exs. 3–11

REASONING Use the diagram to complete the statement.

3. $\triangle ABC \sim$ _?_

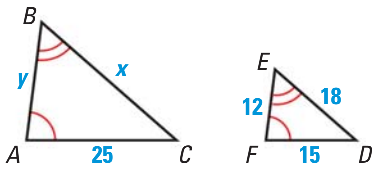
4. $\frac{BA}{?} = \frac{AC}{?} = \frac{CB}{?}$

5. $\frac{25}{?} = \frac{?}{12}$

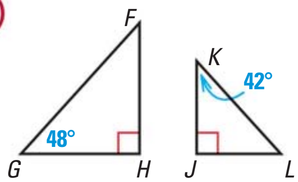
6. $\frac{?}{25} = \frac{18}{?}$

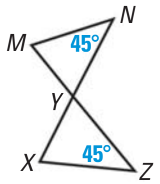
7. $y =$ _?_

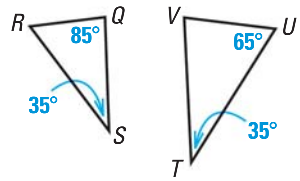
8. $x =$ _?_



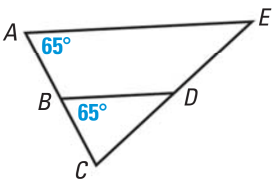
AA SIMILARITY POSTULATE In Exercises 9–14, determine whether the triangles are similar. If they are, write a similarity statement.

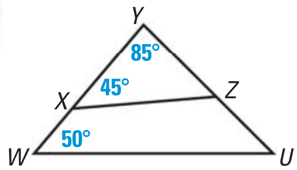
9. 

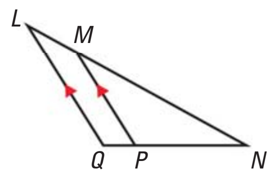
10. 

11. 

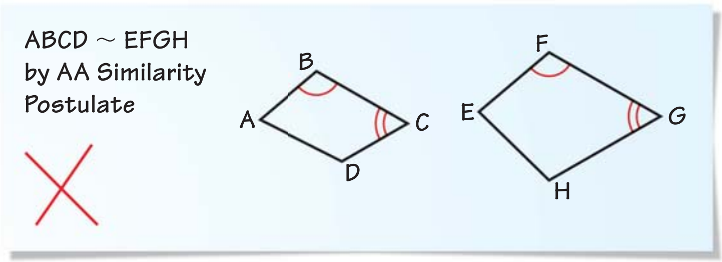
EXAMPLE 2 on p. 382 for Exs. 12–16

12. 

13. 

14. 

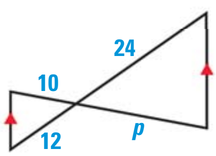
15. **ERROR ANALYSIS** *Explain* why the student's similarity statement is incorrect.



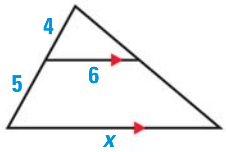
16. ★ **MULTIPLE CHOICE** What is the value of p?

Ⓐ 5 Ⓑ 20

Ⓒ 28.8 Ⓓ Cannot be determined



17. **ERROR ANALYSIS** A student uses the proportion $\frac{4}{6} = \frac{5}{x}$ to find the value of x in the figure. *Explain* why this proportion is incorrect and write a correct proportion.



★ **OPEN-ENDED MATH** **In Exercises 18 and 19, make a sketch that can be used to show that the statement is false.**

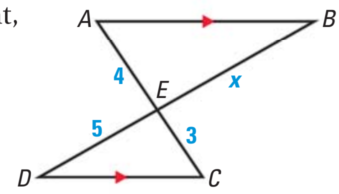
18. If two pairs of sides of two triangles are congruent, then the triangles are similar.

19. If the ratios of two pairs of sides of two triangles are proportional, then the triangles are similar.

20. ★ **MULTIPLE CHOICE** In the figure at the right, find the length of $\overline{BD}$.

Ⓐ $\frac{35}{3}$ Ⓑ $\frac{37}{5}$

Ⓒ $\frac{20}{3}$ Ⓓ $\frac{12}{5}$



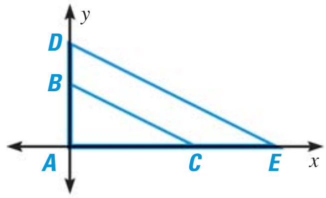
xy ALGEBRA **Find coordinates for point E so that $\triangle ABC \sim \triangle ADE$.**

21. $A(0, 0)$, $B(0, 4)$, $C(8, 0)$, $D(0, 5)$, $E(x, y)$

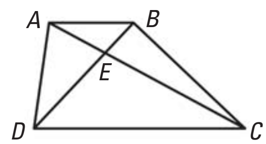
22. $A(0, 0)$, $B(0, 3)$, $C(4, 0)$, $D(0, 7)$, $E(x, y)$

23. $A(0, 0)$, $B(0, 1)$, $C(6, 0)$, $D(0, 4)$, $E(x, y)$

24. $A(0, 0)$, $B(0, 6)$, $C(3, 0)$, $D(0, 9)$, $E(x, y)$



25. **MULTI-STEP PROBLEM** In the diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$, $AE = 6$, $AB = 8$, $CE = 15$, and $DE = 10$.



   a. Copy the diagram and mark all given information.

   b. List two pairs of congruent angles in the diagram.

   c. Name a pair of similar triangles and write a similarity statement.

   d. Find BE and DC.

REASONING **In Exercises 26–29, is it possible for $\triangle JKL$ and $\triangle XYZ$ to be similar? *Explain* why or why not.**

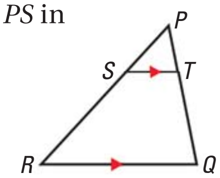
26. $m\angle J = 71°$, $m\angle K = 52°$, $m\angle X = 71°$, and $m\angle Z = 57°$

27. $\triangle JKL$ is a right triangle and $m\angle X + m\angle Y = 150°$.

28. $m\angle J = 87°$ and $m\angle Y = 94°$

29. $m\angle J + m\angle K = 85°$ and $m\angle Y + m\angle Z = 80°$

30. **CHALLENGE** If $PT = x$, $PQ = 3x$, and $SR = \frac{8}{3}x$, find PS in terms of x. *Explain* your reasoning.



PROBLEM SOLVING

EXAMPLE 3 on p. 383 for Exs. 31–32

31. **AIR HOCKEY** An air hockey player returns the puck to his opponent by bouncing the puck off the wall of the table as shown. From physics, the angles that the path of the puck makes with the wall are congruent. What is the distance d between the puck and the wall when the opponent returns it?

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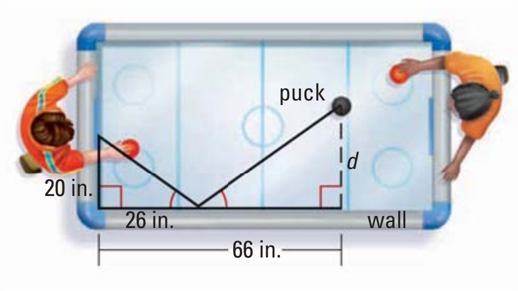
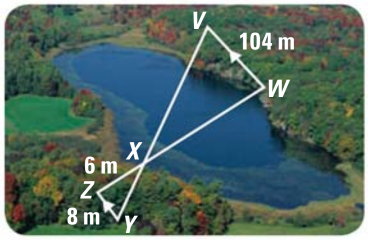


32. **LAKES** You can measure the width of the lake using a surveying technique, as shown in the diagram.

 a. What postulate or theorem can you use to show that the triangles are similar?

 b. Find the width of the lake, WX.

 c. If $XY = 10$ meters, find VX.



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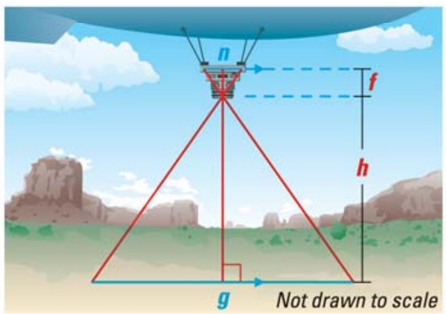
33. ★ **SHORT RESPONSE** *Explain* why all equilateral triangles are similar. Include sketches in your answer.

34. **AERIAL PHOTOGRAPHY** Low-level aerial photos can be taken using a remote-controlled camera suspended from a blimp. You want to take an aerial photo that covers a ground distance g of 50 meters. Use the proportion $\frac{f}{h} = \frac{n}{g}$ to estimate the altitude h that the blimp should fly at to take the photo. In the proportion, use $f = 8$ centimeters and $n = 3$ centimeters. These two variables are determined by the type of camera used.



35. **PROOF** Use the given information to draw a sketch. Then write a proof.

GIVEN ▶ $\triangle STU \sim \triangle PQR$
Point V lies on $\overline{TU}$ so that $\overline{SV}$ bisects $\angle TSU$.
Point N lies on $\overline{QR}$ so that $\overline{PN}$ bisects $\angle QPR$.

PROVE ▶ $\frac{SV}{PN} = \frac{ST}{PQ}$

36. **PROOF** Prove that if an acute angle in one right triangle is congruent to an acute angle in another right triangle, then the triangles are similar.

37. **TECHNOLOGY** Use a graphing calculator or computer.

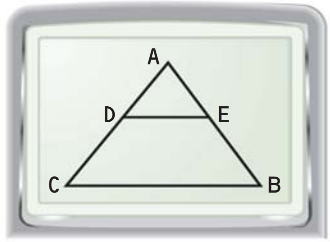
a. Draw $\triangle ABC$. Draw $\overline{DE}$ through two sides of the triangle, parallel to the third side.

b. Measure $\angle ADE$ and $\angle ACB$. Measure $\angle AED$ and $\angle ABC$. What do you notice?

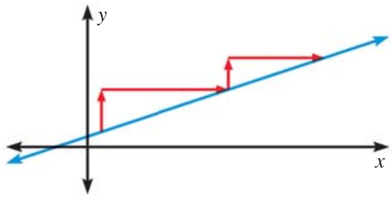
c. What does a postulate in this lesson tell you about $\triangle ADE$ and $\triangle ACB$?

d. Measure all the sides. Show that corresponding side lengths are proportional.

e. Move vertex A to form new triangles. How do your measurements in parts (b) and (d) change? Are the new triangles still similar? *Explain*.



38. ★ **EXTENDED RESPONSE** *Explain* how you could use similar triangles to show that any two points on a line can be used to calculate its slope.



39. **CORRESPONDING LENGTHS** Without using the Corresponding Lengths Property on page 375, prove that the ratio of two corresponding angle bisectors in similar triangles is equal to the scale factor.

40. **CHALLENGE** Prove that if the lengths of two sides of a triangle are a and b respectively, then the lengths of the corresponding altitudes to those sides are in the ratio $\frac{b}{a}$.

MIXED REVIEW

PREVIEW
Prepare for Lesson 6.5 in Exs. 41–44.

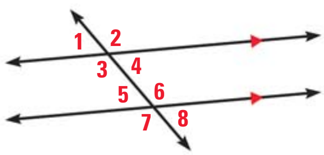
In Exercises 41–44, use the diagram.

41. Name three pairs of corresponding angles. *(p. 147)*

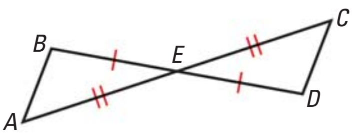
42. Name two pairs of alternate interior angles. *(p. 147)*

43. Name two pairs of alternate exterior angles. *(p. 147)*

44. Find $m\angle 1 + m\angle 7$. *(p. 154)*



45. **CONGRUENCE** Explain why $\triangle ABE \cong \triangle CDE$. *(p. 240)*



Simplify the ratio. *(p. 356)*

46. $\frac{4}{20}$ **47.** $\frac{36}{18}$ **48.** 21 : 63 **49.** 42 : 28

6.5 Prove Triangles Similar by SSS and SAS

Before You used the AA Similarity Postulate to prove triangles similar.

Now You will use the SSS and SAS Similarity Theorems.

Why? So you can show that triangles are similar, as in Ex. 28.

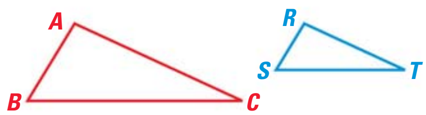
Key Vocabulary
- **ratio,** *p. 356*
- **proportion,** *p. 358*
- **similar polygons,** *p. 372*

In addition to using congruent corresponding angles to show that two triangles are similar, you can use proportional corresponding side lengths.

THEOREM *For Your Notebook*

THEOREM 6.2 Side-Side-Side (SSS) Similarity Theorem

If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

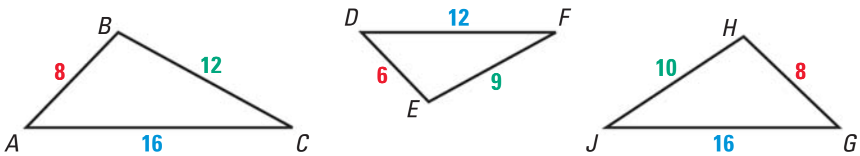


If $\frac{AB}{RS} = \frac{BC}{ST} = \frac{CA}{TR}$, then $\triangle ABC \sim \triangle RST$.

Proof: p. 389

EXAMPLE 1 Use the SSS Similarity Theorem

Is either $\triangle DEF$ or $\triangle GHJ$ similar to $\triangle ABC$?



Solution

APPLY THEOREMS
When using the SSS Similarity Theorem, compare the shortest sides, the longest sides, and then the remaining sides.

Compare $\triangle ABC$ and $\triangle DEF$ by finding ratios of corresponding side lengths.

Shortest sides $\frac{AB}{DE} = \frac{8}{6} = \frac{4}{3}$

Longest sides $\frac{CA}{FD} = \frac{16}{12} = \frac{4}{3}$

Remaining sides $\frac{BC}{EF} = \frac{12}{9} = \frac{4}{3}$

▶ All of the ratios are equal, so $\triangle ABC \sim \triangle DEF$.

Compare $\triangle ABC$ and $\triangle GHJ$ by finding ratios of corresponding side lengths.

Shortest sides $\frac{AB}{GH} = \frac{8}{8} = 1$

Longest sides $\frac{CA}{JG} = \frac{16}{16} = 1$

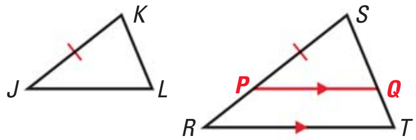
Remaining sides $\frac{BC}{HJ} = \frac{12}{10} = \frac{6}{5}$

▶ The ratios are not all equal, so $\triangle ABC$ and $\triangle GHJ$ are not similar.

PROOF SSS Similarity Theorem

GIVEN ▶ $\frac{RS}{JK} = \frac{ST}{KL} = \frac{TR}{LJ}$

PROVE ▶ $\triangle RST \sim \triangle JKL$



USE AN AUXILIARY LINE

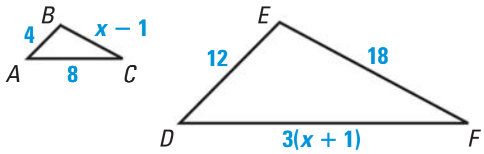
The Parallel Postulate allows you to draw an auxiliary line $\overleftrightarrow{PQ}$ in $\triangle RST$. There is only one line through point P parallel to $\overleftrightarrow{RT}$, so you are able to draw it.

Locate P on $\overline{RS}$ so that $PS = JK$. Draw $\overline{PQ}$ so that $\overline{PQ} \parallel \overline{RT}$. Then $\triangle RST \sim \triangle PSQ$ by the AA Similarity Postulate, and $\frac{RS}{PS} = \frac{ST}{SQ} = \frac{TR}{QP}$.

You can use the given proportion and the fact that $PS = JK$ to deduce that $SQ = KL$ and $QP = LJ$. By the SSS Congruence Postulate, it follows that $\triangle PSQ \cong \triangle JKL$. Finally, use the definition of congruent triangles and the AA Similarity Postulate to conclude that $\triangle RST \sim \triangle JKL$.

EXAMPLE 2 Use the SSS Similarity Theorem

xy ALGEBRA **Find the value of x that makes $\triangle ABC \sim \triangle DEF$.**



Solution

STEP 1 **Find** the value of x that makes corresponding side lengths proportional.

CHOOSE A METHOD

You can use either $\frac{AB}{DE} = \frac{BC}{EF}$ or $\frac{AB}{DE} = \frac{AC}{DF}$ in Step 1.

$\frac{4}{12} = \frac{x-1}{18}$ — **Write proportion.**

$4 \cdot 18 = 12(x - 1)$ — **Cross Products Property**

$72 = 12x - 12$ — **Simplify.**

$7 = x$ — **Solve for x.**

STEP 2 **Check** that the side lengths are proportional when $x = 7$.

$BC = x - 1 = 6$ $\qquad$ $DF = 3(x + 1) = 24$

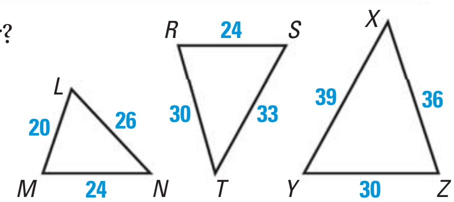
$\frac{AB}{DE} \stackrel{?}{=} \frac{BC}{EF} \Rightarrow \frac{4}{12} = \frac{6}{18}$ ✓ $\qquad$ $\frac{AB}{DE} \stackrel{?}{=} \frac{AC}{DF} \Rightarrow \frac{4}{12} = \frac{8}{24}$ ✓

▶ When $x = 7$, the triangles are similar by the SSS Similarity Theorem.

✓ GUIDED PRACTICE for Examples 1 and 2

1. Which of the three triangles are similar? Write a similarity statement.

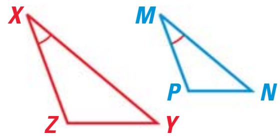
2. The shortest side of a triangle similar to $\triangle RST$ is 12 units long. Find the other side lengths of the triangle.



THEOREM — *For Your Notebook*

THEOREM 6.3 Side-Angle-Side (SAS) Similarity Theorem

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

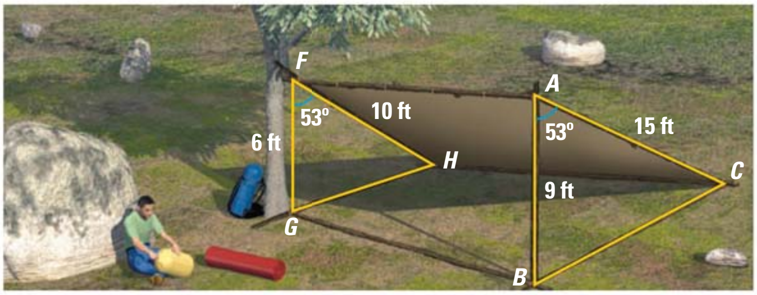


If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

Proof: Ex. 37, p. 395

EXAMPLE 3 Use the SAS Similarity Theorem

LEAN-TO SHELTER You are building a lean-to shelter starting from a tree branch, as shown. Can you construct the right end so it is similar to the left end using the angle measure and lengths shown?



Solution

Both $m\angle A$ and $m\angle F$ equal 53°, so $\angle A \cong \angle F$. Next, compare the ratios of the lengths of the sides that include $\angle A$ and $\angle F$.

Shorter sides $\frac{AB}{FG} = \frac{9}{6} = \frac{3}{2}$ **Longer sides** $\frac{AC}{FH} = \frac{15}{10} = \frac{3}{2}$

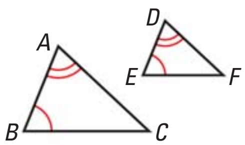
The lengths of the sides that include $\angle A$ and $\angle F$ are proportional.

▶ So, by the SAS Similarity Theorem, $\triangle ABC \sim \triangle FGH$. Yes, you can make the right end similar to the left end of the shelter.

CONCEPT SUMMARY — *For Your Notebook*

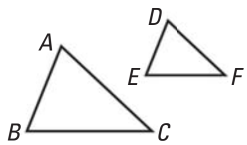
Triangle Similarity Postulate and Theorems

AA Similarity Postulate



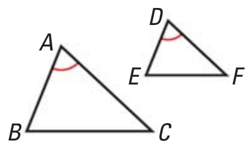
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem



If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

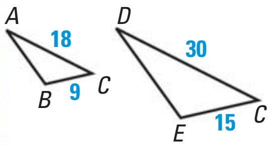
SAS Similarity Theorem



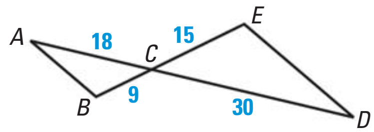
If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

EXAMPLE 4 Choose a method

VISUAL REASONING
To identify corresponding parts, redraw the triangles so that the corresponding parts have the same orientation.



Tell what method you would use to show that the triangles are similar.



Solution

Find the ratios of the lengths of the corresponding sides.

Shorter sides $\frac{BC}{EC} = \frac{9}{15} = \frac{3}{5}$ **Longer sides** $\frac{CA}{CD} = \frac{18}{30} = \frac{3}{5}$

The corresponding side lengths are proportional. The included angles $\angle ACB$ and $\angle DCE$ are congruent because they are vertical angles. So, $\triangle ACB \sim \triangle DCE$ by the SAS Similarity Theorem.

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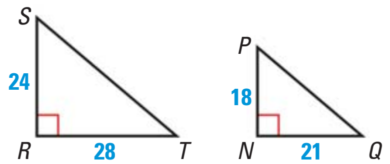
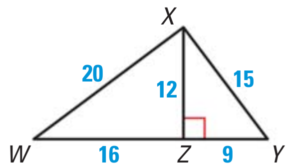
✓ GUIDED PRACTICE for Examples 3 and 4

Explain **how to show that the indicated triangles are similar.**

3. $\triangle SRT \sim \triangle PNQ$



4. $\triangle XZW \sim \triangle YZX$



6.5 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 7, and 31
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 14, 32, 34, and 36

SKILL PRACTICE

1. VOCABULARY You plan to prove that $\triangle ACB$ is similar to $\triangle PXQ$ by the SSS Similarity Theorem. Copy and complete the proportion that is needed to use this theorem: $\frac{AC}{?} = \frac{?}{XQ} = \frac{AB}{?}$.

2. ★ **WRITING** If you know two triangles are similar by the SAS Similarity Theorem, what additional piece(s) of information would you need to know to show that the triangles are congruent?

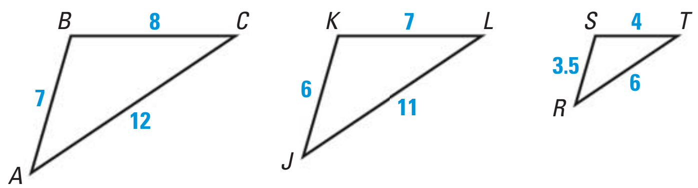
EXAMPLES 1 and 2
on pp. 388–389 for Exs. 3–6

SSS SIMILARITY THEOREM **Verify that $\triangle ABC \sim \triangle DEF$. Find the scale factor of $\triangle ABC$ to $\triangle DEF$.**

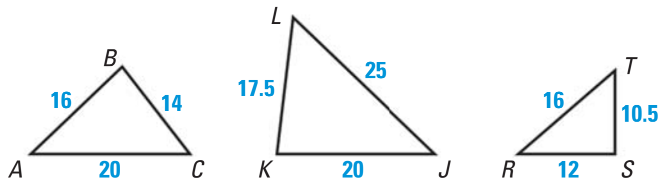
3. $\triangle ABC$: $BC = 18$, $AB = 15$, $AC = 12$
$\triangle DEF$: $EF = 12$, $DE = 10$, $DF = 8$

4. $\triangle ABC$: $AB = 10$, $BC = 16$, $CA = 20$
$\triangle DEF$: $DE = 25$, $EF = 40$, $FD = 50$

5. **SSS SIMILARITY THEOREM** Is either $\triangle JKL$ or $\triangle RST$ similar to $\triangle ABC$?



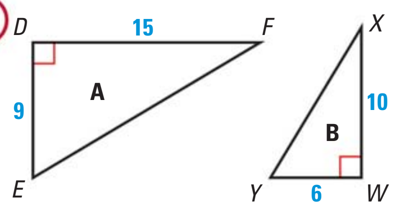
6. **SSS SIMILARITY THEOREM** Is either $\triangle JKL$ or $\triangle RST$ similar to $\triangle ABC$?



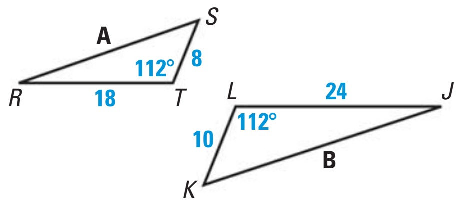
EXAMPLE 3
on p. 390
for Exs. 7–9

SAS SIMILARITY THEOREM **Determine whether the two triangles are similar. If they are similar, write a similarity statement and find the scale factor of Triangle B to Triangle A.**

7.



8.

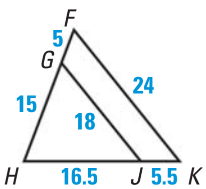


9. **ALGEBRA** Find the value of n that makes $\triangle PQR \sim \triangle XYZ$ when $PQ = 4$, $QR = 5$, $XY = 4(n + 1)$, $YZ = 7n - 1$, and $\angle Q \cong \angle Y$. Include a sketch.

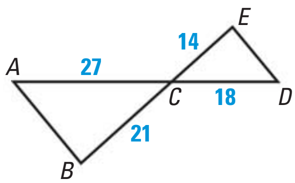
EXAMPLE 4
on p. 391
for Exs. 10–12

SHOWING SIMILARITY **Show that the triangles are similar and write a similarity statement. *Explain* your reasoning.**

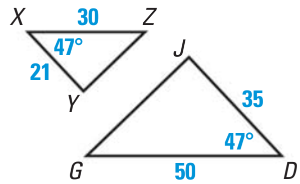
10.



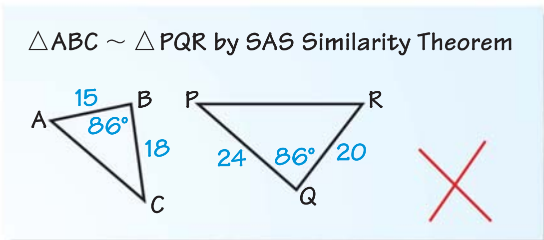
11.



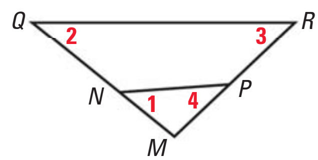
12.



13. **ERROR ANALYSIS** *Describe* and correct the student's error in writing the similarity statement.



14. ★ **MULTIPLE CHOICE** In the diagram, $\frac{MN}{MR} = \frac{MP}{MQ}$. Which of the statements must be true?



Ⓐ $\angle 1 \cong \angle 2$ Ⓑ $\overline{QR} \parallel \overline{NP}$

Ⓒ $\angle 1 \cong \angle 4$ Ⓓ $\triangle MNP \sim \triangle MRQ$

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 ★ = **STANDARDIZED TEST PRACTICE**

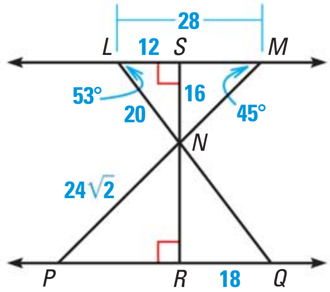
DRAWING TRIANGLES **Sketch the triangles using the given description. *Explain* whether the two triangles can be similar.**

15. In $\triangle XYZ$, $m\angle X = 66°$ and $m\angle Y = 34°$. In $\triangle LMN$, $m\angle M = 34°$ and $m\angle N = 80°$.

16. In $\triangle RST$, $RS = 20$, $ST = 32$, and $m\angle S = 16°$. In $\triangle FGH$, $GH = 30$, $HF = 48$, and $m\angle H = 24°$.

17. The side lengths of $\triangle ABC$ are 24, $8x$, and 54, and the side lengths of $\triangle DEF$ are 15, 25, and $7x$.

FINDING MEASURES **In Exercises 18–23, use the diagram to copy and complete the statements.**



18. $m\angle NQP = \underline{\ ?\ }$

19. $m\angle QPN = \underline{\ ?\ }$

20. $m\angle PNQ = \underline{\ ?\ }$

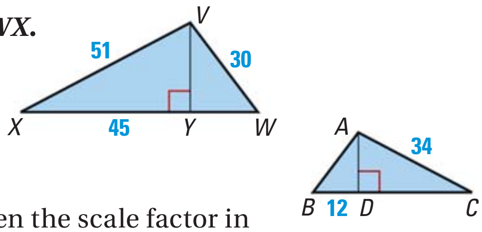
21. $RN = \underline{\ ?\ }$

22. $PQ = \underline{\ ?\ }$

23. $NM = \underline{\ ?\ }$

24. **SIMILAR TRIANGLES** In the diagram at the right, name the three pairs of triangles that are similar.

CHALLENGE **In the figure at the right, $\triangle ABC \sim \triangle VWX$.**



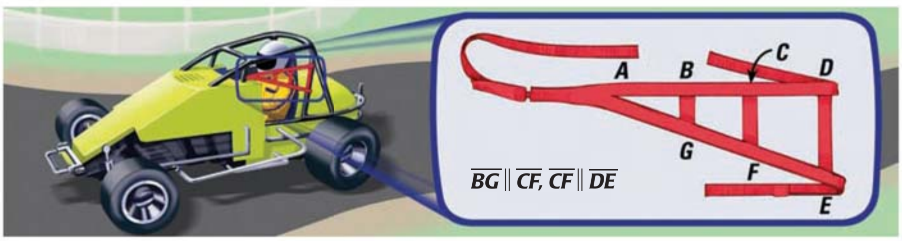
25. Find the scale factor of $\triangle VWX$ to $\triangle ABC$.

26. Find the ratio of the area of $\triangle VWX$ to the area of $\triangle ABC$.

27. Make a conjecture about the relationship between the scale factor in Exercise 25 and the ratio in Exercise 26. *Justify* your conjecture.

PROBLEM SOLVING

28. **RACECAR NET** Which postulate or theorem could you use to show that the three triangles that make up the racecar window net are similar? *Explain*.



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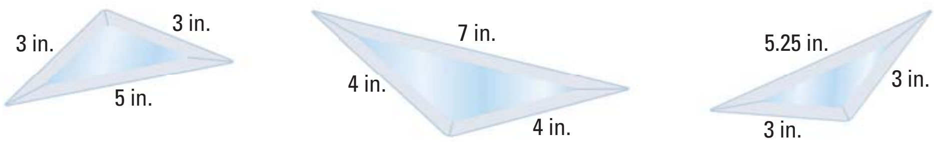
EXAMPLE 1 on p. 388 for Ex. 29

29. **STAINED GLASS** Certain sections of stained glass are sold in triangular *beveled* pieces. Which of the three beveled pieces, if any, are similar?



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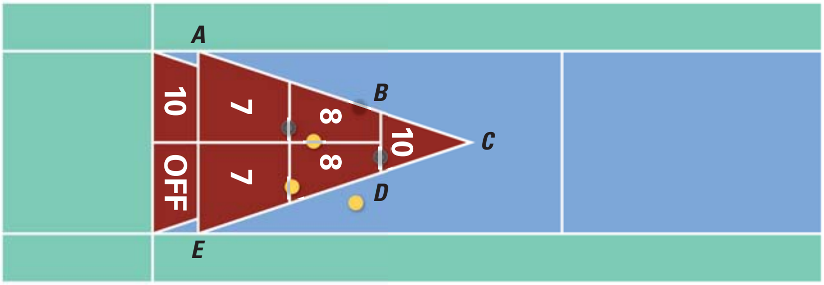
SHUFFLEBOARD In the portion of the shuffleboard court shown, $\frac{BC}{AC} = \frac{BD}{AE}$.



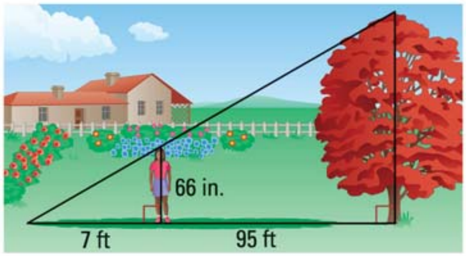
30. What additional piece of information do you need in order to show that $\triangle BCD \sim \triangle ACE$ using the SSS Similarity Theorem?

31. What additional piece of information do you need in order to show that $\triangle BCD \sim \triangle ACE$ using the SAS Similarity Theorem?

32. ★ **OPEN-ENDED MATH** Use a diagram to show why there is no Side-Side-Angle Similarity Postulate.

EXAMPLE 4
on p. 391
for Ex. 33

33. **MULTI-STEP PROBLEM** Ruby is standing in her back yard and she decides to estimate the height of a tree. She stands so that the tip of her shadow coincides with the tip of the tree's shadow, as shown. Ruby is 66 inches tall. The distance from the tree to Ruby is 95 feet and the distance between the tip of the shadows and Ruby is 7 feet.



a. What postulate or theorem can you use to show that the triangles in the diagram are similar?

b. About how tall is the tree, to the nearest foot?

c. **What If?** Curtis is 75 inches tall. At a different time of day, he stands so that the tip of his shadow and the tip of the tree's shadow coincide, as described above. His shadow is 6 feet long. How far is Curtis from the tree?

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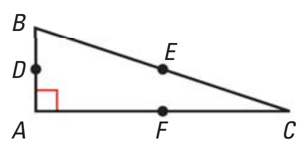
34. ★ **EXTENDED RESPONSE** Suppose you are given two right triangles with one pair of corresponding legs and the pair of corresponding hypotenuses having the same length ratios.

a. The lengths of the given pair of corresponding legs are 6 and 18, and the lengths of the hypotenuses are 10 and 30. Use the Pythagorean Theorem to solve for the lengths of the other pair of corresponding legs. Draw a diagram.

b. Write the ratio of the lengths of the second pair of corresponding legs.

c. Are these triangles similar? Does this suggest a Hypotenuse-Leg Similarity Theorem for right triangles?

35. **PROOF** Given that $\triangle ABC$ is a right triangle and D, E, and F are midpoints, prove that $m\angle DEF = 90°$.



36. ★ **WRITING** Can two triangles have all pairs of corresponding angles in proportion? *Explain*.

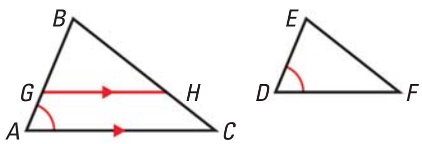
○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

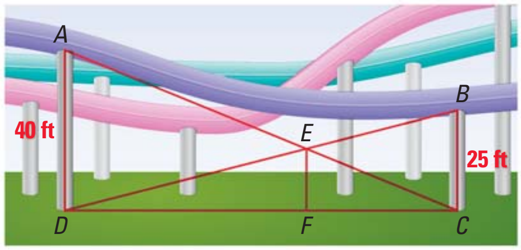
37. **PROVING THEOREM 6.3** Write a paragraph proof of the SAS Similarity Theorem.

GIVEN ▶ $\angle A \cong \angle D$, $\dfrac{AB}{DE} = \dfrac{AC}{DF}$

PROVE ▶ $\triangle ABC \sim \triangle DEF$



38. **CHALLENGE** A portion of a water slide in an amusement park is shown. Find the length of $\overline{EF}$. (*Note:* The posts form right angles with the ground.)

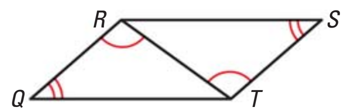


MIXED REVIEW

Find the slope of the line that passes through the given points. *(p. 171)*

39. (0, −8), (4, 16)
40. (−2, −9), (1, −3)
41. (−3, 9), (7, 2)

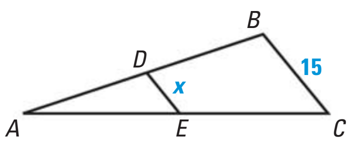
42. State the postulate or theorem you would use to prove the triangles congruent. Then write a congruence statement. *(p. 249)*



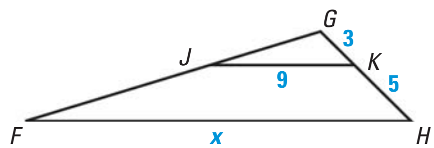
PREVIEW
Prepare for Lesson 6.6 in Exs. 43–44.

Find the value of *x*.

43. $\overline{DE}$ is a midsegment of $\triangle ABC$. *(p. 295)*



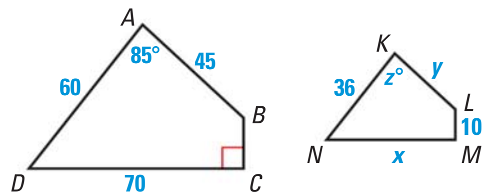
44. $\dfrac{GK}{GH} = \dfrac{JK}{FH}$ *(p. 364)*



QUIZ *for Lessons 6.3–6.5*

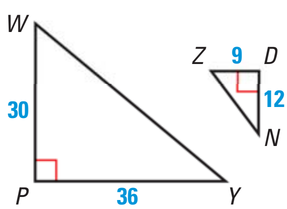
In the diagram, *ABCD* ~ *KLMN*. *(p. 372)*

1. Find the scale factor of *ABCD* to *KLMN*.
2. Find the values of *x*, *y*, and *z*.
3. Find the perimeter of each polygon.

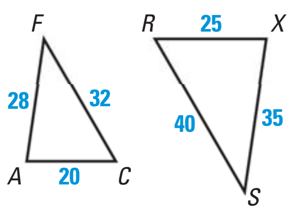


Determine whether the triangles are similar. If they are similar, write a similarity statement. *(pp. 381, 388)*

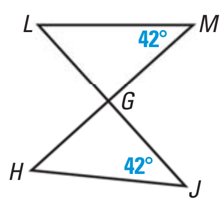
4.



5.



6.



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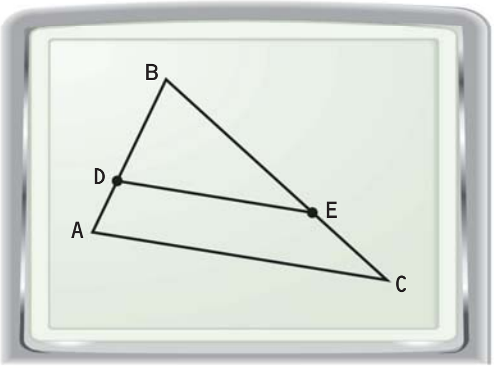
Keystrokes

6.6 Investigate Proportionality

MATERIALS • graphing calculator or computer

QUESTION How can you use geometry drawing software to compare segment lengths in triangles?

EXPLORE 1 Construct a line parallel to a triangle's third side



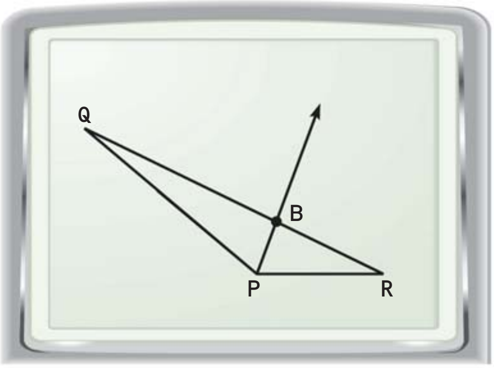
STEP 1 ***Draw a triangle*** Draw a triangle. Label the vertices *A*, *B*, and *C*. Draw a point on $\overline{AB}$. Label the point *D*.

STEP 2 ***Draw a parallel line*** Draw a line through *D* that is parallel to $\overline{AC}$. Label the intersection of the line and $\overline{BC}$ as point *E*.

STEP 3 ***Measure segments*** Measure $\overline{BD}$, $\overline{DA}$, $\overline{BE}$, and $\overline{EC}$. Calculate the ratios $\frac{BD}{DA}$ and $\frac{BE}{EC}$.

STEP 4 ***Compare ratios*** Move one or more of the triangle's vertices to change its shape. *Compare* the ratios from Step 3 as the shape changes. Save as "EXPLORE1."

EXPLORE 2 Construct an angle bisector of a triangle



STEP 1 ***Draw a triangle*** Draw a triangle. Label the vertices *P*, *Q*, and *R*. Draw the angle bisector of $\angle QPR$. Label the intersection of the angle bisector and $\overline{QR}$ as point *B*.

STEP 2 ***Measure segments*** Measure $\overline{BR}$, $\overline{RP}$, $\overline{BQ}$, and $\overline{QP}$. Calculate the ratios $\frac{BR}{BQ}$ and $\frac{RP}{QP}$.

STEP 3 ***Compare ratios*** Move one or more of the triangle's vertices to change its shape. *Compare* the ratios from Step 3. Save as "EXPLORE2."

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Make a conjecture about the ratios of the lengths of the segments formed when two sides of a triangle are cut by a line parallel to the triangle's third side.
2. Make a conjecture about how the ratio of the lengths of two sides of a triangle is related to the ratio of the lengths of the segments formed when an angle bisector is drawn to the third side.

6.6 Use Proportionality Theorems

Before You used proportions with similar triangles.

Now You will use proportions with a triangle or parallel lines.

Why? So you can use perspective drawings, as in Ex. 28.

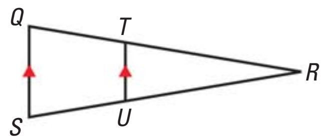
Key Vocabulary
- **corresponding angles,** *p. 147*
- **ratio,** *p. 356*
- **proportion,** *p. 358*

The Midsegment Theorem, which you learned on page 295, is a special case of the Triangle Proportionality Theorem and its converse.

THEOREMS — *For Your Notebook*

THEOREM 6.4 Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

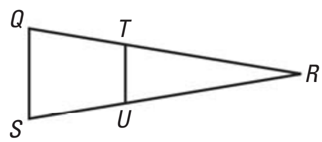


Proof: Ex. 22, p. 402

If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.

THEOREM 6.5 Converse of the Triangle Proportionality Theorem

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

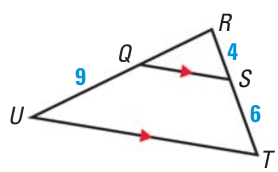


Proof: Ex. 26, p. 402

If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.

EXAMPLE 1 Find the length of a segment

In the diagram, $\overline{QS} \parallel \overline{UT}$, $RS = 4$, $ST = 6$, and $QU = 9$. What is the length of $\overline{RQ}$?



Solution

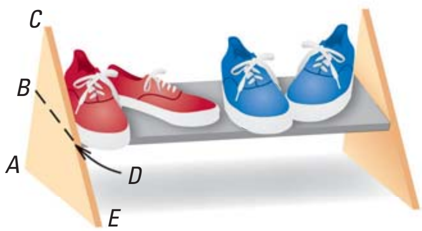
$\frac{RQ}{QU} = \frac{RS}{ST}$	Triangle Proportionality Theorem
$\frac{RQ}{9} = \frac{4}{6}$	Substitute.
$RQ = 6$	Multiply each side by 9 and simplify.

REASONING Theorems 6.4 and 6.5 also tell you that if the lines are *not* parallel, then the proportion is *not* true, and vice-versa.

So if $\overline{TU} \nparallel \overline{QS}$, then $\frac{RT}{TQ} \neq \frac{RU}{US}$. Also, if $\frac{RT}{TQ} \neq \frac{RU}{US}$, then $\overline{TU} \nparallel \overline{QS}$.

EXAMPLE 2 Solve a real-world problem

SHOERACK On the shoerack shown, $AB = 33$ cm, $BC = 27$ cm, $CD = 44$ cm, and $DE = 25$ cm. *Explain* why the gray shelf is not parallel to the floor.



Solution

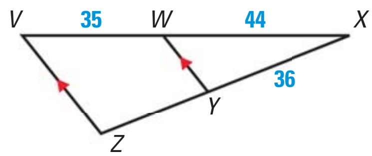
Find and simplify the ratios of lengths determined by the shoerack.

$$\frac{CD}{DE} = \frac{44}{25} \qquad \frac{CB}{BA} = \frac{27}{33} = \frac{9}{11}$$

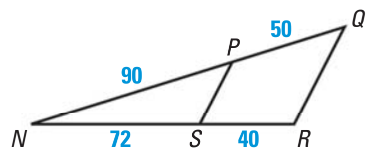
▶ Because $\frac{44}{25} \neq \frac{9}{11}$, $\overline{BD}$ is not parallel to $\overline{AE}$. So, the shelf is not parallel to the floor.

✓ GUIDED PRACTICE for Examples 1 and 2

1. Find the length of $\overline{YZ}$.



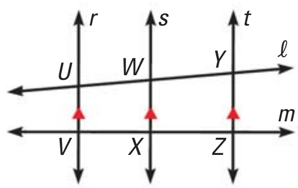
2. Determine whether $\overline{PS} \parallel \overline{QR}$.



THEOREMS *For Your Notebook*

THEOREM 6.6

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

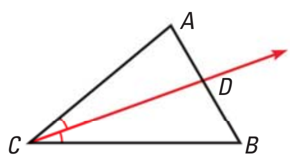


$$\frac{UW}{WY} = \frac{VX}{XZ}$$

Proof: Ex. 23, p. 402

THEOREM 6.7

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

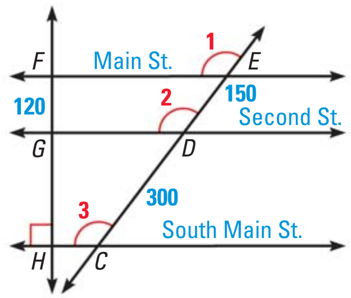


$$\frac{AD}{DB} = \frac{CA}{CB}$$

Proof: Ex. 27, p. 403

EXAMPLE 3 Use Theorem 6.6

CITY TRAVEL In the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are all congruent and $GF = 120$ yards, $DE = 150$ yards, and $CD = 300$ yards. Find the distance HF between Main Street and South Main Street.



ANOTHER WAY

For alternative methods for solving the problem in Example 3, turn to page 404 for the **Problem Solving Workshop**.

Solution

Corresponding angles are congruent, so $\overleftrightarrow{FE}$, $\overleftrightarrow{GD}$, and $\overleftrightarrow{HC}$ are parallel. Use Theorem 6.6.

$\frac{HG}{GF} = \frac{CD}{DE}$ **Parallel lines divide transversals proportionally.**

$\frac{HG + GF}{GF} = \frac{CD + DE}{DE}$ **Property of proportions (Property 4)**

$\frac{HF}{120} = \frac{300 + 150}{150}$ **Substitute.**

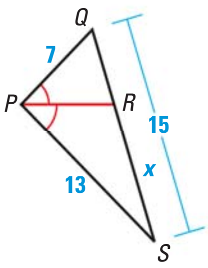
$\frac{HF}{120} = \frac{450}{150}$ **Simplify.**

$HF = 360$ **Multiply each side by 120 and simplify.**

▶ The distance between Main Street and South Main Street is 360 yards.

EXAMPLE 4 Use Theorem 6.7

In the diagram, $\angle QPR \cong \angle RPS$. Use the given side lengths to find the length of $\overline{RS}$.



Solution

Because $\overrightarrow{PR}$ is an angle bisector of $\angle QPS$, you can apply Theorem 6.7. Let $RS = x$. Then $RQ = 15 - x$.

$\frac{RQ}{RS} = \frac{PQ}{PS}$ **Angle bisector divides opposite side proportionally.**

$\frac{15 - x}{x} = \frac{7}{13}$ **Substitute.**

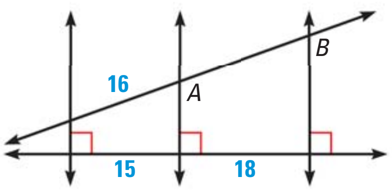
$7x = 195 - 13x$ **Cross Products Property**

$x = 9.75$ **Solve for *x*.**

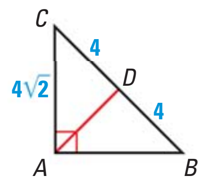
✓ GUIDED PRACTICE for Examples 3 and 4

Find the length of $\overline{AB}$.

3.



4.



6.6 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 9, and 21

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 8, 13, 25, and 28

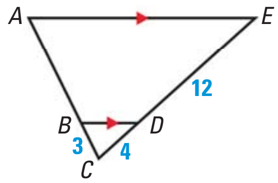
SKILL PRACTICE

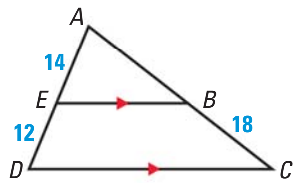
1. **VOCABULARY** State the Triangle Proportionality Theorem. Draw a diagram.

2. ★ **WRITING** *Compare* the Midsegment Theorem (see page 295) and the Triangle Proportionality Theorem. How are they related?

EXAMPLE 1 on p. 397 for Exs. 3–4

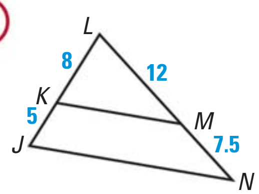
FINDING THE LENGTH OF A SEGMENT **Find the length of $\overline{AB}$.**

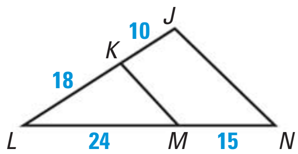
3. 

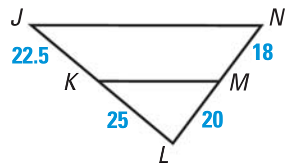
4. 

EXAMPLE 2 on p. 398 for Exs. 5–7

REASONING **Use the given information to determine whether $\overline{KM} \parallel \overline{JN}$. *Explain* your reasoning.**

5. 

6. 

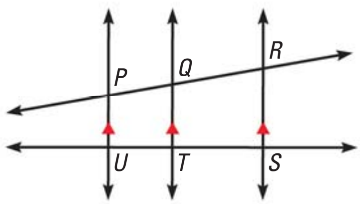
7. 

EXAMPLE 3 on p. 399 for Ex. 8

8. ★ **MULTIPLE CHOICE** For the figure at the right, which statement is *not* necessarily true?

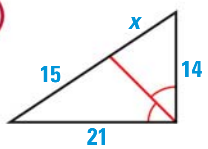
Ⓐ $\dfrac{PQ}{QR} = \dfrac{UT}{TS}$ Ⓑ $\dfrac{TS}{UT} = \dfrac{QR}{PQ}$

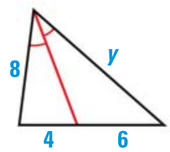
Ⓒ $\dfrac{QR}{RS} = \dfrac{TS}{RS}$ Ⓓ $\dfrac{PQ}{PR} = \dfrac{UT}{US}$

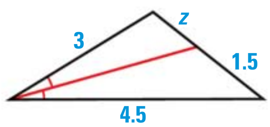


EXAMPLE 4 on p. 399 for Exs. 9–12

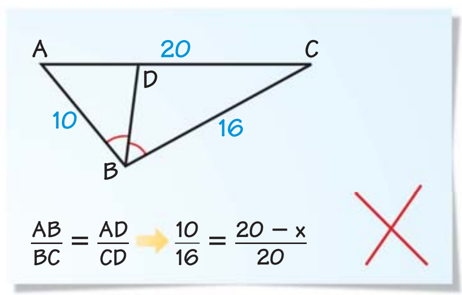
ALGEBRA **Find the value of the variable.**

9. 

10. 

11. 

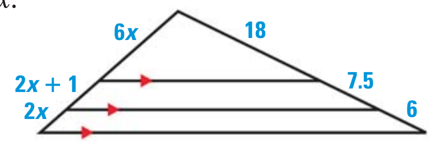
12. **ERROR ANALYSIS** A student begins to solve for the length of $\overline{AD}$ as shown. *Describe* and correct the student's error.



13. ★ **MULTIPLE CHOICE** Find the value of x.

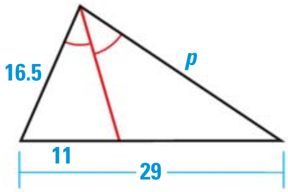
Ⓐ $\frac{1}{2}$ Ⓑ 1

Ⓒ 2 Ⓓ 3



xy ALGEBRA Find the value of the variable.

14.



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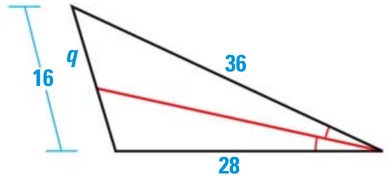
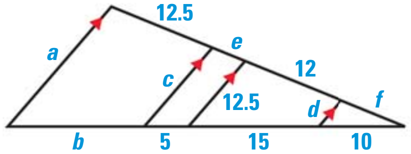


FINDING SEGMENT LENGTHS Use the diagram to find the value of each variable.

16.



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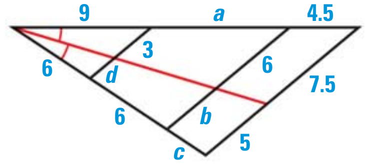
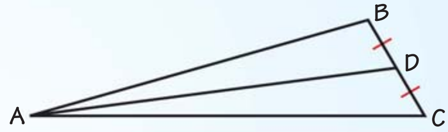


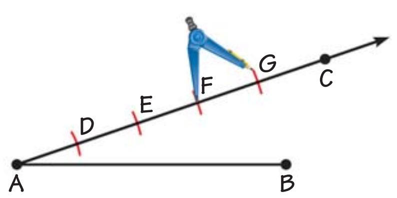
18. ERROR ANALYSIS A student claims that $AB = AC$ using the method shown. *Describe* and correct the student's error.

By Theorem 6.7, $\frac{BD}{CD} = \frac{AB}{AC}$. Because $BD = CD$, it follows that $AB = AC$.



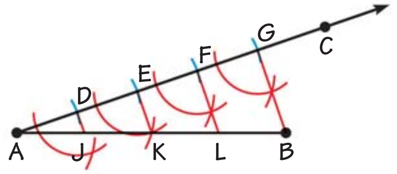
19. CONSTRUCTION Follow the instructions for constructing a line segment that is divided into four equal parts.

a. Draw a line segment that is about 3 inches long, and label its endpoints A and B. Choose any point C not on $\overline{AB}$. Draw $\overrightarrow{AC}$.

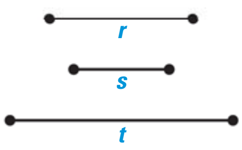


b. Using any length, place the compass point at A and make an arc intersecting $\overrightarrow{AC}$ at D. Using the same compass setting, make additional arcs on $\overrightarrow{AC}$. Label the points E, F, and G so that $AD = DE = EF = FG$.

c. Draw $\overline{GB}$. Construct a line parallel to $\overline{GB}$ through D. Continue constructing parallel lines and label the points as shown. *Explain* why $AJ = JK = KL = LB$.

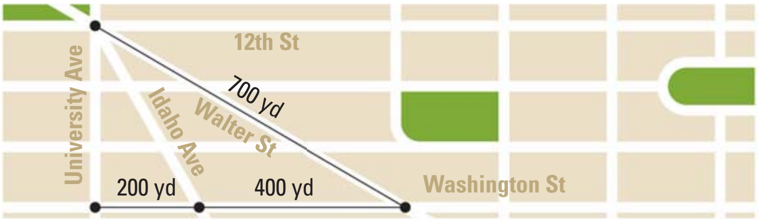


20. CHALLENGE Given segments with lengths r, s, and t, construct a segment of length x, such that $\frac{r}{s} = \frac{t}{x}$.



PROBLEM SOLVING

21. **CITY MAP** On the map below, Idaho Avenue bisects the angle between University Avenue and Walter Street. To the nearest yard, what is the distance along University Avenue from 12th Street to Washington Street?



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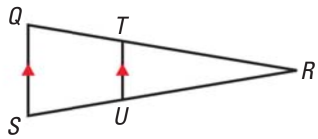
22. **PROVING THEOREM 6.4** Prove the Triangle Proportionality Theorem.

GIVEN ▶ $\overline{QS} \parallel \overline{TU}$

PROVE ▶ $\frac{QT}{TR} = \frac{SU}{UR}$

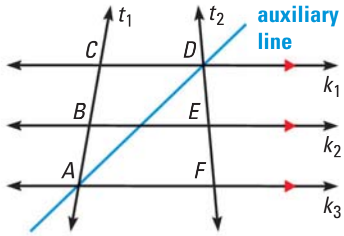


@HomeTutor for problem solving help at classzone.com

23. **PROVING THEOREM 6.6** Use the diagram with the auxiliary line drawn to write a paragraph proof of Theorem 6.6.

GIVEN ▶ $k_1 \parallel k_2$, $k_2 \parallel k_3$

PROVE ▶ $\frac{CB}{BA} = \frac{DE}{EF}$

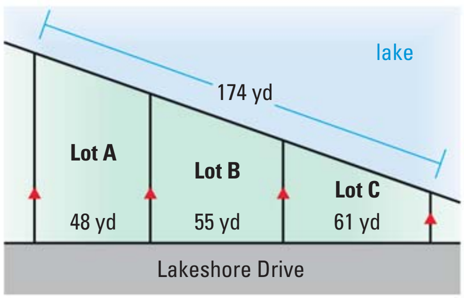


24. **MULTI-STEP PROBLEM** The real estate term *lake frontage* refers to the distance along the edge of a piece of property that touches a lake.

a. Find the lake frontage (to the nearest tenth of a yard) for each lot shown.

b. In general, the more lake frontage a lot has, the higher its selling price. Which of the lots should be listed for the highest price?

c. Suppose that lot prices are in the same ratio as lake frontages. If the least expensive lot is \$100,000, what are the prices of the other lots? *Explain* your reasoning.



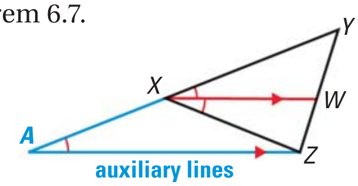
25. ★ **SHORT RESPONSE** Sketch an isosceles triangle. Draw a ray that bisects the angle opposite the base. This ray divides the base into two segments. By Theorem 6.7, the ratio of the legs is proportional to the ratio of these two segments. *Explain* why this ratio is 1 : 1 for an isosceles triangle.

26. **PLAN FOR PROOF** Use the diagram given for the proof of Theorem 6.4 in Exercise 22 to write a plan for proving Theorem 6.5, the Triangle Proportionality Converse.

○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

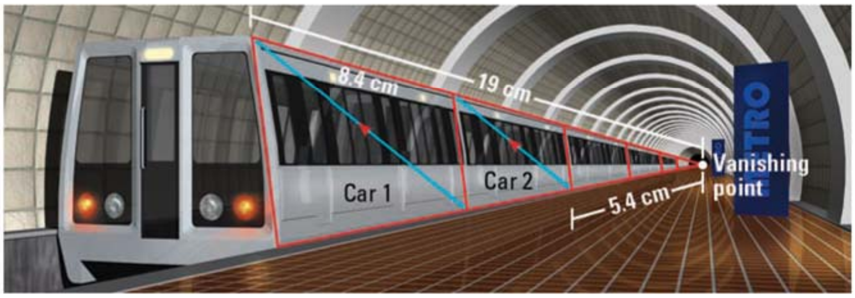
27. PROVING THEOREM 6.7 Use the diagram with the auxiliary lines drawn to write a paragraph proof of Theorem 6.7.



GIVEN ▶ $\angle YXW \cong \angle WXZ$

PROVE ▶ $\frac{YW}{WZ} = \frac{XY}{XZ}$

28. ★ EXTENDED RESPONSE In *perspective drawing*, lines that are parallel in real life must meet at a vanishing point on the horizon. To make the train cars in the drawing appear equal in length, they are drawn so that the lines connecting the opposite corners of each car are parallel.

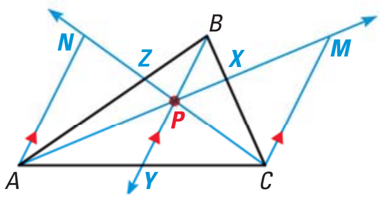


a. Use the dimensions given and the red parallel lines to find the length of the bottom edge of the drawing of Car 2.

b. What other set of parallel lines exist in the figure? *Explain* how these can be used to form a set of similar triangles.

c. Find the length of the top edge of the drawing of Car 2.

29. CHALLENGE Prove *Ceva's Theorem:* If P is any point inside $\triangle ABC$, then $\frac{AY}{YC} \cdot \frac{CX}{XB} \cdot \frac{BZ}{ZA} = 1$. (*Hint:* Draw lines parallel to $\overline{BY}$ through A and C. Apply Theorem 6.4 to $\triangle ACM$. Show that $\triangle APN \sim \triangle MPC$, $\triangle CXM \sim \triangle BXP$, and $\triangle BZP \sim \triangle AZN$.)



MIXED REVIEW

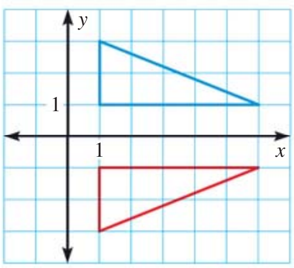
PREVIEW
Prepare for Lesson 6.7 in Exs. 30–36.

Perform the following operations. Then simplify.

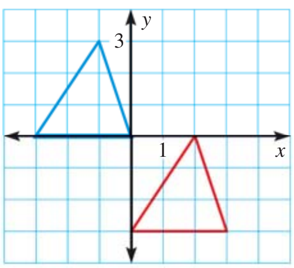
30. $(-3) \cdot \frac{7}{2}$ *(p. 869)* **31.** $\frac{4}{3} \cdot \frac{1}{2}$ *(p. 869)* **32.** $5\left(\frac{1}{2}\right)^2$ *(p. 871)* **33.** $\left(\frac{5}{4}\right)^3$ *(p. 871)*

Describe **the translation in words and write the coordinate rule for the translation.** *(p. 272)*

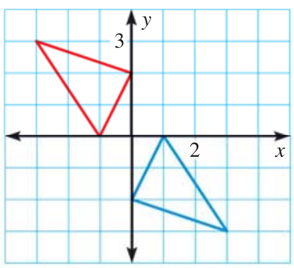
34.



35.



36.



PROBLEM SOLVING WORKSHOP
LESSON 6.6

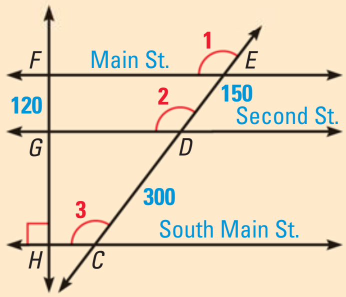
Using ALTERNATIVE METHODS

Another Way to Solve Example 3, page 399

MULTIPLE REPRESENTATIONS In Lesson 6.6, you used proportionality theorems to find lengths of segments formed when transversals intersect two or more parallel lines. Now, you will learn two different ways to solve Example 3 on page 399.

PROBLEM

CITY TRAVEL In the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are all congruent and $GF = 120$ yards, $DE = 150$ yards, and $CD = 300$ yards. Find the distance HF between Main Street and South Main Street.



METHOD 1

Applying a Ratio One alternative approach is to look for ratios in the diagram.

STEP 1 **Read** the problem. Because Main Street, Second Street, and South Main Street are all parallel, the lengths of the segments of the cross streets will be in proportion, so they have the same ratio.

STEP 2 **Apply** a ratio. Notice that on $\overleftrightarrow{CE}$, the distance CD between South Main Street and Second Street is twice the distance DE between Second Street and Main Street. So the same will be true for the distances HG and GF.

$HG = 2 \cdot GF$	**Write equation.**
$= 2 \cdot 120$	**Substitute.**
$= 240$	**Simplify.**

STEP 3 **Calculate** the distance. Line HF is perpendicular to both Main Street and South Main Street, so the distance between Main Street and South Main Street is this perpendicular distance, HF.

$HF = HG + GF$	**Segment Addition Postulate**
$= 120 + 240$	**Substitute.**
$= 360$	**Simplify.**

STEP 4 **Check** page 399 to verify your answer, and confirm that it is the same.

METHOD 2

Writing a Proportion Another alternative approach is to use a graphic organizer to set up a proportion.

STEP 1 **Make** a table to compare the distances.

	$\overleftrightarrow{CE}$	$\overleftrightarrow{HF}$
Total distance	300 + 150, or 450	x
Partial distance	150	120

STEP 2 **Write** and solve a proportion.

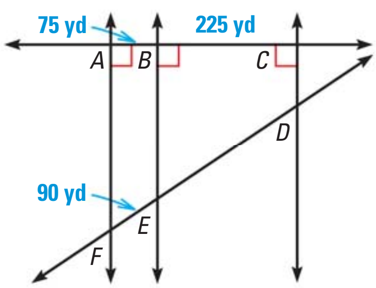
$\frac{450}{150} = \frac{x}{120}$ **Write proportion.**

$360 = x$ **Multiply each side by 12 and simplify.**

▶ The distance is 360 yards.

PRACTICE

1. **MAPS** Use the information on the map.



   a. Find DE.

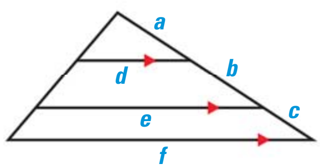
   b. **What If?** Suppose there is an alley one fourth of the way from $\overline{BE}$ to $\overline{CD}$ and parallel to $\overline{BE}$. What is the distance from E to the alley along $\overleftrightarrow{FD}$?

2. **REASONING** Given the diagram below, *explain* why the three given proportions are true.

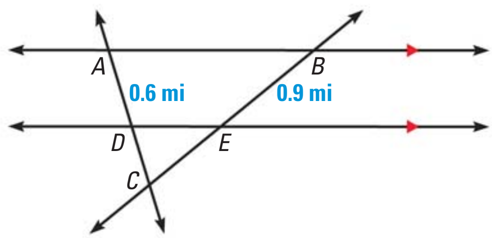
$$\frac{a}{a+b} = \frac{d}{e}$$

$$\frac{a}{a+b+c} = \frac{d}{f}$$

$$\frac{a+b}{a+b+c} = \frac{d}{f}$$

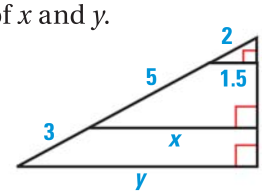


3. **WALKING** Two people leave points A and B at the same time. They intend to meet at point C at the same time. The person who leaves point A walks at a speed of 3 miles per hour. How fast must the person who leaves point B walk?



4. **ERROR ANALYSIS** A student who attempted to solve the problem in Exercise 3 claims that you need to know the length of $\overline{AC}$ to solve the problem. *Describe* and correct the error that the student made.

5. **xy ALGEBRA** Use the diagram to find the values of x and y.



Extension

Use after Lesson 6.6

Fractals

GOAL Explore the properties of fractals.

Key Vocabulary
- fractal
- self-similarity
- iteration

HISTORY NOTE

Computers made it easier to study mathematical iteration by reducing the time needed to perform calculations. Using fractals, mathematicians have been able to create better models of coastlines, clouds, and other natural objects.

A **fractal** is an object that is *self-similar*. An object is **self-similar** if one part of the object can be enlarged to look like the whole object. In nature, fractals can be found in ferns and branches of a river. Scientists use fractals to map out clouds in order to predict rain.

A Mandelbrot fractal

Many fractals are formed by a repetition of a sequence of the steps called **iteration**. The first stage of drawing a fractal is considered Stage 0. Helge van Koch (1870–1924) described a fractal known as the *Koch snowflake*, shown in Example 1.

EXAMPLE 1 Draw a fractal

Use the directions below to draw a Koch snowflake.

Starting with an equilateral triangle, at each stage each side is divided into thirds and a new equilateral triangle is formed using the middle third as the triangle side length.

Solution

STAGE 0 **Draw** an equilateral triangle with a side length of one unit.

STAGE 1 **Replace** the middle third of each side with an equilateral triangle.

STAGE 2 **Repeat** Stage 1 with the six smaller equilateral triangles.

STAGE 3 **Repeat** Stage 1 with the eighteen smaller equilateral triangles.

MEASUREMENT Benoit Mandelbrot (b. 1924) was the first mathematician to formalize the idea of fractals when he observed methods used to measure the lengths of coastlines. Coastlines cannot be measured as straight lines because of the inlets and rocks. Mandelbrot used fractals to model coastlines.

EXAMPLE 2 Find lengths in a fractal

Make a table to study the lengths of the sides of a Koch snowflake at different stages.

Stage number	Edge length	Number of edges	Perimeter
0	1	3	3
1	$\frac{1}{3}$	$3 \cdot 4 = 12$	4
2	$\frac{1}{9}$	$12 \cdot 4 = 48$	$\frac{48}{9} = 5\frac{1}{3}$
3	$\frac{1}{27}$	$48 \cdot 4 = 192$	$\frac{192}{27} = 7\frac{1}{9}$
n	$\frac{1}{3^n}$	$3 \cdot 4^n$	$\frac{4^n}{3^{n-1}}$

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PRACTICE

EXAMPLES 1 and 2 for Exs. 1–3

1. **PERIMETER** Find the ratio of the edge length of the triangle in Stage 0 of a Koch snowflake to the edge length of the triangle in Stage 1. How is the perimeter of the triangle in Stage 0 related to the perimeter of the triangle in Stage 1? *Explain*.

2. **MULTI-STEP PROBLEM** Use the *Cantor set*, which is a fractal whose iteration consists of dividing a segment into thirds and erasing the middle third.
   a. Draw Stage 0 through Stage 5 of the Cantor set. Stage 0 has a length of one unit.
   b. Make a table showing the stage number, number of segments, segment length, and total length of the Cantor set.
   c. What is the total length of the Cantor set at Stage 10? Stage 20? Stage n?

3. **EXTENDED RESPONSE** A *Sierpinski carpet* starts with a square with side length one unit. At each stage, divide the square into nine equal squares with the middle square shaded a different color.
   a. Draw Stage 0 through Stage 3 of a Sierpinski Carpet.
   b. *Explain* why the carpet is said to be *self-similar* by comparing the upper left hand square to the whole square.
   c. Make a table to find the total area of the colored squares at Stage 3.

Investigating Geometry ACTIVITY *Use before Lesson 6.7*

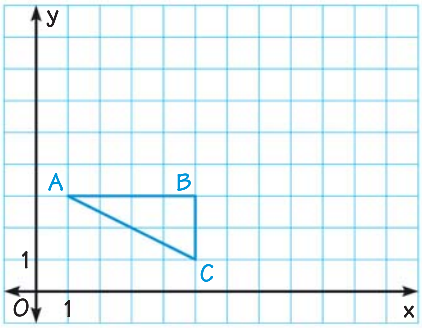
6.7 Dilations

MATERIALS • graph paper • straightedge • compass • ruler

QUESTION How can you construct a similar figure?

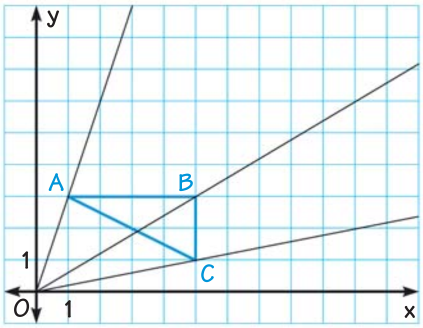
EXPLORE Construct a similar triangle

STEP 1



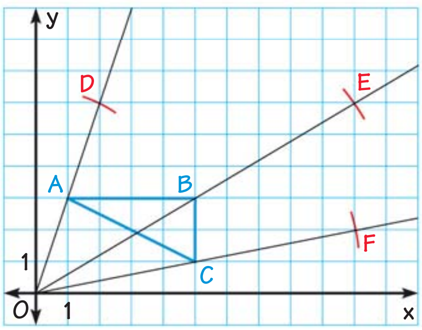
Draw a triangle Plot the points $A(1, 3)$, $B(5, 3)$, and $C(5, 1)$ in a coordinate plane. Draw $\triangle ABC$.

STEP 2



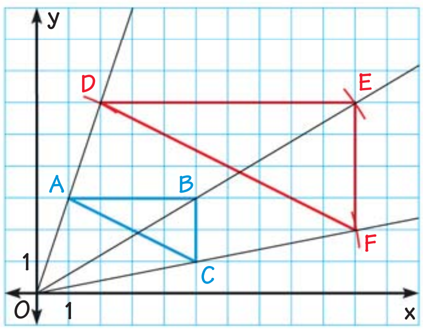
Draw rays Using the origin as an endpoint O, draw $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$.

STEP 3



Draw equal segments Use a compass to mark a point D on $\overrightarrow{OA}$ so $OA = AD$. Mark a point E on $\overrightarrow{OB}$ so $OB = BE$. Mark a point F on $\overrightarrow{OC}$ so $OC = CF$.

STEP 4



Draw the image Connect points D, E, and F to form a right triangle.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Measure $\overline{AB}$, $\overline{BC}$, $\overline{DE}$, and $\overline{EF}$. Calculate the ratios $\frac{DE}{AB}$ and $\frac{EF}{BC}$. Using this information, show that the two triangles are similar.
2. Repeat the steps in the Explore to construct $\triangle GHJ$ so that $3 \cdot OA = AG$, $3 \cdot OB = BH$, and $3 \cdot OC = CJ$.

6.7 Perform Similarity Transformations

Before You performed congruence transformations.

Now You will perform dilations.

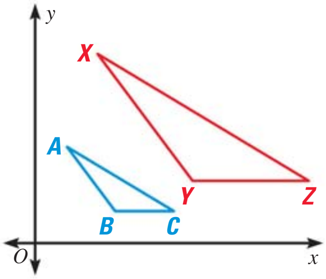
Why? So you can solve problems in art, as in Ex. 26.

Key Vocabulary
- **dilation**
- **center of dilation**
- **scale factor of a dilation**
- **reduction**
- **enlargement**
- **transformation,** *p. 272*

A **dilation** is a transformation that stretches or shrinks a figure to create a similar figure. A dilation is a type of *similarity transformation*.

In a dilation, a figure is enlarged or reduced with respect to a fixed point called the **center of dilation**.

The **scale factor of a dilation** is the ratio of a side length of the image to the corresponding side length of the original figure. In the figure shown, $\triangle XYZ$ is the image of $\triangle ABC$. The center of dilation is (0, 0) and the scale factor is $\frac{XY}{AB}$.



KEY CONCEPT — *For Your Notebook*

Coordinate Notation for a Dilation

You can describe a dilation with respect to the origin with the notation $(x, y) \to (kx, ky)$, where k is the scale factor.

If $0 < k < 1$, the dilation is a **reduction**. If $k > 1$, the dilation is an **enlargement**.

EXAMPLE 1 Draw a dilation with a scale factor greater than 1

READ DIAGRAMS
All of the dilations in this lesson are in the coordinate plane and each center of dilation is the origin.

Draw a dilation of quadrilateral *ABCD* with vertices *A*(2, 1), *B*(4, 1), *C*(4, −1), and *D*(1, −1). Use a scale factor of 2.

Solution

First draw *ABCD*. Find the dilation of each vertex by multiplying its coordinates by 2. Then draw the dilation.

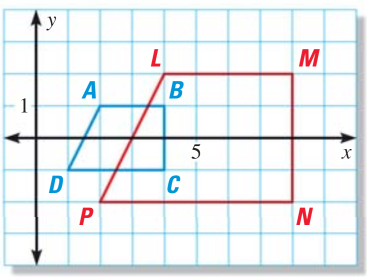
$$\mathbf{(x, y) \to (2x, 2y)}$$

$A(2, 1) \to L(4, 2)$

$B(4, 1) \to M(8, 2)$

$C(4, -1) \to N(8, -2)$

$D(1, -1) \to P(2, -2)$



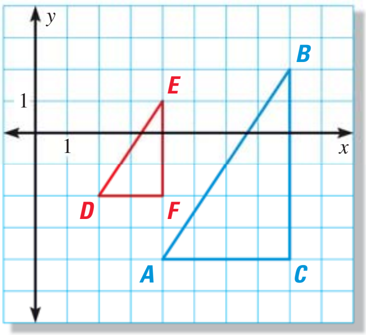
EXAMPLE 2 Verify that a figure is similar to its dilation

A triangle has the vertices $A(4, -4)$, $B(8, 2)$, and $C(8, -4)$. The image of $\triangle ABC$ after a dilation with a scale factor of $\frac{1}{2}$ is $\triangle DEF$.

a. Sketch $\triangle ABC$ and $\triangle DEF$.

b. Verify that $\triangle ABC$ and $\triangle DEF$ are similar.

Solution



a. The scale factor is less than one, so the dilation is a reduction.

$$(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$$

$A(4, -4) \rightarrow D(2, -2)$

$B(8, 2) \rightarrow E(4, 1)$

$C(8, -4) \rightarrow F(4, -2)$

b. Because $\angle C$ and $\angle F$ are both right angles, $\angle C \cong \angle F$. Show that the lengths of the sides that include $\angle C$ and $\angle F$ are proportional. Find the horizontal and vertical lengths from the coordinate plane.

$$\frac{AC}{DF} \stackrel{?}{=} \frac{BC}{EF} \rightarrow \frac{4}{2} = \frac{6}{3} \checkmark$$

So, the lengths of the sides that include $\angle C$ and $\angle F$ are proportional.

▶ Therefore, $\triangle ABC \sim \triangle DEF$ by the SAS Similarity Theorem.

✓ GUIDED PRACTICE for Examples 1 and 2

Find the coordinates of L, M, and N so that $\triangle LMN$ is a dilation of $\triangle PQR$ with a scale factor of k. Sketch $\triangle PQR$ and $\triangle LMN$.

1. $P(-2, -1)$, $Q(-1, 0)$, $R(0, -1)$; $k = 4$
2. $P(5, -5)$, $Q(10, -5)$, $R(10, 5)$; $k = 0.4$

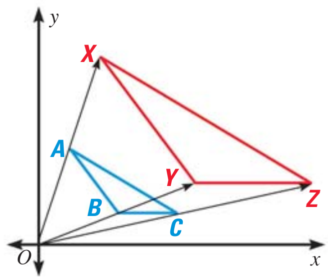
EXAMPLE 3 Find a scale factor

PHOTO STICKERS You are making your own photo stickers. Your photo is 4 inches by 4 inches. The image on the stickers is 1.1 inches by 1.1 inches. What is the scale factor of the reduction?

Solution

The scale factor is the ratio of a side length of the sticker image to a side length of the original photo, or $\frac{1.1 \text{ in.}}{4 \text{ in.}}$. In simplest form, the scale factor is $\frac{11}{40}$.

READING DIAGRAMS Generally, for a center of dilation at the origin, a point of the figure and its image lie on the same ray from the origin. However, if a point of the figure *is* the origin, its image is also the origin.



EXAMPLE 4 Standardized Test Practice

You want to create a quadrilateral *EFGH* that is similar to quadrilateral *PQRS*. What are the coordinates of *H*?

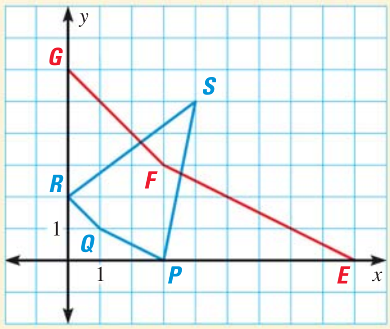
ELIMINATE CHOICES
You can eliminate choice A, because you can tell by looking at the graph that *H* is in Quadrant I. The point (12, −15) is in Quadrant II.

Ⓐ (12, −15)

Ⓑ (7, 8)

Ⓒ (12, 15)

Ⓓ (15, 18)



Solution

Determine if *EFGH* is a dilation of *PQRS* by checking whether the same scale factor can be used to obtain *E*, *F*, and *G* from *P*, *Q*, and *R*.

$(x, y) \to (kx, ky)$

$P(3, 0) \to E(9, 0)$ $k = 3$

$Q(1, 1) \to F(3, 3)$ $k = 3$

$R(0, 2) \to G(0, 6)$ $k = 3$

Because *k* is the same in each case, the image is a dilation with a scale factor of 3. So, you can use the scale factor to find the image *H* of point *S*.

$S(4, 5) \to H(3 \cdot 4, 3 \cdot 5) = H(12, 15)$

▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

CHECK Draw rays from the origin through each point and its image.

✓ GUIDED PRACTICE for Examples 3 and 4

3. **WHAT IF?** In Example 3, what is the scale factor of the reduction if your photo is 5.5 inches by 5.5 inches?

4. Suppose a figure containing the origin is dilated. *Explain* why the corresponding point in the image of the figure is also the origin.

6.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 11, and 27

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 13, 21, 22, 28, 30, and 31

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: In a dilation, the image is _?_ to the original figure.

2. ★ **WRITING** *Explain* how to find the scale factor of a dilation. How do you know whether a dilation is an enlargement or a reduction?

EXAMPLES 1 and 2 on pp. 409–410 for Exs. 3–8

DRAWING DILATIONS **Draw a dilation of the polygon with the given vertices using the given scale factor *k*.**

3. $A(-2, 1)$, $B(-4, 1)$, $C(-2, 4)$; $k = 2$

4. $A(-5, 5)$, $B(-5, -10)$, $C(10, 0)$; $k = \frac{3}{5}$

5. $A(1, 1)$, $B(6, 1)$, $C(6, 3)$; $k = 1.5$

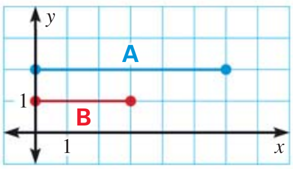
6. $A(2, 8)$, $B(8, 8)$, $C(16, 4)$; $k = 0.25$

7. $A(-8, 0)$, $B(0, 8)$, $C(4, 0)$, $D(0, -4)$; $k = \frac{3}{8}$

8. $A(0, 0)$, $B(0, 3)$, $C(2, 4)$, $D(2, -1)$; $k = \frac{13}{2}$

EXAMPLE 3 on p. 410 for Exs. 9–12

IDENTIFYING DILATIONS **Determine whether the dilation from Figure A to Figure B is a *reduction* or an *enlargement*. Then find its scale factor.**

9.


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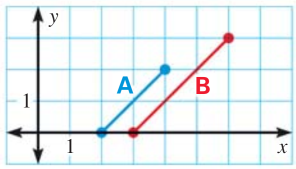

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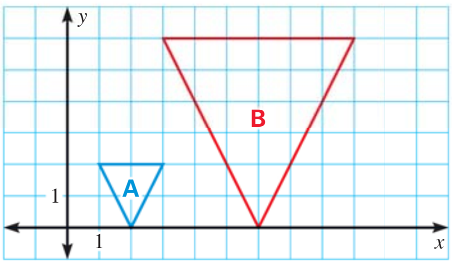

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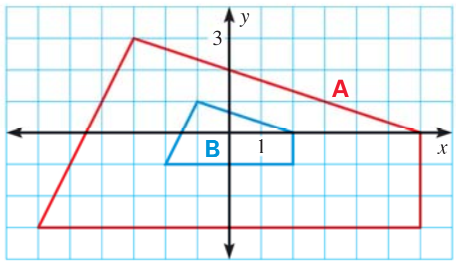
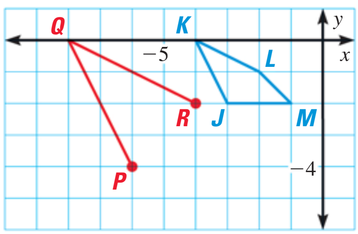


EXAMPLE 4 on p. 411 for Ex. 13

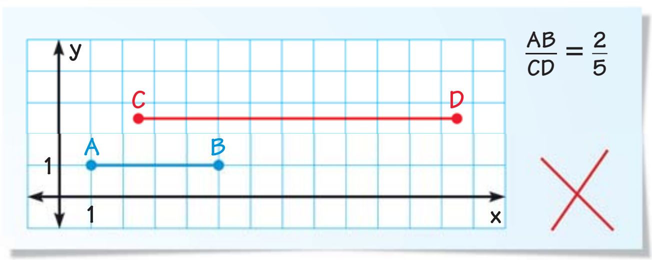
13. ★ **MULTIPLE CHOICE** You want to create a quadrilateral $PQRS$ that is similar to quadrilateral $JKLM$. What are the coordinates of S?

Ⓐ $(2, 4)$ Ⓑ $(4, -2)$

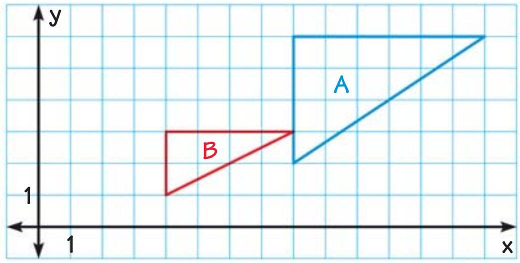
Ⓒ $(-2, -4)$ Ⓓ $(-4, -2)$



14. **ERROR ANALYSIS** A student found the scale factor of the dilation from $\overline{AB}$ to $\overline{CD}$ to be $\frac{2}{5}$. *Describe* and correct the student's error.

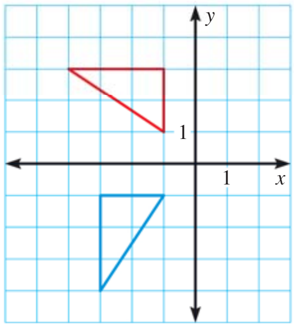


15. **ERROR ANALYSIS** A student says that the figure shown represents a dilation. What is wrong with this statement?

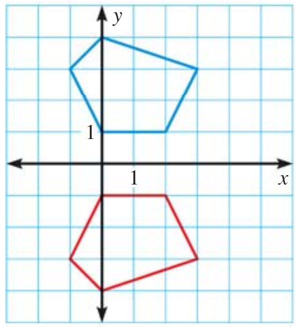


IDENTIFYING TRANSFORMATIONS Determine whether the transformation shown is a *translation*, *reflection*, *rotation*, or *dilation*.

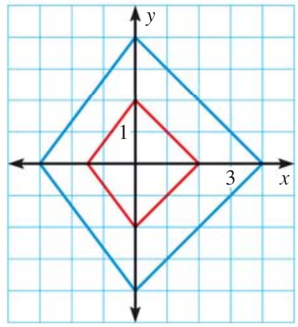
16.



17.

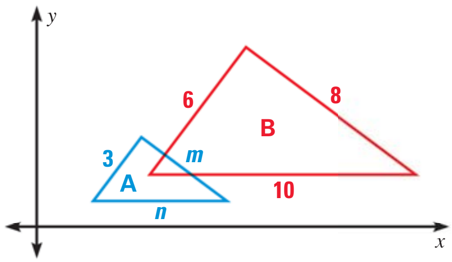


18.

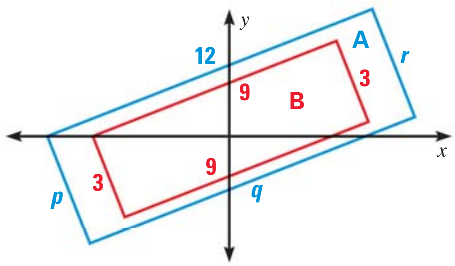


FINDING SCALE FACTORS Find the scale factor of the dilation of Figure A to Figure B. Then give the unknown lengths of Figure A.

19.



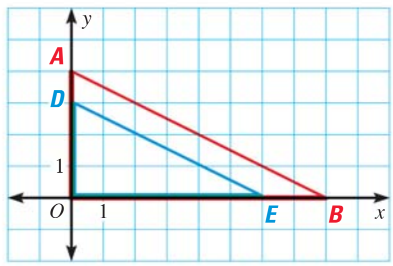
20.



21. ★ **MULTIPLE CHOICE** In the diagram shown, $\triangle ABO$ is a dilation of $\triangle DEO$. The length of a median of $\triangle ABO$ is what percent of the length of the corresponding median of $\triangle DEO$?

Ⓐ 50% Ⓑ 75%

Ⓒ $133\frac{1}{3}\%$ Ⓓ 200%



22. ★ **SHORT RESPONSE** Suppose you dilate a figure using a scale factor of 2. Then, you dilate the image using a scale factor of $\frac{1}{2}$. *Describe* the size and shape of this new image.

CHALLENGE *Describe* the two transformations, the first followed by the second, that combined will transform $\triangle ABC$ into $\triangle DEF$.

23. $A(-3, 3)$, $B(-3, 1)$, $C(0, 1)$
$D(6, 6)$, $E(6, 2)$, $F(0, 2)$

24. $A(6, 0)$, $B(9, 6)$, $C(12, 6)$
$D(0, 3)$, $E(1, 5)$, $F(2, 5)$

PROBLEM SOLVING

EXAMPLE 3 on p. 410 for Exs. 25–27

25. **BILLBOARD ADVERTISEMENT** A billboard advertising agency requires each advertisement to be drawn so that it fits in a 12-inch by 6-inch rectangle. The agency uses a scale factor of 24 to enlarge the advertisement to create the billboard. What are the dimensions of a billboard, in feet?

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26. **POTTERY** Your pottery is used on a poster for a student art show. You want to make postcards using the same image. On the poster, the image is 8 inches in width and 6 inches in height. If the image on the postcard can be 5 inches wide, what scale should you use for the image on the postcard?

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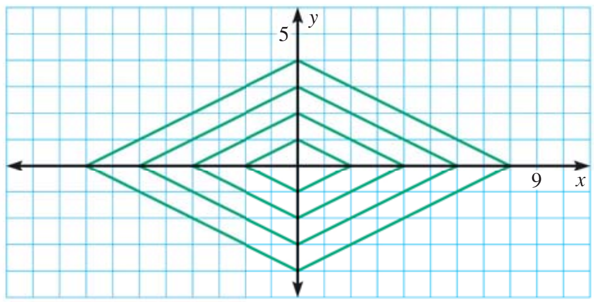
27. **SHADOWS** You and your friend are walking at night. You point a flashlight at your friend, and your friend's shadow is cast on the building behind him. The shadow is an enlargement, and is 15 feet tall. Your friend is 6 feet tall. What is the scale factor of the enlargement?

28. ★ **OPEN-ENDED MATH** *Describe* how you can use dilations to create the figure shown below.



AnimatedGeometry at classzone.com

29. **MULTI-STEP PROBLEM** $\triangle ABC$ has vertices $A(3, -3)$, $B(3, 6)$, and $C(15, 6)$.

    a. Draw a dilation of $\triangle ABC$ using a scale factor of $\frac{2}{3}$.

    b. Find the ratio of the perimeter of the image to the perimeter of the original figure. How does this ratio compare to the scale factor?

    c. Find the ratio of the area of the image to the area of the original figure. How does this ratio compare to the scale factor?

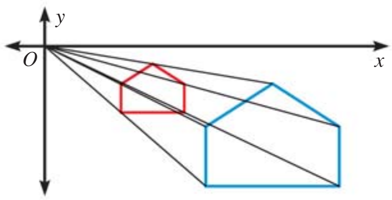
30. ★ **EXTENDED RESPONSE** Look at the coordinate notation for a dilation on page 409. Suppose the definition of dilation allowed $k < 0$.

    a. *Describe* the dilation if $-1 < k < 0$.

    b. *Describe* the dilation if $k < -1$.

    c. Use a rotation to describe a dilation with $k = -1$.

31. ★ **SHORT RESPONSE** *Explain* how you can use dilations to make a perspective drawing with the center of dilation as a vanishing point. Draw a diagram.



32. **MIDPOINTS** Let $\overline{XY}$ be a dilation of $\overline{PQ}$ with scale factor k. Show that the image of the midpoint of $\overline{PQ}$ is the midpoint of $\overline{XY}$.

33. **REASONING** In Exercise 32, show that $\overline{XY} \parallel \overline{PQ}$.

34. **CHALLENGE** A rectangle has vertices $A(0, 0)$, $B(0, 6)$, $C(9, 6)$, and $D(9, 0)$. *Explain* how to dilate the rectangle to produce an image whose area is twice the area of the original rectangle. Make a conjecture about how to dilate any polygon to produce an image whose area is n times the area of the original polygon.

MIXED REVIEW

Simplify the expression. *(p. 873)*

35. $(3x + 2)^2 + (x - 5)^2$ **36.** $4\left(\frac{1}{2}ab\right) + (b - a)^2$ **37.** $(a + b)^2 - (a - b)^2$

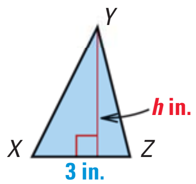
Find the distance between each pair of points. *(p. 15)*

38. $(0, 5)$ and $(4, 3)$ **39.** $(-3, 0)$ and $(2, 4)$ **40.** $(-2, -4)$ and $(3, -2)$

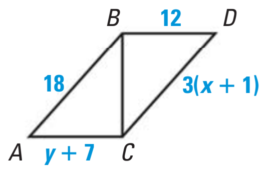
PREVIEW
Prepare for Lesson 7.1 in Exs. 41–43.

Find the value(s) of the variable(s).

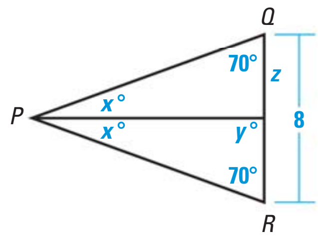
41. Area = 6 in.^{2} *(p. 49)*



42. $\triangle ABC \cong \triangle DCB$ *(p. 256)*



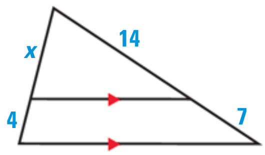
43. $\triangle PQR$ is isosceles. *(p. 303)*



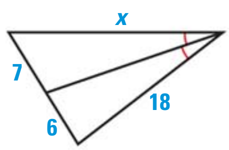
QUIZ for Lessons 6.6–6.7

Find the value of x. *(p. 397)*

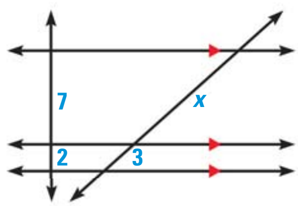
1.



2.



3.



Draw a dilation of $\triangle ABC$ with the given vertices and scale factor k. *(p. 409)*

4. $A(-5, 5)$, $B(-5, -10)$, $C(10, 0)$; $k = 0.4$ **5.** $A(-2, 1)$, $B(-4, 1)$, $C(-2, 4)$; $k = 2.5$

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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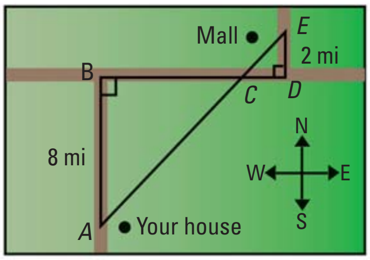
Lessons 6.4–6.7

1. **OPEN-ENDED** The diagram shows the front of a house. What information would you need in order to show that $\triangle WXY \sim \triangle VXZ$ using the SAS Similarity Theorem?

2. **EXTENDED RESPONSE** You leave your house to go to the mall. You drive due north 8 miles, due east 7.5 miles, and due north again 2 miles.

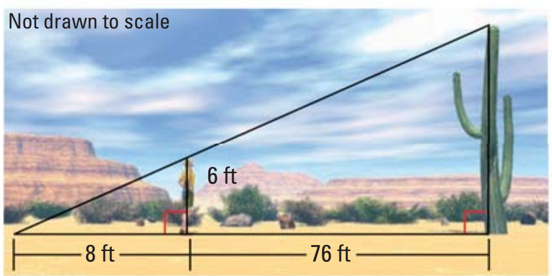


   a. *Explain* how to prove that $\triangle ABC \sim \triangle EDC$.

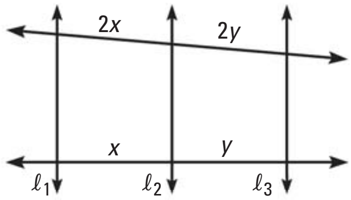
   b. Find CD.

   c. Find AE, the distance between your house and the mall.

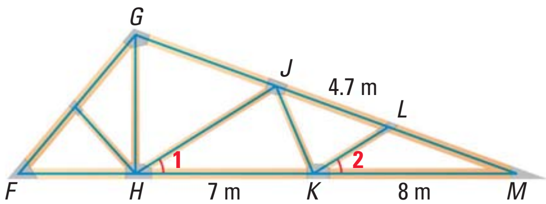
3. **SHORT RESPONSE** The Cardon cactus found in the Sonoran Desert in Mexico is the tallest type of cactus in the world. Marco stands 76 feet from the cactus so that his shadow coincides with the cactus' shadow. Marco is 6 feet tall and his shadow is 8 feet long. How tall is the Cardon cactus? *Explain*.



4. **SHORT RESPONSE** In the diagram, is it *always*, *sometimes*, or *never* true that $l_1 \parallel l_2 \parallel l_3$? *Explain*.



5. **GRIDDED ANSWER** In the diagram of the roof truss, $HK = 7$ meters, $KM = 8$ meters, $JL = 4.7$ meters, and $\angle 1 \cong \angle 2$. Find LM to the nearest tenth of a meter.



6. **GRIDDED ANSWER** You are designing a catalog for a greeting card company. The catalog features a $2\frac{4}{5}$ inch by 2 inch photograph of each card. The actual dimensions of a greeting card are 7 inches by 5 inches. What is the scale factor of the reduction?

7. **MULTI-STEP PROBLEM** Rectangle $ABCD$ has vertices $A(2, 2)$, $B(4, 2)$, $C(4, -4)$, and $D(2, -4)$.

   a. Draw rectangle $ABCD$. Then draw a dilation of rectangle $ABCD$ using a scale factor of $\frac{5}{4}$. Label the image $PQRS$.

   b. Find the ratio of the perimeter of the image to the perimeter of the original figure. How does this ratio compare to the scale factor?

   c. Find the ratio of the area of the image to the area of the original figure. How does this ratio compare to the scale factor?

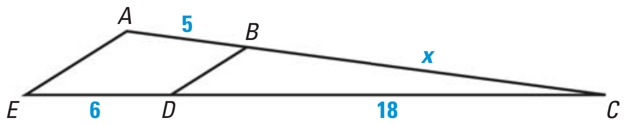
6 CHAPTER SUMMARY

BIG IDEAS *For Your Notebook*

Big Idea 1

Using Ratios and Proportions to Solve Geometry Problems

You can use properties of proportions to solve a variety of algebraic and geometric problems.



For example, in the diagram above, suppose you know that $\frac{AB}{BC} = \frac{ED}{DC}$. Then you can write any of the following relationships.

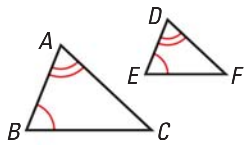
$\frac{5}{x} = \frac{6}{18}$ $\qquad$ $5 \cdot 18 = 6x$ $\qquad$ $\frac{x}{5} = \frac{18}{6}$ $\qquad$ $\frac{5}{6} = \frac{x}{18}$ $\qquad$ $\frac{5 + x}{x} = \frac{6 + 18}{18}$

Big Idea 2

Showing that Triangles are Similar

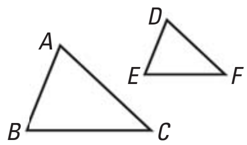
You learned three ways to prove two triangles are similar.

AA Similarity Postulate



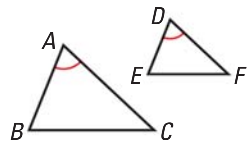
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem



If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

SAS Similarity Theorem

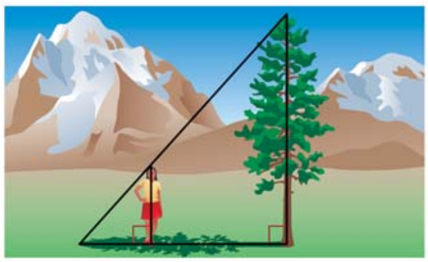


If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

Big Idea 3

Using Indirect Measurement and Similarity

You can use triangle similarity theorems to apply indirect measurement in order to find lengths that would be inconvenient or impossible to measure directly.



Consider the diagram shown. Because the two triangles formed by the person and the tree are similar by the AA Similarity Postulate, you can write the following proportion to find the height of the tree.

$$\frac{\text{height of person}}{\text{length of person's shadow}} = \frac{\text{height of tree}}{\text{length of tree's shadow}}$$

You also learned about dilations, a type of similarity transformation. In a dilation, a figure is either enlarged or reduced in size.

6 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- ratio, *p. 356*
- proportion, *p. 358* means, extremes
- geometric mean, *p. 359*
- scale drawing, *p. 365*
- scale, *p. 365*
- similar polygons, *p. 372*
- scale factor of two similar polygons, *p. 373*
- dilation, *p. 409*
- center of dilation, *p. 409*
- scale factor of a dilation, *p. 409*
- reduction, *p. 409*
- enlargement, *p. 409*

VOCABULARY EXERCISES

Copy and complete the statement.

1. A _?_ is a transformation in which the original figure and its image are similar.

2. If $\triangle PQR \sim \triangle XYZ$, then $\frac{PQ}{XY} = \frac{?}{YZ} = \frac{?}{?}$.

3. **WRITING** *Describe* the relationship between a ratio and a proportion. Give an example of each.

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 6.

6.1 Ratios, Proportions, and the Geometric Mean *pp. 356–363*

EXAMPLE

The measures of the angles in $\triangle ABC$ are in the extended ratio of 3 : 4 : 5. Find the measures of the angles.

Use the extended ratio of 3 : 4 : 5 to label the angle measures as $3x°$, $4x°$, and $5x°$.

$3x° + 4x° + 5x° = 180°$ **Triangle Sum Theorem**

$12x = 180$ **Combine like terms.**

$x = 15$ **Divide each side by 12.**

So, the angle measures are $3(15°) = 45°$, $4(15°) = 60°$, and $5(15°) = 75°$.

EXERCISES

EXAMPLES 1, 3, and 6 on pp. 356–359 for Exs. 4–6

4. The length of a rectangle is 20 meters and the width is 15 meters. Find the ratio of the width to the length of the rectangle. Then simplify the ratio.

5. The measures of the angles in $\triangle UVW$ are in the extended ratio of 1 : 1 : 2. Find the measures of the angles.

6. Find the geometric mean of 8 and 12.

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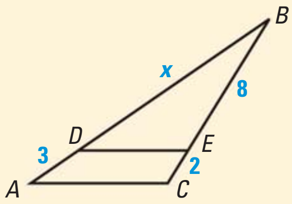
Chapter Review Practice

6.2 Use Proportions to Solve Geometry Problems *pp. 364–370*

EXAMPLE

In the diagram, $\frac{BA}{DA} = \frac{BC}{EC}$. Find BD.



$\frac{x+3}{3} = \frac{8+2}{2}$ — Substitution Property of Equality

$2x + 6 = 30$ — Cross Products Property

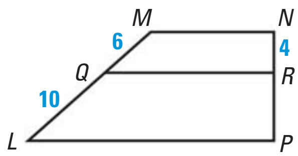
$x = 12$ — Solve for *x*.

EXERCISES

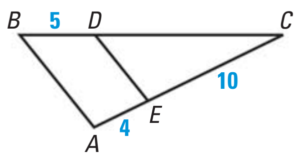
EXAMPLE 2 on p. 365 for Exs. 7–8

Use the diagram and the given information to find the unknown length.

7. Given $\frac{RN}{RP} = \frac{QM}{QL}$, find RP.



8. Given $\frac{CD}{DB} = \frac{CE}{EA}$, find CD.

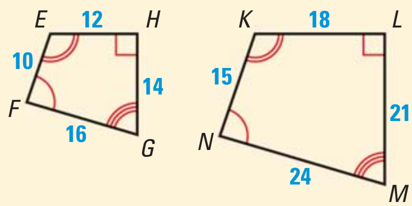


6.3 Use Similar Polygons *pp. 372–379*

EXAMPLE

In the diagram, $EHGF \sim KLMN$. Find the scale factor.

From the diagram, you can see that $\overline{EH}$ and $\overline{KL}$ correspond. So, the scale factor of $EHGF$ to $KLMN$ is $\frac{EH}{KL} = \frac{12}{18} = \frac{2}{3}$.

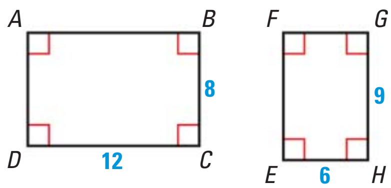


EXERCISES

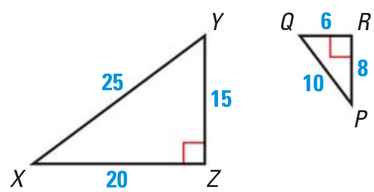
EXAMPLES 2 and 4 on pp. 373–374 for Exs. 9–11

In Exercises 9 and 10, determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor.

9.



10.



11. POSTERS Two similar posters have a scale factor of 4 : 5. The large poster's perimeter is 85 inches. Find the small poster's perimeter.

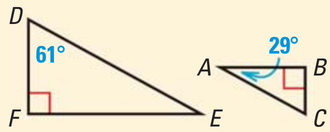
6 CHAPTER REVIEW

6.4 Prove Triangles Similar by AA

pp. 381–387

EXAMPLE

Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.



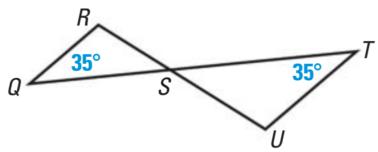
Because they are right angles, $\angle F \cong \angle B$. By the Triangle Sum Theorem, $61° + 90° + m\angle E = 180°$, so $m\angle E = 29°$ and $\angle E \cong \angle A$. Then, two angles of $\triangle DFE$ are congruent to two angles of $\triangle CBA$. So, $\triangle DFE \sim \triangle CBA$.

EXERCISES

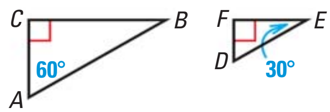
EXAMPLES 2 and 3 on pp. 382–383 for Exs. 12–14

Use the AA Similarity Postulate to show that the triangles are similar.

12.



13.



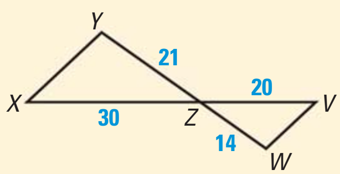
14. **CELL TOWER** A cellular telephone tower casts a shadow that is 72 feet long, while a tree nearby that is 27 feet tall casts a shadow that is 6 feet long. How tall is the tower?

6.5 Prove Triangles Similar by SSS and SAS

pp. 388–395

EXAMPLE

Show that the triangles are similar.



Notice that the lengths of two pairs of corresponding sides are proportional.

$$\frac{WZ}{YZ} = \frac{14}{21} = \frac{2}{3} \qquad \frac{VZ}{XZ} = \frac{20}{30} = \frac{2}{3}$$

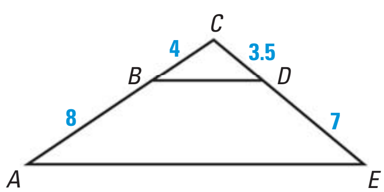
The included angles for these sides, $\angle XZY$ and $\angle VZW$, are vertical angles, so $\angle XZY \cong \angle VZW$. Then $\triangle XYZ \sim \triangle VWZ$ by the SAS Similarity Theorem.

EXERCISES

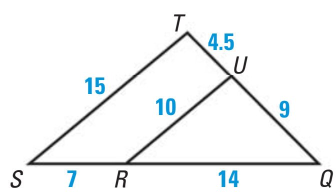
EXAMPLE 4 on p. 391 for Exs. 15–16

Use the SSS Similarity Theorem or SAS Similarity Theorem to show that the triangles are similar.

15.



16.

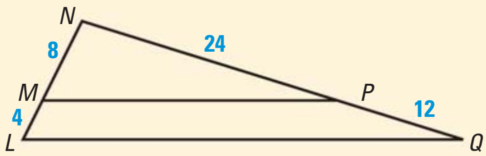


6.6 Use Proportionality Theorems

pp. 397–403

EXAMPLE

Determine whether $\overline{MP} \parallel \overline{LQ}$.



Begin by finding and simplifying ratios of lengths determined by $\overline{MP}$.

$$\frac{NM}{ML} = \frac{8}{4} = \frac{2}{1} \qquad \frac{NP}{PQ} = \frac{24}{12} = \frac{2}{1}$$

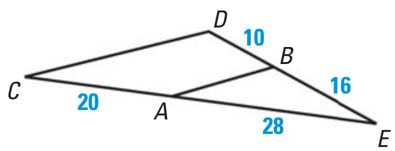
Because $\frac{NM}{ML} = \frac{NP}{PQ}$, $\overline{MP}$ is parallel to $\overline{LQ}$ by Theorem 6.5, the Triangle Proportionality Converse.

EXERCISES

EXAMPLE 2
on p. 398
for Exs. 17–18

Use the given information to determine whether $\overline{AB} \parallel \overline{CD}$.

17.



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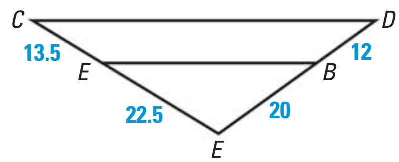


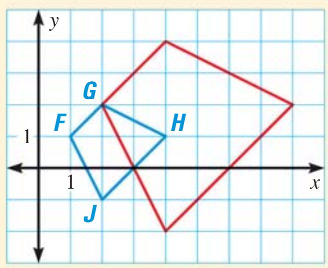
6.7 Perform Similarity Transformations

pp. 409–415

EXAMPLE

Draw a dilation of quadrilateral *FGHJ* with vertices $F(1, 1)$, $G(2, 2)$, $H(4, 1)$, and $J(2, -1)$. Use a scale factor of 2.

First draw *FGHJ*. Find the dilation of each vertex by multiplying its coordinates by 2. Then draw the dilation.



$$(x, y) \rightarrow (2x, 2y)$$

$F(1, 1) \rightarrow (2, 2)$

$G(2, 2) \rightarrow (4, 4)$

$H(4, 1) \rightarrow (8, 2)$

$J(2, -1) \rightarrow (4, -2)$

EXERCISES

EXAMPLES 1 and 2
on pp. 409–410
for Exs. 19–21

Draw a dilation of the polygon with the given vertices using the given scale factor *k*.

19. $T(0, 8)$, $U(6, 0)$, $V(0, 0)$; $k = \frac{3}{2}$

20. $A(6, 0)$, $B(3, 9)$, $C(0, 0)$, $D(3, 1)$; $k = 4$

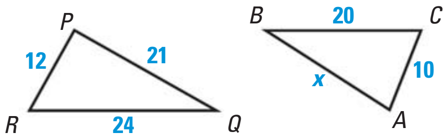
21. $P(8, 2)$, $Q(4, 0)$, $R(3, 1)$, $S(6, 4)$; $k = 0.5$

6 CHAPTER TEST

Solve the proportion.

1. $\frac{6}{x} = \frac{9}{24}$ **2.** $\frac{5}{4} = \frac{y-5}{12}$ **3.** $\frac{3-2b}{4} = \frac{3}{2}$ **4.** $\frac{7}{2a+8} = \frac{1}{a-1}$

In Exercises 5–7, use the diagram where $\triangle PQR \sim \triangle ABC$.



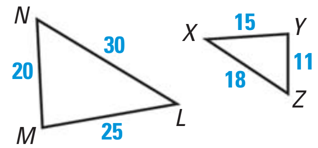
5. List all pairs of congruent angles.

6. Write the ratios of the corresponding sides in a statement of proportionality.

7. Find the value of x.

Determine whether the triangles are similar. If so, write a similarity statement and the postulate or theorem that justifies your answer.

8.



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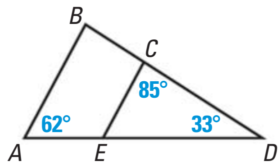

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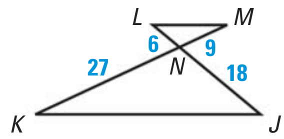
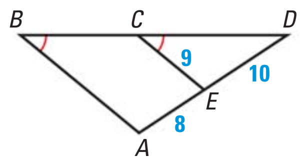


In Exercises 11–13, find the length of $\overline{AB}$.

11.



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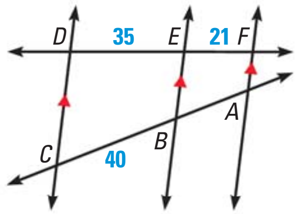

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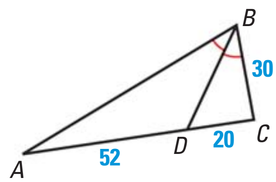
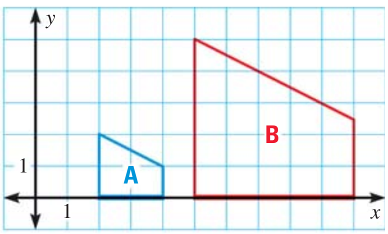


Determine whether the dilation from Figure A to Figure B is a *reduction* or an *enlargement*. Then find its scale factor.

14.



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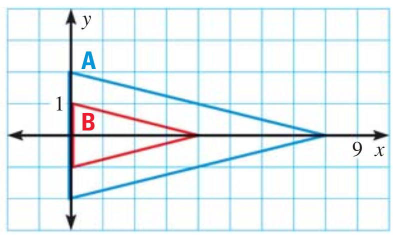
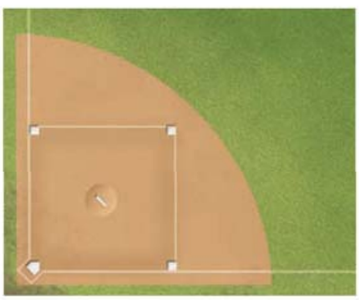


16. SCALE MODEL You are making a scale model of your school's baseball diamond as part of an art project. The distance between two consecutive bases is 90 feet. If you use a scale factor of $\frac{1}{180}$ to build your model, what will be the distance around the bases on your model?



6 ALGEBRA REVIEW

Animated Algebra classzone.com

SOLVE QUADRATIC EQUATIONS AND SIMPLIFY RADICALS

A radical expression is *simplified* when the radicand has no perfect square factor except 1, there is no fraction in the radicand, and there is no radical in a denominator.

EXAMPLE 1 Solve quadratic equations by finding square roots

Solve the equation $4x^2 - 3 = 109$.

$4x^2 - 3 = 109$	Write original equation.
$4x^2 = 112$	Add 3 to each side.
$x^2 = 28$	Divide each side by 4.
$x = \pm\sqrt{28}$	$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, so $\sqrt{28} = \pm\sqrt{4} \cdot \sqrt{7}$.
$x = \pm 2\sqrt{7}$	Simplify.

EXAMPLE 2 Simplify quotients with radicals

Simplify the expression.

a. $\sqrt{\frac{10}{8}}$ **b.** $\sqrt{\frac{1}{5}}$

Solution

a.

$\sqrt{\frac{10}{8}} = \sqrt{\frac{5}{4}}$	Simplify fraction.
$= \frac{\sqrt{5}}{\sqrt{4}}$	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.
$= \frac{\sqrt{5}}{2}$	Simplify.

b.

$\sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$. $\sqrt{1} = 1$.
$= \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$	Multiply numerator and denominator by $\sqrt{5}$.
$= \frac{\sqrt{5}}{5}$	Multiply fractions. $\sqrt{a} \cdot \sqrt{a} = a$.

EXERCISES

EXAMPLE 1 for Exs. 1–9

Solve the equation or write *no solution*.

1. $x^2 + 8 = 108$

2. $2x^2 - 1 = 49$

3. $x^2 - 9 = 8$

4. $5x^2 + 11 = 1$

5. $2(x^2 - 7) = 6$

6. $9 = 21 + 3x^2$

7. $3x^2 - 17 = 43$

8. $56 - x^2 = 20$

9. $-3(-x^2 + 5) = 39$

EXAMPLE 2 for Exs. 10–17

Simplify the expression.

10. $\sqrt{\frac{7}{81}}$

11. $\sqrt{\frac{3}{5}}$

12. $\sqrt{\frac{24}{27}}$

13. $\frac{3\sqrt{7}}{\sqrt{12}}$

14. $\sqrt{\frac{75}{64}}$

15. $\frac{\sqrt{2}}{\sqrt{200}}$

16. $\frac{9}{\sqrt{27}}$

17. $\sqrt{\frac{21}{42}}$

Scoring Rubric

Full Credit
- solution is complete and correct

Partial Credit
- solution is complete but has errors,

or
- solution is without error but is incomplete

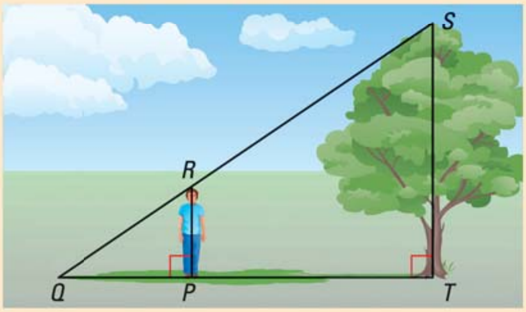
No Credit
- no solution is given,

or
- solution makes no sense

EXTENDED RESPONSE QUESTIONS

PROBLEM

To find the height of a tree, a student 63 inches in height measures the length of the tree's shadow and the length of his own shadow, as shown. The student casts a shadow 81 inches in length and the tree casts a shadow 477 inches in length.



a. Explain why $\triangle PQR \sim \triangle TQS$.

b. Find the height of the tree.

c. Suppose the sun is a little lower in the sky. Can you still use this method to measure the height of the tree? *Explain*.

Below are sample solutions to the problem. Read each solution and the comments in blue to see why the sample represents full credit, partial credit, or no credit.

SAMPLE 1: Full credit solution

The reasoning is complete.

a. Because they are both right angles, $\angle QPR \cong \angle QTS$. Also, $\angle Q \cong \angle Q$ by the Reflexive Property. So, $\triangle PQR \sim \triangle TQS$ by the AA Similarity Postulate.

The proportion and calculations are correct.

b.
$$\frac{PR}{PQ} = \frac{TS}{TQ}$$
$$\frac{63}{81} = \frac{TS}{477}$$
$$63(477) = 81 \cdot TS$$
$$371 = TS$$

The height of the tree is 371 inches.

In part (b), the question is answered correctly.

c. As long as the sun creates two shadows, I can use this method. Angles P and T will always be right angles. The measure of $\angle Q$ will change as the sun's position changes, but the angle will still be congruent to itself. So, $\triangle PQR$ and $\triangle TQS$ will still be similar, and I can write a proportion.

In part (c), the reasoning is complete and correct.

SAMPLE 2: Partial credit solution

In part (a), there is no explanation of why the postulate can be applied.

In part (b), the proportion is incorrect, which leads to an incorrect solution.

In part (c), a partial explanation is given.

a. $\triangle PQR \sim \triangle TQS$ by the Angle-Angle Similarity Postulate.

b. $\frac{PR}{PQ} = \frac{TS}{TP}$

$\frac{63}{81} = \frac{TS}{396}$

$308 = TS$

The height of the tree is 308 inches.

c. As long as the sun creates two shadows, I can use this method because the triangles will always be similar.

SAMPLE 3: No credit solution

The reasoning in part (a) is incomplete.

In part (b), no work is shown.

The answer in part (c) is incorrect.

a. The triangles are similar because the lines are parallel and the angles are congruent.

b. $TS = 371$ inches

c. No. The angles in the triangle will change, so you can't write a proportion.

PRACTICE Apply the Scoring Rubric

1. A student's solution to the problem on the previous page is given below. Score the solution as *full credit*, *partial credit*, or *no credit*. *Explain* your reasoning. If you choose *partial credit* or *no credit*, *explain* how you would change the solution so that it earns a score of full credit.

a. $\angle QPR \cong \angle PTS$, and $\angle Q$ is in both triangles. So, $\triangle PQR \sim \triangle TQS$.

b. $\frac{PR}{PQ} = \frac{QT}{ST}$

$\frac{63}{81} = \frac{477}{x}$

$63x = 81(477)$

$x \approx 613.3$

The tree is about 613.3 inches tall.

c. The method will still work because the triangles will still be similar if the sun changes position. The right angles will stay right angles, and $\angle Q$ is in both triangles, so it does not matter if its measure changes.

6 ★ Standardized TEST PRACTICE

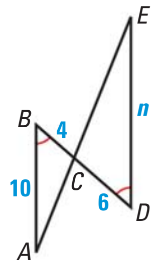
EXTENDED RESPONSE

1. Use the diagram.

a. *Explain* how you know that $\triangle ABC \sim \triangle EDC$.

b. Find the value of n.

c. The perimeter of $\triangle ABC$ is 22. What is the perimeter of $\triangle EDC$? *Justify* your answer.

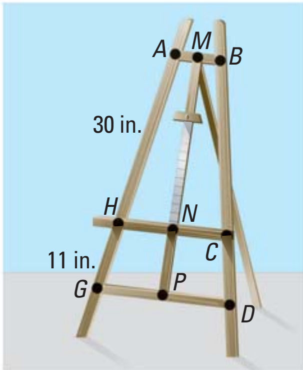


2. On the easel shown at the right, $\overline{AB} \parallel \overline{HC} \parallel \overline{GD}$, and $\overline{AG} \cong \overline{BD}$.

a. Find BD, BC, and CD. *Justify* your answer.

b. On the easel, $\overline{MP}$ is a support bar attached to $\overline{AB}$, $\overline{HC}$, and $\overline{GD}$. On this support bar, $NP = 10$ inches. Find the length of $\overline{MP}$ to the nearest inch. *Justify* your answer.

c. The support bar $\overline{MP}$ bisects $\overline{AB}$, $\overline{HC}$, and $\overline{GD}$. Does this mean that polygons $AMNH$ and $AMPG$ are similar? *Explain*.



3. A handmade rectangular rug is available in two sizes at a rug store. A small rug is 24 inches long and 16 inches wide. A large rug is 36 inches long and 24 inches wide.

a. Are the rugs similar? If so, what is the ratio of their corresponding sides? *Explain*.

b. Find the perimeter and area of each rug. Then find the ratio of the perimeters (large rug to small rug) and the ratio of the areas (large rug to small rug).

c. It takes 250 feet of wool yarn to make 1 square foot of either rug. How many inches of yarn are used for each rug? *Explain*.

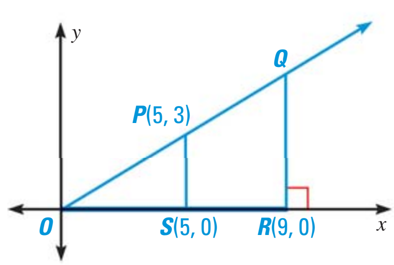
d. The price of a large rug is 1.5 times the price of a small rug. The store owner wants to change the prices for the rugs, so that the price for each rug is based on the amount of yarn used to make the rug. If the owner changes the prices, about how many times as much will the price of a large rug be than the price of a small rug? *Explain*.

4. In the diagram shown at the right, $\overleftrightarrow{OQ}$ passes through the origin.

a. Explain how you know that $\triangle OPS \sim \triangle OQR$.

b. Find the coordinates of point Q. *Justify* your answer.

c. The x-coordinate of a point on $\overleftrightarrow{OQ}$ is a. Write the y-coordinate of this point in terms of a. *Justify* your answer.



MULTIPLE CHOICE

5. If $\triangle PQR \sim \triangle STU$, which proportion is not necessarily true?

Ⓐ $\frac{PQ}{QR} = \frac{ST}{TU}$　Ⓑ $\frac{PQ}{SU} = \frac{PR}{TU}$

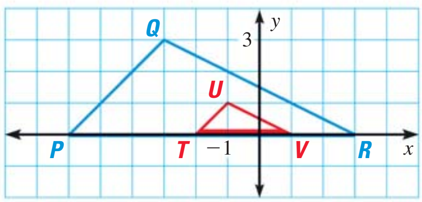
Ⓒ $\frac{PR}{SU} = \frac{QR}{TU}$　Ⓓ $\frac{PQ}{PR} = \frac{ST}{SU}$

6. On a map, the distance between two cities is $2\frac{3}{4}$ inches. The scale on the map is 1 in.:80 mi. What is the actual distance between the two cities?

Ⓐ 160 mi　Ⓑ 180 mi

Ⓒ 200 mi　Ⓓ 220 mi

7. In the diagram, what is the scale factor of the dilation from $\triangle PQR$ to $\triangle TUV$?

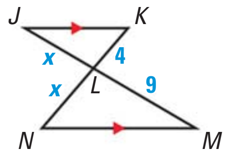


Ⓐ $\frac{1}{2}$　Ⓑ $\frac{1}{3}$

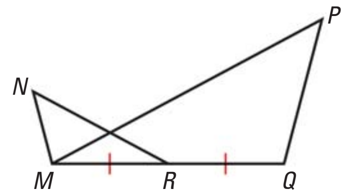
Ⓒ 2　Ⓓ 3

GRIDDED ANSWER

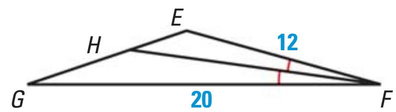
8. Find the value of x.



9. In the diagram below, $\triangle PQM \sim \triangle NMR$, and $\overline{MR} \cong \overline{QR}$. If $NR = 12$, find PM.



10. Given $GE = 10$, find HE.



11. In an acute isosceles triangle, the measures of two of the angles are in the ratio 4 : 1. Find the measure of a base angle in the triangle.

SHORT RESPONSE

12. On a school campus, the gym is 400 feet from the art studio.

a. Suppose you draw a map of the school campus using a scale of $\frac{1}{4}$ inch: 100 feet. How far will the gym be from the art studio on your map?

b. Suppose you draw a map of the school campus using a scale of $\frac{1}{2}$ inch : 100 feet. Will the distance from the gym to the art studio on this map be *greater than* or *less than* the distance on the map in part (a)? *Explain.*

13. Rectangles *ABCD* and *EFGH* are similar, and the ratio of *AB* to *EF* is 1 : 3. In each rectangle, the length is twice the width. The area of *ABCD* is 32 square inches. Find the length, width, and area of *EFGH*. *Explain.*

Find $m\angle 2$ if $\angle 1$ and $\angle 2$ are (a) complementary angles and (b) supplementary angles. *(p. 24)*

1. $m\angle 1 = 57°$ **2.** $m\angle 1 = 23°$ **3.** $m\angle 1 = 88°$ **4.** $m\angle 1 = 46°$

Solve the equation and write a reason for each step. *(p. 105)*

5. $3x - 19 = 47$ **6.** $30 - 4(x - 3) = -x + 18$ **7.** $-5(x + 2) = 25$

State the postulate or theorem that justifies the statement. *(pp. 147, 154)*

8. $\angle 1 \cong \angle 8$

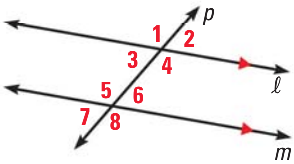
9. $\angle 3 \cong \angle 6$

10. $m\angle 3 + m\angle 5 = 180°$

11. $\angle 3 \cong \angle 7$

12. $\angle 2 \cong \angle 3$

13. $m\angle 7 + m\angle 8 = 180°$



The variable expressions represent the angle measures of a triangle. Find the measure of each angle. Then classify the triangle by its angles. *(p. 217)*

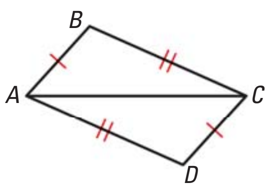
14. $m\angle A = x°$
$m\angle B = 3x°$
$m\angle C = 4x°$

15. $m\angle A = 2x°$
$m\angle B = 2x°$
$m\angle C = (x - 15)°$

16. $m\angle A = (3x - 15)°$
$m\angle B = (x + 5)°$
$m\angle C = (x - 20)°$

Determine whether the triangles are congruent. If so, write a congruence statement and state the postulate or theorem you used. *(pp. 234, 240, 249)*

17.



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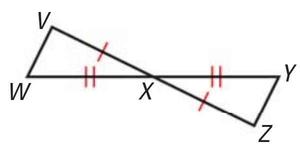

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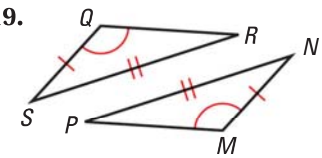
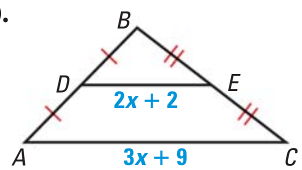


Find the value of x. *(pp. 295, 303, 310)*

20.



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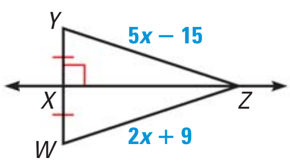

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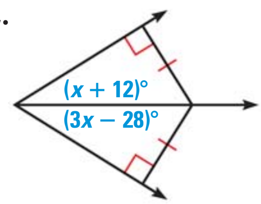
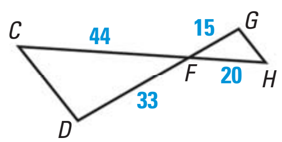


Determine whether the triangles are similar. If they are, write a similarity statement and state the postulate or theorem you used. *(pp. 381, 388)*

23.



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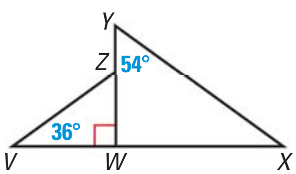

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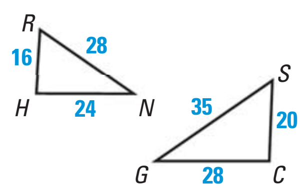
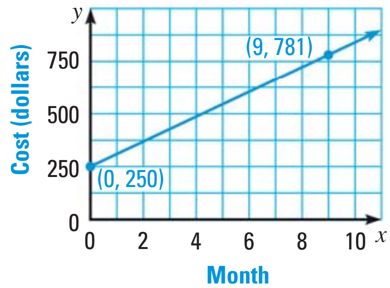


26. **PROFITS** A company's profits for two years are shown in the table. Plot and connect the points (x, y). Use the Midpoint Formula to estimate the company's profits in 2003. (Assume that profits followed a linear pattern.) *(p. 15)*

Years since 2000, x	1	5
Profit, y (in dollars)	21,000	36,250

27. **TENNIS MEMBERSHIP** The graph at the right models the accumulated cost for an individual adult tennis club membership for several months. *(p. 180)*



a. Write an equation of the line.

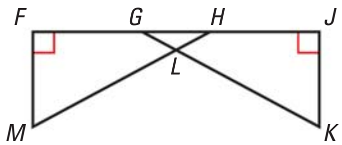
b. Tell what the slope and y-intercept mean in this situation.

c. Find the accumulated cost for one year.

PROOF **Write a two-column proof or a paragraph proof.** *(pp. 234, 240, 249)*

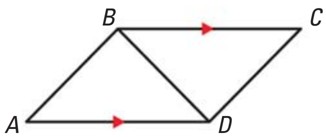
28. **GIVEN** ▶ $\overline{FG} \cong \overline{HJ}$, $\overline{MH} \cong \overline{KG}$, $\overline{MF} \perp \overline{FJ}$, $\overline{KJ} \perp \overline{FJ}$

PROVE ▶ $\triangle FHM \cong \triangle JGK$

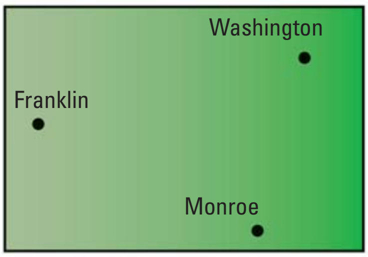


29. **GIVEN** ▶ $\overline{BC} \parallel \overline{AD}$, $\overline{BC} \cong \overline{AD}$

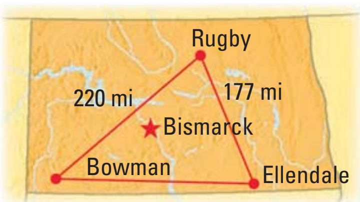
PROVE ▶ $\triangle BCD \cong \triangle DAB$



30. **COMMUNITY CENTER** A building committee needs to choose a site for a new community center. The committee decides that the new center should be located so that it is the same distance from each of the three local schools. Use the diagram to make a sketch of the triangle formed by the three schools. *Explain* how you can use this triangle to locate the site for the new community center. *(p. 303)*



31. **GEOGRAPHY** The map shows the distances between three cities in North Dakota. *Describe* the range of possible distances from Bowman to Ellendale. *(p. 328)*



32. **CALENDAR** You send 12 photos to a company that makes personalized wall calendars. The company enlarges the photos and inserts one for each month on the calendar. Each photo is 4 inches by 6 inches. The image for each photo on the calendar is 10 inches by 15 inches. What is the scale factor of the enlargement? *(p. 409)*