9 Properties of Transformations

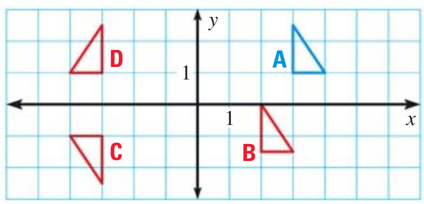
Before

In previous chapters, you learned the following skills, which you'll use in Chapter 9: translating, reflecting, and rotating polygons, and using similar triangles.

Prerequisite Skills

VOCABULARY CHECK

Match the transformation of Triangle A with its graph.



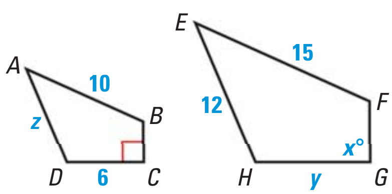
1. Translation of Triangle A
2. Reflection of Triangle A
3. Rotation of Triangle A

SKILLS AND ALGEBRA CHECK

The vertices of *JKLM* are *J*(−1, 6), *K*(2, 5), *L*(2, 2), and *M*(−1, 1). Graph its image after the transformation described. *(Review p. 272 for 9.1, 9.3.)*

4. Translate 3 units left and 1 unit down.
5. Reflect in the *y*-axis.

In the diagram, *ABCD* ~ *EFGH*. *(Review p. 234 for 9.7.)*



6. Find the scale factor of *ABCD* to *EFGH*.
7. Find the values of *x*, *y*, and *z*.

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 9, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 635. You will also use the key vocabulary listed below.

Big Ideas

1. **Performing congruence and similarity transformations**
2. **Making real-world connections to symmetry and tessellations**
3. **Applying matrices and vectors in Geometry**

KEY VOCABULARY

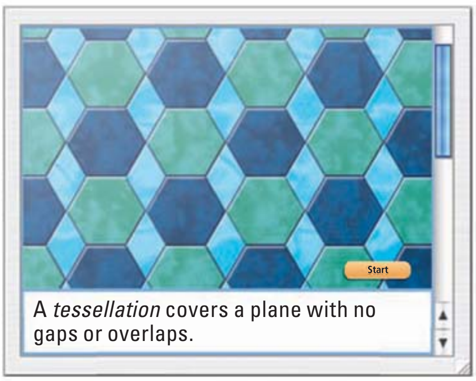
- image, *p. 572*
- preimage, *p. 572*
- isometry, *p. 573*
- vector, *p. 574*
- component form, *p. 574*
- matrix, *p. 580*
- element, *p. 580*
- dimensions, *p. 580*
- line of reflection, *p. 589*
- center of rotation, *p. 598*
- angle of rotation, *p. 598*
- glide reflection, *p. 608*
- composition of transformations, *p. 609*
- line symmetry, *p. 619*
- rotational symmetry, *p. 620*
- scalar multiplication, *p. 627*

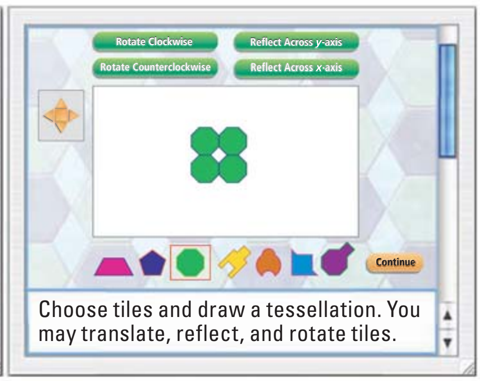
Why?

You can use properties of shapes to determine whether shapes tessellate. For example, you can use angle measurements to determine which shapes can be used to make a tessellation.

Animated Geometry

The animation illustrated below for Example 3 on page 617 helps you answer this question: How can you use tiles to tessellate a floor?





Animated Geometry at classzone.com

Other animations for Chapter 9 : pages 582, 590, 599, 602, 611, 619, and 626

9.1 Translate Figures and Use Vectors

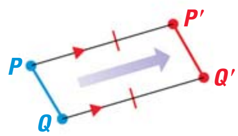
Before	You used a coordinate rule to translate a figure.
Now	You will use a vector to translate a figure.
Why?	So you can find a distance covered on snowshoes, as in Exs. 35–37.

Key Vocabulary
- **image**
- **preimage**
- **isometry**
- **vector**
  initial point, terminal point, horizontal component, vertical component
- **component form**
- **translation,** *p. 272*

In Lesson 4.8, you learned that a *transformation* moves or changes a figure in some way to produce a new figure called an **image**. Another name for the original figure is the **preimage**.

Recall that a *translation* moves every point of a figure the same distance in the same direction. More specifically, a translation maps, or moves, the points P and Q of a plane figure to the points P' (read "P *prime*") and Q', so that one of the following statements is true:

- $PP' = QQ'$ and $\overline{PP'} \parallel \overline{QQ'}$, or
- $PP' = QQ'$ and $\overline{PP'}$ and $\overline{QQ'}$ are collinear.



EXAMPLE 1 Translate a figure in the coordinate plane

Graph quadrilateral $ABCD$ with vertices $A(-1, 2)$, $B(-1, 5)$, $C(4, 6)$, and $D(4, 2)$. Find the image of each vertex after the translation $(x, y) \to (x + 3, y - 1)$. Then graph the image using prime notation.

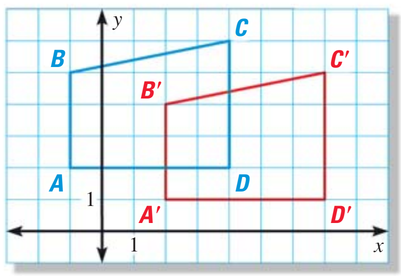
Solution

USE NOTATION
You can use *prime notation* to name an image. For example, if the preimage is $\triangle ABC$, then its image is $\triangle A'B'C'$, read as *"triangle A prime, B prime, C prime."*

First, draw $ABCD$. Find the translation of each vertex by adding 3 to its x-coordinate and subtracting 1 from its y-coordinate. Then graph the image.

$$(x, y) \to (x + 3, y - 1)$$

$A(-1, 2) \to A'(2, 1)$
$B(-1, 5) \to B'(2, 4)$
$C(4, 6) \to C'(7, 5)$
$D(4, 2) \to D'(7, 1)$



✓ GUIDED PRACTICE for Example 1

1. Draw $\triangle RST$ with vertices $R(2, 2)$, $S(5, 2)$, and $T(3, 5)$. Find the image of each vertex after the translation $(x, y) \to (x + 1, y + 2)$. Graph the image using prime notation.

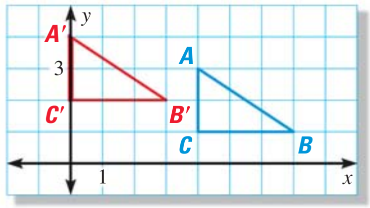
2. The image of $(x, y) \to (x + 4, y - 7)$ is $\overline{P'Q'}$ with endpoints $P'(-3, 4)$ and $Q'(2, 1)$. Find the coordinates of the endpoints of the preimage.

ISOMETRY An **isometry** is a transformation that preserves length and angle measure. Isometry is another word for congruence transformation (page 272).

EXAMPLE 2 Write a translation rule and verify congruence

READ DIAGRAMS
In this book, the preimage is always shown in blue, and the image is always shown in red.

Write a rule for the translation of $\triangle ABC$ to $\triangle A'B'C'$. Then verify that the transformation is an isometry.



Solution

To go from A to A', move 4 units left and 1 unit up. So, a rule for the translation is $(x, y) \rightarrow (x - 4, y + 1)$.

Use the SAS Congruence Postulate. Notice that $CB = C'B' = 3$, and $AC = A'C' = 2$. The slopes of $\overline{CB}$ and $\overline{C'B'}$ are 0, and the slopes of $\overline{CA}$ and $\overline{C'A'}$ are undefined, so the sides are perpendicular. Therefore, $\angle C$ and $\angle C'$ are congruent right angles. So, $\triangle ABC \cong \triangle A'B'C'$. The translation is an isometry.

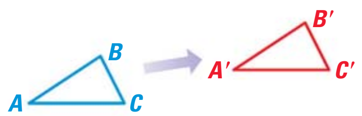
✓ GUIDED PRACTICE for Example 2

3. In Example 2, write a rule to translate $\triangle A'B'C'$ back to $\triangle ABC$.

THEOREM *For Your Notebook*

THEOREM 9.1 Translation Theorem

A translation is an isometry.

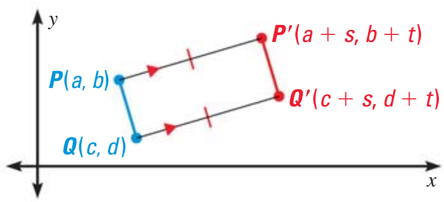


$\triangle ABC \cong \triangle A'B'C'$

Proof: below; Ex. 46, p. 579

PROOF Translation Theorem

A translation is an isometry.



GIVEN ▶ $P(a, b)$ and $Q(c, d)$ are two points on a figure translated by $(x, y) \rightarrow (x + s, y + t)$.

PROVE ▶ $PQ = P'Q'$

The translation maps $P(a, b)$ to $P'(a + s, b + t)$ and $Q(c, d)$ to $Q'(c + s, d + t)$.

Use the Distance Formula to find PQ and $P'Q'$. $PQ = \sqrt{(c - a)^2 + (d - b)^2}$.

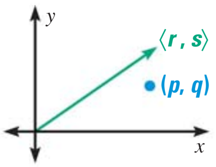
$$P'Q' = \sqrt{[(c + s) - (a + s)]^2 + [(d + t) - (b + t)]^2}$$
$$= \sqrt{(c + s - a - s)^2 + (d + t - b - t)^2}$$
$$= \sqrt{(c - a)^2 + (d - b)^2}$$

Therefore, $PQ = P'Q'$ by the Transitive Property of Equality.

VECTORS Another way to describe a translation is by using a vector. A **vector** is a quantity that has both direction and *magnitude,* or size. A vector is represented in the coordinate plane by an arrow drawn from one point to another.

USE NOTATION

Use brackets to write the component form of the vector $\langle r, s \rangle$. Use parentheses to write the coordinates of the point (p, q).

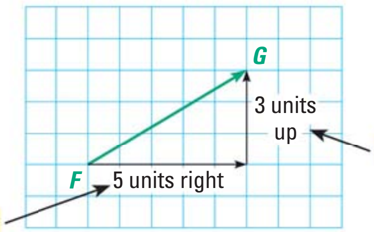


KEY CONCEPT — *For Your Notebook*

Vectors

The diagram shows a vector named $\overrightarrow{FG}$, read as "vector FG."

The **initial point**, or starting point, of the vector is F.



The **terminal point**, or ending point, of the vector is G.

vertical component

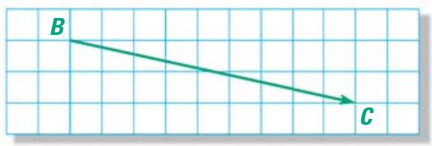
horizontal component

The **component form** of a vector combines the horizontal and vertical components. So, the component form of $\overrightarrow{FG}$ is $\langle 5, 3 \rangle$.

EXAMPLE 3 Identify vector components

Name the vector and write its component form.

a.



b.

Solution

a. The vector is $\overrightarrow{BC}$. From initial point B to terminal point C, you move 9 units right and 2 units down. So, the component form is $\langle 9, -2 \rangle$.

b. The vector is $\overrightarrow{ST}$. From initial point S to terminal point T, you move 8 units left and 0 units vertically. The component form is $\langle -8, 0 \rangle$.

EXAMPLE 4 Use a vector to translate a figure

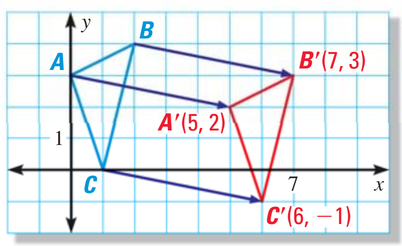
The vertices of $\triangle ABC$ are $A(0, 3)$, $B(2, 4)$, and $C(1, 0)$. Translate $\triangle ABC$ using the vector $\langle 5, -1 \rangle$.

USE VECTORS

Notice that the vector can have different initial points. The vector describes only the direction and magnitude of the translation.

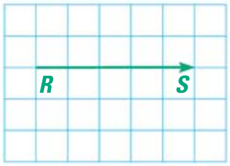
Solution

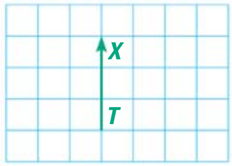
First, graph $\triangle ABC$. Use $\langle 5, -1 \rangle$ to move each vertex 5 units to the right and 1 unit down. Label the image vertices. Draw $\triangle A'B'C'$. Notice that the vectors drawn from preimage to image vertices are parallel.

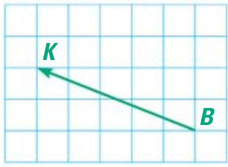


✓ GUIDED PRACTICE for Examples 3 and 4

Name the vector and write its component form.

4. 

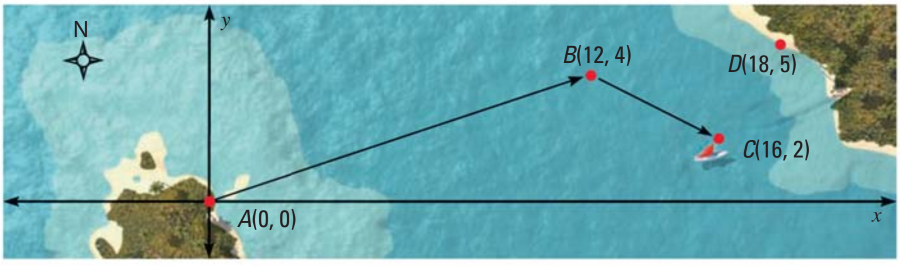
5. 

6. 

7. The vertices of $\triangle LMN$ are $L(2, 2)$, $M(5, 3)$, and $N(9, 1)$. Translate $\triangle LMN$ using the vector $\langle -2, 6 \rangle$.

EXAMPLE 5 Solve a multi-step problem

NAVIGATION A boat heads out from point A on one island toward point D on another. The boat encounters a storm at B, 12 miles east and 4 miles north of its starting point. The storm pushes the boat off course to point C, as shown.



a. Write the component form of $\overrightarrow{AB}$.

b. Write the component form of $\overrightarrow{BC}$.

c. Write the component form of the vector that describes the straight line path from the boat's current position C to its intended destination D.

Solution

a. The component form of the vector from $A(0, 0)$ to $B(12, 4)$ is

$$\overrightarrow{AB} = \langle 12 - 0, 4 - 0 \rangle = \langle 12, 4 \rangle.$$

b. The component form of the vector from $B(12, 4)$ to $C(16, 2)$ is

$$\overrightarrow{BC} = \langle 16 - 12, 2 - 4 \rangle = \langle 4, -2 \rangle.$$

c. The boat is currently at point C and needs to travel to D. The component form of the vector from $C(16, 2)$ to $D(18, 5)$ is

$$\overrightarrow{CD} = \langle 18 - 16, 5 - 2 \rangle = \langle 2, 3 \rangle.$$

✓ GUIDED PRACTICE for Example 5

8. **WHAT IF?** In Example 5, suppose there is no storm. Write the component form of the vector that describes the straight path from the boat's starting point A to its final destination D.

9.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 11, and 35

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 14, and 42

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A _?_ is a quantity that has both _?_ and magnitude.

2. ★ **WRITING** *Describe* the difference between a vector and a ray.

EXAMPLE 1 on p. 572 for Exs. 3–10

IMAGE AND PREIMAGE **Use the translation $(x, y) \rightarrow (x - 8, y + 4)$.**

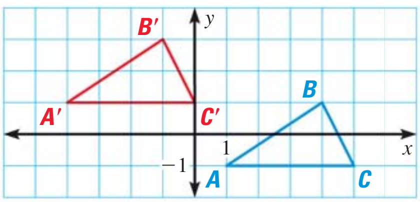
3. What is the image of $A(2, 6)$?
4. What is the image of $B(-1, 5)$?
5. What is the preimage of $C'(-3, -10)$?
6. What is the preimage of $D'(4, -3)$?

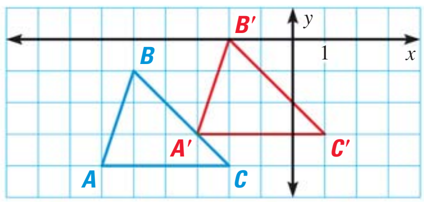
GRAPHING AN IMAGE **The vertices of $\triangle PQR$ are $P(-2, 3)$, $Q(1, 2)$, and $R(3, -1)$. Graph the image of the triangle using prime notation.**

7. $(x, y) \rightarrow (x + 4, y + 6)$
8. $(x, y) \rightarrow (x + 9, y - 2)$
9. $(x, y) \rightarrow (x - 2, y - 5)$
10. $(x, y) \rightarrow (x - 1, y + 3)$

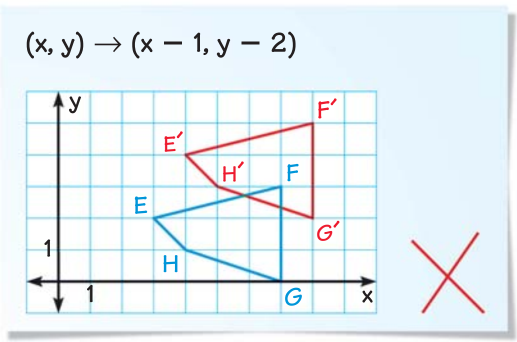
EXAMPLE 2 on p. 573 for Exs. 11–14

WRITING A RULE **$\triangle A'B'C'$ is the image of $\triangle ABC$ after a translation. Write a rule for the translation. Then *verify* that the translation is an isometry.**

11. 

12. 

13. **ERROR ANALYSIS** *Describe* and correct the error in graphing the translation of quadrilateral *EFGH*.

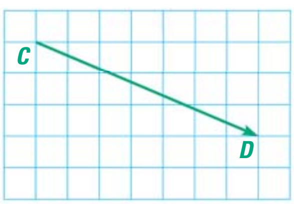


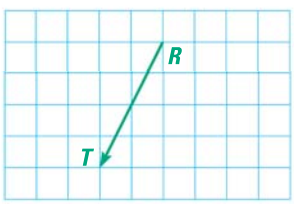
14. ★ **MULTIPLE CHOICE** Translate $Q(0, -8)$ using $(x, y) \rightarrow (x - 3, y + 2)$.

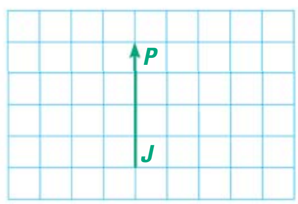
Ⓐ $Q'(-2, 5)$ Ⓑ $Q'(3, -10)$ Ⓒ $Q'(-3, -6)$ Ⓓ $Q'(2, -11)$

EXAMPLE 3 on p. 574 for Exs. 15–23

IDENTIFYING VECTORS **Name the vector and write its component form.**

15. 

16. 

17. 

VECTORS **Use the point $P(-3, 6)$. Find the component form of the vector that describes the translation to P'.**

18. $P'(0, 1)$ **19.** $P'(-4, 8)$ **20.** $P'(-2, 0)$ **21.** $P'(-3, -5)$

TRANSLATIONS **Think of each translation as a vector. *Describe* the vertical component of the vector. *Explain*.**

22.

23.

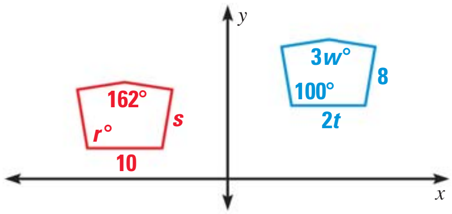
EXAMPLE 4
on p. 574
for Exs. 24–27

TRANSLATING A TRIANGLE **The vertices of $\triangle DEF$ are $D(2, 5)$, $E(6, 3)$, and $F(4, 0)$. Translate $\triangle DEF$ using the given vector. Graph $\triangle DEF$ and its image.**

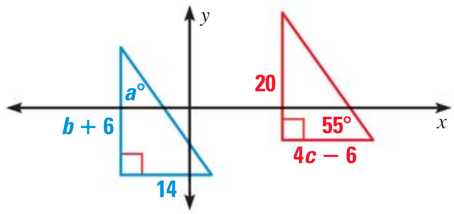
24. $\langle 6, 0\rangle$ **25.** $\langle 5, -1\rangle$ **26.** $\langle -3, -7\rangle$ **27.** $\langle -2, -4\rangle$

xy ALGEBRA **Find the value of each variable in the translation.**

28.



29.



30. **xy ALGEBRA** Translation A maps (x, y) to $(x + n, y + m)$. Translation B maps (x, y) to $(x + s, y + t)$.

a. Translate a point using Translation A, then Translation B. Write a rule for the final image of the point.

b. Translate a point using Translation B, then Translation A. Write a rule for the final image of the point.

c. *Compare* the rules you wrote in parts (a) and (b). Does it matter which translation you do first? *Explain*.

31. **MULTI-STEP PROBLEM** The vertices of a rectangle are $Q(2, -3)$, $R(2, 4)$, $S(5, 4)$, and $T(5, -3)$.

a. Translate $QRST$ 3 units left and 2 units down. Find the areas of $QRST$ $Q'R'S'T'$.

b. *Compare* the areas. Make a conjecture about the areas of a preimage and its image after a translation.

32. **CHALLENGE** The vertices of $\triangle ABC$ are $A(2, 2)$, $B(4, 2)$, and $C(3, 4)$.

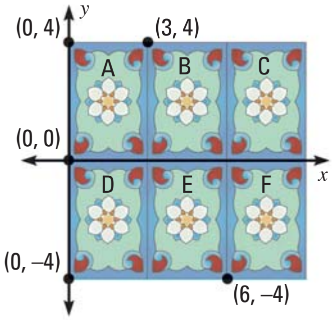
a. Graph the image of $\triangle ABC$ after the transformation $(x, y) \rightarrow (x + y, y)$. Is the transformation an isometry? *Explain*. Are the areas of $\triangle ABC$ and $\triangle A'B'C'$ the same?

b. Graph a new triangle, $\triangle DEF$, and its image after the transformation given in part (a). Are the areas of $\triangle DEF$ and $\triangle D'E'F'$ the same?

PROBLEM SOLVING

EXAMPLE 2 on p. 573 for Exs. 33–34

HOME DESIGN **Designers can use computers to make patterns in fabrics or floors. On the computer, a copy of the design in Rectangle A is used to cover an entire floor. The translation $(x, y) \to (x + 3, y)$ maps Rectangle A to Rectangle B.**



33. Use coordinate notation to describe the translations that map Rectangle A to Rectangles C, D, E, and F.

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34. Write a rule to translate Rectangle F back to Rectangle A.

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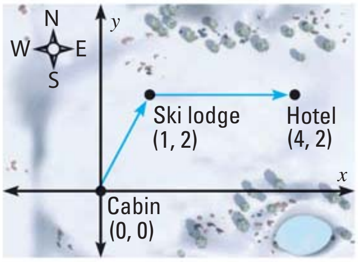
EXAMPLE 5 on p. 575 for Exs. 35–37

SNOWSHOEING **You are snowshoeing in the mountains. The distances in the diagram are in miles. Write the component form of the vector.**

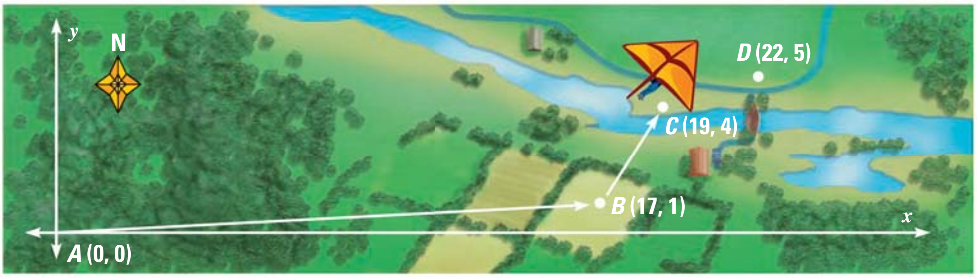


35. From the cabin to the ski lodge

36. From the ski lodge to the hotel

37. From the hotel back to your cabin

HANG GLIDING **A hang glider travels from point A to point D. At point B, the hang glider changes direction, as shown in the diagram. The distances in the diagram are in kilometers.**



38. Write the component form for $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

39. Write the component form of the vector that describes the path from the hang glider's current position C to its intended destination D.

40. What is the total distance the hang glider travels?

41. Suppose the hang glider went straight from A to D. Write the component form of the vector that describes this path. What is this distance?

42. ★ **EXTENDED RESPONSE** Use the equation $2x + y = 4$.

a. Graph the line and its image after the translation $\langle -5, 4\rangle$. What is an equation of the image of the line?

b. *Compare* the line and its image. What are the slopes? the y-intercepts? the x-intercepts?

c. Write an equation of the image of $2x + y = 4$ after the translation $\langle 2, -6\rangle$ *without* using a graph. *Explain* your reasoning.

○ = WORKED-OUT SOLUTIONS on p. WS1

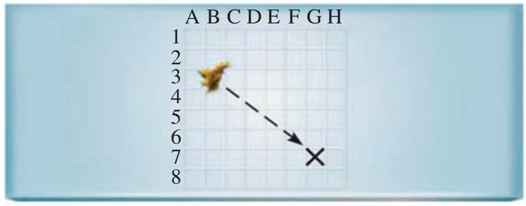
★ = STANDARDIZED TEST PRACTICE

43. **SCIENCE** You are studying an amoeba through a microscope. Suppose the amoeba moves on a grid-indexed microscope slide in a straight line from square B3 to square G7.

a. *Describe* the translation.

b. Each grid square is 2 millimeters on a side. How far does the amoeba travel?

c. Suppose the amoeba moves from B3 to G7 in 24.5 seconds. What is its speed in millimeters per second?



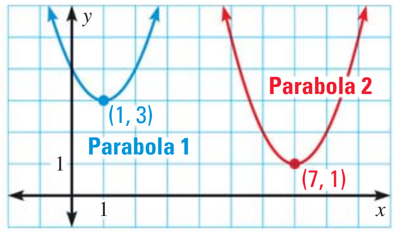
44. **MULTI-STEP PROBLEM** You can write the equation of a parabola in the form $y = (x - h)^2 + k$, where (h, k) is the *vertex* of the parabola. In the graph, an equation of Parabola 1 is $y = (x - 1)^2 + 3$, with vertex (1, 3). Parabola 2 is the image of Parabola 1 after a translation.

a. Write a rule for the translation.

b. Write an equation of Parabola 2.

c. Suppose you translate Parabola 1 using the vector $\langle -4, 8 \rangle$. Write an equation of the image.

d. An equation of Parabola 3 is $y = (x + 5)^2 - 3$. Write a rule for the translation of Parabola 1 to Parabola 3. *Explain* your reasoning.



45. **TECHNOLOGY** The standard form of an exponential equation is $y = a^x$, where $a > 0$ and $a \neq 1$. Use the equation $y = 2^x$.

a. Use a graphing calculator to graph $y = 2^x$ and $y = 2^x - 4$. *Describe* the translation from $y = 2^x$ to $y = 2^x - 4$.

b. Use a graphing calculator to graph $y = 2^x$ and $y = 2^{x-4}$. *Describe* the translation from $y = 2^x$ to $y = 2^{x-4}$.

46. **CHALLENGE** Use properties of congruent triangles to prove part of Theorem 9.1, that a translation preserves angle measure.

MIXED REVIEW

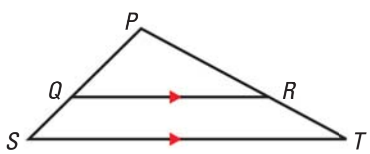
PREVIEW
Prepare for Lesson 9.2 in Exs. 47–50.

Find the sum, difference, product, or quotient. *(p. 869)*

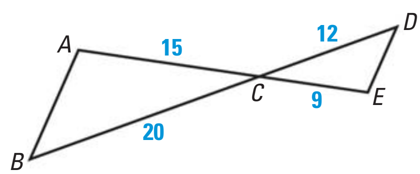
47. $-16 - 7$ 48. $6 + (-12)$ 49. $(13)(-2)$ 50. $16 \div (-4)$

Determine whether the two triangles are similar. If they are, write a similarity statement. *(pp. 381, 388)*

51.



52.



Points *A, B, C,* and *D* are the vertices of a quadrilateral. Give the most specific name for *ABCD*. *Justify* your answer. *(p. 552)*

53. $A(2, 0)$, $B(7, 0)$, $C(4, 4)$, $D(2, 4)$

54. $A(3, 0)$, $B(7, 2)$, $C(3, 4)$, $D(1, 2)$

9.2 Use Properties of Matrices

Before	You performed translations using vectors.
Now	You will perform translations using matrix operations.
Why	So you can calculate the total cost of art supplies, as in Ex. 36.

Key Vocabulary
- **matrix**
- **element**
- **dimensions**

A **matrix** is a rectangular arrangement of numbers in rows and columns. (The plural of matrix is *matrices*.) Each number in a matrix is called an **element**.

$$\begin{array}{rc} & \text{column (3rd)} \\ \text{row (2nd)} & \begin{bmatrix} 5 & 4 & 4 & 9 \\ -3 & 5 & 2 & 6 \\ 3 & -7 & 8 & 7 \end{bmatrix} \end{array}$$

The element in the second row and third column is 2.

READ VOCABULARY
An element of a matrix may also be called an *entry*.

The **dimensions** of a matrix are the numbers of rows and columns. The matrix above has three rows and four columns, so the dimensions of the matrix are 3×4 (read "3 by 4").

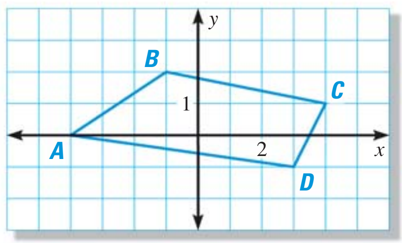
You can represent a figure in the coordinate plane using a matrix with two rows. The first row has the *x*-coordinate(s) of the vertices. The second row has the corresponding *y*-coordinate(s). Each column represents a vertex, so the number of columns depends on the number of vertices of the figure.

EXAMPLE 1 Represent figures using matrices

Write a matrix to represent the point or polygon.

a. Point A

b. Quadrilateral $ABCD$



Solution

AVOID ERRORS
The columns in a polygon matrix follow the consecutive order of the vertices of the polygon.

a. Point matrix for A

$$\begin{bmatrix} -4 \\ 0 \end{bmatrix} \begin{array}{l} \leftarrow x\text{-coordinate} \\ \leftarrow y\text{-coordinate} \end{array}$$

b. Polygon matrix for $ABCD$

$$\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} -4 & -1 & 4 & 3 \\ 0 & 2 & 1 & -1 \end{bmatrix} \end{array} \begin{array}{l} \leftarrow x\text{-coordinates} \\ \leftarrow y\text{-coordinates} \end{array}$$

✓ GUIDED PRACTICE for Example 1

1. Write a matrix to represent $\triangle ABC$ with vertices $A(3, 5)$, $B(6, 7)$ and $C(7, 3)$.

2. How many rows and columns are in a matrix for a hexagon?

ADDING AND SUBTRACTING To add or subtract matrices, you add or subtract corresponding elements. The matrices must have the same dimensions.

EXAMPLE 2 Add and subtract matrices

a. $\begin{bmatrix} 5 & -3 \\ 6 & -6 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 5+1 & -3+2 \\ 6+3 & -6+(-4) \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ 9 & -10 \end{bmatrix}$

b. $\begin{bmatrix} 6 & 8 & 5 \\ 4 & 9 & -1 \end{bmatrix} - \begin{bmatrix} 1 & -7 & 0 \\ 4 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 6-1 & 8-(-7) & 5-0 \\ 4-4 & 9-(-2) & -1-3 \end{bmatrix} = \begin{bmatrix} 5 & 15 & 5 \\ 0 & 11 & -4 \end{bmatrix}$

TRANSLATIONS You can use matrix addition to represent a translation in the coordinate plane. The image matrix for a translation is the sum of the translation matrix and the matrix that represents the preimage.

EXAMPLE 3 Represent a translation using matrices

The matrix $\begin{bmatrix} 1 & 5 & 3 \\ 1 & 0 & -1 \end{bmatrix}$ represents $\triangle ABC$. Find the image matrix that represents the translation of $\triangle ABC$ 1 unit left and 3 units up. Then graph $\triangle ABC$ and its image.

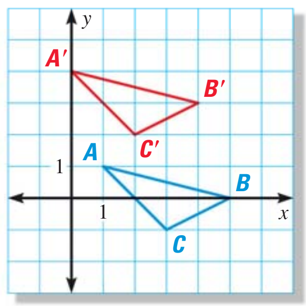
Solution

The translation matrix is $\begin{bmatrix} -1 & -1 & -1 \\ 3 & 3 & 3 \end{bmatrix}$.

Add this to the polygon matrix for the preimage to find the image matrix.

$$\begin{bmatrix} -1 & -1 & -1 \\ 3 & 3 & 3 \end{bmatrix} + \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 1 & 5 & 3 \\ 1 & 0 & -1 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{bmatrix} 0 & 4 & 2 \\ 4 & 3 & 2 \end{bmatrix} \end{array}$$

Translation matrix — Polygon matrix — Image matrix



AVOID ERRORS
In order to add two matrices, they must have the same dimensions, so the translation matrix here must have three columns like the polygon matrix.

✓ GUIDED PRACTICE for Examples 2 and 3

In Exercises 3 and 4, add or subtract.

3. $\begin{bmatrix} -3 & 7 \end{bmatrix} + \begin{bmatrix} 2 & -5 \end{bmatrix}$

4. $\begin{bmatrix} 1 & -4 \\ 3 & -5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 7 & 8 \end{bmatrix}$

5. The matrix $\begin{bmatrix} 1 & 2 & 6 & 7 \\ 2 & -1 & 1 & 3 \end{bmatrix}$ represents quadrilateral *JKLM*. Write the translation matrix and the image matrix that represents the translation of *JKLM* 4 units right and 2 units down. Then graph *JKLM* and its image.

MULTIPLYING MATRICES The product of two matrices A and B is defined only when the number of columns in A is equal to the number of rows in B. If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then the product AB is an $m \times p$ matrix.

USE NOTATION
Recall that the dimensions of a matrix are always written as rows $\times$ columns.

$$\begin{array}{ccccc} A & \cdot & B & = & AB \\ (m \text{ by } n) & \cdot & (n \text{ by } p) & = & (m \text{ by } p) \end{array}$$

equal   dimensions of AB

You will use matrix multiplication in later lessons to represent transformations.

EXAMPLE 4 Multiply matrices

Multiply $\begin{bmatrix} 1 & 0 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 8 \end{bmatrix}$.

Solution

The matrices are both 2×2, so their product is defined. Use the following steps to find the elements of the product matrix.

STEP 1 **Multiply** the numbers in the first row of the first matrix by the numbers in the first column of the second matrix. Put the result in the first row, first column of the product matrix.

$$\begin{bmatrix} 1 & 0 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 0(-1) & ? \\ ? & ? \end{bmatrix}$$

STEP 2 **Multiply** the numbers in the first row of the first matrix by the numbers in the second column of the second matrix. Put the result in the first row, second column of the product matrix.

$$\begin{bmatrix} 1 & 0 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 0(-1) & 1(-3) + 0(8) \\ ? & ? \end{bmatrix}$$

STEP 3 **Multiply** the numbers in the second row of the first matrix by the numbers in the first column of the second matrix. Put the result in the second row, first column of the product matrix.

$$\begin{bmatrix} 1 & 0 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 0(-1) & 1(-3) + 0(8) \\ 4(2) + 5(-1) & ? \end{bmatrix}$$

STEP 4 **Multiply** the numbers in the second row of the first matrix by the numbers in the second column of the second matrix. Put the result in the second row, second column of the product matrix.

$$\begin{bmatrix} 1 & 0 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 0(-1) & 1(-3) + 0(8) \\ 4(2) + 5(-1) & 4(-3) + 5(8) \end{bmatrix}$$

STEP 5 **Simplify** the product matrix.

$$\begin{bmatrix} 1(2) + 0(-1) & 1(-3) + 0(8) \\ 4(2) + 5(-1) & 4(-3) + 5(8) \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 3 & 28 \end{bmatrix}$$

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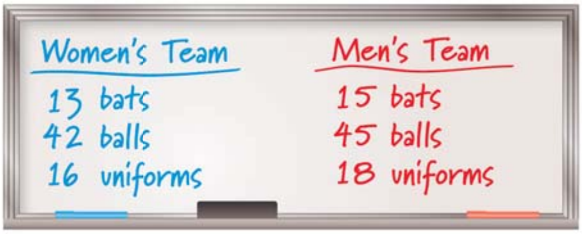
EXAMPLE 5 Solve a real-world problem

SOFTBALL Two softball teams submit equipment lists for the season. A bat costs \$20, a ball costs \$5, and a uniform costs \$40. Use matrix multiplication to find the total cost of equipment for each team.



Solution

> **ANOTHER WAY**
> You could solve this problem arithmetically, multiplying the number of bats by the price of bats, and so on, then adding the costs for each team.

First, write the equipment lists and the costs per item in matrix form. You will use matrix multiplication, so you need to set up the matrices so that the number of columns of the equipment matrix matches the number of rows of the cost per item matrix.

EQUIPMENT · **COST** = **TOTAL COST**

$$\begin{array}{r} \\ \text{Women} \\ \text{Men} \end{array}\begin{array}{c} \begin{array}{ccc}\text{Bats} & \text{Balls} & \text{Uniforms}\end{array} \\ \begin{bmatrix} 13 & 42 & 16 \\ 15 & 45 & 18 \end{bmatrix}\end{array} \cdot \begin{array}{r} \\ \text{Bats} \\ \text{Balls} \\ \text{Uniforms} \end{array}\begin{array}{c} \text{Dollars} \\ \begin{bmatrix} 20 \\ 5 \\ 40 \end{bmatrix}\end{array} = \begin{array}{r} \\ \text{Women} \\ \text{Men} \end{array}\begin{array}{c} \text{Dollars} \\ \begin{bmatrix} ? \\ ? \end{bmatrix}\end{array}$$

You can find the total cost of equipment for each team by multiplying the equipment matrix by the cost per item matrix. The equipment matrix is 2×3 and the cost per item matrix is 3×1, so their product is a 2×1 matrix.

$$\begin{bmatrix} 13 & 42 & 16 \\ 15 & 45 & 18 \end{bmatrix}\begin{bmatrix} 20 \\ 5 \\ 40 \end{bmatrix} = \begin{bmatrix} 13(20) + 42(5) + 16(40) \\ 15(20) + 45(5) + 18(40) \end{bmatrix} = \begin{bmatrix} 1110 \\ 1245 \end{bmatrix}$$

▸ The total cost of equipment for the women's team is \$1110, and the total cost for the men's team is \$1245.

✓ GUIDED PRACTICE for Examples 4 and 5

Use the matrices below. Is the product defined? *Explain.*

$$A = \begin{bmatrix} -3 \\ 4 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 6.7 & 0 \\ -9.3 & 5.2 \end{bmatrix}$$

6. AB **7.** BA **8.** AC

Multiply.

9. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 3 & 8 \\ -4 & 7 \end{bmatrix}$ **10.** $\begin{bmatrix} 5 & 1 \end{bmatrix}\begin{bmatrix} -3 \\ -2 \end{bmatrix}$ **11.** $\begin{bmatrix} 5 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & -4 \\ 5 & 1 \end{bmatrix}$

12. **WHAT IF?** In Example 5, find the total cost if a bat costs \$25, a ball costs \$4, and a uniform costs \$35.

9.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 13, 19, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 24, 25, and 35

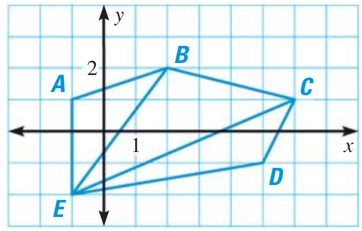
SKILL PRACTICE

1. **VOCABULARY** Copy and complete: To find the sum of two matrices, add corresponding __?__.

2. ★ **WRITING** How can you determine whether two matrices can be added? How can you determine whether two matrices can be multiplied?

EXAMPLE 1 on p. 580 for Exs. 3–6

USING A DIAGRAM **Use the diagram to write a matrix to represent the given polygon.**



3. $\triangle EBC$

4. $\triangle ECD$

5. Quadrilateral $BCDE$

6. Pentagon $ABCDE$

EXAMPLE 2 on p. 581 for Exs. 7–12

MATRIX OPERATIONS **Add or subtract.**

7. $\begin{bmatrix} 3 & 5 \end{bmatrix} + \begin{bmatrix} 9 & 2 \end{bmatrix}$

8. $\begin{bmatrix} -12 & 5 \\ 1 & -4 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 0 & 8 \end{bmatrix}$

9. $\begin{bmatrix} 9 & 8 \\ -2 & 3 \\ 0 & -4 \end{bmatrix} + \begin{bmatrix} 7 & 1 \\ 2 & -3 \\ -5 & 1 \end{bmatrix}$

10. $\begin{bmatrix} 4.6 & 8.1 \end{bmatrix} - \begin{bmatrix} 3.8 & -2.1 \end{bmatrix}$

11. $\begin{bmatrix} -5 & 6 \\ -8 & 9 \end{bmatrix} - \begin{bmatrix} 8 & 10 \\ 4 & -7 \end{bmatrix}$

12. $\begin{bmatrix} 1.2 & 6 \\ 5.3 & 1.1 \end{bmatrix} - \begin{bmatrix} 2.5 & -3.3 \\ 7 & 4 \end{bmatrix}$

EXAMPLE 3 on p. 581 for Exs. 13–17

TRANSLATIONS **Find the image matrix that represents the translation of the polygon. Then graph the polygon and its image.**

13. $\begin{array}{c} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} -2 & 2 & 1 \\ 4 & 1 & -3 \end{bmatrix} \end{array}$; 4 units up

14. $\begin{array}{c} \begin{matrix} F & G & H & J \end{matrix} \\ \begin{bmatrix} 2 & 5 & 8 & 5 \\ 2 & 3 & 1 & -1 \end{bmatrix} \end{array}$; 2 units left and 3 units down

15. $\begin{array}{c} \begin{matrix} L & M & N & P \end{matrix} \\ \begin{bmatrix} 3 & 0 & 2 & 2 \\ -1 & 3 & 3 & -1 \end{bmatrix} \end{array}$; 4 units right and 2 units up

16. $\begin{array}{c} \begin{matrix} Q & R & S \end{matrix} \\ \begin{bmatrix} -5 & 0 & 1 \\ 1 & 4 & 2 \end{bmatrix} \end{array}$; 3 units right and 1 unit down

17. ★ **MULTIPLE CHOICE** The matrix that represents quadrilateral $ABCD$ is $\begin{bmatrix} 3 & 8 & 9 & 7 \\ 3 & 7 & 3 & 1 \end{bmatrix}$. Which matrix represents the image of the quadrilateral after translating it 3 units right and 5 units up?

   Ⓐ $\begin{bmatrix} 6 & 11 & 12 & 10 \\ 8 & 12 & 8 & 6 \end{bmatrix}$ Ⓑ $\begin{bmatrix} 0 & 5 & 6 & 4 \\ 8 & 12 & 8 & 6 \end{bmatrix}$

   Ⓒ $\begin{bmatrix} 6 & 11 & 12 & 10 \\ -2 & 2 & -2 & -4 \end{bmatrix}$ Ⓓ $\begin{bmatrix} 0 & 6 & 6 & 4 \\ -2 & 3 & -2 & -4 \end{bmatrix}$

EXAMPLE 4
on p. 582
for Exs. 18–26

MATRIX OPERATIONS **Multiply.**

18. $\begin{bmatrix} 5 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

19. $\begin{bmatrix} 1.2 & 3 \end{bmatrix}\begin{bmatrix} -2 \\ -1.5 \end{bmatrix}$

20. $\begin{bmatrix} 6 & 7 \\ -5 & 8 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 9 & -3 \end{bmatrix}$

21. $\begin{bmatrix} 0.4 & 6 \\ -6 & 2.3 \end{bmatrix}\begin{bmatrix} 5 & 8 \\ -1 & 2 \end{bmatrix}$

22. $\begin{bmatrix} 4 & 8 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$

23. $\begin{bmatrix} 9 & 1 & 2 \\ 8 & -1 & 4 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$

24. ★ **MULTIPLE CHOICE** Which product is not defined?

Ⓐ $\begin{bmatrix} 1 & 7 \\ 3 & 12 \end{bmatrix}\begin{bmatrix} 6 \\ 15 \end{bmatrix}$ Ⓑ $\begin{bmatrix} 3 & 20 \end{bmatrix}\begin{bmatrix} 9 \\ 30 \end{bmatrix}$ Ⓒ $\begin{bmatrix} 15 \\ -3 \end{bmatrix}\begin{bmatrix} 1 & 6 \\ 4 & 0 \end{bmatrix}$ Ⓓ $\begin{bmatrix} 30 \\ -7 \end{bmatrix}\begin{bmatrix} 5 & 5 \end{bmatrix}$

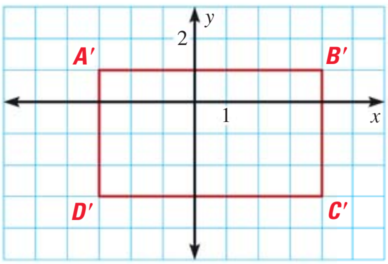
25. ★ **OPEN-ENDED MATH** Write two matrices that have a defined product. Then find the product.

26. **ERROR ANALYSIS** *Describe* and correct the error in the computation.

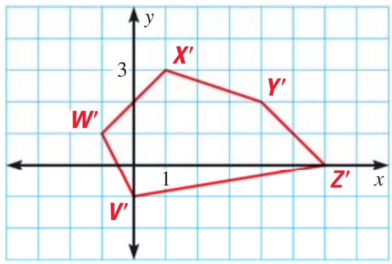
$$\begin{bmatrix} 9 & -2 \\ 4 & 10 \end{bmatrix}\begin{bmatrix} -6 & 12 \\ 3 & -6 \end{bmatrix} = \begin{bmatrix} 9(-6) & -2(12) \\ 4(3) & 10(-6) \end{bmatrix}$$

TRANSLATIONS **Use the described translation and the graph of the image to find the matrix that represents the preimage.**

27. 4 units right and 2 units down



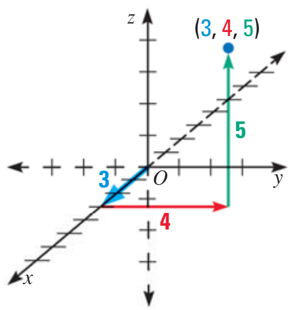
28. 6 units left and 5 units up



29. **MATRIX EQUATION** Use the description of a translation of a triangle to find the value of each variable. *Explain* your reasoning. What are the coordinates of the vertices of the image triangle?

$$\begin{bmatrix} 12 & 12 & w \\ -7 & v & -7 \end{bmatrix} + \begin{bmatrix} 9 & a & b \\ 6 & -2 & c \end{bmatrix} = \begin{bmatrix} m & 20 & -8 \\ n & -9 & 13 \end{bmatrix}$$

30. **CHALLENGE** A point in space has three coordinates (x, y, z), as shown at the right. From the origin, a point can be forward or back on the x-axis, left or right on the y-axis, and up or down on the z-axis.



a. You translate a point three units forward, four units right, and five units up. Write a translation matrix for the point.

b. You translate a figure that has five vertices. Write a translation matrix to move the figure five units back, ten units left, and six units down.

PROBLEM SOLVING

EXAMPLE 5 on p. 583 for Ex. 31

31. **COMPUTERS** Two computer labs submit equipment lists. A mouse costs \$10, a package of CDs costs \$32, and a keyboard costs \$15. Use matrix multiplication to find the total cost of equipment for each lab.

Lab 1
25 Mice
10 CDs
18 Keyboards

Lab 2
15 Mice
20 CDs
12 Keyboards

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32. **SWIMMING** Two swim teams submit equipment lists. The women's team needs 30 caps and 26 goggles. The men's team needs 15 caps and 25 goggles. A cap costs \$10 and goggles cost \$15.

a. Use matrix addition to find the total number of caps and the total number of goggles for each team.

b. Use matrix multiplication to find the total equipment cost for each team.

c. Find the total cost for both teams.

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MATRIX PROPERTIES In Exercises 33–35, use matrices *A*, *B*, and *C*.

$$A = \begin{bmatrix} 5 & 1 \\ 10 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 4 \\ -5 & 1 \end{bmatrix}$$

33. **MULTI-STEP PROBLEM** Use the 2×2 matrices above to explore the Commutative Property of Multiplication.

a. What does it mean that multiplication is *commutative*?

b. Find and *compare* AB and BA.

c. Based on part (b), make a conjecture about whether matrix multiplication is commutative.

34. **MULTI-STEP PROBLEM** Use the 2×2 matrices above to explore the Associative Property of Multiplication.

a. What does it mean that multiplication is *associative*?

b. Find and *compare* $A(BC)$ and $(AB)C$.

c. Based on part (b), make a conjecture about whether matrix multiplication is associative.

35. ★ **SHORT RESPONSE** Find and *compare* $A(B + C)$ and $AB + AC$. Make a conjecture about matrices and the Distributive Property.

36. **ART** Two art classes are buying supplies. A brush is \$4 and a paint set is \$10. Each class has only \$225 to spend. Use matrix multiplication to find the maximum number of brushes Class A can buy and the maximum number of paint sets Class B can buy. *Explain*.

Class A
x brushes
12 paint sets

Class B
18 brushes
y paint sets

37. CHALLENGE The total United States production of corn was 8,967 million bushels in 2002, and 10,114 million bushels in 2003. The table shows the percents of the total grown by four states.

a. Use matrix multiplication to find the number of bushels (in millions) harvested in each state each year.

b. How many bushels (in millions) were harvested in these two years in Iowa?

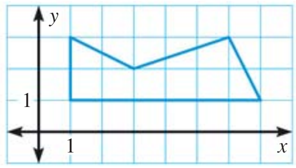
c. The price for a bushel of corn in Nebraska was \$2.32 in 2002, and \$2.45 in 2003. Use matrix multiplication to find the total value of corn harvested in Nebraska in these two years.

	2002	2003
Iowa	21.5%	18.6%
Illinois	16.4%	17.9%
Nebraska	10.5%	11.1%
Minnesota	11.7%	9.6%

MIXED REVIEW

PREVIEW
Prepare for Lesson 9.3 in Exs. 38–39.

Copy the figure and draw its image after the reflection. *(p. 272)*

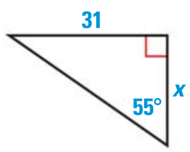


38. Reflect the figure in the x-axis.

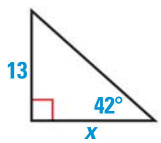
39. Reflect the figure in the y-axis.

Find the value of x to the nearest tenth. *(p. 466)*

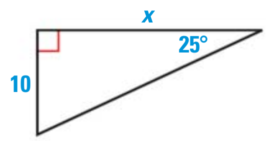
40.



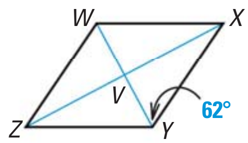
41.



42.



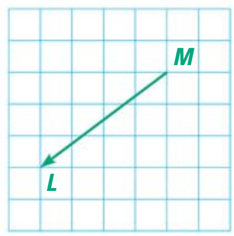
The diagonals of rhombus $WXYZ$ intersect at V. Given that $m\angle XYW = 62°$, find the indicated measure. *(p. 533)*



43. $m\angle ZYW = \underline{\ ?\ }$ **44.** $m\angle WXY = \underline{\ ?\ }$ **45.** $m\angle XVY = \underline{\ ?\ }$

QUIZ for Lessons 9.1–9.2

1. In the diagram shown, name the vector and write its component form. *(p. 572)*



Use the translation $(x, y) \rightarrow (x + 3, y - 2)$. *(p. 572)*

2. What is the image of $(-1, 5)$?

3. What is the image of $(6, 3)$?

4. What is the preimage of $(-4, -1)$?

Add, subtract, or multiply. *(p. 580)*

5. $\begin{bmatrix} 5 & -3 \\ 8 & -2 \end{bmatrix} + \begin{bmatrix} -9 & 6 \\ 4 & -7 \end{bmatrix}$ **6.** $\begin{bmatrix} -6 & 1 \\ 3 & 12 \end{bmatrix} - \begin{bmatrix} 4 & 15 \\ -7 & 8 \end{bmatrix}$ **7.** $\begin{bmatrix} 7 & -6 & 2 \\ 8 & 3 & 5 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ -9 & 0 \\ 3 & -7 \end{bmatrix}$

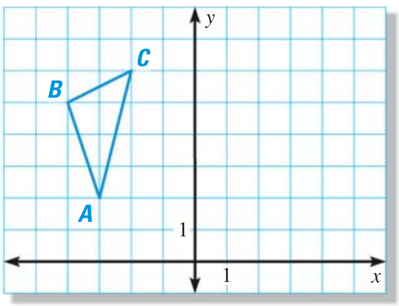
9.3 Reflections in the Plane

MATERIALS • graph paper • straightedge

QUESTION What is the relationship between the line of reflection and the segment connecting a point and its image?

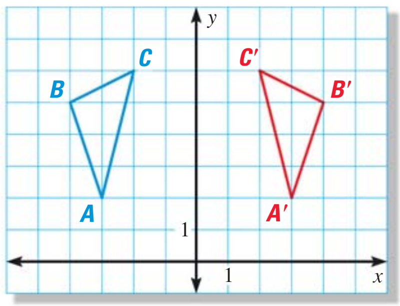
EXPLORE Graph a reflection of a triangle

STEP 1



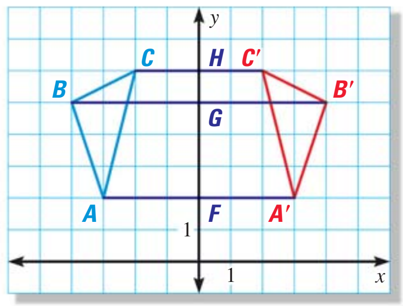
Draw a triangle Graph $A(-3, 2)$, $B(-4, 5)$, and $C(-2, 6)$. Connect the points to form $\triangle ABC$.

STEP 2



Graph a reflection Reflect $\triangle ABC$ in the y-axis. Label points A', B', and C' appropriately.

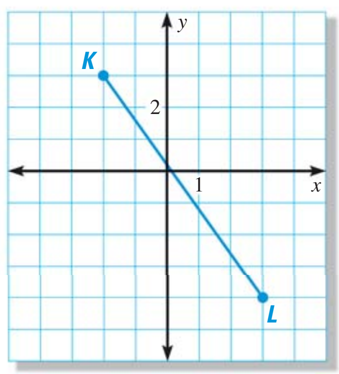
STEP 3



Draw segments Draw $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$. Label the points where these segments intersect the y-axis as F, G, and H, respectively.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Find the lengths of $\overline{CH}$ and $\overline{HC'}$, $\overline{BG}$ and $\overline{GB'}$, and $\overline{AF}$ and $\overline{FA'}$. *Compare* the lengths of each pair of segments.
2. Find the measures of $\angle CHG$, $\angle BGF$, and $\angle AFG$. *Compare* the angle measures.
3. How is the y-axis related to $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$?
4. Use the graph at the right.
   a. $\overline{K'L'}$ is the reflection of $\overline{KL}$ in the x-axis. Copy the diagram and draw $\overline{K'L'}$.
   b. Draw $\overline{KK'}$ and $\overline{LL'}$. Label the points where the segments intersect the x-axis as J and M.
   c. How is the x-axis related to $\overline{KK'}$ and $\overline{LL'}$?
5. How is the line of reflection related to the segment connecting a point and its image?



9.3 Perform Reflections

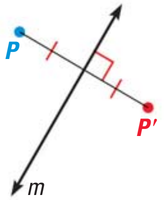
Before	You reflected a figure in the *x*- or *y*-axis.
Now	You will reflect a figure in any given line.
Why?	So you can identify reflections, as in Exs. 31–33.

Key Vocabulary
- **line of reflection**
- **reflection,** *p. 272*

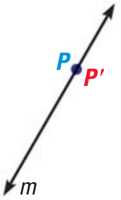
In Lesson 4.8, you learned that a *reflection* is a transformation that uses a line like a mirror to reflect an image. The mirror line is called the **line of reflection**.

A reflection in a line m maps every point P in the plane to a point P', so that for each point one of the following properties is true:

- If P is not on m, then m is the perpendicular bisector of $\overline{PP'}$, or
- If P is on m, then $P = P'$.



Point *P* not on *m*



Point *P* on *m*

EXAMPLE 1 Graph reflections in horizontal and vertical lines

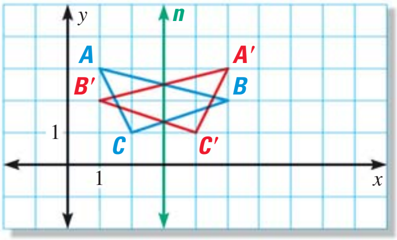
The vertices of $\triangle ABC$ are $A(1, 3)$, $B(5, 2)$, and $C(2, 1)$. Graph the reflection of $\triangle ABC$ described.

a. In the line n: $x = 3$

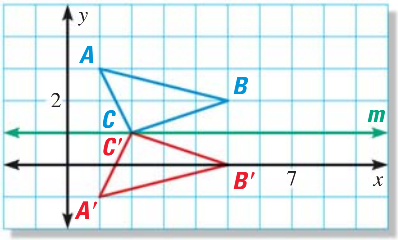
b. In the line m: $y = 1$

Solution

a. Point A is 2 units left of n, so its reflection A' is 2 units right of n at $(5, 3)$. Also, B' is 2 units left of n at $(1, 2)$, and C' is 1 unit right of n at $(4, 1)$.



b. Point A is 2 units above m, so A' is 2 units below m at $(1, -1)$. Also, B' is 1 unit below m at $(5, 0)$. Because point C is on line m, you know that $C = C'$.



✓ GUIDED PRACTICE for Example 1

Graph a reflection of $\triangle ABC$ from Example 1 in the given line.

1. $y = 4$

2. $x = -3$

3. $y = 2$

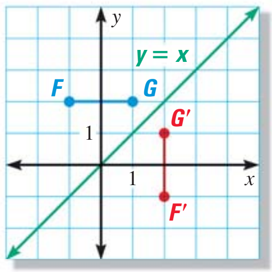
EXAMPLE 2 Graph a reflection in $y = x$

The endpoints of $\overline{FG}$ are $F(-1, 2)$ and $G(1, 2)$. Reflect the segment in the line $y = x$. Graph the segment and its image.

REVIEW SLOPE
The product of the slopes of perpendicular lines is -1.

Solution

The slope of $y = x$ is 1. The segment from F to its image, $\overline{FF'}$, is perpendicular to the line of reflection $y = x$, so the slope of $\overline{FF'}$ will be -1 (because $1(-1) = -1$). From F, move 1.5 units right and 1.5 units down to $y = x$. From that point, move 1.5 units right and 1.5 units down to locate $F'(3, -1)$.



The slope of $\overline{GG'}$ will also be -1. From G, move 0.5 units right and 0.5 units down to $y = x$. Then move 0.5 units right and 0.5 units down to locate $G'(2, 1)$.

COORDINATE RULES You can use coordinate rules to find the images of points reflected in four special lines.

KEY CONCEPT *For Your Notebook*

Coordinate Rules for Reflections

- If (a, b) is reflected in the x-axis, its image is the point $(a, -b)$.
- If (a, b) is reflected in the y-axis, its image is the point $(-a, b)$.
- If (a, b) is reflected in the line $y = x$, its image is the point (b, a).
- If (a, b) is reflected in the line $y = -x$, its image is the point $(-b, -a)$.

EXAMPLE 3 Graph a reflection in $y = -x$

Reflect $\overline{FG}$ from Example 2 in the line $y = -x$. Graph $\overline{FG}$ and its image.

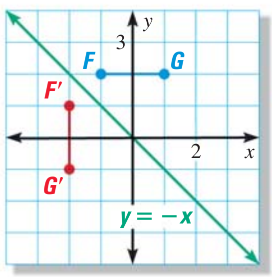
Solution

Use the coordinate rule for reflecting in $y = -x$.

$$(a, b) \to (-b, -a)$$

$F(-1, 2) \to F'(-2, 1)$

$G(1, 2) \to G'(-2, -1)$



AnimatedGeometry at classzone.com

GUIDED PRACTICE for Examples 2 and 3

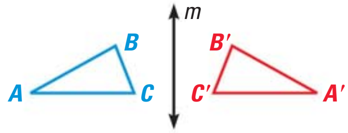
4. Graph $\triangle ABC$ with vertices $A(1, 3)$, $B(4, 4)$, and $C(3, 1)$. Reflect $\triangle ABC$ in the lines $y = -x$ and $y = x$. Graph each image.
5. In Example 3, *verify* that $\overline{FF'}$ is perpendicular to $y = -x$.

REFLECTION THEOREM You saw in Lesson 9.1 that the image of a translation is congruent to the original figure. The same is true for a reflection.

THEOREM *For Your Notebook*

THEOREM 9.2 Reflection Theorem

A reflection is an isometry.



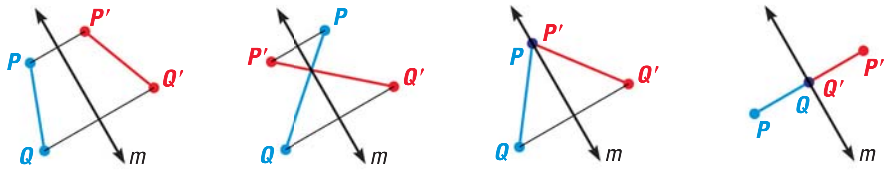
$\triangle ABC \cong \triangle A'B'C'$

Proof: Exs. 35–38, p. 595

WRITE PROOFS

Some theorems, such as the Reflection Theorem, have more than one case. To prove this type of theorem, each case must be proven.

PROVING THE THEOREM To prove the Reflection Theorem, you need to show that a reflection preserves the length of a segment. Consider a segment $\overline{PQ}$ that is reflected in a line m to produce $\overline{P'Q'}$. There are four cases to prove:



Case 1 P and Q are on the same side of m.

Case 2 P and Q are on opposite sides of m.

Case 3 P lies on m, and $\overline{PQ}$ is not $\perp$ to m.

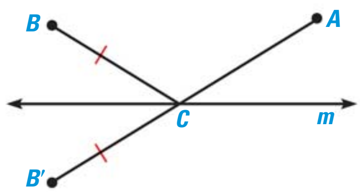
Case 4 Q lies on m, and $\overline{PQ} \perp m$.

EXAMPLE 4 Find a minimum distance

PARKING You are going to buy books. Your friend is going to buy CDs. Where should you park to minimize the distance you both will walk?

Solution

Reflect B in line m to obtain B'. Then draw $\overline{AB'}$. Label the intersection of $\overline{AB'}$ and m as C. Because AB' is the shortest distance between A and B' and $BC = B'C$, park at point C to minimize the combined distance, $AC + BC$, you both have to walk.



GUIDED PRACTICE for Example 4

6. Look back at Example 4. Answer the question by using a reflection of point A instead of point B.

REFLECTION MATRIX You can find the image of a polygon reflected in the x-axis or y-axis using matrix multiplication. Write the reflection matrix to the *left* of the polygon matrix, then multiply.

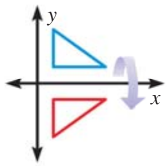
Notice that because matrix multiplication is not commutative, the order of the matrices in your product is important. The reflection matrix must be first followed by the polygon matrix.

KEY CONCEPT *For Your Notebook*

Reflection Matrices

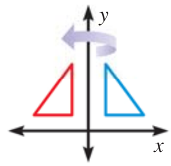
Reflection in the x-axis

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



Reflection in the y-axis

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$



EXAMPLE 5 Use matrix multiplication to reflect a polygon

The vertices of $\triangle DEF$ are $D(1, 2)$, $E(3, 3)$, and $F(4, 0)$. Find the reflection of $\triangle DEF$ in the y-axis using matrix multiplication. Graph $\triangle DEF$ and its image.

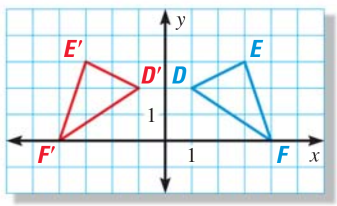
Solution

STEP 1 **Multiply** the polygon matrix by the matrix for a reflection in the y-axis.

$$\underset{\text{Reflection matrix}}{\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}} \underset{\text{Polygon matrix}}{\begin{array}{c} \begin{array}{ccc} D & E & F \end{array} \\ \begin{bmatrix} 1 & 3 & 4 \\ 2 & 3 & 0 \end{bmatrix} \end{array}} = \begin{bmatrix} -1(1) + 0(2) & -1(3) + 0(3) & -1(4) + 0(0) \\ 0(1) + 1(2) & 0(3) + 1(3) & 0(4) + 1(0) \end{bmatrix}$$

$$= \begin{array}{c} \begin{array}{ccc} D' & E' & F' \end{array} \\ \begin{bmatrix} -1 & -3 & -4 \\ 2 & 3 & 0 \end{bmatrix} \end{array} \quad \text{Image matrix}$$

STEP 2 **Graph** $\triangle DEF$ and $\triangle D'E'F'$.



✓ GUIDED PRACTICE for Example 5

The vertices of $\triangle LMN$ are $L(-3, 3)$, $M(1, 2)$, and $N(-2, 1)$. Find the described reflection using matrix multiplication.

7. Reflect $\triangle LMN$ in the x-axis.

8. Reflect $\triangle LMN$ in the y-axis.

9.3 EXERCISES

HOMEWORK KEY
◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 13, and 33
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 12, 25, and 40

SKILL PRACTICE

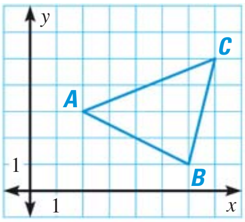
1. VOCABULARY What is a *line of reflection*?

2. ★ WRITING *Explain* how to find the distance from a point to its image if you know the distance from the point to the line of reflection.

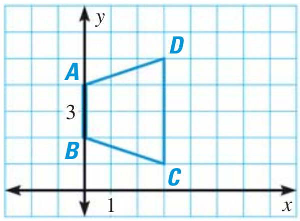
EXAMPLE 1 on p. 589 for Exs. 3–8

REFLECTIONS **Graph the reflection of the polygon in the given line.**

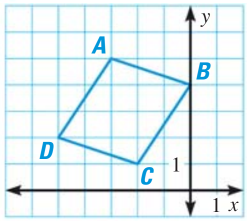
3. x-axis



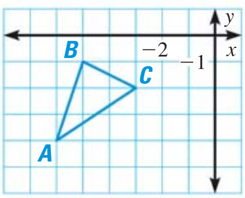
4. y-axis



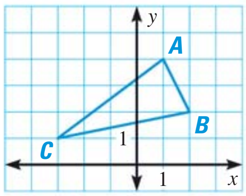
5. $y = 2$



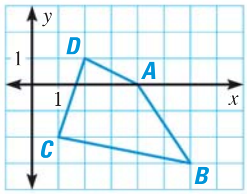
6. $x = -1$



7. y-axis

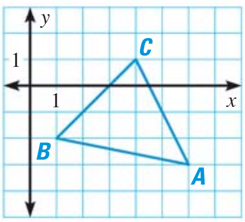


8. $y = -3$

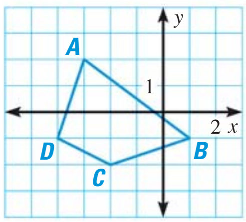


EXAMPLES 2 and 3 on p. 590 for Exs. 9–12

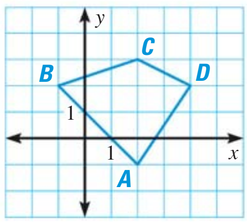
9. $y = x$



10. $y = -x$



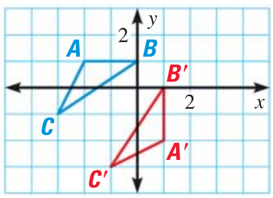
11. $y = x$



12. ★ MULTIPLE CHOICE What is the line of reflection for $\triangle ABC$ and its image?

Ⓐ $y = 0$ (the x-axis) Ⓑ $y = -x$

Ⓒ $x = 1$ Ⓓ $y = x$



EXAMPLE 5 on p. 592 for Exs. 13–17

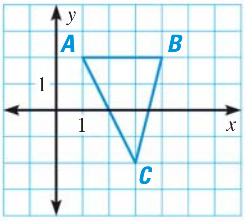
USING MATRIX MULTIPLICATION **Use matrix multiplication to find the image. Graph the polygon and its image.**

13. Reflect $\begin{array}{c} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} -2 & 3 & 4 \\ 5 & -3 & 6 \end{bmatrix} \end{array}$ in the x-axis.

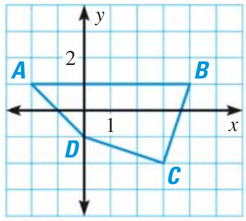
14. Reflect $\begin{array}{c} \begin{matrix} P & Q & R & S \end{matrix} \\ \begin{bmatrix} 2 & 6 & 5 & 2 \\ -2 & -3 & -8 & -5 \end{bmatrix} \end{array}$ in the y-axis.

FINDING IMAGE MATRICES **Write a matrix for the polygon. Then find the image matrix that represents the polygon after a reflection in the given line.**

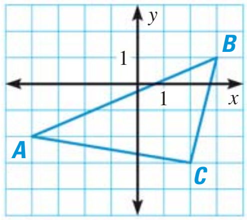
15. y-axis



16. x-axis



17. y-axis



18. **ERROR ANALYSIS** *Describe* and correct the error in finding the image matrix of $\triangle PQR$ reflected in the y-axis.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -5 & 4 & -2 \\ 4 & 8 & -1 \end{bmatrix} = \begin{bmatrix} -5 & 4 & -2 \\ -4 & -8 & -1 \end{bmatrix}$$

MINIMUM DISTANCE **Find point C on the x-axis so $AC + BC$ is a minimum.**

19. $A(1, 4)$, $B(6, 1)$

20. $A(4, -3)$, $B(12, -5)$

21. $A(-8, 4)$, $B(-1, 3)$

TWO REFLECTIONS **The vertices of $\triangle FGH$ are $F(3, 2)$, $G(1, 5)$, and $H(-1, 2)$. Reflect $\triangle FGH$ in the first line. Then reflect $\triangle F'G'H'$ in the second line. Graph $\triangle F'G'H'$ and $\triangle F''G''H''$.**

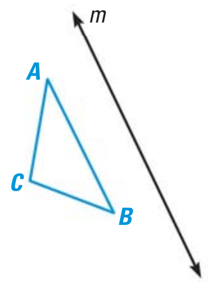
22. In $y = 2$, then in $y = -1$

23. In $y = -1$, then in $x = 2$

24. In $y = x$, then in $x = -3$

25. ★ **SHORT RESPONSE** Use your graphs from Exercises 22–24. What do you notice about the order of vertices in the preimages and images?

26. **CONSTRUCTION** Use these steps to construct a reflection of $\triangle ABC$ in line m using a straightedge and a compass.



STEP 1 **Draw** $\triangle ABC$ and line m.

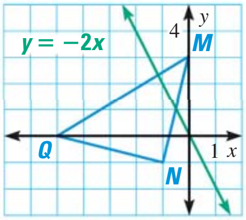
STEP 2 **Use** one compass setting to find two points that are equidistant from A on line m. Use the same compass setting to find a point on the other side of m that is the same distance from line m. Label that point A'.

STEP 3 **Repeat** Step 2 to find points B' and C'. Draw $\triangle A'B'C'$.

27. *xy* **ALGEBRA** The line $y = 3x + 2$ is reflected in the line $y = -1$. What is the equation of the image?

28. *xy* **ALGEBRA** Reflect the graph of the quadratic equation $y = 2x^2 - 5$ in the x-axis. What is the equation of the image?

29. **REFLECTING A TRIANGLE** Reflect $\triangle MNQ$ in the line $y = -2x$.



30. **CHALLENGE** Point $B'(1, 4)$ is the image of $B(3, 2)$ after a reflection in line c. Write an equation of line c.

PROBLEM SOLVING

REFLECTIONS **Identify the case of the Reflection Theorem represented.**

31.

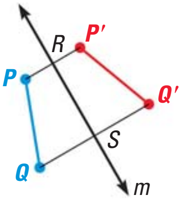
32.

33.

EXAMPLE 4
on p. 591
for Ex. 34

34. DELIVERING PIZZA You park at some point K on line n. You deliver a pizza to house H, go back to your car, and deliver a pizza to house J. Assuming that you can cut across both lawns, how can you determine the parking location K that minimizes the total walking distance?

35. PROVING THEOREM 9.2 Prove Case 1 of the Reflection Theorem.



Case 1 The segment does not intersect the line of reflection.

GIVEN ▶ A reflection in m maps P to P' and Q to Q'.

PROVE ▶ $PQ = P'Q'$

Plan for Proof

a. Draw $\overline{PP'}$, $\overline{QQ'}$, $\overline{RQ}$, and $\overline{RQ'}$. Prove that $\triangle RSQ \cong \triangle RSQ'$.

b. Use the properties of congruent triangles and perpendicular bisectors to prove that $PQ = P'Q'$.

PROVING THEOREM 9.2 **In Exercises 36–38, write a proof for the given case of the Reflection Theorem. (Refer to the diagrams on page 591.)**

36. Case 2 The segment intersects the line of reflection.

GIVEN ▶ A reflection in m maps P to P' and Q to Q'.
Also, $\overline{PQ}$ intersects m at point R.

PROVE ▶ $PQ = P'Q'$

37. Case 3 One endpoint is on the line of reflection, and the segment is not perpendicular to the line of reflection.

GIVEN ▶ A reflection in m maps P to P' and Q to Q'.
Also, P lies on line m, and $\overline{PQ}$ is not perpendicular to m.

PROVE ▶ $PQ = P'Q'$

38. Case 4 One endpoint is on the line of reflection, and the segment is perpendicular to the line of reflection.

GIVEN ▶ A reflection in m maps P to P' and Q to Q'.
Also, Q lies on line m, and $\overline{PQ}$ is perpendicular to line m.

PROVE ▶ $PQ = P'Q'$

39. REFLECTING POINTS Use $C(1, 3)$.

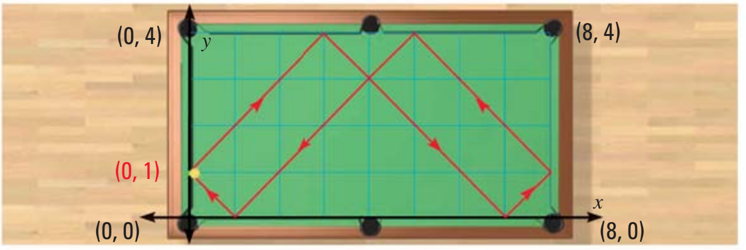
a. Point A has coordinates $(-1, 1)$. Find point B on $\overrightarrow{AC}$ so $AC = CB$.

b. The endpoints of $\overline{FG}$ are $F(2, 0)$ and $G(3, 2)$. Find point H on $\overrightarrow{FC}$ so $FC = CH$. Find point J on $\overrightarrow{GC}$ so $GC = CJ$.

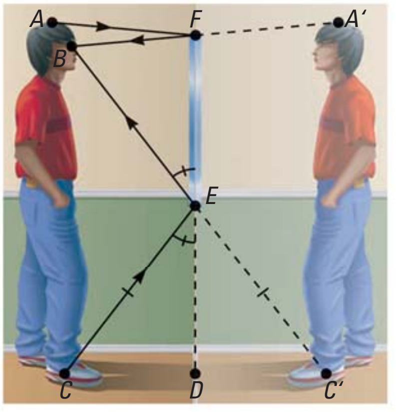
c. Explain why parts (a) and (b) can be called *reflection in a point*.

PHYSICS The Law of Reflection states that the angle of incidence is congruent to the angle of reflection. Use this information in Exercises 40 and 41.

40. ★ SHORT RESPONSE Suppose a billiard table has a coordinate grid on it. If a ball starts at the point $(0, 1)$ and rolls at a $45°$ angle, it will eventually return to its starting point. Would this happen if the ball started from other points on the y-axis between $(0, 0)$ and $(0, 4)$? *Explain.*



41. CHALLENGE Use the diagram to prove that you can see your full self in a mirror that is only half of your height. Assume that you and the mirror are both perpendicular to the floor.



a. Think of a light ray starting at your foot and reflected in a mirror. Where does it have to hit the mirror in order to reflect to your eye?

b. Think of a light ray starting at the top of your head and reflected in a mirror. Where does it have to hit the mirror in order to reflect to your eye?

c. Show that the distance between the points you found in parts (a) and (b) is half your height.

MIXED REVIEW

PREVIEW
Prepare for Lesson 9.4 in Exs. 42–43.

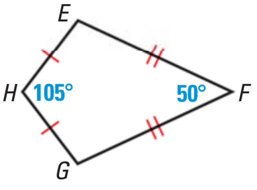
Tell whether the lines through the given points are *parallel*, *perpendicular*, or *neither*. *Justify* your answer. *(p. 171)*

42. Line 1: $(3, 7)$ and $(9, 7)$
Line 2: $(-2, 8)$ and $(-2, 1)$

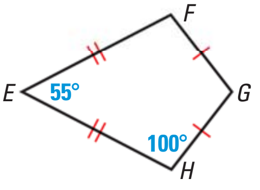
43. Line 1: $(-4, -1)$ and $(-8, -4)$
Line 2: $(1, -3)$ and $(5, 0)$

Quadrilateral *EFGH* is a kite. Find $m\angle G$. *(p. 542)*

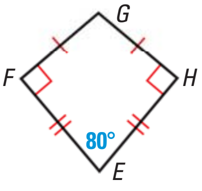
44.



45.



46.



MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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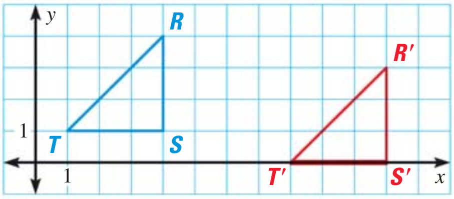
Lessons 9.1–9.3

1. MULTI-STEP PROBLEM $\triangle R'S'T'$ is the image of $\triangle RST$ after a translation.

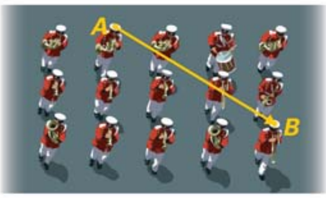


a. Write a rule for the translation.

b. *Verify* that the transformation is an isometry.

c. Suppose $\triangle R'S'T'$ is translated using the rule $(x, y) \to (x + 4, y - 2)$. What are the coordinates of the vertices of $\triangle R''S''T''$?

2. SHORT RESPONSE During a marching band routine, a band member moves directly from point A to point B. Write the component form of the vector $\overrightarrow{AB}$. *Explain* your answer.



3. SHORT RESPONSE Trace the picture below. Reflect the image in line m. How is the distance from X to line m related to the distance from X' to line m? Write the property that makes this true.

4. SHORT RESPONSE The endpoints of $\overline{AB}$ are $A(2, 4)$ and $B(4, 0)$. The endpoints of $\overline{CD}$ are $C(3, 3)$ and $D(7, -1)$. Is the transformation from $\overline{AB}$ to $\overline{CD}$ an isometry? *Explain.*

5. GRIDDED ANSWER The vertices of $\triangle FGH$ are $F(-4, 3)$, $G(3, -1)$, and $H(1, -2)$. The coordinates of F' are $(-1, 4)$ after a translation. What is the x-coordinate of G'?

6. OPEN-ENDED Draw a triangle in a coordinate plane. Reflect the triangle in an axis. Write the reflection matrix that would yield the same result.

7. EXTENDED RESPONSE Two cross-country teams submit equipment lists for a season. A pair of running shoes costs \$60, a pair of shorts costs \$18, and a shirt costs \$15.

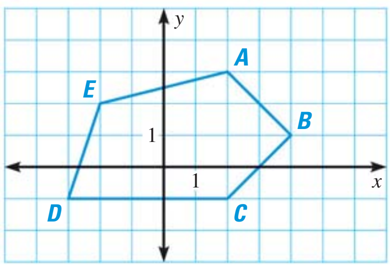
Women's Team	Men's Team
14 pairs of shoes	10 pairs of shoes
16 pairs of shorts	13 pairs of shorts
16 shirts	13 shirts

a. Use matrix multiplication to find the total cost of equipment for each team.

b. How much money will the teams need to raise if the school gives each team \$200?

c. Repeat parts (a) and (b) if a pair of shoes costs \$65 and a shirt costs \$10. Does the change in prices change which team needs to raise more money? *Explain*.

8. MULTI-STEP PROBLEM Use the polygon as the preimage.



a. Reflect the preimage in the y-axis.

b. Reflect the preimage in the x-axis.

c. *Compare* the order of vertices in the preimage with the order in each image.

9.4 Perform Rotations

Before You rotated figures about the origin.

Now You will rotate figures about a point.

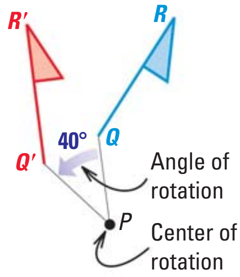
Why? So you can classify transformations, as in Exs. 3–5.

Key Vocabulary
- **center of rotation**
- **angle of rotation**
- **rotation,** *p. 272*

Recall from Lesson 4.8 that a *rotation* is a transformation in which a figure is turned about a fixed point called the **center of rotation**. Rays drawn from the center of rotation to a point and its image form the **angle of rotation**.

A rotation about a point P through an angle of $x°$ maps every point Q in the plane to a point Q' so that one of the following properties is true:

- If Q is not the center of rotation P, then $QP = Q'P$ and $m\angle QPQ' = x°$, or
- If Q is the center of rotation P, then the image of Q is Q.



A 40° counterclockwise rotation is shown at the right. Rotations can be *clockwise* or *counterclockwise*. In this chapter, all rotations are counterclockwise.

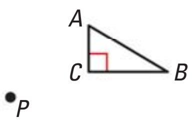
DIRECTION OF ROTATION

clockwise

counterclockwise

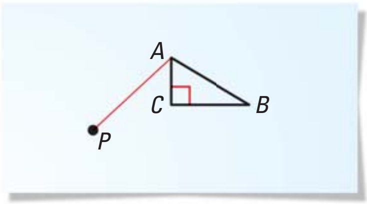
EXAMPLE 1 Draw a rotation

Draw a 120° rotation of $\triangle ABC$ about P.

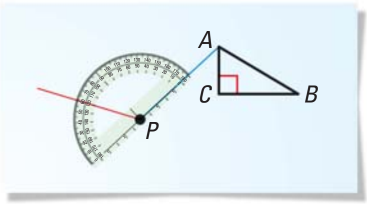


Solution

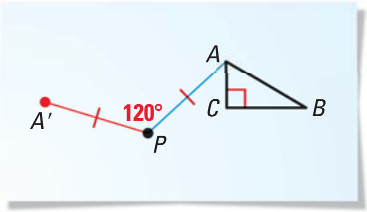
STEP 1 **Draw** a segment from A to P.



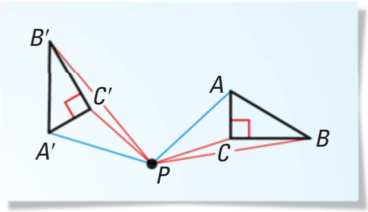
STEP 2 **Draw** a ray to form a 120° angle with $\overline{PA}$.



STEP 3 **Draw** A' so that $PA' = PA$.



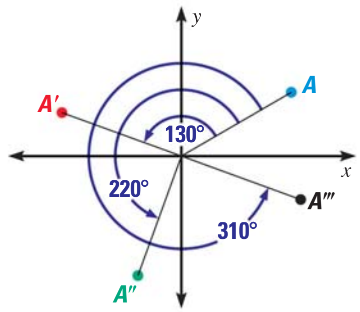
STEP 4 **Repeat** Steps 1–3 for each vertex. Draw $\triangle A'B'C'$.



USE ROTATIONS

You can rotate a figure more than 360°. However, the effect is the same as rotating the figure by the angle minus 360°.

ROTATIONS ABOUT THE ORIGIN You can rotate a figure more than 180°. The diagram shows rotations of point A 130°, 220°, and 310° about the origin. A rotation of 360° returns a figure to its original coordinates.



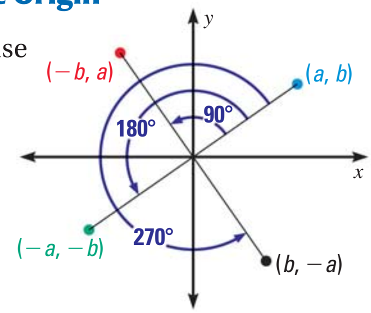
There are coordinate rules that can be used to find the coordinates of a point after rotations of 90°, 180°, or 270° about the origin.

KEY CONCEPT *For Your Notebook*

Coordinate Rules for Rotations about the Origin

When a point (a, b) is rotated counterclockwise about the origin, the following are true:

1. For a rotation of 90°, $(a, b) \to (-b, a)$.
2. For a rotation of 180°, $(a, b) \to (-a, -b)$.
3. For a rotation of 270°, $(a, b) \to (b, -a)$.



EXAMPLE 2 Rotate a figure using the coordinate rules

Graph quadrilateral $RSTU$ with vertices $R(3, 1)$, $S(5, 1)$, $T(5, -3)$, and $U(2, -1)$. Then rotate the quadrilateral 270° about the origin.

ANOTHER WAY

For an alternative method for solving the problem in Example 2, turn to page 606 for the **Problem Solving Workshop**.

Solution

Graph $RSTU$. Use the coordinate rule for a 270° rotation to find the images of the vertices.

$$(a, b) \to (b, -a)$$

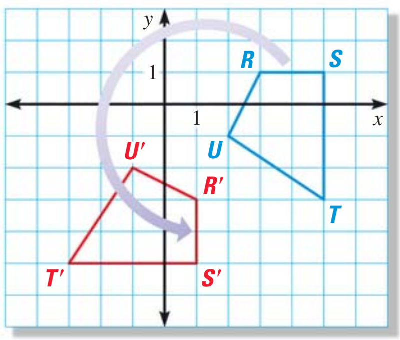
$R(3, 1) \to R'(1, -3)$

$S(5, 1) \to S'(1, -5)$

$T(5, -3) \to T'(-3, -5)$

$U(2, -1) \to U'(-1, -2)$

Graph the image $R'S'T'U'$.



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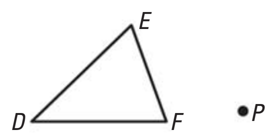
✓ GUIDED PRACTICE for Examples 1 and 2

1. Trace $\triangle DEF$ and P. Then draw a 50° rotation of $\triangle DEF$ about P.

2. Graph $\triangle JKL$ with vertices $J(3, 0)$, $K(4, 3)$, and $L(6, 0)$. Rotate the triangle 90° about the origin.



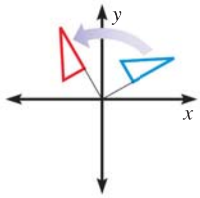
USING MATRICES You can find certain images of a polygon rotated about the origin using matrix multiplication. Write the rotation matrix to the left of the polygon matrix, then multiply.

KEY CONCEPT *For Your Notebook*

Rotation Matrices (Counterclockwise)

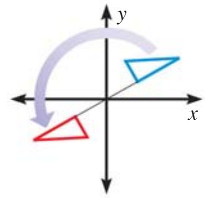
90° rotation

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$



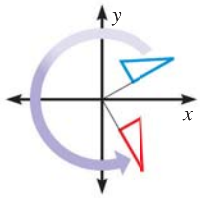
180° rotation

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$



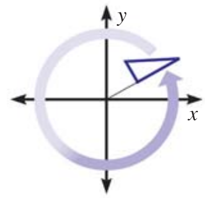
270° rotation

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$



360° rotation

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



READ VOCABULARY

Notice that a 360° rotation returns the figure to its original position. Multiplying by the matrix that represents this rotation gives you the polygon matrix you started with, which is why it is also called the *identity matrix*.

EXAMPLE 3 Use matrices to rotate a figure

Trapezoid *EFGH* has vertices *E*(−3, 2), *F*(−3, 4), *G*(1, 4), and *H*(2, 2). Find the image matrix for a 180° rotation of *EFGH* about the origin. Graph *EFGH* and its image.

Solution

STEP 1 **Write** the polygon matrix:

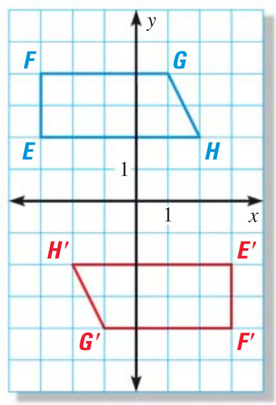
$$\begin{array}{c} \begin{array}{cccc} E & F & G & H \end{array} \\ \begin{bmatrix} -3 & -3 & 1 & 2 \\ 2 & 4 & 4 & 2 \end{bmatrix} \end{array}$$

STEP 2 **Multiply** by the matrix for a 180° rotation.

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{array}{c} \begin{array}{cccc} E & F & G & H \end{array} \\ \begin{bmatrix} -3 & -3 & 1 & 2 \\ 2 & 4 & 4 & 2 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} E' & F' & G' & H' \end{array} \\ \begin{bmatrix} 3 & 3 & -1 & -2 \\ -2 & -4 & -4 & -2 \end{bmatrix} \end{array}$$

Rotation matrix | **Polygon matrix** | **Image matrix**

STEP 3 **Graph** the preimage *EFGH*. Graph the image *E′F′G′H′*.



AVOID ERRORS

Because matrix multiplication is not commutative, you should always write the rotation matrix first, then the polygon matrix.

✓ GUIDED PRACTICE for Example 3

Use the quadrilateral *EFGH* in Example 3. Find the image matrix after the rotation about the origin. Graph the image.

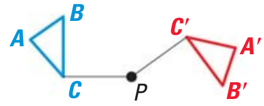
3. 90° **4.** 270° **5.** 360°

THEOREM — For Your Notebook

THEOREM 9.3 Rotation Theorem

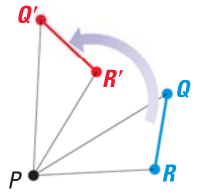
A rotation is an isometry.

Proof: Exs. 33–35, p. 604

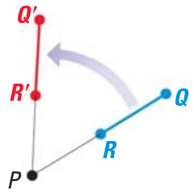


$\triangle ABC \cong \triangle A'B'C'$

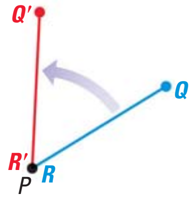
CASES OF THEOREM 9.3 To prove the Rotation Theorem, you need to show that a rotation preserves the length of a segment. Consider a segment $\overline{QR}$ rotated about point P to produce $\overline{Q'R'}$. There are three cases to prove:



Case 1 R, Q, and P are noncollinear.



Case 2 R, Q, and P are collinear.



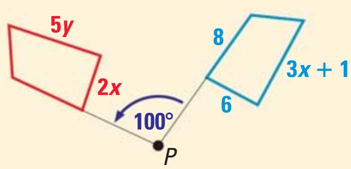
Case 3 P and R are the same point.

EXAMPLE 4 Standardized Test Practice

The quadrilateral is rotated about P. What is the value of y?

Ⓐ $\frac{8}{5}$ Ⓑ 2

Ⓒ 3 Ⓓ 10



Solution

By Theorem 9.3, the rotation is an isometry, so corresponding side lengths are equal. Then $2x = 6$, so $x = 3$. Now set up an equation to solve for y.

$5y = 3x + 1$ **Corresponding lengths in an isometry are equal.**

$5y = 3(3) + 1$ **Substitute 3 for x.**

$y = 2$ **Solve for y.**

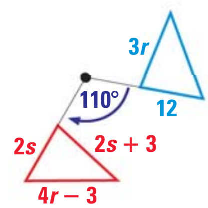
▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Example 4

6. Find the value of r in the rotation of the triangle.

Ⓐ 3 Ⓑ 5

Ⓒ 6 Ⓓ 15



9.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 13, 15, and 29

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 20, 21, 23, 24, and 37

SKILL PRACTICE

1. **VOCABULARY** What is a *center of rotation*?

2. ★ **WRITING** *Compare* the coordinate rules and the rotation matrices for a rotation of 90°.

EXAMPLE 1 on p. 598 for Exs. 3–11

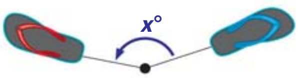
IDENTIFYING TRANSFORMATIONS **Identify the type of transformation, *translation*, *reflection*, or *rotation*, in the photo. *Explain* your reasoning.**

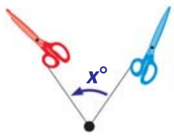
3. 

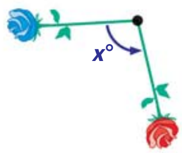
4. 

5. 

ANGLE OF ROTATION **Match the diagram with the angle of rotation.**

6. 

7. 

8. 

A. 70° **B.** 100° **C.** 150°

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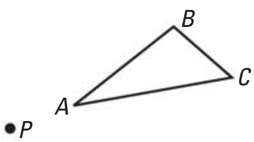
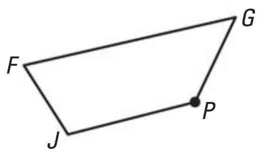
ROTATING A FIGURE **Trace the polygon and point *P* on paper. Then draw a rotation of the polygon the given number of degrees about *P*.**

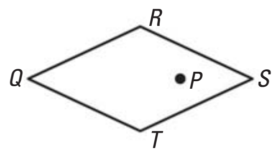
9. 30°



10. 150°



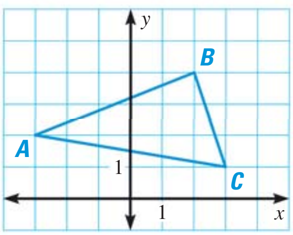
11. 130°



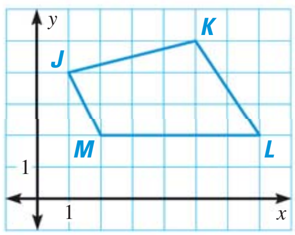
EXAMPLE 2 on p. 599 for Exs. 12–14

USING COORDINATE RULES **Rotate the figure the given number of degrees about the origin. List the coordinates of the vertices of the image.**

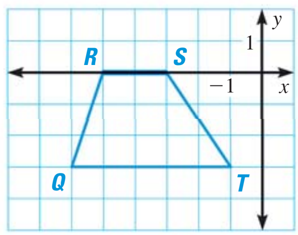
12. 90°



13. 180°



14. 270°



EXAMPLE 3
on p. 600
for Exs. 15–19

USING MATRICES Find the image matrix that represents the rotation of the polygon about the origin. Then graph the polygon and its image.

15. $\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 1 & 5 & 4 \\ 4 & 6 & 3 \end{bmatrix} \end{array}$; 90°

16. $\begin{array}{c} \begin{array}{ccc} J & K & L \end{array} \\ \begin{bmatrix} 1 & 2 & 0 \\ 1 & -1 & -3 \end{bmatrix} \end{array}$; 180°

17. $\begin{array}{c} \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{bmatrix} -4 & 2 & 2 & -4 \\ -4 & -2 & -5 & -7 \end{bmatrix} \end{array}$; 270°

ERROR ANALYSIS The endpoints of $\overline{AB}$ are $A(-1, 1)$ and $B(2, 3)$. *Describe* and correct the error in setting up the matrix multiplication for a 270° rotation about the origin.

18.

270° rotation of $\overline{AB}$

$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}$

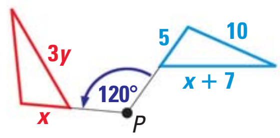
19.

270° rotation of $\overline{AB}$

$\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

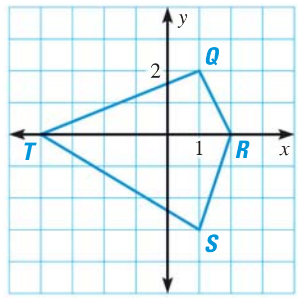
EXAMPLE 4
on p. 601
for Exs. 20–21

20. ★ **MULTIPLE CHOICE** What is the value of y in the rotation of the triangle about P?



Ⓐ 4 Ⓑ 5 Ⓒ $\frac{17}{3}$ Ⓓ 10

21. ★ **MULTIPLE CHOICE** Suppose quadrilateral $QRST$ is rotated 180° about the origin. In which quadrant is Q'?



Ⓐ I Ⓑ II Ⓒ III Ⓓ IV

22. **FINDING A PATTERN** The vertices of $\triangle ABC$ are $A(2, 0)$, $B(3, 4)$, and $C(5, 2)$. Make a table to show the vertices of each image after a 90°, 180°, 270°, 360°, 450°, 540°, 630°, and 720° rotation. What would be the coordinates of A' after a rotation of 1890°? *Explain.*

23. ★ **MULTIPLE CHOICE** A rectangle has vertices at (4, 0), (4, 2), (7, 0), and (7, 2). Which image has a vertex at the origin?

Ⓐ Translation right 4 units and down 2 units

Ⓑ Rotation of 180° about the origin

Ⓒ Reflection in the line $x = 4$

Ⓓ Rotation of 180° about the point (2, 0)

24. ★ **SHORT RESPONSE** Rotate the triangle in Exercise 12 90° about the origin. Show that corresponding sides of the preimage and image are perpendicular. *Explain.*

25. **VISUAL REASONING** A point in space has three coordinates (x, y, z). What is the image of point (3, 2, 0) rotated 180° about the origin in the xz-plane? (*See* Exercise 30, page 585.)

CHALLENGE Rotate the line the given number of degrees (a) about the x-intercept and (b) about the y-intercept. Write the equation of each image.

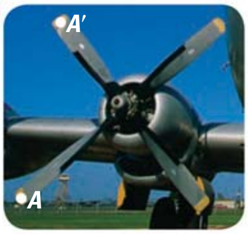
26. $y = 2x - 3$; 90°

27. $y = -x + 8$; 180°

28. $y = \frac{1}{2}x + 5$; 270°

PROBLEM SOLVING

ANGLE OF ROTATION **Use the photo to find the angle of rotation that maps *A* onto *A*′. *Explain* your reasoning.**

29. 

30. 

31. 

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32. **REVOLVING DOOR** You enter a revolving door and rotate the door 180°. What does this mean in the context of the situation? Now, suppose you enter a revolving door and rotate the door 360°. What does this mean in the context of the situation? *Explain*.

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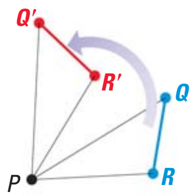
33. **PROVING THEOREM 9.3** Copy and complete the proof of Case 1.



Case 1 The segment is noncollinear with the center of rotation.

GIVEN ▶ A rotation about P maps Q to Q' and R to R'.

PROVE ▶ $QR = Q'R'$

STATEMENTS	REASONS
1. $PQ = PQ'$, $PR = PR'$, $m\angle QPQ' = m\angle RPR'$	1. Definition of ?
2. $m\angle QPQ' = m\angle QPR' + m\angle R'PQ'$ $m\angle RPR' = m\angle RPQ + m\angle QPR'$	2. ?
3. $m\angle QPR' + m\angle R'PQ' = m\angle RPQ + m\angle QPR'$	3. ? Property of Equality
4. $m\angle QPR = m\angle Q'PR'$	4. ? Property of Equality
5. ? $\cong$?	5. SAS Congruence Postulate
6. $\overline{QR} \cong \overline{Q'R'}$	6. ?
7. $QR = Q'R'$	7. ?

PROVING THEOREM 9.3 **Write a proof for Case 2 and Case 3. (Refer to the diagrams on page 601.)**

34. **Case 2** The segment is collinear with the center of rotation.

GIVEN ▶ A rotation about P maps Q to Q' and R to R'. P, Q, and R are collinear.

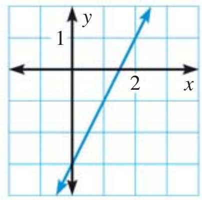
PROVE ▶ $QR = Q'R'$

35. **Case 3** The center of rotation is one endpoint of the segment.

GIVEN ▶ A rotation about P maps Q to Q' and R to R'. P and R are the same point.

PROVE ▶ $QR = Q'R'$

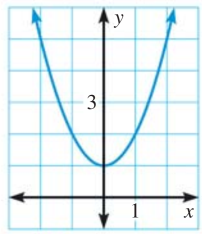
36. **MULTI-STEP PROBLEM** Use the graph of $y = 2x - 3$.



a. Rotate the line 90°, 180°, 270°, and 360° about the origin. *Describe* the relationship between the equation of the preimage and each image.

b. Do you think that the relationships you described in part (a) are true for *any* line? *Explain* your reasoning.

37. ★ **EXTENDED RESPONSE** Use the graph of the quadratic equation $y = x^2 + 1$ at the right.



a. Rotate the *parabola* by replacing y with x and x with y in the original equation, then graph this new equation.

b. What is the angle of rotation?

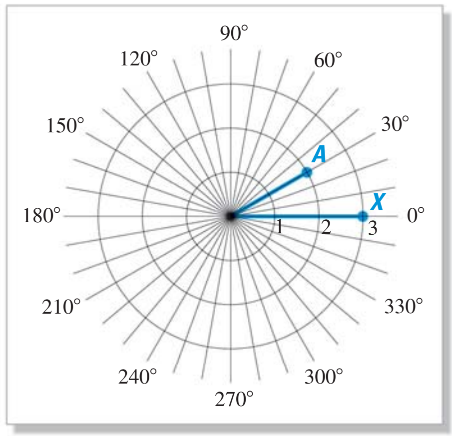
c. Are the image and the preimage both functions? *Explain*.

TWO ROTATIONS **The endpoints of $\overline{FG}$ are $F(1, 2)$ and $G(3, 4)$. Graph $\overline{F'G'}$ and $\overline{F''G''}$ after the given rotations.**

38. **Rotation:** 90° about the origin
Rotation: 180° about (0, 4)

39. **Rotation:** 270° about the origin
Rotation: 90° about (−2, 0)

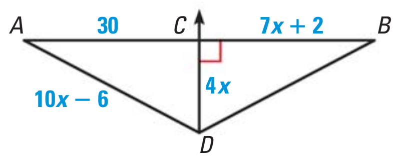
40. **CHALLENGE** A polar coordinate system locates a point in a plane by its distance from the origin O and by the measure of an angle with its vertex at the origin. For example, the point $A(2, 30°)$ at the right is 2 units from the origin and $m\angle XOA = 30°$. What are the polar coordinates of the image of point A after a 90° rotation? 180° rotation? 270° rotation? *Explain*.



MIXED REVIEW

PREVIEW
Prepare for Lesson 9.5 in Exs. 41–43.

In the diagram, $\overrightarrow{DC}$ is the perpendicular bisector of $\overline{AB}$. *(p. 303)*



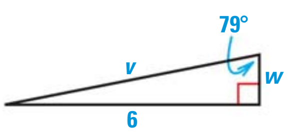
41. What segment lengths are equal?

42. What is the value of x?

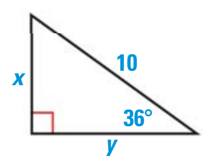
43. Find BD. *(p. 433)*

Use a sine or cosine ratio to find the value of each variable. Round decimals to the nearest tenth. *(p. 473)*

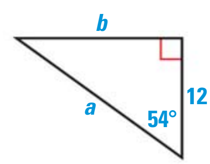
44.



45.



46.



PROBLEM SOLVING WORKSHOP
LESSON 9.4

Using ALTERNATIVE METHODS

Another Way to Solve Example 2, page 599

MULTIPLE REPRESENTATIONS In Example 2 on page 599, you saw how to use a coordinate rule to rotate a figure. You can also *use tracing paper* and move a copy of the figure around the coordinate plane.

PROBLEM

Graph quadrilateral *RSTU* with vertices $R(3, 1)$, $S(5, 1)$, $T(5, -3)$, and $U(2, -1)$. Then rotate the quadrilateral 270° about the origin.

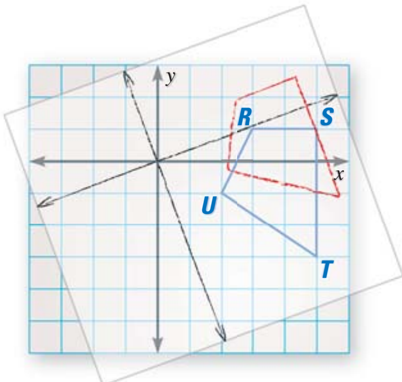
METHOD

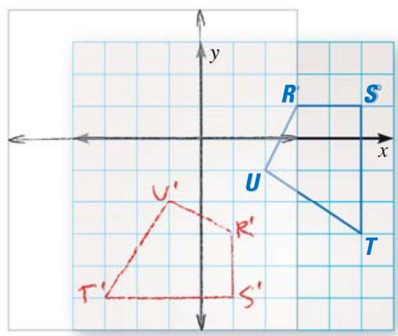
Using Tracing Paper You can use tracing paper to rotate a figure.

STEP 1 **Graph** the original figure in the coordinate plane.

STEP 2 **Trace** the quadrilateral and the axes on tracing paper.

STEP 3 **Rotate** the tracing paper 270°. Then transfer the resulting image onto the graph paper.





PRACTICE

1. **GRAPH** Graph quadrilateral *ABCD* with vertices $A(2, -2)$, $B(5, -3)$, $C(4, -5)$, and $D(2, -4)$. Then rotate the quadrilateral 180° about the origin using tracing paper.

2. **GRAPH** Graph $\triangle RST$ with vertices $R(0, 6)$, $S(1, 4)$, and $T(-2, 3)$. Then rotate the triangle 270° about the origin using tracing paper.

3. **SHORT RESPONSE** *Explain* why rotating a figure 90° clockwise is the same as rotating the figure 270° counterclockwise.

4. **SHORT RESPONSE** *Explain* how you could use tracing paper to do a reflection.

5. **REASONING** If you rotate the point (3, 4) 90° about the origin, what happens to the *x*-coordinate? What happens to the *y*-coordinate?

6. **GRAPH** Graph $\triangle JKL$ with vertices $J(4, 8)$, $K(4, 6)$, and $L(2, 6)$. Then rotate the triangle 90° about the point $(-1, 4)$ using tracing paper.

Investigating Geometry ACTIVITY *Use before Lesson 9.5*

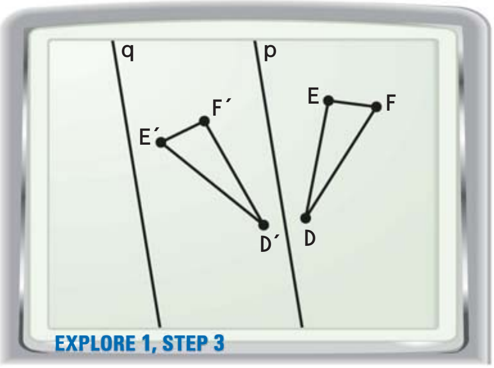
@HomeTutor classzone.com Keystrokes

9.5 Double Reflections

MATERIALS • graphing calculator or computer

QUESTION What happens when you reflect a figure in two lines in a plane?

EXPLORE 1 Double reflection in parallel lines



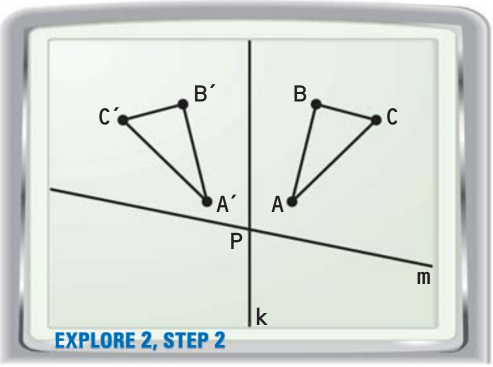
STEP 1 ***Draw a scalene triangle*** Construct a scalene triangle like the one at the right. Label the vertices D, E, and F.

STEP 2 ***Draw parallel lines*** Construct two parallel lines p and q on one side of the triangle. Make sure that the lines do not intersect the triangle. Save as "EXPLORE1".

STEP 3 ***Reflect triangle*** Reflect $\triangle DEF$ in line p. Reflect $\triangle D'E'F'$ in line q. How is $\triangle D''E''F''$ related to $\triangle DEF$?

STEP 4 ***Make conclusion*** Drag line q. Does the relationship appear to be true if p and q are not on the same side of the figure?

EXPLORE 2 Double reflection in intersecting lines



STEP 1 ***Draw intersecting lines*** Follow Step 1 in Explore 1 for $\triangle ABC$. Change Step 2 from parallel lines to intersecting lines k and m. Make sure that the lines do not intersect the triangle. Label the point of intersection of lines k and m as P. Save as "EXPLORE2".

STEP 2 ***Reflect triangle*** Reflect $\triangle ABC$ in line k. Reflect $\triangle A'B'C'$ in line m. How is $\triangle A''B''C''$ related to $\triangle ABC$?

STEP 3 ***Measure angles*** Measure $\angle APA''$ and the acute angle formed by lines k and m. What is the relationship between these two angles? Does this relationship remain true when you move lines k and m?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. What other transformation maps a figure onto the same image as a reflection in two parallel lines?
2. What other transformation maps a figure onto the same image as a reflection in two intersecting lines?

9.5 Apply Compositions of Transformations

Before You performed rotations, reflections, or translations.

Now You will perform combinations of two or more transformations.

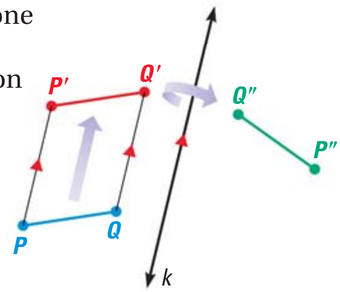
Why? So you can describe the transformations that represent a rowing crew, as in Ex. 30.

Key Vocabulary
- **glide reflection**
- **composition of transformations**

A translation followed by a reflection can be performed one after the other to produce a *glide reflection*. A translation can be called a glide. A **glide reflection** is a transformation in which every point P is mapped to a point P'' by the following steps.

STEP 1 **First,** a translation maps P to P'.

STEP 2 **Then,** a reflection in a line k parallel to the direction of the translation maps P' to P''.



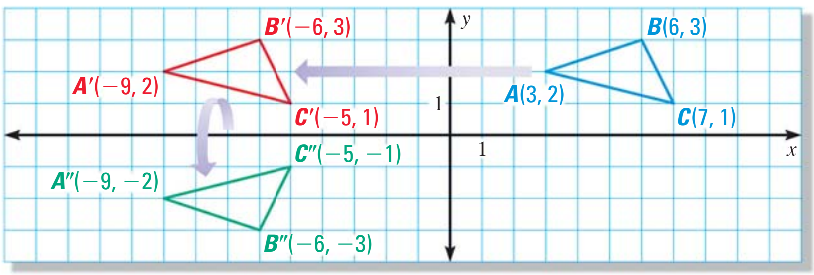
EXAMPLE 1 Find the image of a glide reflection

The vertices of $\triangle ABC$ are $A(3, 2)$, $B(6, 3)$, and $C(7, 1)$. Find the image of $\triangle ABC$ after the glide reflection.

Translation: $(x, y) \to (x - 12, y)$
Reflection: in the x-axis

Solution

Begin by graphing $\triangle ABC$. Then graph $\triangle A'B'C'$ after a translation 12 units left. Finally, graph $\triangle A''B''C''$ after a reflection in the x-axis.



AVOID ERRORS
The line of reflection must be parallel to the direction of the translation to be a glide reflection.

✓ GUIDED PRACTICE for Example 1

1. Suppose $\triangle ABC$ in Example 1 is translated 4 units down, then reflected in the y-axis. What are the coordinates of the vertices of the image?

2. In Example 1, *describe* a glide reflection from $\triangle A''B''C''$ to $\triangle ABC$.

COMPOSITIONS When two or more transformations are combined to form a single transformation, the result is a **composition of transformations**. A glide reflection is an example of a composition of transformations.

In this lesson, a composition of transformations uses isometries, so the final image is congruent to the preimage. This suggests the Composition Theorem.

THEOREM *For Your Notebook*

THEOREM 9.4 Composition Theorem

The composition of two (or more) isometries is an isometry.

Proof: Exs. 35–36, p. 614

EXAMPLE 2 Find the image of a composition

The endpoints of $\overline{RS}$ are $R(1, -3)$ and $S(2, -6)$. Graph the image of $\overline{RS}$ after the composition.

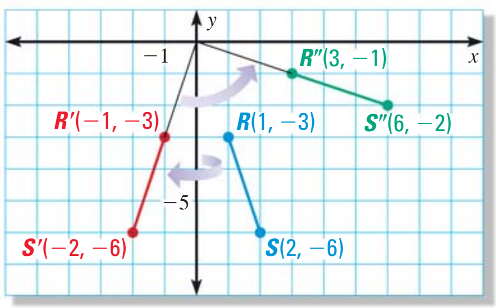
Reflection: in the y-axis
Rotation: 90° about the origin

Solution

STEP 1 **Graph** $\overline{RS}$.

STEP 2 **Reflect** $\overline{RS}$ in the y-axis. $\overline{R'S'}$ has endpoints $R'(-1, -3)$ and $S'(-2, -6)$.

STEP 3 **Rotate** $\overline{R'S'}$ 90° about the origin. $\overline{R''S''}$ has endpoints $R''(3, -1)$ and $S''(6, -2)$.



AVOID ERRORS
Unless you are told otherwise, do the transformations in the order given.

TWO REFLECTIONS Compositions of two reflections result in either a translation or a rotation, as described in Theorems 9.5 and 9.6.

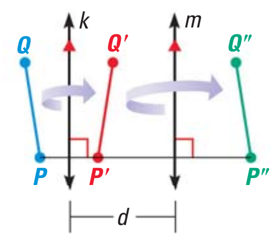
THEOREM *For Your Notebook*

THEOREM 9.5 Reflections in Parallel Lines Theorem

If lines k and m are parallel, then a reflection in line k followed by a reflection in line m is the same as a translation.

If P'' is the image of P, then:

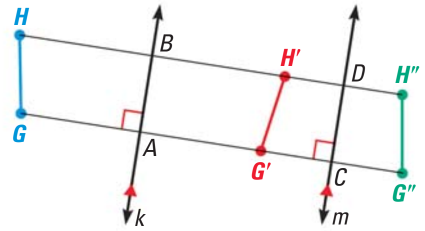
1. $\overline{PP''}$ is perpendicular to k and m, and
2. $PP'' = 2d$, where d is the distance between k and m.



Proof: Ex. 37, p. 614

EXAMPLE 3 Use Theorem 9.5

In the diagram, a reflection in line k maps $\overline{GH}$ to $\overline{G'H'}$. A reflection in line m maps $\overline{G'H'}$ to $\overline{G''H''}$. Also, $HB = 9$ and $DH'' = 4$.



a. Name any segments congruent to each segment: $\overline{HG}$, $\overline{HB}$, and $\overline{GA}$.

b. Does $AC = BD$? Explain.

c. What is the length of $\overline{GG''}$?

Solution

a. $\overline{HG} \cong \overline{H'G'}$, and $\overline{HG} \cong \overline{H''G''}$. $\overline{HB} \cong \overline{H'B}$. $\overline{GA} \cong \overline{G'A}$.

b. Yes, $AC = BD$ because $\overline{GG''}$ and $\overline{HH''}$ are perpendicular to both k and m, so $\overline{BD}$ and $\overline{AC}$ are opposite sides of a rectangle.

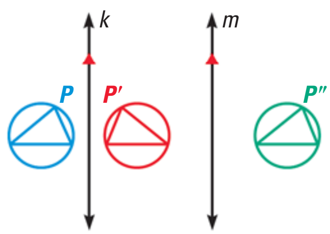
c. By the properties of reflections, $H'B = 9$ and $H'D = 4$. Theorem 9.5 implies that $GG'' = HH'' = 2 \cdot BD$, so the length of $\overline{GG''}$ is $2(9 + 4)$, or 26 units.

✓ GUIDED PRACTICE for Examples 2 and 3

3. Graph $\overline{RS}$ from Example 2. Do the rotation first, followed by the reflection. Does the order of the transformations matter? *Explain.*

4. In Example 3, part (c), *explain* how you know that $GG'' = HH''$.

Use the figure below for Exercises 5 and 6. The distance between line k and line m is 1.6 centimeters.



5. The preimage is reflected in line k, then in line m. *Describe* a single transformation that maps the blue figure to the green figure.

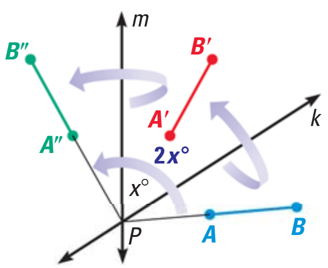
6. What is the distance between P and P''? If you draw $\overline{PP'}$, what is its relationship with line k? *Explain.*

THEOREM *For Your Notebook*

THEOREM 9.6 Reflections in Intersecting Lines Theorem

If lines k and m intersect at point P, then a reflection in k followed by a reflection in m is the same as a rotation about point P.

The angle of rotation is $2x°$, where $x°$ is the measure of the acute or right angle formed by k and m.

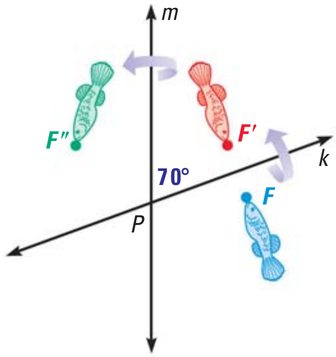


$m\angle BPB'' = 2x°$

Proof: Ex. 38, p. 614

EXAMPLE 4 Use Theorem 9.6

In the diagram, the figure is reflected in line *k*. The image is then reflected in line *m*. Describe a single transformation that maps *F* to *F*″.



Solution

The measure of the acute angle formed between lines *k* and *m* is 70°. So, by Theorem 9.6, a single transformation that maps *F* to *F*″ is a 140° rotation about point *P*.

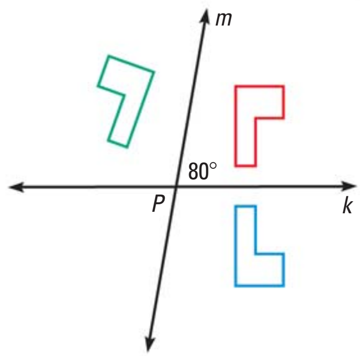
You can check that this is correct by tracing lines *k* and *m* and point *F*, then rotating the point 140°.

Animated Geometry at classzone.com

✓ GUIDED PRACTICE for Example 4

7. In the diagram at the right, the preimage is reflected in line *k*, then in line *m*. *Describe* a single transformation that maps the blue figure onto the green figure.

8. A rotation of 76° maps *C* to *C*′. To map *C* to *C*′ using two reflections, what is the angle formed by the intersecting lines of reflection?



9.5 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 17, and 27
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 25, 29, and 34

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: In a glide reflection, the direction of the translation must be __?__ to the line of reflection.

2. ★ **WRITING** *Explain* why a glide reflection is an isometry.

EXAMPLE 1 on p. 608 for Exs. 3–6

GLIDE REFLECTION **The endpoints of $\overline{CD}$ are $C(2, -5)$ and $D(4, 0)$. Graph the image of $\overline{CD}$ after the glide reflection.**

3. **Translation:** $(x, y) \to (x, y - 1)$
   Reflection: in the y-axis

4. **Translation:** $(x, y) \to (x - 3, y)$
   Reflection: in $y = -1$

5. **Translation:** $(x, y) \to (x, y + 4)$
   Reflection: in $x = 3$

6. **Translation:** $(x, y) \to (x + 2, y + 2)$
   Reflection: in $y = x$

EXAMPLE 2
on p. 609
for Exs. 7–14

GRAPHING COMPOSITIONS **The vertices of $\triangle PQR$ are $P(2, 4)$, $Q(6, 0)$, and $R(7, 2)$. Graph the image of $\triangle PQR$ after a composition of the transformations in the order they are listed.**

7. **Translation:** $(x, y) \rightarrow (x, y - 5)$
Reflection: in the y-axis

8. **Translation:** $(x, y) \rightarrow (x - 3, y + 2)$
Rotation: 90° about the origin

9. **Translation:** $(x, y) \rightarrow (x + 12, y + 4)$
Translation: $(x, y) \rightarrow (x - 5, y - 9)$

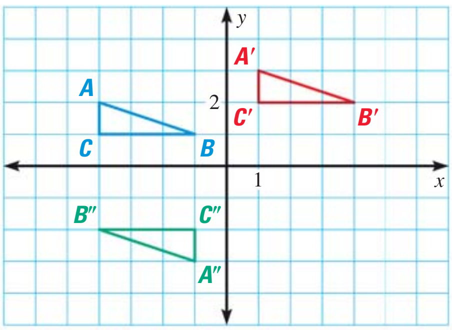
10. **Reflection:** in the x-axis
Rotation: 90° about the origin

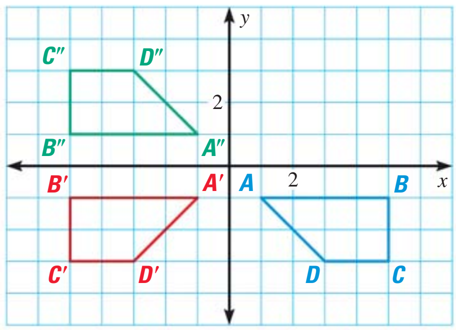
REVERSING ORDERS **Graph $\overline{F''G''}$ after a composition of the transformations in the order they are listed. Then perform the transformations in reverse order. Does the order affect the final image $\overline{F''G''}$?**

11. $F(-5, 2)$, $G(-2, 4)$
Translation: $(x, y) \rightarrow (x + 3, y - 8)$
Reflection: in the x-axis

12. $F(-1, -8)$, $G(-6, -3)$
Reflection: in the line $y = 2$
Rotation: 90° about the origin

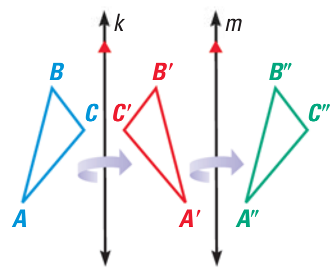
DESCRIBING COMPOSITIONS ***Describe*** **the composition of transformations.**

13.


14.


EXAMPLE 3
on p. 610
for Exs. 15–19

USING THEOREM 9.5 **In the diagram, $k \parallel m$, $\triangle ABC$ is reflected in line k, and $\triangle A'B'C'$ is reflected in line m.**



15. A translation maps $\triangle ABC$ onto which triangle?

16. Which lines are perpendicular to $\overleftrightarrow{AA''}$?

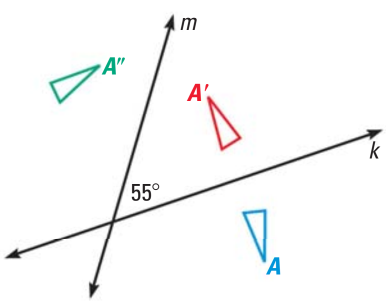
17. Name two segments parallel to $\overleftrightarrow{BB''}$.

18. If the distance between k and m is 2.6 inches, what is the length of $\overleftrightarrow{CC''}$?

19. Is the distance from B' to m the same as the distance from B'' to m? *Explain.*

EXAMPLE 4
on p. 611
for Exs. 20–21

USING THEOREM 9.6 **Find the angle of rotation that maps A onto A''.**

20.


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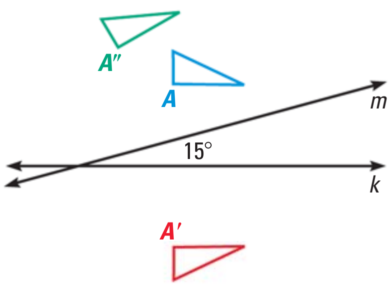
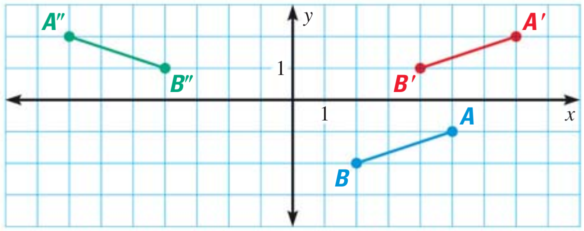


= **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

22. **ERROR ANALYSIS** A student described the translation of $\overline{AB}$ to $\overline{A'B'}$ followed by the reflection of $\overline{A'B'}$ to $\overline{A''B''}$ in the y-axis as a glide reflection. *Describe* and correct the student's error.



USING MATRICES The vertices of $\triangle PQR$ are $P(1, 4)$, $Q(3, -2)$, and $R(7, 1)$. Use matrix operations to find the image matrix that represents the composition of the given transformations. Then graph $\triangle PQR$ and its image.

23. **Translation:** $(x, y) \to (x, y + 5)$
Reflection: in the y-axis

24. **Reflection:** in the x-axis
Translation: $(x, y) \to (x - 9, y - 4)$

25. ★ **OPEN-ENDED MATH** Sketch a polygon. Apply three transformations of your choice on the polygon. What can you say about the congruence of the preimage and final image after multiple transformations? *Explain*.

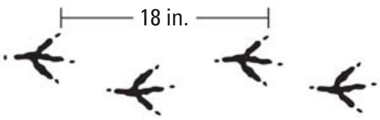
26. **CHALLENGE** The vertices of $\triangle JKL$ are $J(1, -3)$, $K(2, 2)$, and $L(3, 0)$. Find the image of the triangle after a 180° rotation about the point $(-2, 2)$, followed by a reflection in the line $y = -x$.

PROBLEM SOLVING

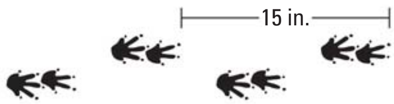
EXAMPLE 1
on p. 608
for Exs. 27–30

ANIMAL TRACKS The left and right prints in the set of animal tracks can be related by a glide reflection. Copy the tracks and *describe* a translation and reflection that combine to create the glide reflection.

27. bald eagle (2 legs)



28. armadillo (4 legs)



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29. ★ **MULTIPLE CHOICE** Which is *not* a glide reflection?

Ⓐ The teeth of a closed zipper

Ⓑ The tracks of a walking duck

Ⓒ The keys on a computer keyboard

Ⓓ The red squares on two adjacent rows of a checkerboard

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30. **ROWING** *Describe* the transformations that are combined to represent an eight-person rowing shell.

SWEATER PATTERNS **In Exercises 31–33, *describe* the transformations that are combined to make each sweater pattern.**

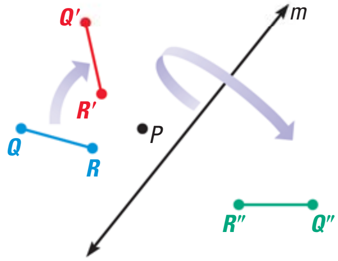
31. 

32. 

33. 

34. ★ **SHORT RESPONSE** Use Theorem 9.5 to *explain* how you can make a glide reflection using three reflections. How are the lines of reflection related?

35. **PROVING THEOREM 9.4** Write a plan for proof for one case of the Composition Theorem.

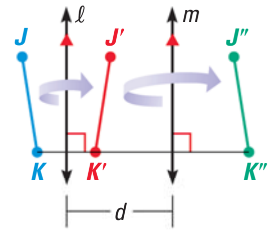


GIVEN ▶ A rotation about P maps Q to Q' and R to R'. A reflection in m maps Q' to Q'' and R' to R''.

PROVE ▶ $QR = Q''R''$

36. **PROVING THEOREM 9.4** A composition of a rotation and a reflection, as in Exercise 35, is one case of the Composition Theorem. List all possible cases, and prove the theorem for another pair of compositions.

37. **PROVING THEOREM 9.5** Prove the Reflection in Parallel Lines Theorem.

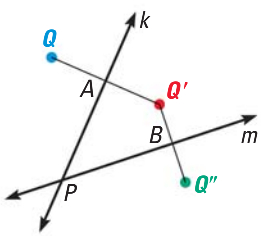


GIVEN ▶ A reflection in line ℓ maps $\overline{JK}$ to $\overline{J'K'}$, a reflection in line m maps $\overline{J'K'}$ to $\overline{J''K''}$, and $\ell \parallel m$.

PROVE ▶ **a.** $\overleftrightarrow{KK''}$ is perpendicular to ℓ and m.

b. $KK'' = 2d$, where d is the distance between ℓ and m.

38. **PROVING THEOREM 9.6** Prove the Reflection in Intersecting Lines Theorem.



GIVEN ▶ Lines k and m intersect at point P. Q is any point not on k or m.

PROVE ▶ **a.** If you reflect point Q in k, and then reflect its image Q' in m, Q'' is the image of Q after a rotation about point P.

b. $m\angle QPQ'' = 2(m\angle APB)$

Plan for Proof First show $k \perp \overline{QQ'}$ and $\overline{QA} \cong \overline{Q'A}$. Then show $\triangle QAP \cong \triangle Q'AP$. In the same way, show $\triangle Q'BP \cong \triangle Q''BP$. Use congruent triangles and substitution to show that $\overline{QP} \cong \overline{Q''P}$. That proves part (a) by the definition of a rotation. Then use congruent triangles to prove part (b).

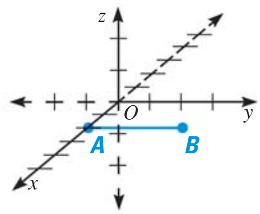
39. **VISUAL REASONING** You are riding a bicycle along a flat street.

a. What two transformations does the wheel's motion use?

b. *Explain* why this is not a composition of transformations.

★ = STANDARDIZED TEST PRACTICE

40. **MULTI-STEP PROBLEM** A point in space has three coordinates (x, y, z). From the origin, a point can be forward or back on the x-axis, left or right on the y-axis, and up or down on the z-axis. The endpoints of segment $\overline{AB}$ in space are $A(2, 0, 0)$ and $B(2, 3, 0)$, as shown at the right.



a. Rotate $\overline{AB}$ 90° about the x-axis with center of rotation A. What are the coordinates of $\overline{A'B'}$?

b. Translate $\overline{A'B'}$ using the vector $\langle 4, 0, -1\rangle$. What are the coordinates of $\overline{A''B''}$?

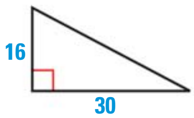
41. **CHALLENGE** *Justify* the following conjecture or provide a counterexample.

Conjecture When performing a composition of two transformations of the *same type*, order does not matter.

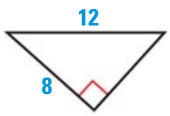
MIXED REVIEW

Find the unknown side length. Write your answer in simplest radical form. *(p. 433)*

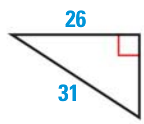
42.



43.



44.



PREVIEW
Prepare for Lesson 9.6 in Exs. 45–48.

The coordinates of $\triangle PQR$ are $P(3, 1)$, $Q(3, 3)$, and $R(6, 1)$. Graph the image of the triangle after the translation. *(p. 572)*

45. $(x, y) \to (x + 3, y)$

46. $(x, y) \to (x - 3, y)$

47. $(x, y) \to (x, y + 2)$

48. $(x, y) \to (x + 3, y + 2)$

QUIZ *for Lessons 9.3–9.5*

The vertices of $\triangle ABC$ are $A(7, 1)$, $B(3, 5)$, and $C(10, 7)$. Graph the reflection in the line. *(p. 589)*

1. y-axis

2. $x = -4$

3. $y = -x$

Find the coordinates of the image of $P(2, -3)$ after the rotation about the origin. *(p. 598)*

4. 180° rotation

5. 90° rotation

6. 270° rotation

The vertices of $\triangle PQR$ are $P(-8, 8)$, $Q(-5, 0)$, and $R(-1, 3)$. Graph the image of $\triangle PQR$ after a composition of the transformations in the order they are listed. *(p. 608)*

7. **Translation:** $(x, y) \to (x + 6, y)$
Reflection: in the y-axis

8. **Reflection:** in the line $y = -2$
Rotation: 90° about the origin

9. **Translation:** $(x, y) \to (x - 5, y)$
Translation: $(x, y) \to (x + 2, y + 7)$

10. **Rotation:** 180° about the origin
Translation: $(x, y) \to (x + 4, y - 3)$

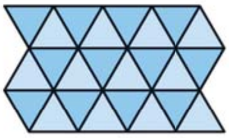
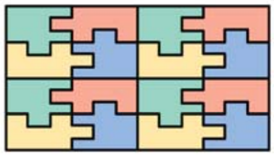
Extension
Use after Lesson 9.5

Tessellations

GOAL Make tessellations and discover their properties.

Key Vocabulary
- tessellation

A **tessellation** is a collection of figures that cover a plane with no gaps or overlaps. You can use transformations to make tessellations.

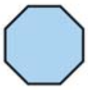



A *regular tessellation* is a tessellation of congruent regular polygons. In the figures above, the tessellation of equilateral triangles is a regular tessellation.

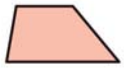
EXAMPLE 1 Determine whether shapes tessellate

Does the shape tessellate? If so, tell whether the tessellation is regular.

a. Regular octagon



b. Trapezoid

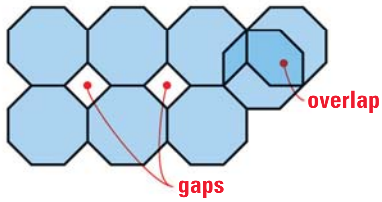


c. Regular hexagon

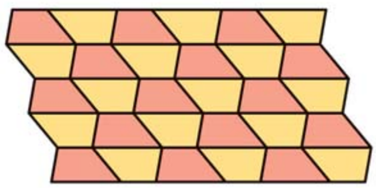
Solution

AVOID ERRORS
The sum of the angles surrounding every vertex of a tessellation is 360°. This means that no regular polygon with more than six sides can be used in a *regular* tesssellation.

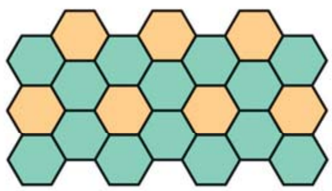
a. A regular octagon does not tessellate.



b. The trapezoid tessellates. The tessellation is not regular because the trapezoid is not a regular polygon.



c. A regular hexagon tessellates using translations. The tessellation is regular because it is made of congruent regular hexagons.



EXAMPLE 2 Draw a tessellation using one shape

Change a triangle to make a tessellation.

Solution

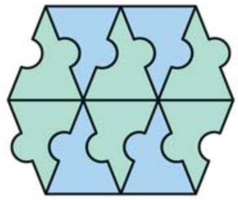
STEP 1

Cut a piece from the triangle.

STEP 2

Slide the piece to another side.

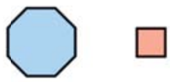
STEP 3



Translate and reflect the figure to make a tessellation.

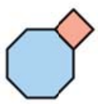
EXAMPLE 3 Draw a tessellation using two shapes

Draw a tessellation using the given floor tiles.



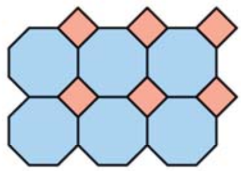
Solution

STEP 1



Combine one octagon and one square by connecting sides of the same length.

STEP 2



Translate the pair of polygons to make a tessellation

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READ VOCABULARY

Notice that in the tessellation in Example 3, the same combination of regular polygons meet at each vertex. This type of tessellation is called *semi-regular*.

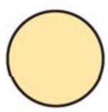
PRACTICE

EXAMPLE 1
on p. 616
for Exs. 1–4

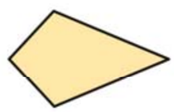
REGULAR TESSELLATIONS **Does the shape tessellate? If so, tell whether the tessellation is regular.**

1. Equilateral triangle

2. Circle



3. Kite



4. ★ **OPEN-ENDED MATH** Draw a rectangle. Use the rectangle to make two different tessellations.

5. **MULTI-STEP PROBLEM** Choose a tessellation and measure the angles at three vertices.

   a. What is the sum of the measures of the angles? What can you conclude?

   b. *Explain* how you know that any *quadrilateral* will tessellate.

EXAMPLE 2
on p. 617
for Exs. 6–9

DRAWING TESSELLATIONS **In Exercises 6–8, use the steps in Example 2 to make a figure that will tessellate.**

6. Make a tessellation using a triangle as the base figure.

7. Make a tessellation using a square as the base figure. Change both pairs of opposite sides.

8. Make a tessellation using a hexagon as the base figure. Change all three pairs of opposite sides.

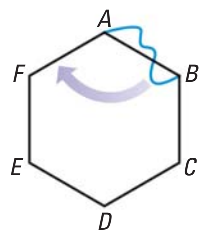
9. **ROTATION TESSELLATION** Use these steps to make another tessellation based on a regular hexagon *ABCDEF*.

   a. Connect points *A* and *B* with a curve. Rotate the curve 120° about *A* so that *B* coincides with *F*.

   b. Connect points *E* and *F* with a curve. Rotate the curve 120° about *E* so that *F* coincides with *D*.

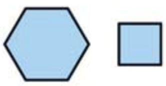
   c. Connect points *C* and *D* with a curve. Rotate the curve 120° about *C* so that *D* coincides with *B*.

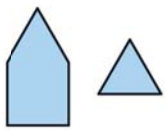
   d. Use this figure to draw a tessellation.

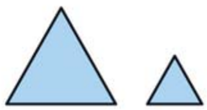


EXAMPLE 3
on p. 617
for Exs. 10–12

USING TWO POLYGONS **Draw a tessellation using the given polygons.**

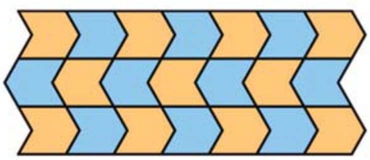
10. 

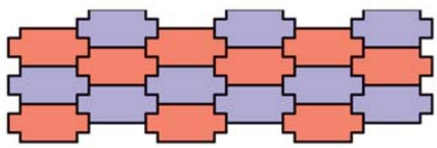
11. 

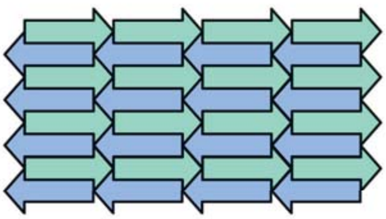
12. 

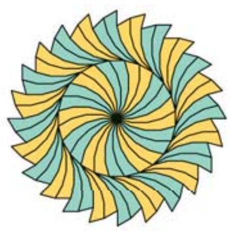
13. ★ **OPEN-ENDED MATH** Draw a tessellation using three different polygons.

TRANSFORMATIONS ***Describe*** **the transformation(s) used to make the tessellation.**

14. 

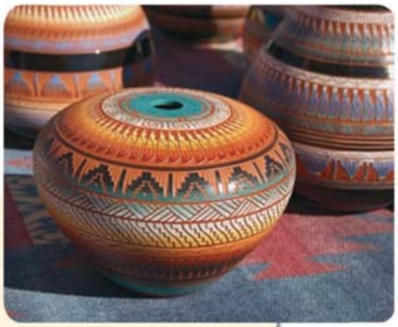
15. 

16. 

17. 

18. **USING SHAPES** On graph paper, outline a capital H. Use this shape to make a tessellation. What transformations did you use?

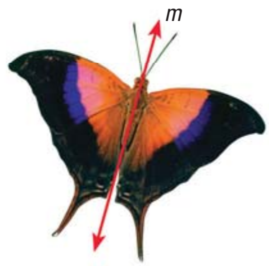
9.6 Identify Symmetry



Before	You reflected or rotated figures.
Now	You will identify line and rotational symmetries of a figure.
Why?	So you can identify the symmetry in a bowl, as in Ex. 11.

Key Vocabulary
- **line symmetry**
- **line of symmetry**
- **rotational symmetry**
- **center of symmetry**

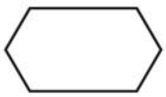
A figure in the plane has **line symmetry** if the figure can be mapped onto itself by a reflection in a line. This line of reflection is a **line of symmetry**, such as line m at the right. A figure can have more than one line of symmetry.



EXAMPLE 1 Identify lines of symmetry

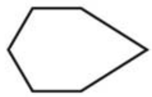
How many lines of symmetry does the hexagon have?

a.



b.

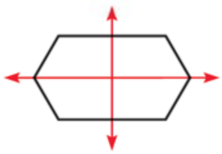
c.



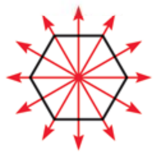
Solution

REVIEW REFLECTION
Notice that the lines of symmetry are also lines of reflection.

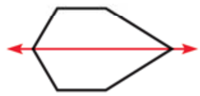
a. Two lines of symmetry



b. Six lines of symmetry



c. One line of symmetry



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✓ GUIDED PRACTICE for Example 1

How many lines of symmetry does the object appear to have?

1.

2.

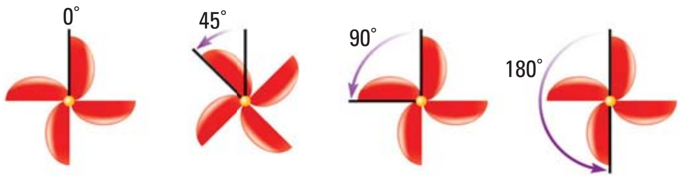
3.

4. Draw a hexagon with no lines of symmetry.

ROTATIONAL SYMMETRY A figure in a plane has **rotational symmetry** if the figure can be mapped onto itself by a rotation of 180° or less about the center of the figure. This point is the **center of symmetry**. Note that the rotation can be either clockwise or counterclockwise.

REVIEW ROTATION
For a figure with rotational symmetry, the *angle of rotation* is the smallest angle that maps the figure onto itself.

For example, the figure below has rotational symmetry, because a rotation of either 90° or 180° maps the figure onto itself (although a rotation of 45° does not).



The figure above also has *point symmetry*, which is 180° rotational symmetry.

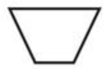
EXAMPLE 2 Identify rotational symmetry

Does the figure have rotational symmetry? If so, describe any rotations that map the figure onto itself.

a. Parallelogram

b. Regular octagon

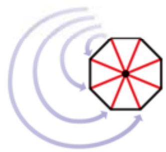
c. Trapezoid



Solution

a. The parallelogram has rotational symmetry. The center is the intersection of the diagonals. A 180° rotation about the center maps the parallelogram onto itself.

b. The regular octagon has rotational symmetry. The center is the intersection of the diagonals. Rotations of 45°, 90°, 135°, or 180° about the center all map the octagon onto itself.

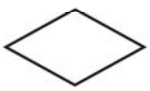


c. The trapezoid does not have rotational symmetry because no rotation of 180° or less maps the trapezoid onto itself.

✓ GUIDED PRACTICE for Example 2

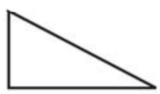
Does the figure have rotational symmetry? If so, *describe* any rotations that map the figure onto itself.

5. Rhombus



6. Octagon

7. Right triangle



★ EXAMPLE 3 Standardized Test Practice

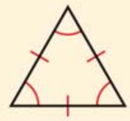
Identify the line symmetry and rotational symmetry of the equilateral triangle at the right.

Ⓐ 3 lines of symmetry, 60° rotational symmetry

Ⓑ 3 lines of symmetry, 120° rotational symmetry

Ⓒ 1 line of symmetry, 180° rotational symmetry

Ⓓ 1 line of symmetry, no rotational symmetry

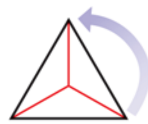


Solution

ELIMINATE CHOICES
An equilateral triangle can be mapped onto itself by reflecting over any of three different lines. So, you can eliminate choices C and D.

The triangle has line symmetry. Three lines of symmetry can be drawn for the figure.

For a figure with s lines of symmetry, the smallest rotation that maps the figure onto itself has the measure $\frac{360°}{s}$. So, the equilateral triangle has $\frac{360°}{3}$, or 120° rotational symmetry.



▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Example 3

8. *Describe* the lines of symmetry and rotational symmetry of a non-equilateral isosceles triangle.

9.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 13, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 13, 14, 21, and 23

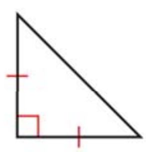
SKILL PRACTICE

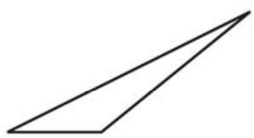
1. **VOCABULARY** What is a *center of symmetry*?

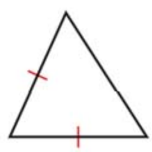
2. ★ **WRITING** Draw a figure that has one line of symmetry and does not have rotational symmetry. Can a figure have two lines of symmetry and no rotational symmetry?

EXAMPLE 1
on p. 619
for Exs. 3–5

LINE SYMMETRY **How many lines of symmetry does the triangle have?**

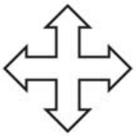
3.


4.


5.


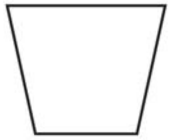
EXAMPLE 2
on p. 620
for Exs. 6–9

ROTATIONAL SYMMETRY **Does the figure have rotational symmetry? If so, *describe* any rotations that map the figure onto itself.**

6.


7.


8.

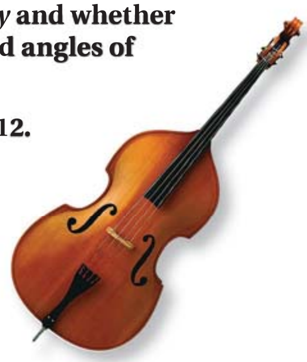

9.


EXAMPLE 3
on p. 621
for Exs. 10–16

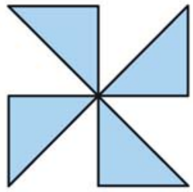
SYMMETRY **Determine whether the figure has *line symmetry* and whether it has *rotational symmetry*. Identify all lines of symmetry and angles of rotation that map the figure onto itself.**

10.


11.


12.


13. ★ **MULTIPLE CHOICE** Identify the line symmetry and rotational symmetry of the figure at the right.



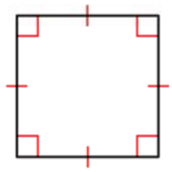
Ⓐ 1 line of symmetry, no rotational symmetry

Ⓑ 1 line of symmetry, 180° rotational symmetry

Ⓒ No lines of symmetry, 90° rotational symmetry

Ⓓ No lines of symmetry, 180° rotational symmetry

14. ★ **MULTIPLE CHOICE** Which statement best describes the rotational symmetry of a square?



Ⓐ The square has no rotational symmetry.

Ⓑ The square has 90° rotational symmetry.

Ⓒ The square has point symmetry.

Ⓓ Both B and C are correct.

ERROR ANALYSIS ***Describe* and correct the error made in describing the symmetry of the figure.**

15.


The figure has 1 line of symmetry and 180° rotational symmetry.

16.


The figure has 1 line of symmetry and 180° rotational symmetry.

DRAWING FIGURES **In Exercises 17–20, use the description to draw a figure. If not possible, write *not possible*.**

17. A quadrilateral with no line of symmetry

18. An octagon with exactly two lines of symmetry

19. A hexagon with no point symmetry

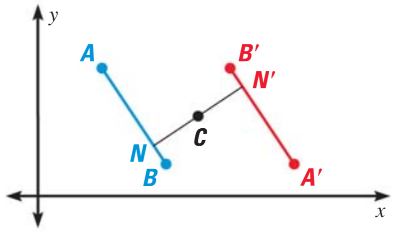
20. A trapezoid with rotational symmetry

○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

21. ★ **OPEN-ENDED MATH** Draw a polygon with 180° rotational symmetry and with exactly two lines of symmetry.

22. **POINT SYMMETRY** In the graph, $\overline{AB}$ is reflected in the point C to produce the image $\overline{A'B'}$. To make a reflection in a point C for each point N on the preimage, locate N' so that $N'C = NC$ and N' is on $\overleftrightarrow{NC}$. *Explain* what kind of rotation would produce the same image. What kind of symmetry does quadrilateral $AB'A'B$ have?



23. ★ **SHORT RESPONSE** A figure has more than one line of symmetry. Can two of the lines of symmetry be parallel? *Explain*.

24. **REASONING** How many lines of symmetry does a circle have? How many angles of rotational symmetry does a circle have? *Explain*.

25. **VISUAL REASONING** How many planes of symmetry does a cube have?

26. **CHALLENGE** What can you say about the rotational symmetry of a regular polygon with n sides? *Explain*.

PROBLEM SOLVING

EXAMPLES 1 and 2 on pp. 619–620 for Exs. 27–30

WORDS Identify the line symmetry and rotational symmetry (if any) of each word.

27. MOW 28. RADAR 29. OHIO 30. pod

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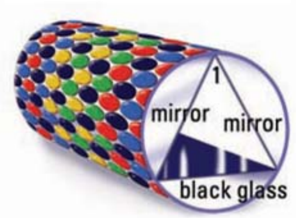
KALEIDOSCOPES In Exercises 31–33, use the following information about kaleidoscopes.

Inside a kaleidoscope, two mirrors are placed next to each other to form a V, as shown at the right. The angle between the mirrors determines the number of lines of symmetry in the image. Use the formula $n(m\angle 1) = 180°$ to find the measure of $\angle 1$ between the mirrors or the number n of lines of symmetry in the image.



Calculate the angle at which the mirrors must be placed for the image of a kaleidoscope to make the design shown.

31.

32.

33.

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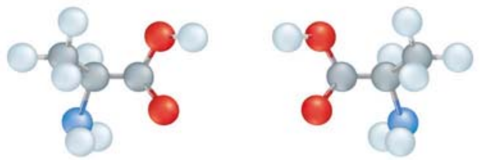
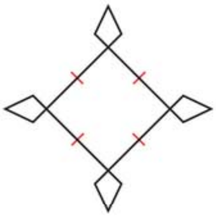
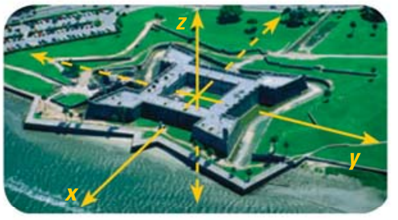
34. CHEMISTRY The diagram at the right shows two forms of the amino acid *alanine*. One form is laevo-alanine and the other is dextro-alanine. How are the structures of these two molecules related? *Explain*.



35. MULTI-STEP PROBLEM The *Castillo de San Marcos* in St. Augustine, Florida, has the shape shown.

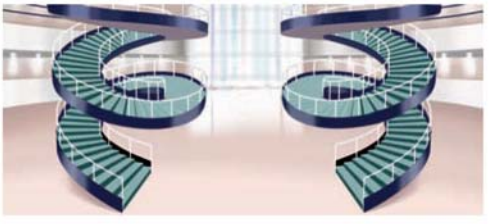




a. What kind(s) of symmetry does the shape of the building show?

b. Imagine the building on a three-dimensional coordinate system. Copy and complete the following statement: The lines of symmetry in part (a) are now described as __?__ of symmetry and the rotational symmetry about the center is now described as rotational symmetry about the __?__.

36. CHALLENGE Spirals have a type of symmetry called spiral, or helical, symmetry. *Describe* the two transformations involved in a spiral staircase. Then *explain* the difference in transformations between the two staircases at the right.



MIXED REVIEW

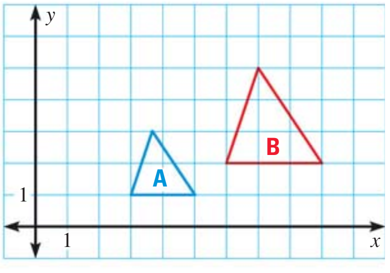
PREVIEW
Prepare for Lesson 9.7 in Exs. 37–39.

Solve the proportion. *(p. 356)*

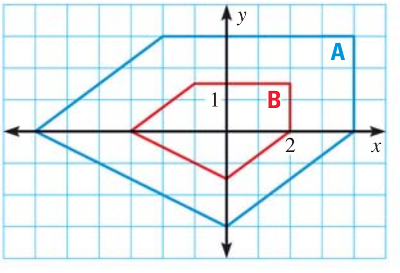
37. $\frac{5}{x} = \frac{15}{27}$ **38.** $\frac{a+4}{7} = \frac{49}{56}$ **39.** $\frac{5}{2b-3} = \frac{1}{3b+1}$

Determine whether the dilation from Figure A to Figure B is a *reduction* or an *enlargement*. Then find its scale factor. *(p. 409)*

40.



41.



Write a matrix to represent the given polygon. *(p. 580)*

42. Triangle A in Exercise 40 **43.** Triangle B in Exercise 40

44. Pentagon A in Exercise 41 **45.** Pentagon B in Exercise 41

9.7 Investigate Dilations

MATERIALS • straightedge • compass • ruler

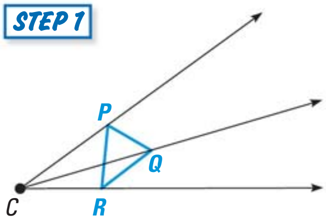
QUESTION How do you construct a dilation of a figure?

Recall from Lesson 6.7 that a dilation enlarges or reduces a figure to make a similar figure. You can use construction tools to make enlargement dilations.

EXPLORE Construct an enlargement dilation

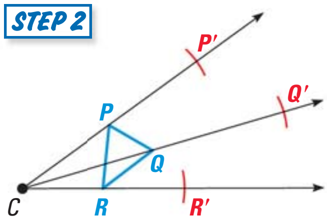
Use a compass and straightedge to construct a dilation of $\triangle PQR$ with a scale factor of 2, using a point C outside the triangle as the center of dilation.

STEP 1



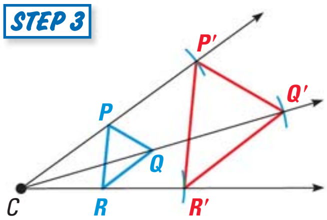
Draw a triangle Draw $\triangle PQR$ and choose the center of the dilation C outside the triangle. Draw lines from C through the vertices of the triangle.

STEP 2



Use a compass Use a compass to locate P' on $\overrightarrow{CP}$ so that $CP' = 2(CP)$. Locate Q' and R' in the same way.

STEP 3



Connect points Connect points P', Q', and R' to form $\triangle P'Q'R'$.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Find the ratios of corresponding side lengths of $\triangle PQR$ and $\triangle P'Q'R'$. Are the triangles similar? *Explain.*
2. Draw $\triangle DEF$. Use a compass and straightedge to construct a dilation with a scale factor of 3, using point D on the triangle as the center of dilation.
3. Find the ratios of corresponding side lengths of $\triangle DEF$ and $\triangle D'E'F'$. Are the triangles similar? *Explain.*
4. Draw $\triangle JKL$. Use a compass and straightedge to construct a dilation with a scale factor of 2, using a point A inside the triangle as the center of dilation.
5. Find the ratios of corresponding side lengths of $\triangle JKL$ and $\triangle J'K'L'$. Are the triangles similar? *Explain.*
6. What can you conclude about the corresponding angles measures of a triangle and an enlargement dilation of the triangle?

9.7 Identify and Perform Dilations

Before You used a coordinate rule to draw a dilation.

Now You will use drawing tools and matrices to draw dilations.

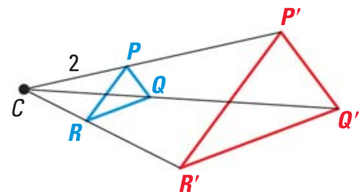
Why? So you can determine the scale factor of a photo, as in Ex. 37.

Key Vocabulary
- **scalar multiplication**
- **dilation,** *p. 409*
- **reduction,** *p. 409*
- **enlargement,** *p. 409*

Recall from Lesson 6.7 that a dilation is a transformation in which the original figure and its image are similar.

A dilation with center C and scale factor k maps every point P in a figure to a point P' so that one of the following statements is true:

- If P is not the center point C, then the image point P' lies on $\overrightarrow{CP}$. The scale factor k is a positive number such that $k = \frac{CP'}{CP}$ and $k \neq 1$, or



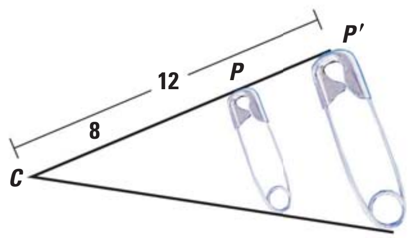
- If P is the center point C, then $P = P'$.

As you learned in Lesson 6.7, the dilation is a *reduction* if $0 < k < 1$ and it is an *enlargement* if $k > 1$.

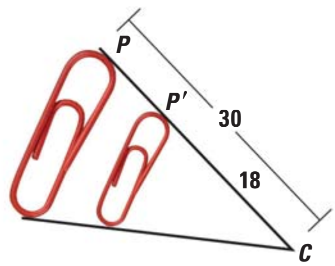
EXAMPLE 1 Identify dilations

Find the scale factor of the dilation. Then tell whether the dilation is a *reduction* or an *enlargement*.

a.



b.



Solution

a. Because $\frac{CP'}{CP} = \frac{12}{8}$, the scale factor is $k = \frac{3}{2}$. The image P' is an enlargement.

b. Because $\frac{CP'}{CP} = \frac{18}{30}$, the scale factor is $k = \frac{3}{5}$. The image P' is a reduction.

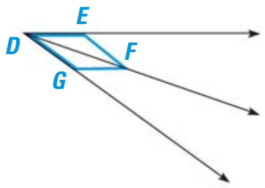
Animated Geometry at classzone.com

EXAMPLE 2 Draw a dilation

Draw and label $\square DEFG$. Then construct a dilation of $\square DEFG$ with point D as the center of dilation and a scale factor of 2.

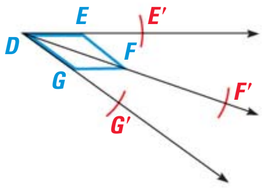
Solution

STEP 1



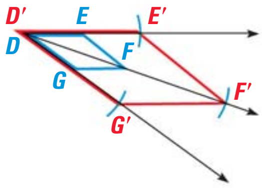
Draw $DEFG$. Draw rays from D through vertices E, F, and G.

STEP 2



Open the compass to the length of $\overline{DE}$. Locate E' on $\overrightarrow{DE}$ so $DE' = 2(DE)$. Locate F' and G' the same way.

STEP 3



Add a second label D' to point D. Draw the sides of $D'E'F'G'$.

✓ GUIDED PRACTICE for Examples 1 and 2

1. In a dilation, $CP' = 3$ and $CP = 12$. Tell whether the dilation is a *reduction* or an *enlargement* and find its scale factor.
2. Draw and label $\triangle RST$. Then construct a dilation of $\triangle RST$ with R as the center of dilation and a scale factor of 3.

MATRICES **Scalar multiplication** is the process of multiplying each element of a matrix by a real number or *scalar*.

EXAMPLE 3 Scalar multiplication

Simplify the product: $4\begin{bmatrix} 3 & 0 & 1 \\ 2 & -1 & -3 \end{bmatrix}$.

Solution

$$4\begin{bmatrix} 3 & 0 & 1 \\ 2 & -1 & -3 \end{bmatrix} = \begin{bmatrix} 4(3) & 4(0) & 4(1) \\ 4(2) & 4(-1) & 4(-3) \end{bmatrix}$$ Multiply each element in the matrix by 4.

$$= \begin{bmatrix} 12 & 0 & 4 \\ 8 & -4 & -12 \end{bmatrix}$$ Simplify.

✓ GUIDED PRACTICE for Example 3

Simplify the product.

3. $5\begin{bmatrix} 2 & 1 & -10 \\ 3 & -4 & 7 \end{bmatrix}$

4. $-2\begin{bmatrix} -4 & 1 & 0 \\ 9 & -5 & -7 \end{bmatrix}$

DILATIONS USING MATRICES You can use scalar multiplication to represent a dilation centered at the origin in the coordinate plane. To find the image matrix for a dilation centered at the origin, use the scale factor as the scalar.

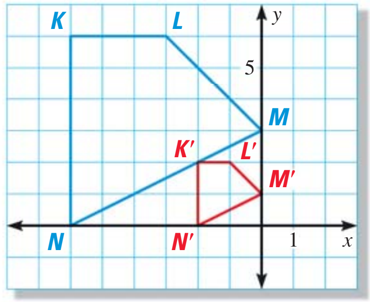
EXAMPLE 4 Use scalar multiplication in a dilation

The vertices of quadrilateral $KLMN$ are $K(-6, 6)$, $L(-3, 6)$, $M(0, 3)$, and $N(-6, 0)$. Use scalar multiplication to find the image of $KLMN$ after a dilation with its center at the origin and a scale factor of $\frac{1}{3}$. Graph $KLMN$ and its image.

Solution

$$\frac{1}{3}\begin{array}{c} \begin{matrix} K & L & M & N \end{matrix} \\ \begin{bmatrix} -6 & -3 & 0 & -6 \\ 6 & 6 & 3 & 0 \end{bmatrix} \end{array} = \begin{array}{c} \begin{matrix} K' & L' & M' & N' \end{matrix} \\ \begin{bmatrix} -2 & -1 & 0 & -2 \\ 2 & 2 & 1 & 0 \end{bmatrix} \end{array}$$

Scale factor Polygon matrix Image matrix



EXAMPLE 5 Find the image of a composition

The vertices of $\triangle ABC$ are $A(-4, 1)$, $B(-2, 2)$, and $C(-2, 1)$. Find the image of $\triangle ABC$ after the given composition.

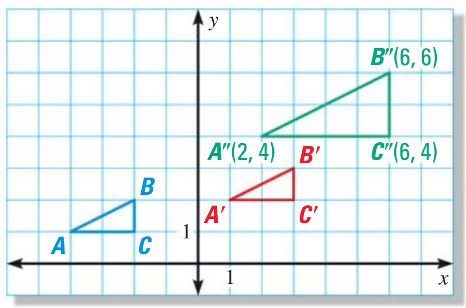
Translation: $(x, y) \to (x + 5, y + 1)$
Dilation: centered at the origin with a scale factor of 2

Solution

STEP 1 **Graph** the preimage $\triangle ABC$ on the coordinate plane.

STEP 2 **Translate** $\triangle ABC$ 5 units to the right and 1 unit up. Label it $\triangle A'B'C'$.

STEP 3 **Dilate** $\triangle A'B'C'$ using the origin as the center and a scale factor of 2 to find $\triangle A''B''C''$.



✓ GUIDED PRACTICE for Examples 4 and 5

5. The vertices of $\triangle RST$ are $R(1, 2)$, $S(2, 1)$, and $T(2, 2)$. Use scalar multiplication to find the vertices of $\triangle R'S'T'$ after a dilation with its center at the origin and a scale factor of 2.

6. A segment has the endpoints $C(-1, 1)$ and $D(1, 1)$. Find the image of $\overline{CD}$ after a $90°$ rotation about the origin followed by a dilation with its center at the origin and a scale factor of 2.

9.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 19, and 35

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 24, 25, 27, 29, and 38

SKILL PRACTICE

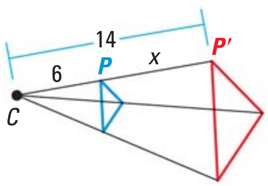
1. VOCABULARY What is a *scalar*?

2. ★ **WRITING** If you know the scale factor, *explain* how to determine if an image is larger or smaller than the preimage.

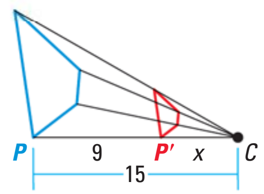
EXAMPLE 1 on p. 626 for Exs. 3–6

IDENTIFYING DILATIONS Find the scale factor. Tell whether the dilation is a *reduction* or an *enlargement*. Find the value of *x*.

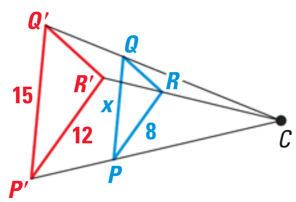
3.



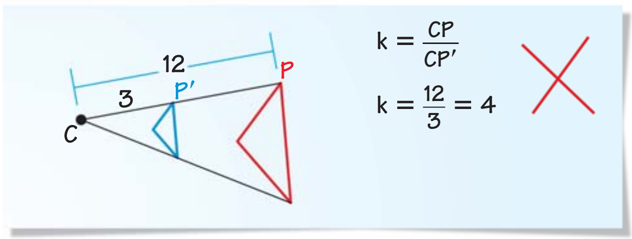
4.



5.



6. ERROR ANALYSIS *Describe* and correct the error in finding the scale factor k of the dilation.



EXAMPLE 2 on p. 627 for Exs. 7–14

CONSTRUCTION Copy the diagram. Then draw the given dilation.

7. Center H; $k = 2$

8. Center H; $k = 3$

9. Center J; $k = 2$

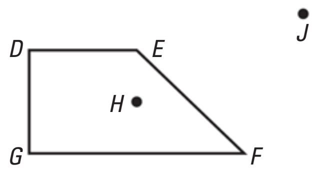
10. Center F; $k = 2$

11. Center J; $k = \frac{1}{2}$

12. Center F; $k = \frac{3}{2}$

13. Center D; $k = \frac{3}{2}$

14. Center G; $k = \frac{1}{2}$



EXAMPLE 3 on p. 627 for Exs. 15–17

SCALAR MULTIPLICATION Simplify the product.

15. $4\begin{bmatrix} 3 & 7 & 4 \\ 0 & 9 & -1 \end{bmatrix}$

16. $-5\begin{bmatrix} -2 & -5 & 7 & 3 \\ 1 & 4 & 0 & -1 \end{bmatrix}$

17. $9\begin{bmatrix} 0 & 3 & 2 \\ -1 & 7 & 0 \end{bmatrix}$

EXAMPLE 4 on p. 628 for Exs. 18–20

DILATIONS WITH MATRICES Find the image matrix that represents a dilation of the polygon centered at the origin with the given scale factor. Then graph the polygon and its image.

18. $\begin{array}{c} \begin{matrix} D & E & F \end{matrix} \\ \begin{bmatrix} 2 & 3 & 5 \\ 1 & 6 & 4 \end{bmatrix} \end{array}; k = 2$

19. $\begin{array}{c} \begin{matrix} G & H & J \end{matrix} \\ \begin{bmatrix} -2 & 0 & 6 \\ -4 & 2 & -2 \end{bmatrix} \end{array}; k = \frac{1}{2}$

20. $\begin{array}{c} \begin{matrix} J & L & M & N \end{matrix} \\ \begin{bmatrix} -6 & -3 & 3 & 3 \\ 0 & 3 & 0 & -3 \end{bmatrix} \end{array}; k = \frac{2}{3}$

EXAMPLE 5
on p. 628
for Exs. 21–23

COMPOSING TRANSFORMATIONS **The vertices of $\triangle FGH$ are $F(-2, -2)$, $G(-2, -4)$, and $H(-4, -4)$. Graph the image of the triangle after a composition of the transformations in the order they are listed.**

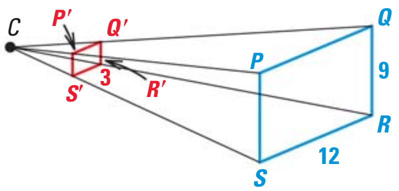
21. **Translation:** $(x, y) \rightarrow (x + 3, y + 1)$
Dilation: centered at the origin with a scale factor of 2

22. **Dilation:** centered at the origin with a scale factor of $\frac{1}{2}$
Reflection: in the y-axis

23. **Rotation:** 90° about the origin
Dilation: centered at the origin with a scale factor of 3

24. ★ **WRITING** Is a composition of transformations that includes a dilation ever an isometry? *Explain.*

25. ★ **MULTIPLE CHOICE** In the diagram, the center of the dilation of ▱$PQRS$ is point C. The length of a side of ▱$P'Q'R'S'$ is what percent of the length of the corresponding side of ▱$PQRS$?



Ⓐ 25% Ⓑ 33% Ⓒ 300% Ⓓ 400%

26. **REASONING** The distance from the center of dilation to the image of a point is shorter than the distance from the center of dilation to the preimage. Is the dilation a *reduction* or an *enlargement*? *Explain.*

27. ★ **SHORT RESPONSE** Graph a triangle in the coordinate plane. Rotate the triangle, then dilate it. Then do the same dilation first, followed by the rotation. In this composition of transformations, does it matter in which order the triangle is dilated and rotated? *Explain* your answer.

28. **REASONING** A dilation maps $A(5, 1)$ to $A'(2, 1)$ and $B(7, 4)$ to $B'(6, 7)$.

a. Find the scale factor of the dilation.

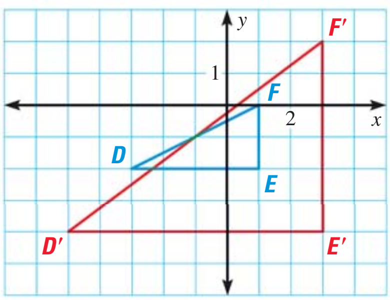
b. Find the center of the dilation.

29. ★ **MULTIPLE CHOICE** Which transformation of (x, y) is a dilation?

Ⓐ $(3x, y)$ Ⓑ $(-x, 3y)$ Ⓒ $(3x, 3y)$ Ⓓ $(x + 3, y + 3)$

30. *xy* **ALGEBRA** Graph parabolas of the form $y = ax^2$ using three different values of a. Describe the effect of changing the value of a. Is this a dilation? *Explain.*

31. **REASONING** In the graph at the right, determine whether $\triangle D'E'F'$ is a dilation of $\triangle DEF$. *Explain.*



32. **CHALLENGE** $\triangle ABC$ has vertices $A(4, 2)$, $B(4, 6)$, and $C(7, 2)$. Find the vertices that represent a dilation of $\triangle ABC$ centered at (4, 0) with a scale factor of 2.

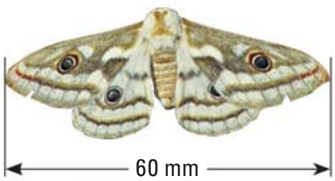
○ = WORKED-OUT SOLUTIONS on p. WS1   ★ = STANDARDIZED TEST PRACTICE

PROBLEM SOLVING

EXAMPLE 1
on p. 626
for Exs. 33–35

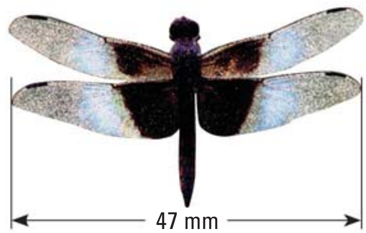
SCIENCE You are using magnifying glasses. Use the length of the insect and the magnification level to determine the length of the image seen through the magnifying glass.

33. Emperor moth
magnification 5x



34. Ladybug
magnification 10x

35. Dragonfly
magnification 20x



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36. MURALS A painter sketches plans for a mural. The plans are 2 feet by 4 feet. The actual mural will be 25 feet by 50 feet. What is the scale factor? Is this a dilation? *Explain*.

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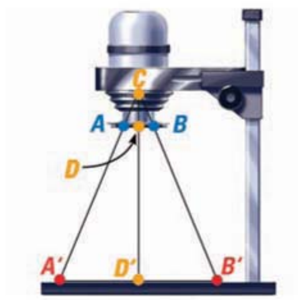
37. PHOTOGRAPHY By adjusting the distance between the negative and the enlarged print in a photographic enlarger, you can make prints of different sizes. In the diagram shown, you want the enlarged print to be 9 inches wide ($A'B'$). The negative is 1.5 inches wide (AB), and the distance between the light source and the negative is 1.75 inches (CD).

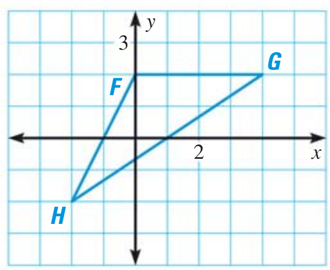


a. What is the scale factor of the enlargement?

b. What is the distance between the negative and the enlarged print?

38. ★ **OPEN-ENDED MATH** Graph a polygon in a coordinate plane. Draw a figure that is similar but not congruent to the polygon. What is the scale factor of the dilation you drew? What is the center of the dilation?

39. MULTI-STEP PROBLEM Use the figure at the right.



a. Write a polygon matrix for the figure. Multiply the matrix by the scalar -2.

b. Graph the polygon represented by the new matrix.

c. Repeat parts (a) and (b) using the scalar $-\frac{1}{2}$.

d. Make a conjecture about the effect of multiplying a polygon matrix by a negative scale factor.

40. AREA You have an 8 inch by 10 inch photo.

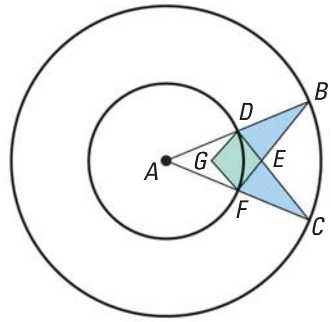
a. What is the area of the photo?

b. You photocopy the photo at 50%. What are the dimensions of the image? What is the area of the image?

c. How many images of this size would you need to cover the original photo?

41. REASONING You put a reduction of a page on the original page. *Explain* why there is a point that is in the same place on both pages.

42. CHALLENGE Draw two concentric circles with center A. Draw $\overline{AB}$ and $\overline{AC}$ to the larger circle to form a $45°$ angle. Label points D and F, where $\overline{AB}$ and $\overline{AC}$ intersect the smaller circle. Locate point E at the intersection of $\overline{BF}$ and $\overline{CD}$. Choose a point G and draw quadrilateral $DEFG$. Use A as the center of the dilation and a scale factor of $\frac{1}{2}$. Dilate $DEFG$, $\triangle DBE$, and $\triangle CEF$ two times. Sketch each image on the circles. *Describe* the result.

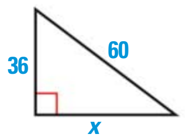


MIXED REVIEW

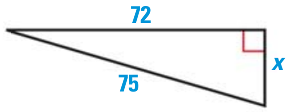
PREVIEW
Prepare for Lesson 10.1 in Exs. 43–45.

Find the unknown leg length x. *(p. 433)*

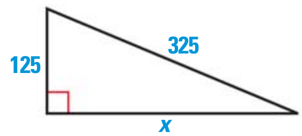
43.



44.



45.



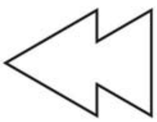
Find the sum of the measures of the interior angles of the indicated convex polygon. *(p. 507)*

46. Hexagon **47.** 13-gon **48.** 15-gon **49.** 18-gon

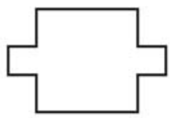
QUIZ *for Lessons 9.6–9.7*

Determine whether the figure has *line symmetry* and/or *rotational symmetry*. Identify the number of lines of symmetry and/or the rotations that map the figure onto itself. *(p. 619)*

1.



2.

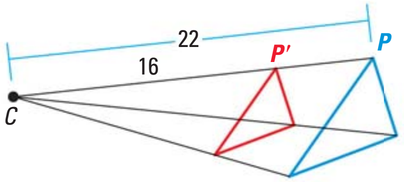


3.

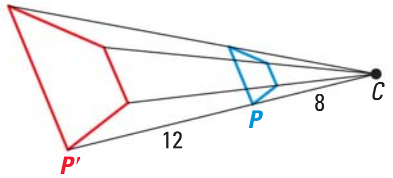
4.

Tell whether the dilation is a *reduction* or an *enlargement* and find its scale factor. *(p. 626)*

5.



6.



7. The vertices of $\triangle RST$ are $R(3, 1)$, $S(0, 4)$, and $T(-2, 2)$. Use scalar multiplication to find the image of the triangle after a dilation centered at the origin with scale factor $4\frac{1}{2}$. *(p. 626)*

 EXTRA PRACTICE for Lesson 9.7, p. 913 **ONLINE QUIZ** at classzone.com

Technology **ACTIVITY** *Use after Lesson 9.7*

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Keystrokes

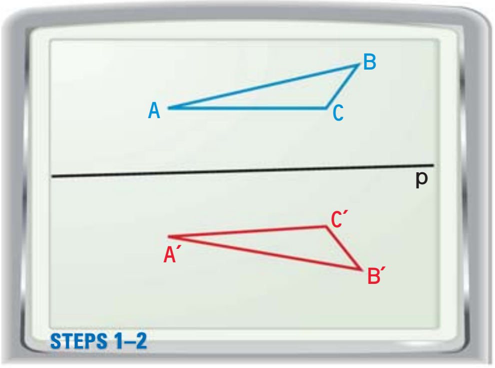
9.7 Compositions With Dilations

MATERIALS • graphing calculator or computer

QUESTION **How can you graph compositions with dilations?**

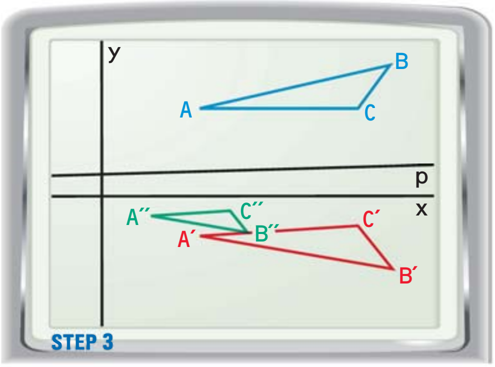
You can use geometry drawing software to perform compositions with dilations.

EXAMPLE **Perform a reflection and dilation**



STEP 1 ***Draw triangle*** Construct a scalene triangle like $\triangle ABC$ at the right. Label the vertices A, B, and C. Construct a line that does not intersect the triangle. Label the line p.

STEP 2 ***Reflect triangle*** Select Reflection from the F4 menu. To reflect $\triangle ABC$ in line p, choose the triangle, then the line.



STEP 3 ***Dilate triangle*** Select Hide/Show from the F5 menu and show the axes. To set the scale factor, select Alpha-Num from the F5 menu, press ENTER when the cursor is where you want the number, and then enter 0.5 for the scale factor.

Next, select Dilation from the F4 menu. Choose the image of $\triangle ABC$, then choose the origin as the center of dilation, and finally choose 0.5 as the scale factor to dilate the triangle. Save this as “DILATE”.

PRACTICE

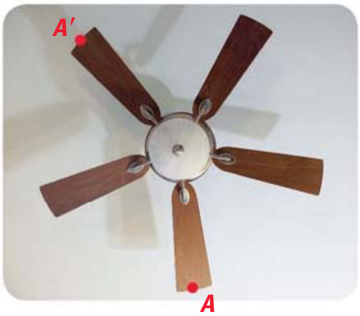
1. Move the line of reflection. How does the final image change?
2. To change the scale factor, select the Alpha-Num tool. Place the cursor over the scale factor. Press ENTER, then DELETE. Enter a new scale. How does the final image change?
3. Dilate with a center not at the origin. How does the final image change?
4. Use $\triangle ABC$ and line p, and the dilation and reflection from the Example. Dilate the triangle first, then reflect it. How does the final image change?

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
classzone.com

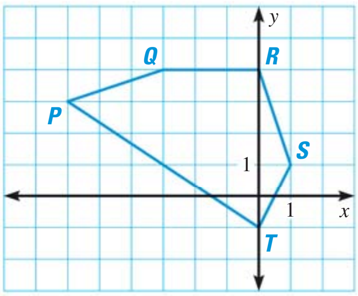
Lessons 9.4–9.7

1. **GRIDDED ANSWER** What is the angle of rotation, in degrees, that maps A to A' in the photo of the ceiling fan below?



2. **SHORT RESPONSE** The vertices of $\triangle DEF$ are $D(-3, 2)$, $E(2, 3)$, and $F(3, -1)$. Graph $\triangle DEF$. Rotate $\triangle DEF$ 90° about the origin. Compare the slopes of corresponding sides of the preimage and image. What do you notice?

3. **MULTI-STEP PROBLEM** Use pentagon $PQRST$ shown below.

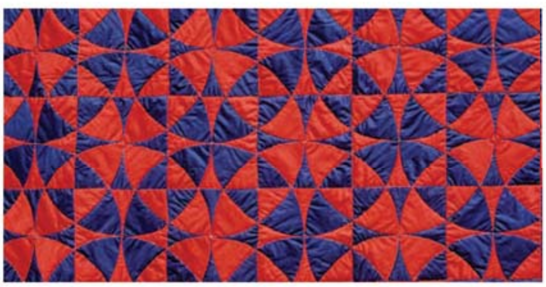


   a. Write the polygon matrix for $PQRST$.

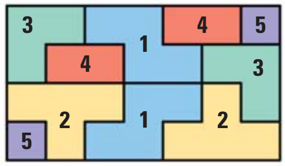
   b. Find the image matrix for a 270° rotation about the origin.

   c. Graph the image.

4. **SHORT RESPONSE** *Describe* the transformations that can be found in the quilt pattern below.



5. **MULTI-STEP PROBLEM** The diagram shows the pieces of a puzzle.



   a. Which pieces are translated?

   b. Which pieces are reflected?

   c. Which pieces are glide reflected?

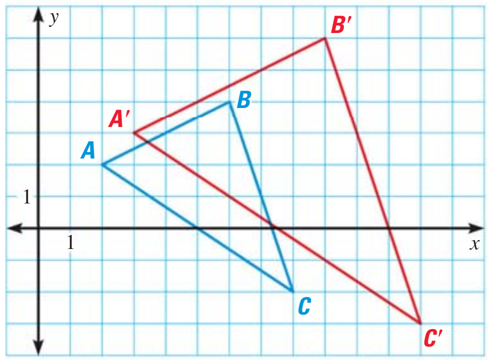
6. **OPEN-ENDED** Draw a figure that has the given type(s) of symmetry.

   a. Line symmetry only

   b. Rotational symmetry only

   c. Both line symmetry and rotational symmetry

7. **EXTENDED RESPONSE** In the graph below, $\triangle A'B'C'$ is a dilation of $\triangle ABC$.



   a. Is the dilation a *reduction* or an *enlargement*?

   b. What is the scale factor? *Explain* your steps.

   c. What is the polygon matrix? What is the image matrix?

   d. When you perform a composition of a dilation and a translation on a figure, does order matter? *Justify* your answer using the translation $(x, y) \to (x + 3, y - 1)$ and the dilation of $\triangle ABC$.

9 CHAPTER SUMMARY

BIG IDEAS

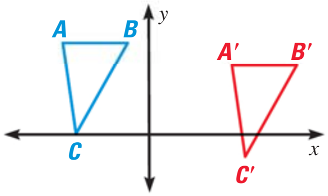
For Your Notebook

Big Idea 1

Performing Congruence and Similarity Transformations

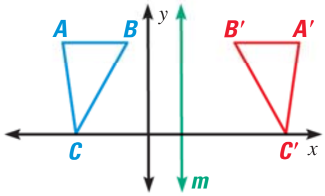
Translation

Translate a figure right or left, up or down.



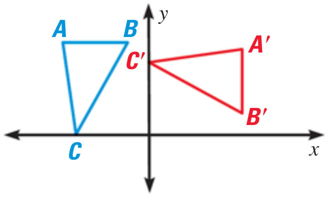
Reflection

Reflect a figure in a line.



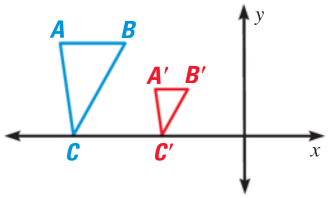
Rotation

Rotate a figure about a point.



Dilation

Dilate a figure to change the size but not the shape.

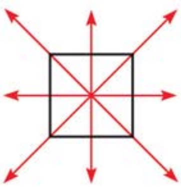


You can combine congruence and similarity transformations to make a composition of transformations, such as a glide reflection.

Big Idea 2

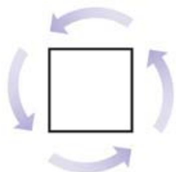
Making Real-World Connections to Symmetry and Tessellations

Line symmetry



4 lines of symmetry

Rotational symmetry



90° rotational symmetry

Big Idea 3

Applying Matrices and Vectors in Geometry

You can use matrices to represent points and polygons in the coordinate plane. Then you can use matrix addition to represent translations, matrix multiplication to represent reflections and rotations, and scalar multiplication to represent dilations. You can also use vectors to represent translations.

9 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- image, *p. 572*
- preimage, *p. 572*
- isometry, *p. 573*
- vector, *p. 574*
  initial point, terminal point, horizontal component, vertical component
- component form, *p. 574*
- matrix, *p. 580*
- element, *p. 580*
- dimensions, *p. 580*
- line of reflection, *p. 589*
- center of rotation, *p. 598*
- angle of rotation, *p. 598*
- glide reflection, *p. 608*
- composition of transformations, *p. 609*
- line symmetry, *p. 619*
- line of symmetry, *p. 619*
- rotational symmetry, *p. 620*
- center of symmetry, *p. 620*
- scalar multiplication, *p. 627*

VOCABULARY EXERCISES

1. Copy and complete: A(n) __?__ is a transformation that preserves lengths.

2. Draw a figure with exactly one line of symmetry.

3. **WRITING** *Explain* how to identify the dimensions of a matrix. Include an example with your explanation.

Match the point with the appropriate name on the vector.

4. T **A.** Initial point

5. H **B.** Terminal point

REVIEW EXAMPLES AND EXERCISES

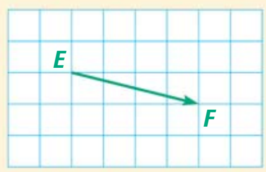
Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 9.

9.1 Translate Figures and Use Vectors *pp. 572–579*

EXAMPLE

Name the vector and write its component form.

The vector is $\overrightarrow{EF}$. From initial point E to terminal point F, you move 4 units right and 1 unit down. So, the component form is $\langle 4, 1\rangle$.



EXERCISES

EXAMPLES 1 and 4 on pp. 572, 574 for Exs. 6–7

6. The vertices of $\triangle ABC$ are $A(2, 3)$, $B(1, 0)$, and $C(-2, 4)$. Graph the image of $\triangle ABC$ after the translation $(x, y) \rightarrow (x + 3, y - 2)$.

7. The vertices of $\triangle DEF$ are $D(-6, 7)$, $E(-5, 5)$, and $F(-8, 4)$. Graph the image of $\triangle DEF$ after the translation using the vector $\langle -1, 6\rangle$.

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Chapter Review Practice

9.2 Use Properties of Matrices *pp. 580–587*

EXAMPLE

Add $\begin{bmatrix} -9 & 12 \\ 5 & -4 \end{bmatrix} + \begin{bmatrix} 20 & 18 \\ 11 & 25 \end{bmatrix}$.

These two matrices have the same dimensions, so you can perform the addition. To add matrices, you add corresponding elements.

$$\begin{bmatrix} -9 & 12 \\ 5 & -4 \end{bmatrix} + \begin{bmatrix} 20 & 18 \\ 11 & 25 \end{bmatrix} = \begin{bmatrix} -9+20 & 12+18 \\ 5+11 & -4+25 \end{bmatrix} = \begin{bmatrix} 11 & 30 \\ 16 & 21 \end{bmatrix}$$

EXERCISES

EXAMPLE 3
on p. 581
for Exs. 8–9

Find the image matrix that represents the translation of the polygon. Then graph the polygon and its image.

8. $\begin{array}{c} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 2 & 8 & 1 \\ 4 & 3 & 2 \end{bmatrix} \end{array}$; 5 units up and 3 units left

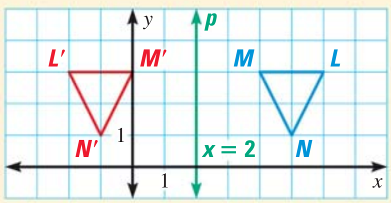
9. $\begin{array}{c} \begin{matrix} D & E & F & G \end{matrix} \\ \begin{bmatrix} -2 & 3 & 4 & -1 \\ 3 & 6 & 4 & -1 \end{bmatrix} \end{array}$; 2 units down

9.3 Perform Reflections *pp. 589–596*

EXAMPLE

The vertices of $\triangle MLN$ are $M(4, 3)$, $L(6, 3)$, and $N(5, 1)$. Graph the reflection of $\triangle MLN$ in the line p with equation $x = 2$.

Point M is 2 units to the right of p, so its reflection M' is 2 units to the left of p at $(0, 3)$. Similarly, L' is 4 units to the left of p at $(-2, 3)$ and N' is 3 units to the left of p at $(-1, 1)$.

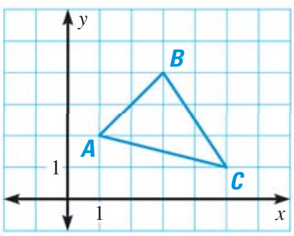


EXERCISES

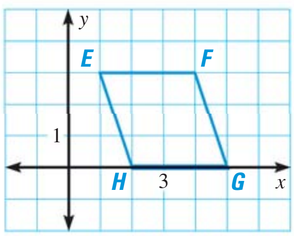
EXAMPLES 1 and 2
on pp. 589–590
for Exs. 10–12

Graph the reflection of the polygon in the given line.

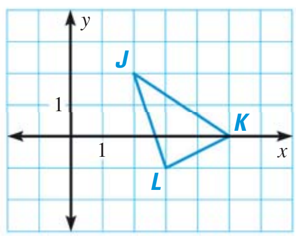
10. $x = 4$



11. $y = 3$



12. $y = x$



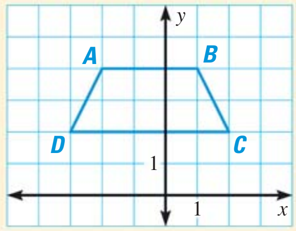
9 CHAPTER REVIEW

9.4 Perform Rotations

pp. 598–605

EXAMPLE

Find the image matrix that represents the 90° rotation of *ABCD* about the origin.



The polygon matrix for *ABCD* is $\begin{bmatrix} -2 & 1 & 2 & -3 \\ 4 & 4 & 2 & 2 \end{bmatrix}$.

Multiply by the matrix for a 90° rotation.

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} -2 & 1 & 2 & -3 \\ 4 & 4 & 2 & 2 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} A' & B' & C' & D' \end{array} \\ \begin{bmatrix} -4 & -4 & -2 & -2 \\ -2 & 1 & 2 & -3 \end{bmatrix} \end{array}$$

EXERCISES

EXAMPLE 3 on p. 600 for Exs. 13–14

Find the image matrix that represents the given rotation of the polygon about the origin. Then graph the polygon and its image.

13. $\begin{array}{c} \begin{array}{ccc} Q & R & S \end{array} \\ \begin{bmatrix} 3 & 4 & 1 \\ 0 & 5 & -2 \end{bmatrix} \end{array}$; 180°

14. $\begin{array}{c} \begin{array}{cccc} L & M & N & P \end{array} \\ \begin{bmatrix} -1 & 3 & 5 & -2 \\ 6 & 5 & 0 & -3 \end{bmatrix} \end{array}$; 270°

9.5 Apply Compositions of Transformations

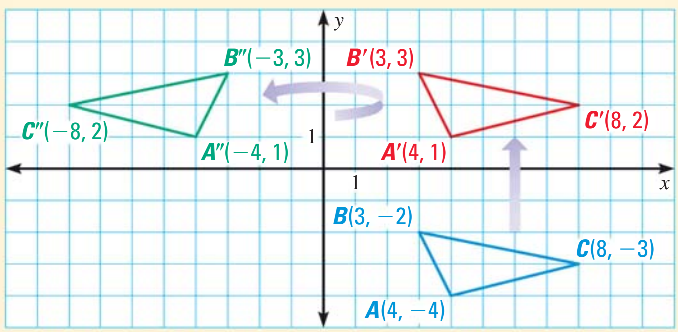
pp. 608–615

EXAMPLE

The vertices of $\triangle ABC$ are $A(4, -4)$, $B(3, -2)$, and $C(8, -3)$. Graph the image of $\triangle ABC$ after the glide reflection.

Translation: $(x, y) \to (x, y + 5)$
Reflection: in the y-axis

Begin by graphing $\triangle ABC$. Then graph the image $\triangle A'B'C'$ after a translation of 5 units up. Finally, graph the image $\triangle A''B''C''$ after a reflection in the y-axis.



EXERCISES

EXAMPLE 1 on p. 608 for Exs. 15–16

Graph the image of $H(-4, 5)$ after the glide reflection.

15. Translation: $(x, y) \to (x + 6, y - 2)$
Reflection: in $x = 3$

16. Translation: $(x, y) \to (x - 4, y - 5)$
Reflection: in $y = x$

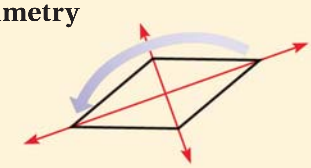
9.6 Identify Symmetry

pp. 619–624

EXAMPLE

Determine whether the rhombus has *line symmetry* and/or *rotational symmetry*. Identify the number of lines of symmetry and/or the rotations that map the figure onto itself.

The rhombus has two lines of symmetry. It also has rotational symmetry, because a 180° rotation maps the rhombus onto itself.



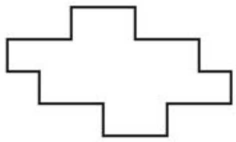
EXERCISES

EXAMPLES 1 and 2 on pp. 619–620 for Exs. 17–19

Determine whether the figure has *line symmetry* and/or *rotational symmetry*. Identify the number of lines of symmetry and/or the rotations that map the figure onto itself.

17.

18.



19.

9.7 Identify and Perform Dilations

pp. 626–632

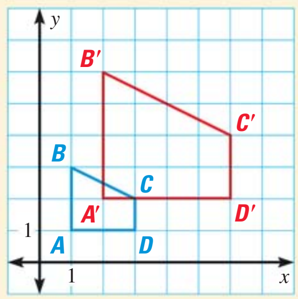
EXAMPLE

Quadrilateral *ABCD* has vertices $A(0, 0)$, $B(0, 3)$, $C(2, 2)$, and $D(2, 0)$. Use scalar multiplication to find the image of *ABCD* after a dilation with its center at the origin and a scale factor of 2. Graph *ABCD* and its image.

To find the image matrix, multiply each element of the polygon matrix by the scale factor.

$$\begin{array}{c} \\ 2 \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 1 & 3 & 3 \\ 1 & 3 & 2 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ = \end{array} \begin{array}{c} \begin{array}{cccc} A' & B' & C' & D' \end{array} \\ \begin{bmatrix} 2 & 2 & 6 & 6 \\ 2 & 6 & 4 & 2 \end{bmatrix} \end{array}$$

Scale factor **Polygon matrix** **Image matrix**



EXERCISES

EXAMPLE 4 on p. 628 for Exs. 20–21

Find the image matrix that represents a dilation of the polygon centered at the origin with the given scale factor. Then graph the polygon and its image.

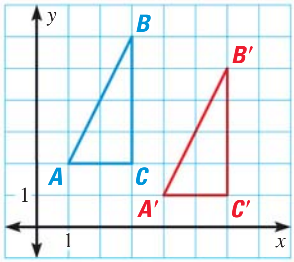
20. $\begin{array}{c} \begin{array}{ccc} Q & R & S \end{array} \\ \begin{bmatrix} 2 & 4 & 8 \\ 2 & 4 & 2 \end{bmatrix} \end{array}; k = \frac{1}{4}$

21. $\begin{array}{c} \begin{array}{ccc} L & M & N \end{array} \\ \begin{bmatrix} -1 & 1 & 2 \\ -2 & 3 & 4 \end{bmatrix} \end{array}; k = 3$

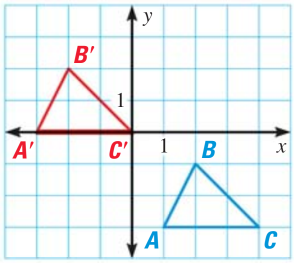
9 CHAPTER TEST

Write a rule for the translation of $\triangle ABC$ to $\triangle A'B'C'$. Then verify that the translation is an isometry.

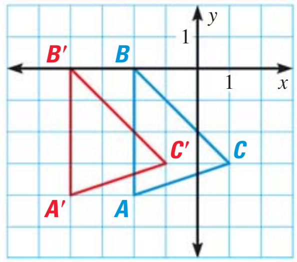
1.



2.



3.



Add, subtract, or multiply.

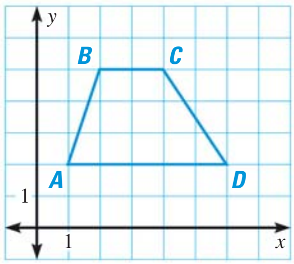
4. $\begin{bmatrix} 3 & -8 \\ 9 & 4.3 \end{bmatrix} + \begin{bmatrix} -10 & 2 \\ 5.1 & -5 \end{bmatrix}$

5. $\begin{bmatrix} -2 & 2.6 \\ 0.8 & 4 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ -1 & 3 \end{bmatrix}$

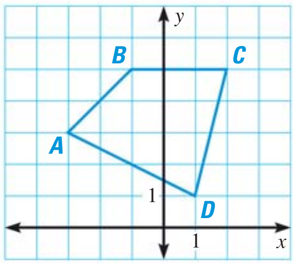
6. $\begin{bmatrix} 7 & -3 & 2 \\ 5 & 1 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$

Graph the image of the polygon after the reflection in the given line.

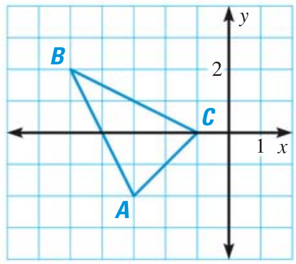
7. x-axis



8. $y = 3$



9. $y = -x$



Find the image matrix that represents the rotation of the polygon. Then graph the polygon and its image.

10. $\triangle ABC$: $\begin{bmatrix} 2 & 4 & 6 \\ 2 & 5 & 1 \end{bmatrix}$; 90° rotation

11. $KLMN$: $\begin{bmatrix} -5 & -2 & -3 & -5 \\ 0 & 3 & -1 & -3 \end{bmatrix}$; 180° rotation

The vertices of $\triangle PQR$ are $P(-5, 1)$, $Q(-4, 6)$, and $R(-2, 3)$. Graph $\triangle P''Q''R''$ after a composition of the transformations in the order they are listed.

12. **Translation:** $(x, y) \rightarrow (x - 8, y)$
Dilation: centered at the origin, $k = 2$

13. **Reflection:** in the y-axis
Rotation: 90° about the origin

Determine whether the flag has *line symmetry* and/or *rotational symmetry*. Identify all lines of symmetry and/or angles of rotation that map the figure onto itself.

14.

15.

16.

9 ALGEBRA REVIEW

Animated Algebra classzone.com

MULTIPLY BINOMIALS AND USE QUADRATIC FORMULA

EXAMPLE 1 Multiply binomials

Find the product $(2x + 3)(x - 7)$.

Solution

Use the **FOIL** pattern: Multiply the **F**irst, **O**uter, **I**nner, and **L**ast terms.

$$\begin{aligned}(2x+3)(x-7) &= \overset{\text{First}}{2x(x)} + \overset{\text{Outer}}{2x(-7)} + \overset{\text{Inner}}{3(x)} + \overset{\text{Last}}{3(-7)} && \text{Write the products of terms.}\\ &= 2x^2 - 14x + 3x - 21 && \text{Multiply.}\\ &= 2x^2 - 11x - 21 && \text{Combine like terms.}\end{aligned}$$

EXAMPLE 2 Solve a quadratic equation using the quadratic formula

Solve $2x^2 + 1 = 5x$.

Solution

Write the equation in standard form to be able to use the quadratic formula.

$$\begin{aligned} 2x^2 + 1 &= 5x && \text{Write the original equation.}\\ 2x^2 - 5x + 1 &= 0 && \text{Write in standard form.}\\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{Write the quadratic formula.}\\ x &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(1)}}{2(2)} && \text{Substitute values in the quadratic formula: } a = 2,\ b = -5, \text{ and } c = 1.\\ x &= \frac{5 \pm \sqrt{25 - 8}}{4} = \frac{5 \pm \sqrt{17}}{4} && \text{Simplify.}\end{aligned}$$

▶ The solutions are $\frac{5 + \sqrt{17}}{4} \approx 2.28$ and $\frac{5 - \sqrt{17}}{4} \approx 0.22$.

EXERCISES

EXAMPLE 1 for Exs. 1–9

Find the product.

1. $(x + 3)(x - 2)$
2. $(x - 8)^2$
3. $(x + 4)(x - 4)$
4. $(x - 5)(x - 1)$
5. $(7x + 6)^2$
6. $(3x - 1)(x + 9)$
7. $(2x + 1)(2x - 1)$
8. $(-3x + 1)^2$
9. $(x + y)(2x + y)$

EXAMPLE 2 for Exs. 10–18

Use the quadratic formula to solve the equation.

10. $3x^2 - 2x - 5 = 0$
11. $x^2 - 7x + 12 = 0$
12. $x^2 + 5x - 2 = 0$
13. $4x^2 + 9x + 2 = 0$
14. $3x^2 + 4x - 10 = 0$
15. $x^2 + x = 7$
16. $3x^2 = 5x - 1$
17. $x^2 = -11x - 4$
18. $5x^2 + 6 = 17x$

Scoring Rubric

Full Credit
- solution is complete and correct

Partial Credit
- solution is complete but has errors,

or
- solution is without error but incomplete

No Credit
- no solution is given,

or
- solution makes no sense

SHORT RESPONSE QUESTIONS

PROBLEM

The vertices of $\triangle PQR$ are $P(1, -1)$, $Q(4, -1)$, and $R(0, -3)$. What are the coordinates of the image of $\triangle PQR$ after the given composition? *Describe* your steps. Include a graph with your answer.

Translation: $(x, y) \to (x - 6, y)$
Reflection: in the x-axis

Below are sample solutions to the problem. Read each solution and the comments in blue to see why the sample represents full credit, partial credit, or no credit.

SAMPLE 1: Full credit solution

The reasoning is correct, and the graphs are correct.

First, graph $\triangle PQR$. Next, to translate $\triangle PQR$ 6 units left, subtract 6 from the x-coordinate of each vertex.

$P(1, -1) \to P'(-5, -1)$

$Q(4, -1) \to Q'(-2, -1)$

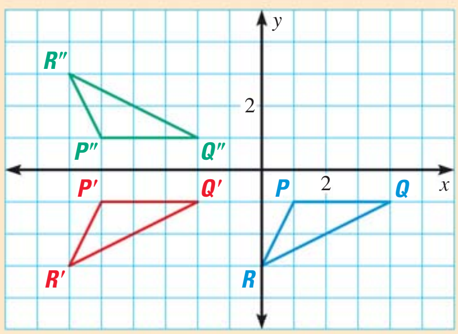
$R(0, -3) \to R'(-6, -3)$

Finally, reflect $\triangle P'Q'R'$ in the x-axis by multiplying the y-coordinates by -1.

$P'(-5, -1) \to P''(-5, 1)$

$Q'(-2, -1) \to Q''(-2, 1)$

$R'(-6, -3) \to R''(-6, 3)$

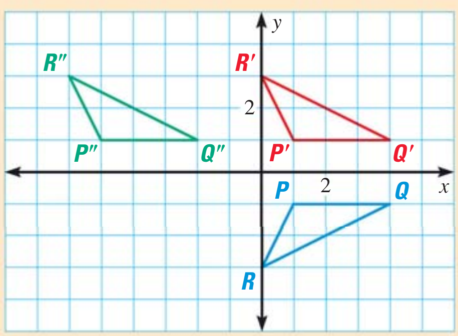


SAMPLE 2: Partial credit solution

Each transformation is performed correctly. However, the transformations are not performed in the order given in the problem.

First, graph $\triangle PQR$. Next, reflect $\triangle PQR$ over the x-axis by multiplying each y-coordinate by -1. Finally, to translate $\triangle P'Q'R'$ 6 units left, subtract 6 from each x-coordinate.

The coordinates of the image of $\triangle PQR$ after the composition are $P''(-2, 1)$, $Q''(-5, 1)$, and $R''(-6, 3)$.



SAMPLE 3: Partial credit solution

The reasoning is correct, but the student does not show a graph.

First subtract 6 from each x-coordinate. So, $P'(1 - 6, -1) = P'(-5, -1)$, $Q'(4 - 6, -1) = Q'(-2, -1)$, and $R'(0 - 6, -3) = R'(-6, -3)$. Then reflect the triangle in the x-axis by multiplying each y-coordinate by -1. So, $P''(-5, -1 \cdot (-1)) = P''(-5, 1)$, $Q''(-2, -1 \cdot (-1)) = Q''(-2, 1)$, and $R''(-6, -1 \cdot (-3)) = R''(-6, 3)$.

SAMPLE 4: No credit solution

The reasoning is incorrect, and the student does not show a graph.

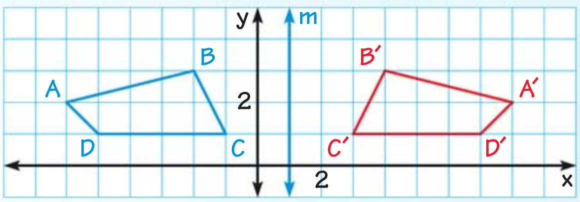
Translate $\triangle PQR$ 6 units by adding 6 to each x-coordinate. Then multiply each x-coordinate by -1 to reflect the image over the x-axis. The resulting $\triangle P'Q'R'$ has vertices $P'(-7, -1)$, $Q'(-10, -1)$, and $R'(-6, -3)$.

PRACTICE Apply Scoring Rubric

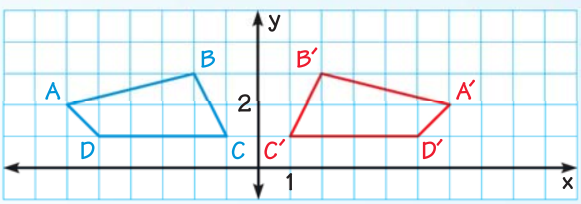
Use the rubric on page 642 to score the solution to the problem below as *full credit*, *partial credit*, or *no credit*. *Explain* your reasoning.

PROBLEM The vertices of $ABCD$ are $A(-6, 2)$, $B(-2, 3)$, $C(-1, 1)$, and $D(-5, 1)$. Graph the reflection of $ABCD$ in line m with equation $x = 1$.

1. First, graph ABCD. Because m is a vertical line, the reflection will not change the y-coordinates. A is 7 units left of m, so A′ is 7 units right of m, at A′(8, 2). Since B is 3 units left of m, B′ is 3 units right of m, at B′(4, 3). The images of C and D are C′(3, 1) and D′(7, 1).



2. First, graph ABCD. The reflection is in a vertical line, so only the x-coordinates change. Multiply the x-coordinates in ABCD by −1 to get A′(6, 2), B′(2, 3), C′(−1, 1), and D′(5, 1). Graph A′B′C′D′.



9 ★ *Standardized* TEST PRACTICE

SHORT RESPONSE

1. Use the square window shown below.

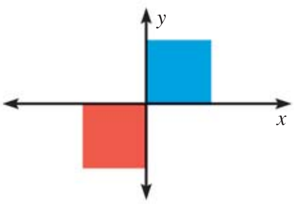
a. Draw a sketch showing all the lines of symmetry in the window design.

b. Does the design have rotational symmetry? If so, *describe* the rotations that map the design onto itself.

2. The vertices of a triangle are $A(0, 2)$, $B(2, 0)$, and $C(-2, 0)$. What are the coordinates of the image of $\triangle ABC$ after the given composition? Include a graph with your answer.

Dilation: $(x, y) \to (3x, 3x)$
Translation: $(x, y) \to (x - 2, y - 2)$

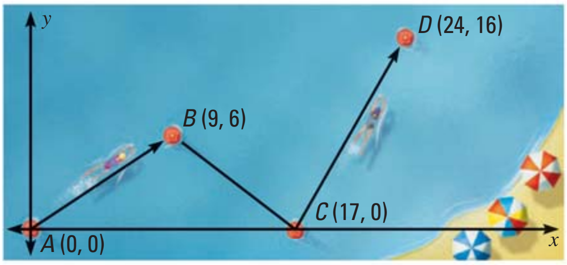
3. The red square is the image of the blue square after a single transformation. *Describe* three different transformations that could produce the image.



4. At a stadium concession stand, a hotdog costs \$3.25, a soft drink costs \$2.50, and a pretzel costs \$3. The Johnson family buys 5 hotdogs, 3 soft drinks, and 1 pretzel. The Scott family buys 4 hotdogs, 4 soft drinks, and 2 pretzels. Use matrix multiplication to find the total amount spent by each family. Which family spends more money? *Explain*.

5. The design below is made of congruent isosceles trapezoids. Find the measures of the four interior angles of one of the trapezoids. *Explain* your reasoning.

6. Two swimmers design a race course near a beach. The swimmers must move from point A to point B. Then they swim from point B to point C. Finally, they swim from point C to point D. Write the component form of the vectors shown in the diagram, $\overrightarrow{AB}$, $\overrightarrow{BC}$, and $\overrightarrow{CD}$. Then write the component form of $\overrightarrow{AD}$.



7. A polygon is reflected in the x-axis and then reflected in the y-axis. *Explain* how you can use a rotation to obtain the same result as this composition of transformations. Draw an example.

8. In rectangle $PQRS$, one side is twice as long as the other side. Rectangle $P'Q'R'S'$ is the image of $PQRS$ after a dilation centered at P with a scale factor of 0.5. The area of $P'Q'R'S'$ is 32 square inches.

a. Find the lengths of the sides of $PQRS$. *Explain*.

b. Find the ratio of the area of $PQRS$ to the area of $P'Q'R'S'$.

MULTIPLE CHOICE

9. Which matrix product is equivalent to the product $\begin{bmatrix} 3 & -1 \end{bmatrix}\begin{bmatrix} 7 \\ 4 \end{bmatrix}$?

Ⓐ $\begin{bmatrix} -3 & 1 \end{bmatrix}\begin{bmatrix} -7 \\ 4 \end{bmatrix}$

Ⓑ $\begin{bmatrix} 1 & 3 \end{bmatrix}\begin{bmatrix} -4 \\ 7 \end{bmatrix}$

Ⓒ $\begin{bmatrix} -1 & 3 \end{bmatrix}\begin{bmatrix} 7 \\ 4 \end{bmatrix}$

Ⓓ $\begin{bmatrix} 1 & -3 \end{bmatrix}\begin{bmatrix} 4 \\ -7 \end{bmatrix}$

10. Which transformation is *not* an isometry?

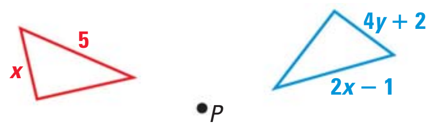
Ⓐ Translation　Ⓑ Reflection

Ⓒ Rotation　Ⓓ Dilation

GRIDDED ANSWER

11. Line p passes through points $J(2, 5)$ and $K(-4, 13)$. Line q is the image of line p after line p is reflected in the x-axis. Find the slope of line q.

12. The red triangle is the image of the blue triangle after it is rotated about point P. What is the value of y?



13. The vertices of $\triangle PQR$ are $P(1, 4)$, $Q(2, 0)$, and $R(4, 5)$. What is the x-coordinate of Q' after the given composition?

Translation: $(x, y) \rightarrow (x - 2, y + 1)$
Dilation: centered at $(0, 0)$ with $k = 2$

EXTENDED RESPONSE

14. An equation of line ℓ is $y = 3x$.

a. Graph line ℓ. Then graph the image of line ℓ after it is reflected in the line $y = x$.

b. Find the equation of the image.

c. Suppose a line has an equation of the form $y = ax$. Make a conjecture about the equation of the image of that line when it is reflected in the line $y = x$. Use several examples to support your conjecture.

15. The vertices of $\triangle EFG$ are $E(4, 2)$, $F(-2, 1)$, and $G(0, -3)$.

a. Find the coordinates of the vertices of $\triangle E'F'G'$, the image of $\triangle EFG$ after a dilation centered at the origin with a scale factor of 2. Graph $\triangle EFG$ and $\triangle E'F'G'$ in the same coordinate plane.

b. Find the coordinates of the vertices of $\triangle E''F''G''$, the image of $\triangle E'F'G'$ after a dilation centered at the origin with a scale factor of 2.5. Graph $\triangle E''F''G''$ in the same coordinate plane you used in part (a).

c. What is the dilation that maps $\triangle EFG$ to $\triangle E''F''G''$?

d. What is the scale factor of a dilation that is equivalent to the composition of two dilations described below? *Explain.*

Dilation: centered at $(0, 0)$ with a scale factor of a
Dilation: centered at $(0, 0)$ with a scale factor of b

CUMULATIVE REVIEW Chapters 1–9

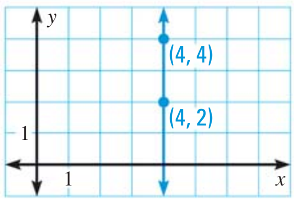
Tell whether the lines through the given points are *parallel*, *perpendicular*, or *neither*. *(p. 171)*

1. Line 1: (3, 5), (−2, 6)
Line 2: (−3, 5), (−4, 10)

2. Line 1: (2, −10), (9, −8)
Line 2: (8, 6), (1, 4)

Write an equation of the line shown. *(p. 180)*

3.



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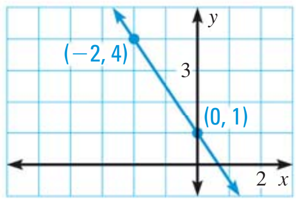

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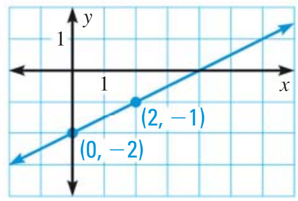
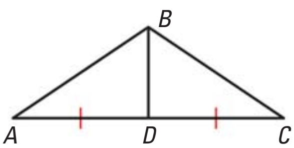
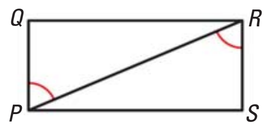


State the third congruence that must be given to prove that the triangles are congruent using the given postulate or theorem. *(pp. 234, 240, and 249)*

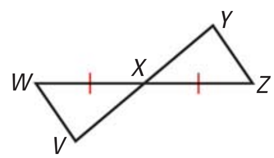
6. SSS Congruence Post.



7. SAS Congruence Post.

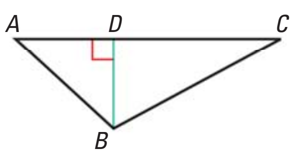


8. AAS Congruence Thm

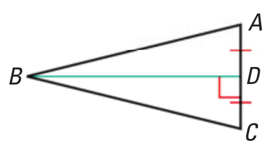


Determine whether $\overline{BD}$ is a *perpendicular bisector*, *median*, or *altitude* of $\triangle ABC$. *(p. 319)*

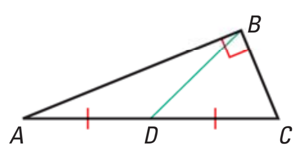
9.



10.



11.



Determine whether the segment lengths form a triangle. If so, would the triangle be *acute*, *right*, or *obtuse*? *(pp. 328 and 441)*

12. 11, 11, 15

13. 33, 44, 55

14. 9, 9, 13

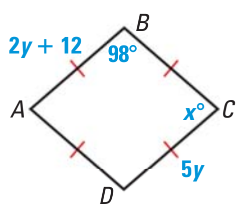
15. 7, 8, 16

16. 9, 40, 41

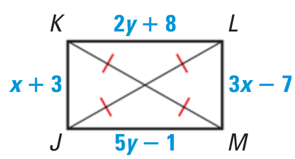
17. 0.5, 1.2, 1.3

Classify the special quadrilateral. *Explain* your reasoning. Then find the values of x and y. *(p. 533)*

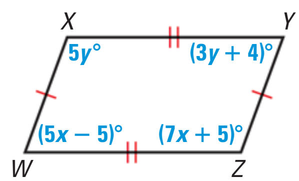
18.



19.



20.

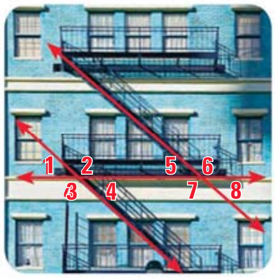


Graph the image of the triangle after the composition of the transformations in the order they are listed. ***(p. 608)***

21. $P(-5, 2)$, $Q(-2, 4)$, $R(0, 0)$
Translation: $(x, y) \rightarrow (x - 2, y + 5)$
Reflection: in the x-axis

22. $F(-1, -8)$, $G(-6, -3)$, $R(0, 0)$
Reflection: in the line $x = 2$
Rotation: $90°$ about the origin

FIRE ESCAPE In the diagram, the staircases on the fire escape are parallel. The measure of $\angle 1$ is $48°$. ***(p. 154)***



23. Identify the angle(s) congruent to $\angle 1$.

24. Identify the angle(s) congruent to $\angle 2$.

25. What is $m\angle 2$?

26. What is $m\angle 6$?

27. BAHAMA ISLANDS The map of some of the Bahamas has a scale of $\frac{1}{2}$ inch : 60 miles. Use a ruler to estimate the actual distance from Freeport to Nassau. ***(p. 364)***

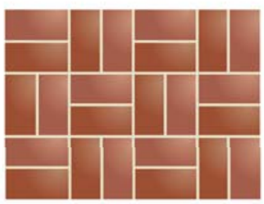
28. ANGLE OF ELEVATION You are standing 12 feet away from your house and the angle of elevation is $65°$ from your foot. How tall is your house? Round to the nearest foot. ***(p. 473)***

29. PURSE You are decorating 8 trapezoid-shaped purses to sell at a craft show. You want to decorate the front of each purse with a string of beads across the midsegment. On each purse, the length of the bottom is 5.5 inches and the length of the top is 9 inches. If the beading costs \$1.59 per foot, how much will it cost to decorate the 8 purses? ***(p. 542)***

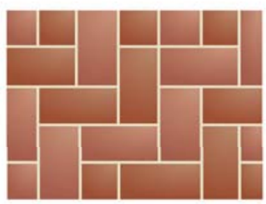
TILE PATTERNS ***Describe*** **the transformations that are combined to make the tile pattern.** ***(p. 607)***

30.

31.



32.



10 Properties of Circles

Before

In previous chapters, you learned the following skills, which you'll use in Chapter 10: classifying triangles, finding angle measures, and solving equations.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

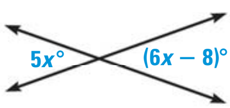
1. Two similar triangles have congruent corresponding angles and _?_ corresponding sides.
2. Two angles whose sides form two pairs of opposite rays are called _?_.
3. The _?_ of an angle is all of the points between the sides of the angle.

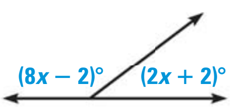
SKILLS AND ALGEBRA CHECK

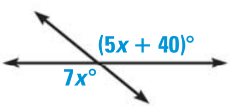
Use the Converse of the Pythagorean Theorem to classify the triangle. *(Review p. 441 for 10.1.)*

4. 0.6, 0.8, 0.9
5. 11, 12, 17
6. 1.5, 2, 2.5

Find the value of the variable. *(Review pp. 24, 35 for 10.2, 10.4.)*

7.


8.


9.


@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 10, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 707. You will also use the key vocabulary listed below.

Big Ideas

1. Using properties of segments that intersect circles
2. Applying angle relationships in circles
3. Using circles in the coordinate plane

KEY VOCABULARY

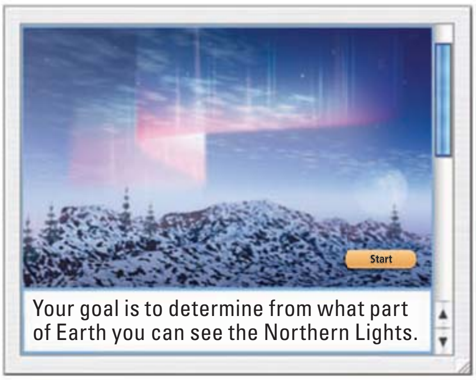
- circle, *p. 651*
  center, radius, diameter
- chord, *p. 651*
- secant, *p. 651*
- tangent, *p. 651*
- central angle, *p. 659*
- minor arc, *p. 659*
- major arc, *p. 659*
- semicircle, *p. 659*
- congruent circles, *p. 660*
- congruent arcs, *p. 660*
- inscribed angle, *p. 672*
- intercepted arc, *p. 672*
- standard equation of a circle, *p. 699*

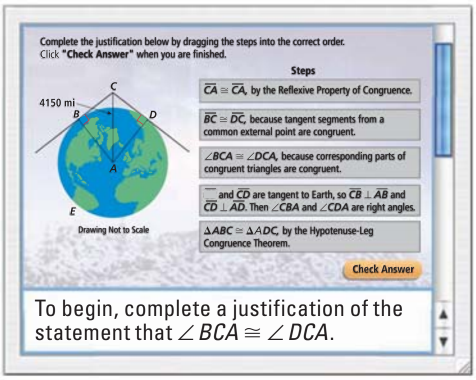
Why?

Circles can be used to model a wide variety of natural phenomena. You can use properties of circles to investigate the Northern Lights.

Animated Geometry

The animation illustrated below for Example 4 on page 682 helps you answer this question: From what part of Earth are the Northern Lights visible?





Animated Geometry at classzone.com

Other animations for Chapter 10: pages 655, 661, 671, 691, and 701

Investigating Geometry **ACTIVITY** *Use before Lesson 10.1*

10.1 Explore Tangent Segments

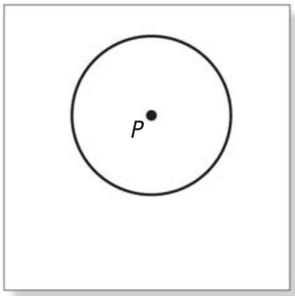
MATERIALS • compass • ruler

QUESTION How are the lengths of tangent segments related?

A line can intersect a circle at 0, 1, or 2 points. If a line is in the plane of a circle and intersects the circle at 1 point, the line is a *tangent*.

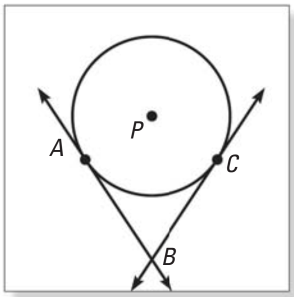
EXPLORE Draw tangents to a circle

STEP 1



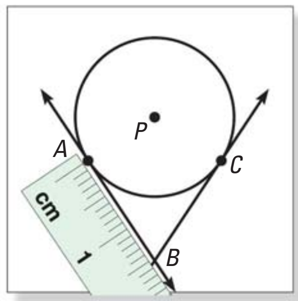
Draw a circle Use a compass to draw a circle. Label the center P.

STEP 2



Draw tangents Draw lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CB}$ so that they intersect $\odot P$ only at A and C, respectively. These lines are called *tangents*.

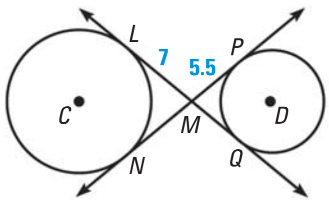
STEP 3



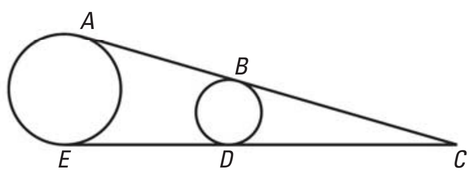
Measure segments $\overline{AB}$ and $\overline{CB}$ are called *tangent segments*. Measure and compare the lengths of the tangent segments.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Repeat Steps 1–3 with three different circles.
2. Use your results from Exercise 1 to make a conjecture about the lengths of tangent segments that have a common endpoint.
3. In the diagram, L, Q, N, and P are points of tangency. Use your conjecture from Exercise 2 to find LQ and NP if $LM = 7$ and $MP = 5.5$.



4. In the diagram below, A, B, D, and E are points of tangency. Use your conjecture from Exercise 2 to explain why $\overline{AB} \cong \overline{ED}$.



10.1 Use Properties of Tangents

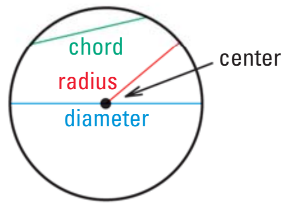
Before You found the circumference and area of circles.

Now You will use properties of a tangent to a circle.

Why? So you can find the range of a GPS satellite, as in Ex. 37.

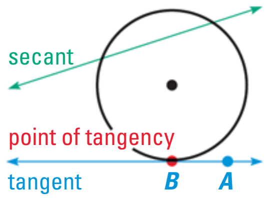
Key Vocabulary
- **circle**
  center, radius, diameter
- **chord**
- **secant**
- **tangent**

A **circle** is the set of all points in a plane that are equidistant from a given point called the **center** of the circle. A circle with center P is called "circle P" and can be written $\odot P$. A segment whose endpoints are the center and any point on the circle is a **radius**.



A **chord** is a segment whose endpoints are on a circle. A **diameter** is a chord that contains the center of the circle.

A **secant** is a line that intersects a circle in two points. A **tangent** is a line in the plane of a circle that intersects the circle in exactly one point, the *point of tangency*. The *tangent ray* $\overrightarrow{AB}$ and the *tangent segment* $\overline{AB}$ are also called tangents.

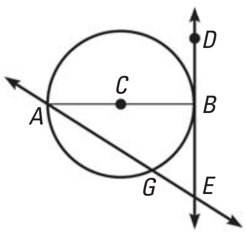


EXAMPLE 1 Identify special segments and lines

Tell whether the line, ray, or segment is best described as a *radius, chord, diameter, secant,* or *tangent* of $\odot C$.

a. $\overline{AC}$ **b.** $\overline{AB}$

c. $\overrightarrow{DE}$ **d.** $\overleftrightarrow{AE}$



Solution

a. $\overline{AC}$ is a radius because C is the center and A is a point on the circle.

b. $\overline{AB}$ is a diameter because it is a chord that contains the center C.

c. $\overrightarrow{DE}$ is a tangent ray because it is contained in a line that intersects the circle at only one point.

d. $\overleftrightarrow{AE}$ is a secant because it is a line that intersects the circle in two points.

✓ GUIDED PRACTICE for Example 1

1. In Example 1, what word best describes $\overline{AG}$? $\overline{CB}$?

2. In Example 1, name a tangent and a tangent segment.

READ VOCABULARY
The plural of radius is *radii.* All radii of a circle are congruent.

RADIUS AND DIAMETER The words *radius* and *diameter* are used for lengths as well as segments. For a given circle, think of *a radius* and *a diameter* as segments and *the radius* and *the diameter* as lengths.

EXAMPLE 2 Find lengths in circles in a coordinate plane

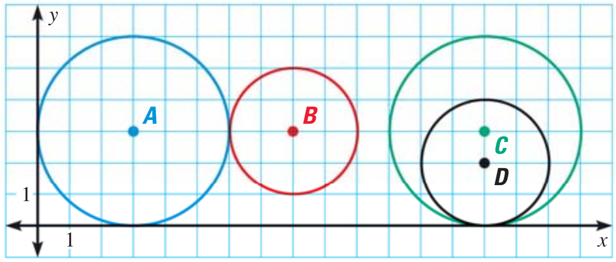
Use the diagram to find the given lengths.

a. Radius of $\odot A$

b. Diameter of $\odot A$

c. Radius of $\odot B$

d. Diameter of $\odot B$



Solution

a. The radius of $\odot A$ is 3 units.

b. The diameter of $\odot A$ is 6 units.

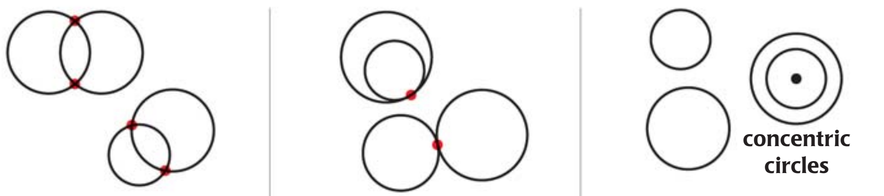
c. The radius of $\odot B$ is 2 units.

d. The diameter of $\odot B$ is 4 units.

✓ GUIDED PRACTICE for Example 2

3. Use the diagram in Example 2 to find the radius and diameter of $\odot C$ and $\odot D$.

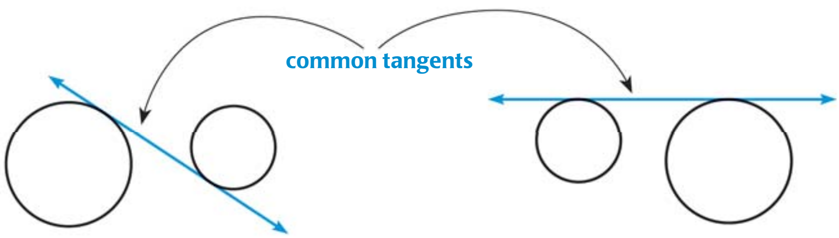
COPLANAR CIRCLES Two circles can intersect in two points, one point, or no points. Coplanar circles that intersect in one point are called *tangent circles*. Coplanar circles that have a common center are called *concentric*.



2 points of intersection | **1 point of intersection (tangent circles)** | **no points of intersection**

READ VOCABULARY
A line that intersects a circle in exactly one point is said to be *tangent* to the circle.

COMMON TANGENTS A line, ray, or segment that is tangent to two coplanar circles is called a *common tangent*.

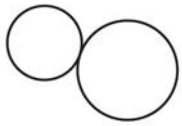


EXAMPLE 3 Draw common tangents

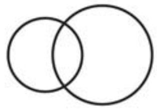
Tell how many common tangents the circles have and draw them.

a.

b.

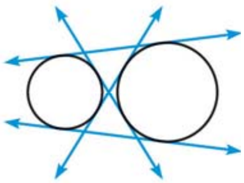


c.

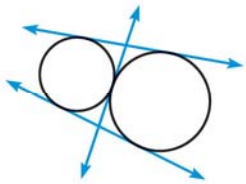


Solution

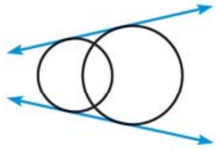
a. 4 common tangents



b. 3 common tangents



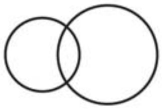
c. 2 common tangents



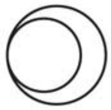
✓ GUIDED PRACTICE for Example 3

Tell how many common tangents the circles have and draw them.

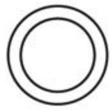
4.



5.



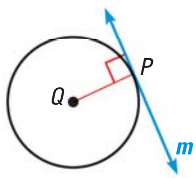
6.



THEOREM *For Your Notebook*

THEOREM 10.1

In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.

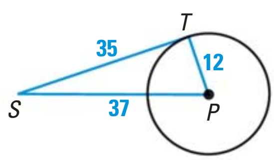


Line *m* is tangent to $\odot Q$ if and only if $m \perp \overline{QP}$.

Proof: Exs. 39–40, p. 658

EXAMPLE 4 Verify a tangent to a circle

In the diagram, $\overline{PT}$ is a radius of $\odot P$. Is $\overline{ST}$ tangent to $\odot P$?

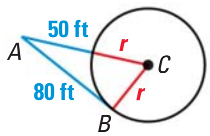


Solution

Use the Converse of the Pythagorean Theorem. Because $12^2 + 35^2 = 37^2$, $\triangle PST$ is a right triangle and $\overline{ST} \perp \overline{PT}$. So, $\overline{ST}$ is perpendicular to a radius of $\odot P$ at its endpoint on $\odot P$. By Theorem 10.1, $\overline{ST}$ is tangent to $\odot P$.

EXAMPLE 5 Find the radius of a circle

In the diagram, B is a point of tangency. Find the radius r of $\odot C$.



Solution

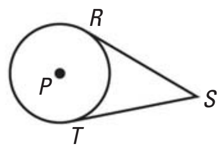
You know from Theorem 10.1 that $\overline{AB} \perp \overline{BC}$, so $\triangle ABC$ is a right triangle. You can use the Pythagorean Theorem.

$AC^2 = BC^2 + AB^2$	**Pythagorean Theorem**
$(r + 50)^2 = r^2 + 80^2$	**Substitute.**
$r^2 + 100r + 2500 = r^2 + 6400$	**Multiply.**
$100r = 3900$	**Subtract from each side.**
$r = 39$ ft	**Divide each side by 100.**

THEOREM *For Your Notebook*

THEOREM 10.2

Tangent segments from a common external point are congruent.

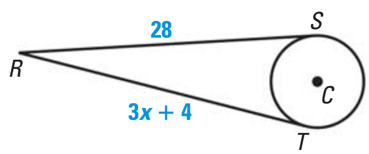


If $\overline{SR}$ and $\overline{ST}$ are tangent segments, then $\overline{SR} \cong \overline{ST}$.

Proof: Ex. 41, p. 658

EXAMPLE 6 Find the radius of a circle

$\overline{RS}$ is tangent to $\odot C$ at S and $\overline{RT}$ is tangent to $\odot C$ at T. Find the value of x.

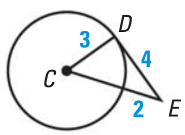


Solution

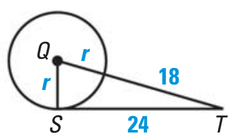
$RS = RT$	**Tangent segments from the same point are $\cong$.**
$28 = 3x + 4$	**Substitute.**
$8 = x$	**Solve for x.**

✓ GUIDED PRACTICE for Examples 4, 5, and 6

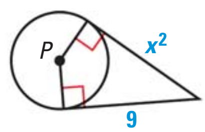
7. Is $\overline{DE}$ tangent to $\odot C$?



8. $\overline{ST}$ is tangent to $\odot Q$. Find the value of r.



9. Find the value(s) of x.



10.1 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 19, and 37
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 29, 33, and 38

SKILL PRACTICE

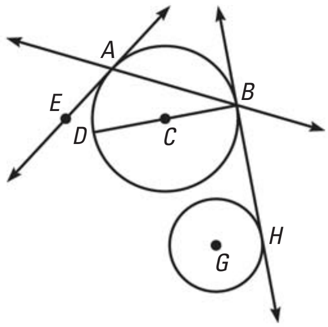
1. **VOCABULARY** Copy and complete: The points A and B are on $\odot C$. If C is a point on $\overline{AB}$, then $\overline{AB}$ is a _?_.

2. ★ **WRITING** Explain how you can determine from the context whether the words *radius* and *diameter* are referring to a segment or a length.

EXAMPLE 1 on p. 651 for Exs. 3–11

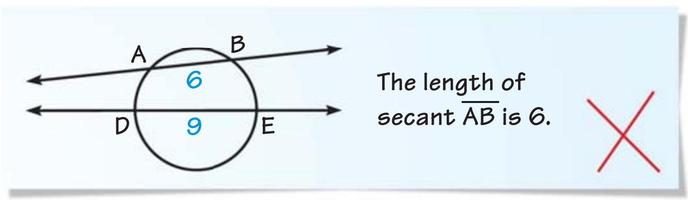
MATCHING TERMS **Match the notation with the term that best describes it.**

3. B	A. Center
4. $\overleftrightarrow{BH}$	B. Radius
5. $\overline{AB}$	C. Chord
6. $\overleftrightarrow{AB}$	D. Diameter
7. $\overleftrightarrow{AE}$	E. Secant
8. G	F. Tangent
9. $\overline{CD}$	G. Point of tangency
10. $\overline{BD}$	H. Common tangent



Animated Geometry at classzone.com

11. **ERROR ANALYSIS** *Describe* and correct the error in the statement about the diagram.



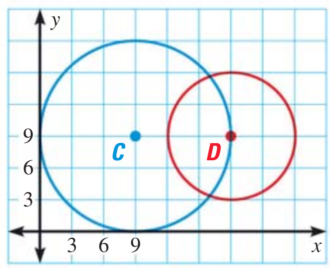
EXAMPLES 2 and 3 on pp. 652–653 for Exs. 12–17

COORDINATE GEOMETRY **Use the diagram at the right.**

12. What are the radius and diameter of $\odot C$?

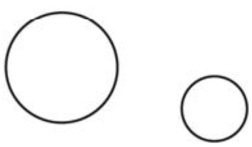
13. What are the radius and diameter of $\odot D$?

14. Copy the circles. Then draw all the common tangents of the two circles.



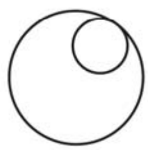
DRAWING TANGENTS **Copy the diagram. Tell how many common tangents the circles have and draw them.**

15.



16.

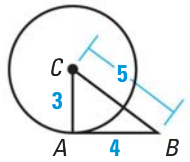
17.



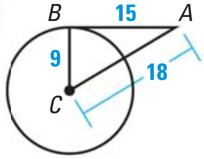
EXAMPLE 4 on p. 653 for Exs. 18–20

DETERMINING TANGENCY **Determine whether $\overline{AB}$ is tangent to $\odot C$. *Explain.***

18.



19.



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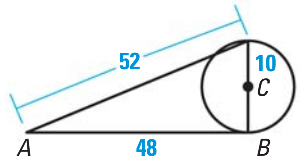
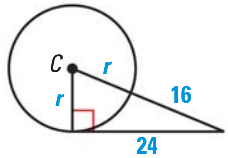


EXAMPLES 5 and 6 on p. 654 for Exs. 21–26

ALGEBRA **Find the value(s) of the variable. In Exercises 24–26, *B* and *D* are points of tangency.**

21.



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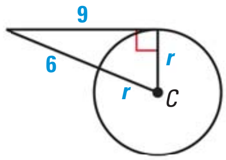

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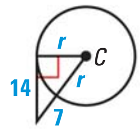

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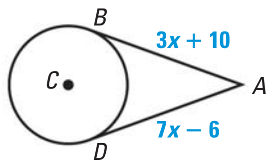

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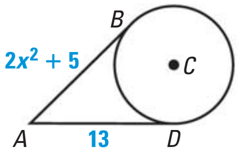

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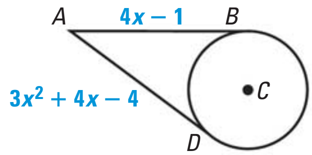
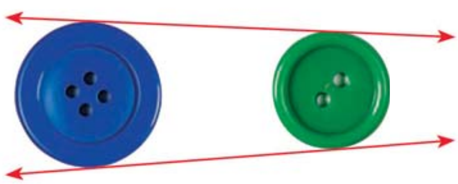


COMMON TANGENTS **A *common internal tangent* intersects the segment that joins the centers of two circles. A *common external* tangent does not intersect the segment that joins the centers of the two circles. Determine whether the common tangents shown are *internal* or *external*.**

27.



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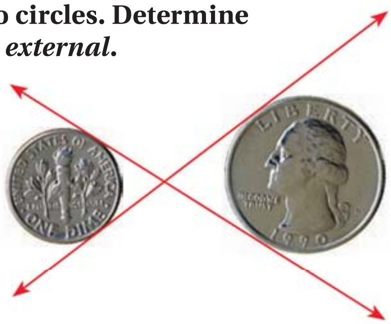


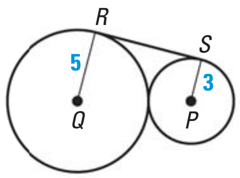
29. ★ **MULTIPLE CHOICE** In the diagram, $\odot P$ and $\odot Q$ are tangent circles. $\overline{RS}$ is a common tangent. Find RS.

Ⓐ $-2\sqrt{15}$

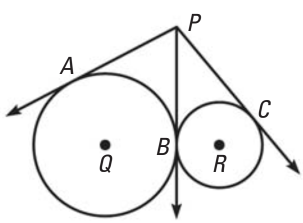
Ⓑ 4

Ⓒ $2\sqrt{15}$

Ⓓ 8



30. **REASONING** In the diagram, $\overrightarrow{PB}$ is tangent to $\odot Q$ and $\odot R$. *Explain* why $\overline{PA} \cong \overline{PB} \cong \overline{PC}$ even though the radius of $\odot Q$ is not equal to the radius of $\odot R$.

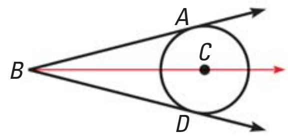


31. **TANGENT LINES** When will two lines tangent to the same circle not intersect? Use Theorem 10.1 to *explain* your answer.

○ = **WORKED-OUT SOLUTIONS** on p. WS1

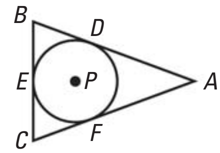
★ = **STANDARDIZED TEST PRACTICE**

32. **ANGLE BISECTOR** In the diagram at right, A and D are points of tangency on $\odot C$. *Explain* how you know that $\overrightarrow{BC}$ bisects $\angle ABD$. (*Hint*: Use Theorem 5.6, page 310.)



33. ★ **SHORT RESPONSE** For any point outside of a circle, is there ever only one tangent to the circle that passes through the point? Are there ever more than two such tangents? *Explain* your reasoning.

34. **CHALLENGE** In the diagram at the right, $AB = AC = 12$, $BC = 8$, and all three segments are tangent to $\odot P$. What is the radius of $\odot P$?



PROBLEM SOLVING

BICYCLES On modern bicycles, rear wheels usually have *tangential spokes*. Occasionally, front wheels have *radial spokes*. Use the definitions of *tangent* and *radius* to determine if the wheel shown has *tangential spokes* or *radial spokes*.

35.

36.

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EXAMPLE 4
on p. 653
for Ex. 37

37. **GLOBAL POSITIONING SYSTEM (GPS)** GPS satellites orbit about 11,000 miles above Earth. The mean radius of Earth is about 3959 miles. Because GPS signals cannot travel through Earth, a satellite can transmit signals only as far as points A and C from point B, as shown. Find BA and BC to the nearest mile.

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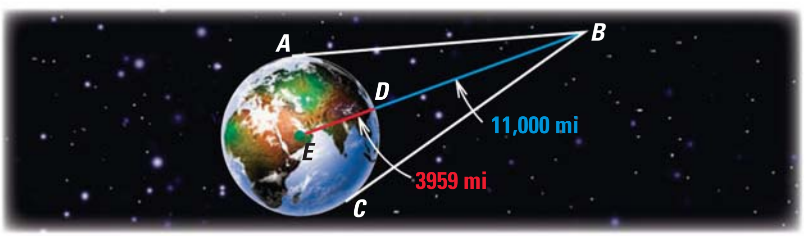
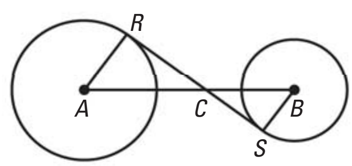
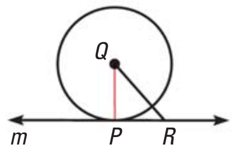


38. ★ **SHORT RESPONSE** In the diagram, $\overline{RS}$ is a common internal tangent (see Exercises 27–28) to $\odot A$ and $\odot B$. Use similar triangles to *explain* why $\frac{AC}{BC} = \frac{RC}{SC}$.



39. **PROVING THEOREM 10.1** Use parts (a)–(c) to prove indirectly that if a line is tangent to a circle, then it is perpendicular to a radius.



GIVEN ▶ Line m is tangent to $\odot Q$ at P.

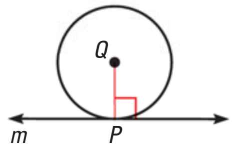
PROVE ▶ $m \perp \overline{QP}$

a. Assume m is not perpendicular to $\overline{QP}$. Then the perpendicular segment from Q to m intersects m at some other point R. Because m is a tangent, R cannot be inside $\odot Q$. *Compare* the length QR to QP.

b. Because $\overline{QR}$ is the perpendicular segment from Q to m, $\overline{QR}$ is the shortest segment from Q to m. Now *compare* QR to QP.

c. Use your results from parts (a) and (b) to complete the indirect proof.

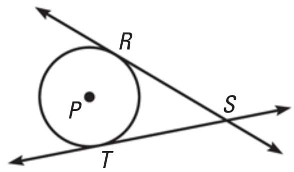
40. **PROVING THEOREM 10.1** Write an indirect proof that if a line is perpendicular to a radius at its endpoint, the line is a tangent.



GIVEN ▶ $m \perp \overline{QP}$

PROVE ▶ Line m is tangent to $\odot Q$.

41. **PROVING THEOREM 10.2** Write a proof that tangent segments from a common external point are congruent.

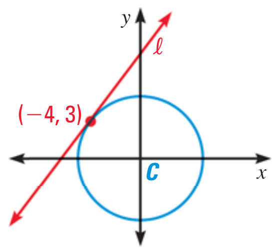


GIVEN ▶ $\overline{SR}$ and $\overline{ST}$ are tangent to $\odot P$.

PROVE ▶ $\overline{SR} \cong \overline{ST}$

Plan for Proof Use the Hypotenuse–Leg Congruence Theorem to show that $\triangle SRP \cong \triangle STP$.

42. **CHALLENGE** Point C is located at the origin. Line ℓ is tangent to $\odot C$ at $(-4, 3)$. Use the diagram at the right to complete the problem.



a. Find the slope of line ℓ.

b. Write the equation for ℓ.

c. Find the radius of $\odot C$.

d. Find the distance from ℓ to $\odot C$ along the y-axis.

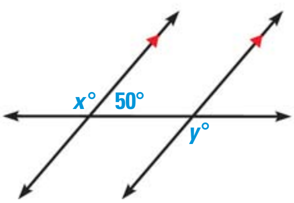
MIXED REVIEW

PREVIEW
Prepare for Lesson 10.2 in Ex. 43.

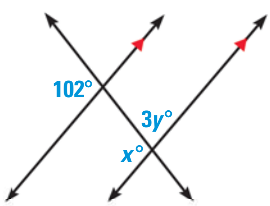
43. D is in the interior of $\angle ABC$. If $m\angle ABD = 25°$ and $m\angle ABC = 70°$, find $m\angle DBC$. *(p. 24)*

Find the values of x and y. *(p. 154)*

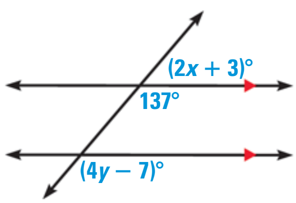
44.



45.



46.



47. A triangle has sides of lengths 8 and 13. Use an inequality to describe the possible length of the third side. What if two sides have lengths 4 and 11? *(p. 328)*

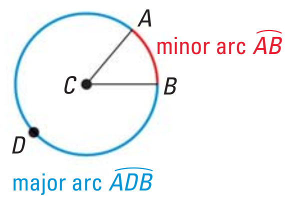
10.2 Find Arc Measures

Before	You found angle measures.
Now	You will use angle measures to find arc measures.
Why?	So you can describe the arc made by a bridge, as in Ex. 22.

Key Vocabulary
- **central angle**
- **minor arc**
- **major arc**
- **semicircle**
- **measure**
  minor arc, major arc
- **congruent circles**
- **congruent arcs**

A **central angle** of a circle is an angle whose vertex is the center of the circle. In the diagram, $\angle ACB$ is a central angle of $\odot C$.

If $m\angle ACB$ is less than 180°, then the points on $\odot C$ that lie in the interior of $\angle ACB$ form a **minor arc** with endpoints A and B. The points on $\odot C$ that do not lie on minor arc $\overset{\frown}{AB}$ form a **major arc** with endpoints A and B. A **semicircle** is an arc with endpoints that are the endpoints of a diameter.



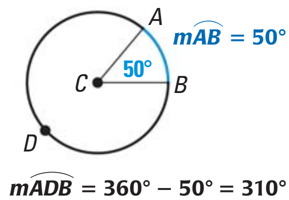
NAMING ARCS Minor arcs are named by their endpoints. The minor arc associated with $\angle ACB$ is named $\overset{\frown}{AB}$. Major arcs and semicircles are named by their endpoints and a point on the arc. The major arc associated with $\angle ACB$ can be named $\overset{\frown}{ADB}$.

KEY CONCEPT — *For Your Notebook*

Measuring Arcs

The **measure of a minor arc** is the measure of its central angle. The expression $m\overset{\frown}{AB}$ is read as "the measure of arc AB."

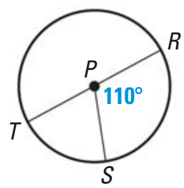
The measure of the entire circle is 360°. The **measure of a major arc** is the difference between 360° and the measure of the related minor arc. The measure of a semicircle is 180°.



EXAMPLE 1 Find measures of arcs

Find the measure of each arc of $\odot P$, where $\overline{RT}$ is a diameter.

a. $\overset{\frown}{RS}$ **b.** $\overset{\frown}{RTS}$ **c.** $\overset{\frown}{RST}$



Solution

a. $\overset{\frown}{RS}$ is a minor arc, so $m\overset{\frown}{RS} = m\angle RPS = 110°$.

b. $\overset{\frown}{RTS}$ is a major arc, so $m\overset{\frown}{RTS} = 360° - 110° = 250°$.

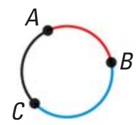
c. $\overline{RT}$ is a diameter, so $\overset{\frown}{RST}$ is a semicircle, and $m\overset{\frown}{RST} = 180°$.

ADJACENT ARCS Two arcs of the same circle are *adjacent* if they have a common endpoint. You can add the measures of two adjacent arcs.

POSTULATE — *For Your Notebook*

POSTULATE 23 Arc Addition Postulate

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

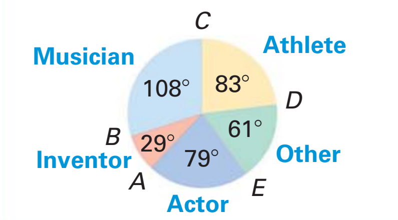


$m\widehat{ABC} = m\widehat{AB} + m\widehat{BC}$

EXAMPLE 2 Find measures of arcs

SURVEY A recent survey asked teenagers if they would rather meet a famous musician, athlete, actor, inventor, or other person. The results are shown in the circle graph. Find the indicated arc measures.

Whom Would You Rather Meet?



a. $m\widehat{AC}$ **b.** $m\widehat{ACD}$

c. $m\widehat{ADC}$ **d.** $m\widehat{EBD}$

Solution

ARC MEASURES
The measure of a minor arc is less than 180°. The measure of a major arc is greater than 180°.

a. $m\widehat{AC} = m\widehat{AB} + m\widehat{BC}$
$= 29° + 108°$
$= 137°$

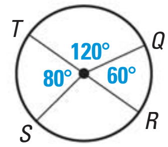
b. $m\widehat{ACD} = m\widehat{AC} + m\widehat{CD}$
$= 137° + 83°$
$= 220°$

c. $m\widehat{ADC} = 360° - m\widehat{AC}$
$= 360° - 137°$
$= 223°$

d. $m\widehat{EBD} = 360° - m\widehat{ED}$
$= 360° - 61°$
$= 299°$

✓ GUIDED PRACTICE for Examples 1 and 2

Identify the given arc as a *major arc, minor arc,* or *semicircle,* and find the measure of the arc.



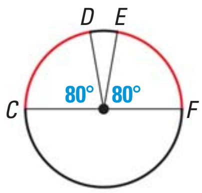
1. $\widehat{TQ}$ **2.** $\widehat{QRT}$ **3.** $\widehat{TQR}$

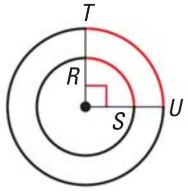
4. $\widehat{QS}$ **5.** $\widehat{TS}$ **6.** $\widehat{RST}$

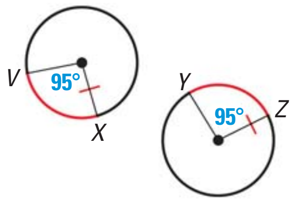
CONGRUENT CIRCLES AND ARCS Two circles are **congruent circles** if they have the same radius. Two arcs are **congruent arcs** if they have the same measure and they are arcs of the same circle or of congruent circles. If $\odot C$ is congruent to $\odot D$, then you can write $\odot C \cong \odot D$.

EXAMPLE 3 Identify congruent arcs

Tell whether the red arcs are congruent. Explain why or why not.

a.


b.


c.


Solution

a. $\overset{\frown}{CD} \cong \overset{\frown}{EF}$ because they are in the same circle and $m\overset{\frown}{CD} = m\overset{\frown}{EF}$.

b. $\overset{\frown}{RS}$ and $\overset{\frown}{TU}$ have the same measure, but are not congruent because they are arcs of circles that are not congruent.

c. $\overset{\frown}{VX} \cong \overset{\frown}{YZ}$ because they are in congruent circles and $m\overset{\frown}{VX} = m\overset{\frown}{YZ}$.

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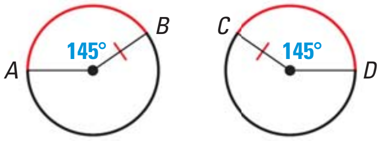
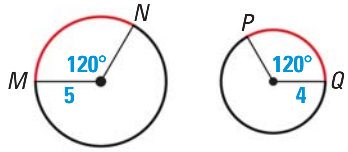
✓ GUIDED PRACTICE for Example 3

Tell whether the red arcs are congruent. *Explain* why or why not.

7.


8.


10.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 13, and 23

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 11, 17, 18, and 24

SKILL PRACTICE

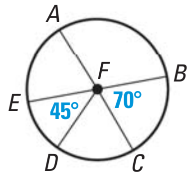
1. **VOCABULARY** Copy and complete: If $\angle ACB$ and $\angle DCE$ are congruent central angles of $\odot C$, then $\overset{\frown}{AB}$ and $\overset{\frown}{DE}$ are __?__.

2. ★ **WRITING** What do you need to know about two circles to show that they are congruent? *Explain.*

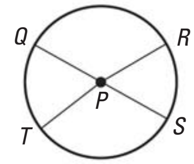
EXAMPLES 1 and 2 on pp. 659–660 for Exs. 3–11

MEASURING ARCS $\overline{AC}$ **and** $\overline{BE}$ **are diameters of** $\odot F$**. Determine whether the arc is a *minor arc*, a *major arc*, or a *semicircle* of** $\odot F$**. Then find the measure of the arc.**

3. $\overset{\frown}{BC}$
4. $\overset{\frown}{DC}$
5. $\overset{\frown}{DB}$
6. $\overset{\frown}{AE}$
7. $\overset{\frown}{AD}$
8. $\overset{\frown}{ABC}$
9. $\overset{\frown}{ACD}$
10. $\overset{\frown}{EAC}$



11. ★ **MULTIPLE CHOICE** In the diagram, $\overline{QS}$ is a diameter of $\odot P$. Which arc represents a semicircle?



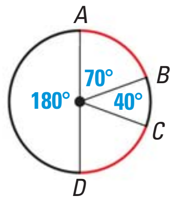
Ⓐ $\overset{\frown}{QR}$ Ⓑ $\overset{\frown}{RQT}$

Ⓒ $\overset{\frown}{QRS}$ Ⓓ $\overset{\frown}{QRT}$

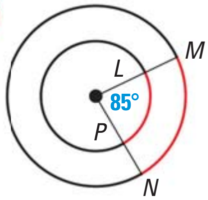
EXAMPLE 3
on p. 661
for Exs. 12–14

CONGRUENT ARCS **Tell whether the red arcs are congruent.** ***Explain*** **why or why not.**

12.



13.



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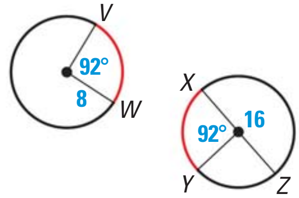
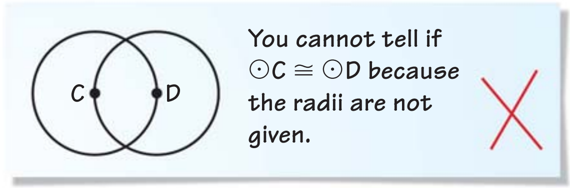
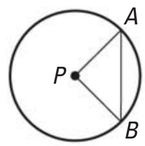


15. **ERROR ANALYSIS** *Explain* what is wrong with the statement.



16. **ARCS** Two diameters of $\odot P$ are $\overline{AB}$ and $\overline{CD}$. If $m\overset{\frown}{AD} = 20°$, find $m\overset{\frown}{ACD}$ and $m\overset{\frown}{AC}$.

17. ★ **MULTIPLE CHOICE** $\odot P$ has a radius of 3 and $\overset{\frown}{AB}$ has a measure of 90°. What is the length of $\overline{AB}$?



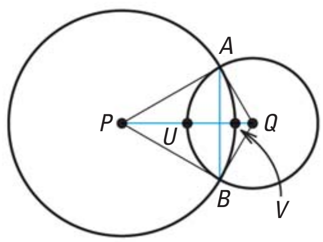
Ⓐ $3\sqrt{2}$ Ⓑ $3\sqrt{3}$

Ⓒ 6 Ⓓ 9

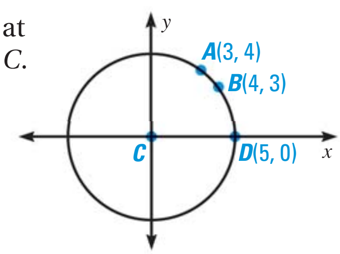
18. ★ **SHORT RESPONSE** On $\odot C$, $m\overset{\frown}{EF} = 100°$, $m\overset{\frown}{FG} = 120°$, and $m\overset{\frown}{EFG} = 220°$. If H is on $\odot C$ so that $m\overset{\frown}{GH} = 150°$, *explain* why H must be on $\overset{\frown}{EF}$.

19. **REASONING** In $\odot R$, $m\overset{\frown}{AB} = 60°$, $m\overset{\frown}{BC} = 25°$, $m\overset{\frown}{CD} = 70°$, and $m\overset{\frown}{DE} = 20°$. Find two possible values for $m\overset{\frown}{AE}$.

20. **CHALLENGE** In the diagram shown, $\overline{PQ} \perp \overline{AB}$, $\overline{QA}$ is tangent to $\odot P$, and $m\overset{\frown}{AVB} = 60°$. What is $m\overset{\frown}{AUB}$?



21. **CHALLENGE** In the coordinate plane shown, C is at the origin. Find the following arc measures on $\odot C$.



a. $m\overset{\frown}{BD}$

b. $m\overset{\frown}{AD}$

c. $m\overset{\frown}{AB}$

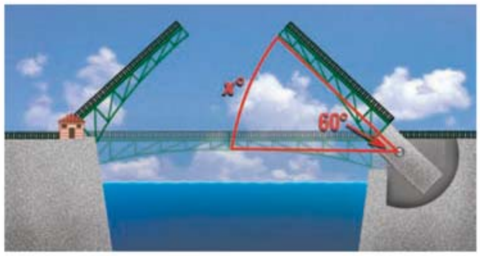
◯ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

EXAMPLE 1 on p. 659 for Ex. 22

22. BRIDGES The deck of a bascule bridge creates an arc when it is moved from the closed position to the open position. Find the measure of the arc.



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23. DARTS On a regulation dartboard, the outermost circle is divided into twenty congruent sections. What is the measure of each arc in this circle?

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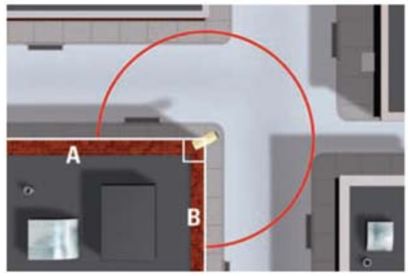
24. ★ **EXTENDED RESPONSE** A surveillance camera is mounted on a corner of a building. It rotates clockwise and counterclockwise continuously between Wall A and Wall B at a rate of 10° per minute.



a. What is the measure of the arc surveyed by the camera?

b. How long does it take the camera to survey the entire area once?

c. If the camera is at an angle of 85° from Wall B while rotating counterclockwise, how long will it take for the camera to return to that same position?

d. The camera is rotating counterclockwise and is 50° from Wall A. Find the location of the camera after 15 minutes.

25. CHALLENGE A clock with hour and minute hands is set to 1:00 P.M.

a. After 20 minutes, what will be the measure of the minor arc formed by the hour and minute hands?

b. At what time before 2:00 P.M., to the nearest minute, will the hour and minute hands form a diameter?

MIXED REVIEW

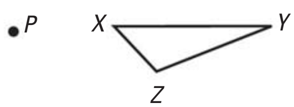
PREVIEW Prepare for Lesson 10.3 in Exs. 26–27.

Determine if the lines with the given equations are parallel. *(p. 180)*

26. $y = 5x + 2,\ y = 5(1 - x)$

27. $2y + 2x = 5,\ y = 4 - x$

28. Trace $\triangle XYZ$ and point P. Draw a counterclockwise rotation of $\triangle XYZ$ 145° about P. *(p. 598)*



Find the product. *(p. 641)*

29. $(x + 2)(x + 3)$

30. $(2y - 5)(y + 7)$

31. $(x + 6)(x - 6)$

32. $(z - 3)^2$

33. $(3x + 7)(5x + 4)$

34. $(z - 1)(z - 4)$

10.3 Apply Properties of Chords

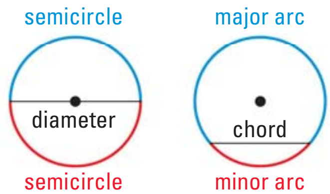
Before You used relationships of central angles and arcs in a circle.

Now You will use relationships of arcs and chords in a circle.

Why? So you can design a logo for a company, as in Ex. 25.

Key Vocabulary
- **chord,** *p. 651*
- **arc,** *p. 659*
- **semicircle,** *p. 659*

Recall that a *chord* is a segment with endpoints on a circle. Because its endpoints lie on the circle, any chord divides the circle into two arcs. A diameter divides a circle into two semicircles. Any other chord divides a circle into a minor arc and a major arc.

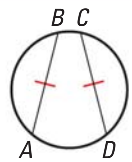


THEOREM *For Your Notebook*

THEOREM 10.3

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

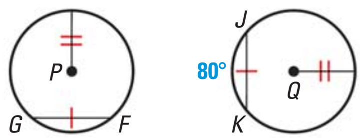
Proof: Exs. 27–28, p. 669



$\overset{\frown}{AB} \cong \overset{\frown}{CD}$ if and only if $\overline{AB} \cong \overline{CD}$.

EXAMPLE 1 Use congruent chords to find an arc measure

In the diagram, $\odot P \cong \odot Q$, $\overline{FG} \cong \overline{JK}$, and $m\overset{\frown}{JK} = 80°$. Find $m\overset{\frown}{FG}$.



Solution

Because $\overline{FG}$ and $\overline{JK}$ are congruent chords in congruent circles, the corresponding minor arcs $\overset{\frown}{FG}$ and $\overset{\frown}{JK}$ are congruent.

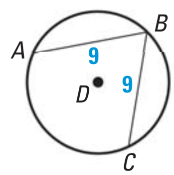
▶ So, $m\overset{\frown}{FG} = m\overset{\frown}{JK} = 80°$.

✓ GUIDED PRACTICE for Example 1

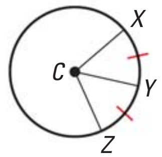
Use the diagram of $\odot D$.

1. If $m\overset{\frown}{AB} = 110°$, find $m\overset{\frown}{BC}$.

2. If $m\overset{\frown}{AC} = 150°$, find $m\overset{\frown}{AB}$.



BISECTING ARCS If $\overset{\frown}{XY} \cong \overset{\frown}{YZ}$, then the point Y, and any line, segment, or ray that contains Y, *bisects* $\overset{\frown}{XYZ}$.



$\overline{CY}$ bisects $\overset{\frown}{XYZ}$.

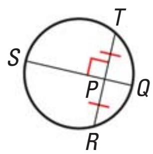
THEOREMS — For Your Notebook

THEOREM 10.4

If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.

If $\overline{QS}$ is a perpendicular bisector of $\overline{TR}$, then $\overline{QS}$ is a diameter of the circle.

Proof: Ex. 31, p. 670

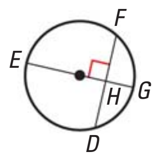


THEOREM 10.5

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

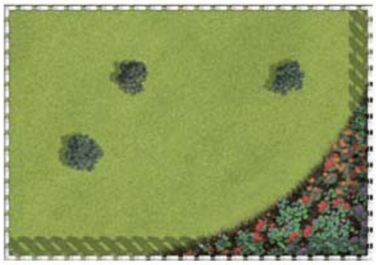
If $\overline{EG}$ is a diameter and $\overline{EG} \perp \overline{DF}$, then $\overline{HD} \cong \overline{HF}$ and $\overset{\frown}{GD} \cong \overset{\frown}{GF}$.

Proof: Ex. 32, p. 670



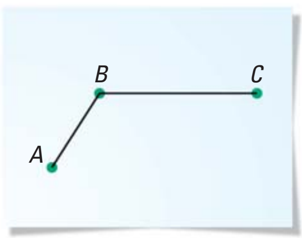
EXAMPLE 2 Use perpendicular bisectors

GARDENING Three bushes are arranged in a garden as shown. Where should you place a sprinkler so that it is the same distance from each bush?



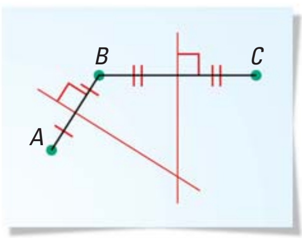
Solution

STEP 1



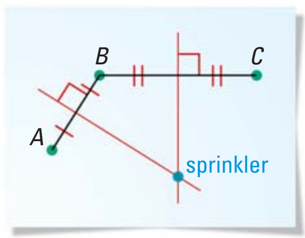
Label the bushes A, B, and C, as shown. Draw segments $\overline{AB}$ and $\overline{BC}$.

STEP 2



Draw the perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$. By Theorem 10.4, these are diameters of the circle containing A, B, and C.

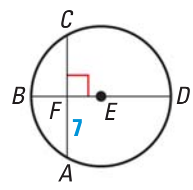
STEP 3



Find the point where these bisectors intersect. This is the center of the circle through A, B, and C, and so it is equidistant from each point.

EXAMPLE 3 Use a diameter

Use the diagram of $\odot E$ to find the length of $\overline{AC}$. Tell what theorem you use.

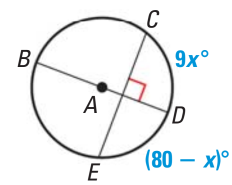


Solution

Diameter $\overline{BD}$ is perpendicular to $\overline{AC}$. So, by Theorem 10.5, $\overline{BD}$ bisects $\overline{AC}$, and $CF = AF$. Therefore, $AC = 2(AF) = 2(7) = 14$.

✓ GUIDED PRACTICE for Examples 2 and 3

Find the measure of the indicated arc in the diagram.

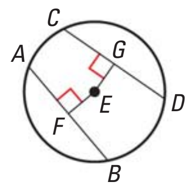


3. $\overset{\frown}{CD}$ **4.** $\overset{\frown}{DE}$ **5.** $\overset{\frown}{CE}$

THEOREM *For Your Notebook*

THEOREM 10.6

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

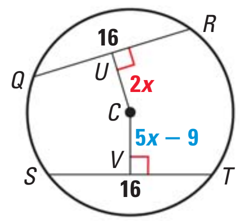


$\overline{AB} \cong \overline{CD}$ if and only if $EF = EG$.

Proof: Ex. 33, p. 670

EXAMPLE 4 Use Theorem 10.6

In the diagram of $\odot C$, $QR = ST = 16$. Find CU.



Solution

Chords $\overline{QR}$ and $\overline{ST}$ are congruent, so by Theorem 10.6 they are equidisant from C. Therefore, $CU = CV$.

$CU = CV$	**Use Theorem 10.6.**
$2x = 5x - 9$	**Substitute.**
$x = 3$	**Solve for *x*.**

▶ So, $CU = 2x = 2(3) = 6$.

✓ GUIDED PRACTICE for Example 4

In the diagram in Example 4, suppose $ST = 32$, and $CU = CV = 12$. Find the given length.

6. QR **7.** QU **8.** The radius of $\odot C$

10.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 9, and 25

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 22, and 26

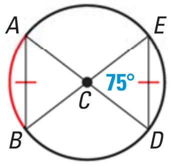
SKILL PRACTICE

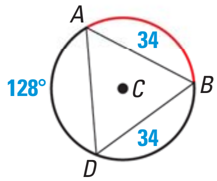
1. **VOCABULARY** *Describe* what it means to *bisect* an arc.

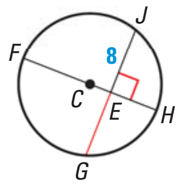
2. ★ **WRITING** Two chords of a circle are perpendicular and congruent. Does one of them have to be a diameter? *Explain* your reasoning.

EXAMPLES 1 and 3 on pp. 664, 666 for Exs. 3–5

FINDING ARC MEASURES **Find the measure of the red arc or chord in $\odot C$.**

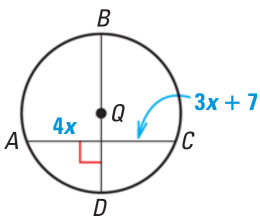
3. 

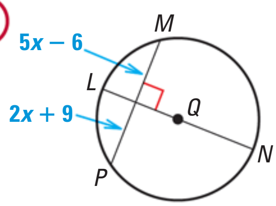
4. 

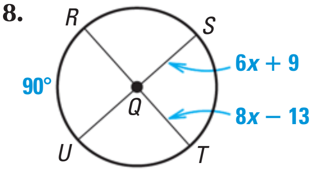
5. 

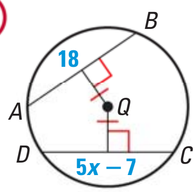
EXAMPLES 3 and 4 on p. 666 for Exs. 6–11

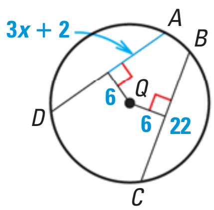
ALGEBRA **Find the value of x in $\odot Q$. *Explain* your reasoning.**

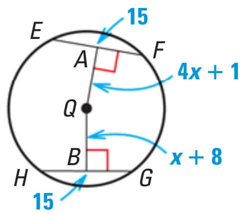
6. 

7. 

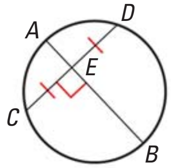
8. 

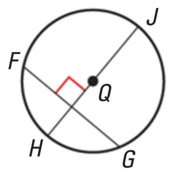
9. 

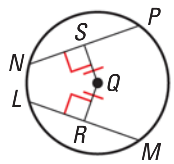
10. 

11. 

REASONING **In Exercises 12–14, what can you conclude about the diagram shown? State a theorem that justifies your answer.**

12. 

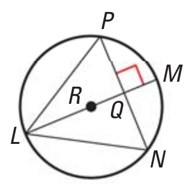
13. 

14. 

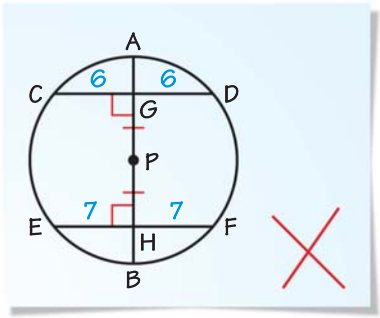
15. ★ **MULTIPLE CHOICE** In the diagram of $\odot R$, which congruence relation is not necessarily true?

(A) $\overline{PQ} \cong \overline{QN}$ (B) $\overline{NL} \cong \overline{LP}$

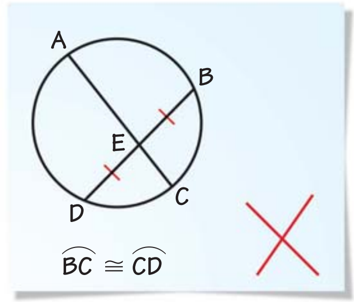
(C) $\overparen{MN} \cong \overparen{MP}$ (D) $\overline{PN} \cong \overline{PL}$



16. **ERROR ANALYSIS** *Explain* what is wrong with the diagram of $\odot P$.

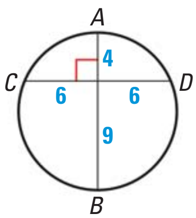


17. **ERROR ANALYSIS** *Explain* why the congruence statement is wrong.

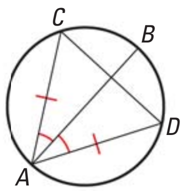


IDENTIFYING DIAMETERS **Determine whether $\overline{AB}$ is a diameter of the circle. *Explain* your reasoning.**

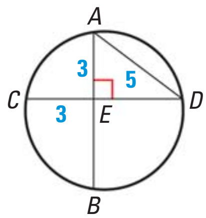
18.



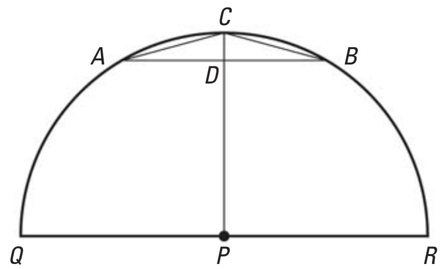
19.



20.



21. **REASONING** In the diagram of semicircle $\overset{\frown}{QCR}$, $\overline{PC} \cong \overline{AB}$ and $m\overset{\frown}{AC} = 30°$. *Explain* how you can conclude that $\triangle ADC \cong \triangle BDC$.

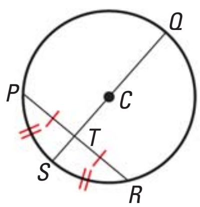


22. ★ **WRITING** Theorem 10.4 is nearly the converse of Theorem 10.5.

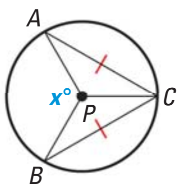
a. Write the converse of Theorem 10.5. *Explain* how it is different from Theorem 10.4.

b. Copy the diagram of $\odot C$ and draw auxiliary segments $\overline{PC}$ and $\overline{RC}$. Use congruent triangles to prove the converse of Theorem 10.5.

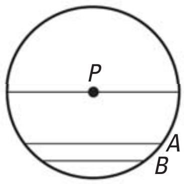
c. Use the converse of Theorem 10.5 to show that $QP = QR$ in the diagram of $\odot C$.



23. **xy ALGEBRA** In $\odot P$ below, $\overline{AC}$, $\overline{BC}$, and all arcs have integer measures. Show that x must be even.

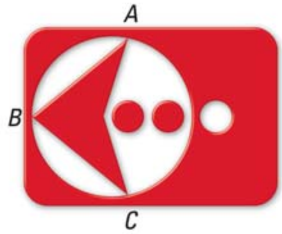


24. **CHALLENGE** In $\odot P$ below, the lengths of the parallel chords are 20, 16, and 12. Find $m\overset{\frown}{AB}$.



PROBLEM SOLVING

25. **LOGO DESIGN** The owner of a new company would like the company logo to be a picture of an arrow inscribed in a circle, as shown. For symmetry, she wants $\overset{\frown}{AB}$ to be congruent to $\overset{\frown}{BC}$. How should $\overline{AB}$ and $\overline{BC}$ be related in order for the logo to be exactly as desired?



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EXAMPLE 2 on p. 665 for Ex. 26

26. ★ **OPEN-ENDED MATH** In the cross section of the submarine shown, the control panels are parallel and the same length. *Explain* two ways you can find the center of the cross section.

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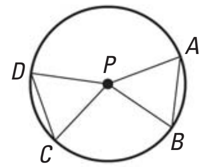
PROVING THEOREM 10.3 **In Exercises 27 and 28, prove Theorem 10.3.**

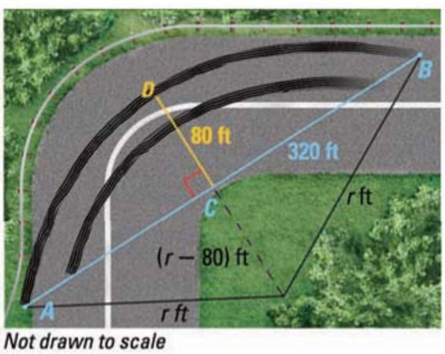
27. **GIVEN** ▶ $\overline{AB}$ and $\overline{CD}$ are congruent chords.
PROVE ▶ $\overset{\frown}{AB} \cong \overset{\frown}{CD}$

28. **GIVEN** ▶ $\overline{AB}$ and $\overline{CD}$ are chords and $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.
PROVE ▶ $\overline{AB} \cong \overline{CD}$



29. **CHORD LENGTHS** Make and prove a conjecture about chord lengths.
   a. Sketch a circle with two noncongruent chords. Is the *longer* chord or the *shorter* chord closer to the center of the circle? Repeat this experiment several times.
   b. Form a conjecture related to your experiment in part (a).
   c. Use the Pythagorean Theorem to prove your conjecture.

30. **MULTI-STEP PROBLEM** If a car goes around a turn too quickly, it can leave tracks that form an arc of a circle. By finding the radius of the circle, accident investigators can estimate the speed of the car.
   a. To find the radius, choose points A and B on the tire marks. Then find the midpoint C of $\overline{AB}$. Measure $\overline{CD}$, as shown. Find the radius r of the circle.
   b. The formula $S = 3.86\sqrt{fr}$ can be used to estimate a car's speed in miles per hours, where f is the *coefficient of friction* and r is the radius of the circle in feet. The coefficient of friction measures how slippery a road is. If $f = 0.7$, estimate the car's speed in part (a).



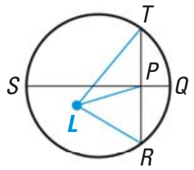
Not drawn to scale

PROVING THEOREMS 10.4 AND 10.5 **Write proofs.**

31. **GIVEN** ▶ $\overline{QS}$ is the perpendicular bisector of $\overline{RT}$.

PROVE ▶ $\overline{QS}$ is a diameter of $\odot L$.

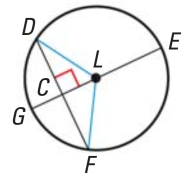
Plan for Proof Use indirect reasoning. Assume center L is not on $\overline{QS}$. Prove that $\triangle RLP \cong \triangle TLP$, so $\overline{PL} \perp \overline{RT}$. Then use the Perpendicular Postulate.



32. **GIVEN** ▶ $\overline{EG}$ is a diameter of $\odot L$. $\overline{EG} \perp \overline{DF}$

PROVE ▶ $\overline{CD} \cong \overline{CF}$, $\overset{\frown}{DG} \cong \overset{\frown}{FG}$

Plan for Proof Draw $\overline{LD}$ and $\overline{LF}$. Use congruent triangles to show $\overline{CD} \cong \overline{CF}$ and $\angle DLG \cong \angle FLG$. Then show $\overset{\frown}{DG} \cong \overset{\frown}{FG}$.



33. **PROVING THEOREM 10.6** For Theorem 10.6, prove both cases of the biconditional. Use the diagram shown for the theorem on page 666.

34. **CHALLENGE** A car is designed so that the rear wheel is only partially visible below the body of the car, as shown. The bottom panel is parallel to the ground. Prove that the point where the tire touches the ground bisects $\overset{\frown}{AB}$.

MIXED REVIEW

PREVIEW
Prepare for Lesson 10.4 in Exs. 35–37.

35. The measures of the interior angles of a quadrilateral are 100°, 140°, $(x + 20)°$, and $(2x + 10)°$. Find the value of x. *(p. 507)*

Quadrilateral *JKLM* is a parallelogram. Graph ▱*JKLM*. Decide whether it is best described as a *rectangle*, a *rhombus*, or a *square*. *(p. 552)*

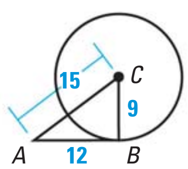
36. $J(-3, 5)$, $K(2, 5)$, $L(2, -1)$, $M(-3, -1)$

37. $J(-5, 2)$, $K(1, 1)$, $L(2, -5)$, $M(-4, -4)$

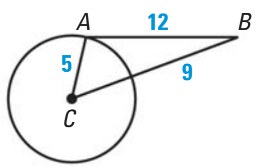
QUIZ *for Lessons 10.1–10.3*

Determine whether $\overline{AB}$ is tangent to $\odot C$. *Explain* your reasoning. *(p. 651)*

1.



2.



3. If $m\overset{\frown}{EFG} = 195°$, and $m\overset{\frown}{EF} = 80°$, find $m\overset{\frown}{FG}$ and $m\overset{\frown}{EG}$. *(p. 659)*

4. The points A, B, and D are on $\odot C$, $\overline{AB} \cong \overline{BD}$, and $m\overset{\frown}{ABD} = 194°$. What is the measure of $\overset{\frown}{AB}$? *(p. 664)*

Investigating Geometry **ACTIVITY** *Use before Lesson 10.4*

10.4 Explore Inscribed Angles

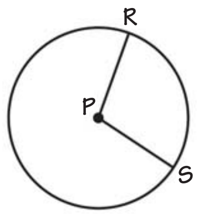
MATERIALS • compass • straightedge • protractor

QUESTION **How are inscribed angles related to central angles?**

The vertex of a central angle is at the center of the circle. The vertex of an *inscribed angle* is on the circle, and its sides form chords of the circle.

EXPLORE **Construct inscribed angles of a circle**

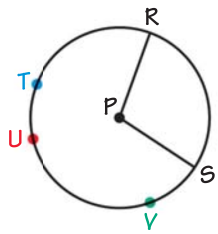
STEP 1



Draw a central angle Use a compass to draw a circle. Label the center P. Use a straightedge to draw a central angle. Label it $\angle RPS$.

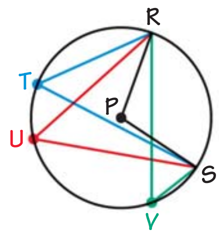
Animated Geometry at classzone.com

STEP 2



Draw points Locate three points on $\odot P$ in the exterior of $\angle RPS$ and label them T, U, and V.

STEP 3



Measure angles Draw $\angle RTS$, $\angle RUS$, and $\angle RVS$. These are called *inscribed angles*. Measure each angle.

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Copy and complete the table.

	Central angle	Inscribed angle 1	Inscribed angle 2	Inscribed angle 3
Name	$\angle RPS$	$\angle RTS$	$\angle RUS$	$\angle RVS$
Measure	?	?	?	?

2. Draw two more circles. Repeat Steps 1–3 using different central angles. Record the measures in a table similar to the one above.

3. Use your results to make a conjecture about how the measure of an inscribed angle is related to the measure of the corresponding central angle.

10.4 Use Inscribed Angles and Polygons

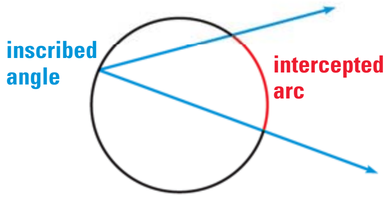
Before You used central angles of circles.

Now You will use inscribed angles of circles.

Why? So you can take a picture from multiple angles, as in Example 4.

Key Vocabulary
- **inscribed angle**
- **intercepted arc**
- **inscribed polygon**
- **circumscribed circle**

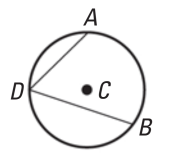
An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the **intercepted arc** of the angle.



THEOREM *For Your Notebook*

THEOREM 10.7 Measure of an Inscribed Angle Theorem

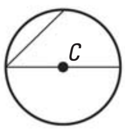
The measure of an inscribed angle is one half the measure of its intercepted arc.



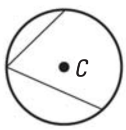
$m\angle ADB = \frac{1}{2}m\overset{\frown}{AB}$

Proof: Exs. 31–33, p. 678

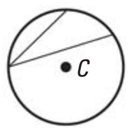
The proof of Theorem 10.7 in Exercises 31–33 involves three cases.



Case 1 Center C is on a side of the inscribed angle.



Case 2 Center C is inside the inscribed angle.

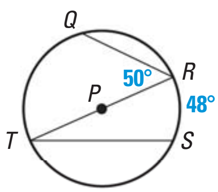


Case 3 Center C is outside the inscribed angle.

EXAMPLE 1 Use inscribed angles

Find the indicated measure in $\odot P$.

a. $m\angle T$ **b.** $m\overset{\frown}{QR}$



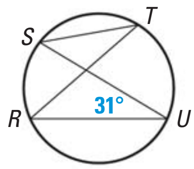
Solution

a. $m\angle T = \frac{1}{2}m\overset{\frown}{RS} = \frac{1}{2}(48°) = 24°$

b. $m\overset{\frown}{TQ} = 2m\angle R = 2 \cdot 50° = 100°$. Because $\overset{\frown}{TQR}$ is a semicircle, $m\overset{\frown}{QR} = 180° - m\overset{\frown}{TQ} = 180° - 100° = 80°$. So, $m\overset{\frown}{QR} = 80°$.

EXAMPLE 2 Find the measure of an intercepted arc

Find $m\overset{\frown}{RS}$ and $m\angle STR$. What do you notice about $\angle STR$ and $\angle RUS$?



Solution

From Theorem 10.7, you know that $m\overset{\frown}{RS} = 2m\angle RUS = 2(31°) = 62°$.

Also, $m\angle STR = \frac{1}{2}m\overset{\frown}{RS} = \frac{1}{2}(62°) = 31°$. So, $\angle STR \cong \angle RUS$.

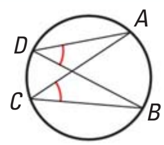
INTERCEPTING THE SAME ARC Example 2 suggests Theorem 10.8.

THEOREM *For Your Notebook*

THEOREM 10.8

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

Proof: Ex. 34, p. 678



$\angle ADB \cong \angle ACB$

EXAMPLE 3 Standardized Test Practice

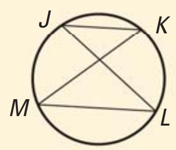
Name two pairs of congruent angles in the figure.

Ⓐ $\angle JKM \cong \angle KJL$, $\angle JLM \cong \angle KML$

Ⓑ $\angle JLM \cong \angle KJL$, $\angle JKM \cong \angle KML$

Ⓒ $\angle JKM \cong \angle JLM$, $\angle KJL \cong \angle KML$

Ⓓ $\angle JLM \cong \angle KJL$, $\angle JLM \cong \angle JKM$



Solution

ELIMINATE CHOICES You can eliminate choices A and B, because they do not include the pair $\angle JKM \cong \angle JLM$.

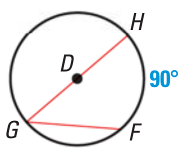
Notice that $\angle JKM$ and $\angle JLM$ intercept the same arc, and so $\angle JKM \cong \angle JLM$ by Theorem 10.8. Also, $\angle KJL$ and $\angle KML$ intercept the same arc, so they must also be congruent. Only choice C contains both pairs of angles.

▶ So, by Theorem 10.8, the correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

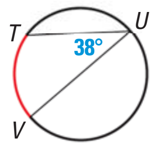
GUIDED PRACTICE for Examples 1, 2, and 3

Find the measure of the red arc or angle.

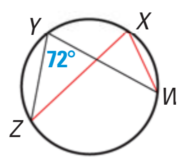
1.



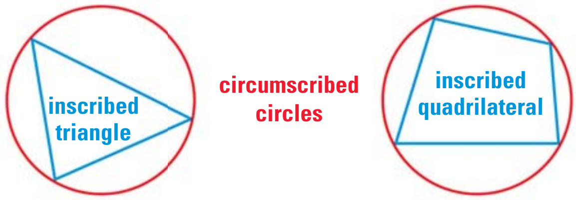
2.



3.



POLYGONS A polygon is an **inscribed polygon** if all of its vertices lie on a circle. The circle that contains the vertices is a **circumscribed circle**.

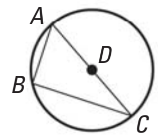


THEOREM *For Your Notebook*

THEOREM 10.9

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

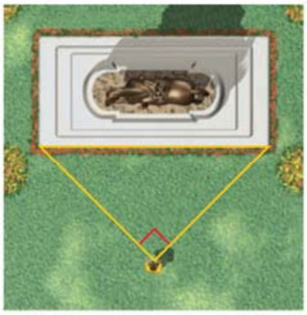
Proof: Ex. 35, p. 678



$m\angle ABC = 90°$ if and only if $\overline{AC}$ is a diameter of the circle.

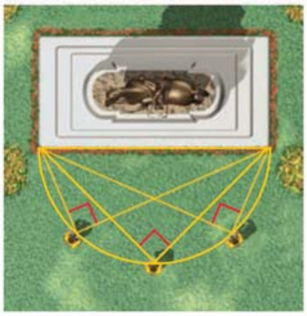
EXAMPLE 4 Use a circumscribed circle

PHOTOGRAPHY Your camera has a 90° field of vision and you want to photograph the front of a statue. You move to a spot where the statue is the only thing captured in your picture, as shown. You want to change your position. Where else can you stand so that the statue is perfectly framed in this way?



Solution

From Theorem 10.9, you know that if a right triangle is inscribed in a circle, then the hypotenuse of the triangle is a diameter of the circle. So, draw the circle that has the front of the statue as a diameter. The statue fits perfectly within your camera's 90° field of vision from any point on the semicircle in front of the statue.



✓ GUIDED PRACTICE for Example 4

4. **WHAT IF?** In Example 4, *explain* how to find locations if you want to frame the front and left side of the statue in your picture.

INSCRIBED QUADRILATERAL Only certain quadrilaterals can be inscribed in a circle. Theorem 10.10 describes these quadrilaterals.

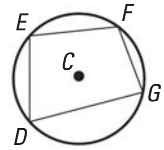
THEOREM *For Your Notebook*

THEOREM 10.10

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

D, E, F, and G lie on $\odot C$ if and only if $m\angle D + m\angle F = m\angle E + m\angle G = 180°$.

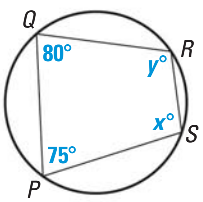
Proof: Ex. 30, p. 678; p. 938



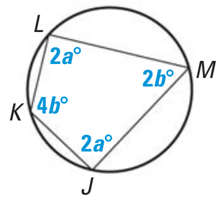
EXAMPLE 5 Use Theorem 10.10

Find the value of each variable.

a.



b.



Solution

a. *PQRS* is inscribed in a circle, so opposite angles are supplementary.

$m\angle P + m\angle R = 180°$ $\qquad$ $m\angle Q + m\angle S = 180°$

$75° + y° = 180°$ $\qquad$ $80° + x° = 180°$

$y = 105$ $\qquad$ $x = 100$

b. *JKLM* is inscribed in a circle, so opposite angles are supplementary.

$m\angle J + m\angle L = 180°$ $\qquad$ $m\angle K + m\angle M = 180°$

$2a° + 2a° = 180°$ $\qquad$ $4b° + 2b° = 180°$

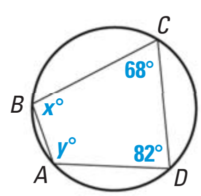
$4a = 180$ $\qquad$ $6b = 180$

$a = 45$ $\qquad$ $b = 30$

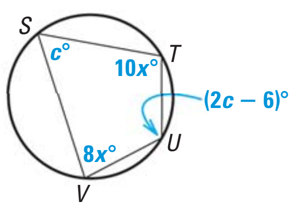
✓ GUIDED PRACTICE for Example 5

Find the value of each variable.

5.



6.



10.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 13, and 29

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 18, 29, and 36

SKILL PRACTICE

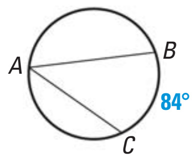
1. **VOCABULARY** Copy and complete: If a circle is circumscribed about a polygon, then the polygon is _?_ in the circle.

2. ★ **WRITING** *Explain* why the diagonals of a rectangle inscribed in a circle are diameters of the circle.

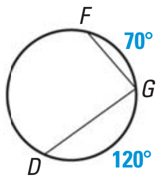
EXAMPLES 1 and 2 on pp. 672–673 for Exs. 3–9

INSCRIBED ANGLES **Find the indicated measure.**

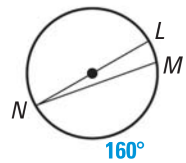
3. $m\angle A$



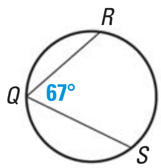
4. $m\angle G$



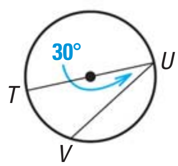
5. $m\angle N$



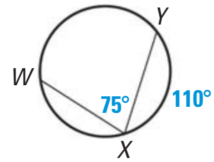
6. $m\widehat{RS}$



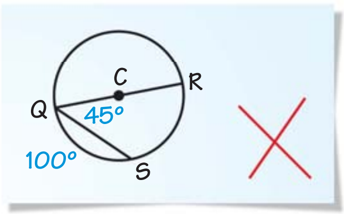
7. $m\widehat{VU}$



8. $m\widehat{WX}$



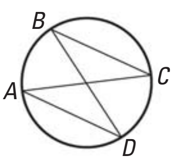
9. **ERROR ANALYSIS** *Describe* the error in the diagram of $\odot C$. Find two ways to correct the error.



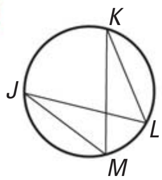
EXAMPLE 3 on p. 673 for Exs. 10–12

CONGRUENT ANGLES **Name two pairs of congruent angles.**

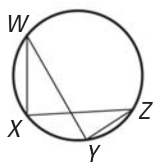
10.



11.



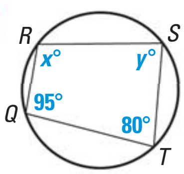
12.



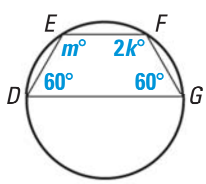
EXAMPLE 5 on p. 675 for Exs. 13–15

ALGEBRA **Find the values of the variables.**

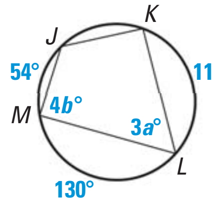
13.



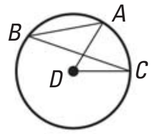
14.



15.



16. ★ **MULTIPLE CHOICE** In the diagram, $\angle ADC$ is a central angle and $m\angle ADC = 60°$. What is $m\angle ABC$?



Ⓐ 15° Ⓑ 30°

Ⓒ 60° Ⓓ 120°

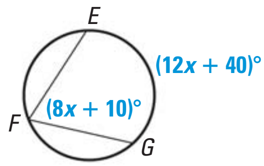
17. **INSCRIBED ANGLES** In each star below, all of the inscribed angles are congruent. Find the measure of an inscribed angle for each star. Then find the sum of all the inscribed angles for each star.

a. 

b. 

c. 

18. ★ **MULTIPLE CHOICE** What is the value of x?



Ⓐ 5 Ⓑ 10

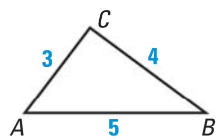
Ⓒ 13 Ⓓ 15

19. **PARALLELOGRAM** Parallelogram $QRST$ is inscribed in $\odot C$. Find $m\angle R$.

REASONING Determine whether the quadrilateral can always be inscribed in a circle. *Explain* your reasoning.

20. Square
21. Rectangle
22. Parallelogram
23. Kite
24. Rhombus
25. Isosceles trapezoid

26. **CHALLENGE** In the diagram, $\angle C$ is a right angle. If you draw the smallest possible circle through C and tangent to $\overline{AB}$, the circle will intersect $\overline{AC}$ at J and $\overline{BC}$ at K. Find the exact length of $\overline{JK}$.



PROBLEM SOLVING

27. **ASTRONOMY** Suppose three moons A, B, and C orbit 100,000 kilometers above the surface of a planet. Suppose $m\angle ABC = 90°$, and the planet is 20,000 kilometers in diameter. Draw a diagram of the situation. How far is moon A from moon C?

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EXAMPLE 4
on p. 674
for Ex. 28

28. **CARPENTER** A *carpenter's square* is an L-shaped tool used to draw right angles. You need to cut a circular piece of wood into two semicircles. How can you use a carpenter's square to draw a diameter on the circular piece of wood?

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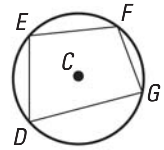
29. ★ **WRITING** A right triangle is inscribed in a circle and the radius of the circle is given. *Explain* how to find the length of the hypotenuse.

30. **PROVING THEOREM 10.10** Copy and complete the proof that opposite angles of an inscribed quadrilateral are supplementary.



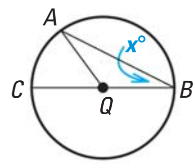
GIVEN ▶ $\odot C$ with inscribed quadrilateral $DEFG$

PROVE ▶ $m\angle D + m\angle F = 180°$, $m\angle E + m\angle G = 180°$.

By the Arc Addition Postulate, $m\widehat{EFG}$ + __?__ = 360° and $m\widehat{FGD} + m\widehat{DEF} = 360°$. Using the __?__ Theorem, $m\widehat{EDG} = 2m\angle F$, $m\widehat{EFG} = 2m\angle D$, $m\widehat{DEF} = 2m\angle G$, and $m\widehat{FGD} = 2m\angle E$. By the Substitution Property, $2m\angle D$ + __?__ = 360°, so __?__ . Similarly, __?__.

PROVING THEOREM 10.7 If an angle is inscribed in $\odot Q$, the center Q can be on a side of the angle, in the interior of the angle, or in the exterior of the angle. In Exercises 31–33, you will prove Theorem 10.7 for each of these cases.

31. **Case 1** Prove Case 1 of Theorem 10.7.

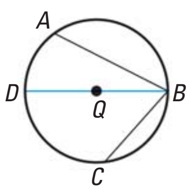


GIVEN ▶ $\angle B$ is inscribed in $\odot Q$. Let $m\angle B = x°$. Point Q lies on $\overline{BC}$.

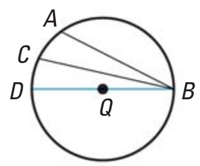
PROVE ▶ $m\angle B = \frac{1}{2}m\widehat{AC}$

Plan for Proof Show that $\triangle AQB$ is isosceles. Use the Base Angles Theorem and the Exterior Angles Theorem to show that $m\angle AQC = 2x°$. Then, show that $m\widehat{AC} = 2x°$. Solve for x, and show that $m\angle B = \frac{1}{2}m\widehat{AC}$.

32. **Case 2** Use the diagram and auxiliary line to write GIVEN and PROVE statements for Case 2 of Theorem 10.7. Then write a plan for proof.



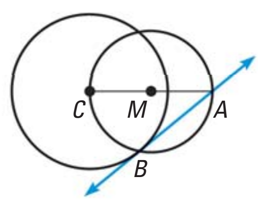
33. **Case 3** Use the diagram and auxiliary line to write GIVEN and PROVE statements for Case 3 of Theorem 10.7. Then write a plan for proof.



34. **PROVING THEOREM 10.8** Write a paragraph proof of Theorem 10.8. First draw a diagram and write GIVEN and PROVE statements.

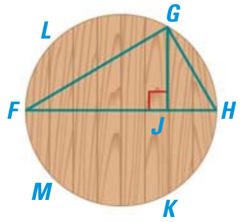
35. **PROVING THEOREM 10.9** Theorem 10.9 is written as a conditional statement and its converse. Write a plan for proof of each statement.

36. ★ **EXTENDED RESPONSE** In the diagram, $\odot C$ and $\odot M$ intersect at B, and $\overline{AC}$ is a diameter of $\odot M$. *Explain* why $\overleftrightarrow{AB}$ is tangent to $\odot C$.



CHALLENGE **In Exercises 37 and 38, use the following information.**

You are making a circular cutting board. To begin, you glue eight 1 inch by 2 inch boards together, as shown at the right. Then you draw and cut a circle with an 8 inch diameter from the boards.



37. $\overline{FH}$ is a diameter of the circular cutting board. Write a proportion relating GJ and JH. State a theorem to justify your answer.

38. Find FJ, JH, and JG. What is the length of the cutting board seam labeled $\overline{GK}$?

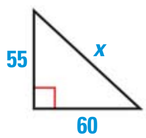
39. **SPACE SHUTTLE** To maximize thrust on a NASA space shuttle, engineers drill an 11-point star out of the solid fuel that fills each booster. They begin by drilling a hole with radius 2 feet, and they would like each side of the star to be 1.5 feet. Is this possible if the fuel cannot have angles greater than 45° at its points?

MIXED REVIEW

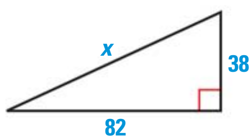
PREVIEW
Prepare for Lesson 10.5 in Exs. 40–42.

Find the approximate length of the hypotenuse. Round your answer to the nearest tenth. *(p. 433)*

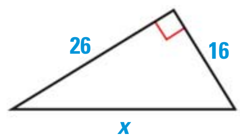
40.



41.

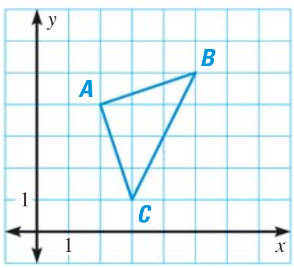


42.

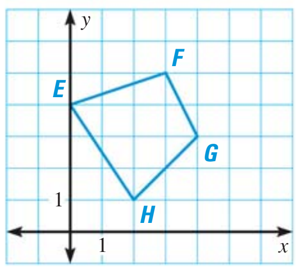


Graph the reflection of the polygon in the given line. *(p. 589)*

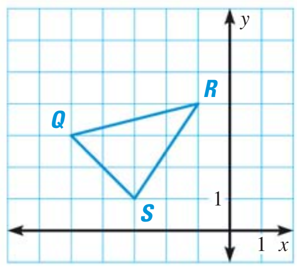
43. y-axis



44. $x = 3$



45. $y = 2$



Sketch the image of $A(3, -4)$ after the described glide reflection. *(p. 608)*

46. **Translation:** $(x, y) \to (x, y - 2)$
Reflection: in the y-axis

47. **Translation:** $(x, y) \to (x + 1, y + 4)$
Reflection: in $y = 4x$

10.5 Apply Other Angle Relationships in Circles

Before You found the measures of angles formed on a circle.

Now You will find the measures of angles inside or outside a circle.

Why So you can determine the part of Earth seen from a hot air balloon, as in Ex. 25.

Key Vocabulary
- **chord,** *p. 651*
- **secant,** *p. 651*
- **tangent,** *p. 651*

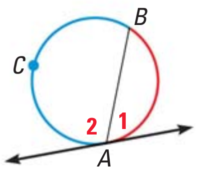
You know that the measure of an inscribed angle is half the measure of its intercepted arc. This is true even if one side of the angle is tangent to the circle.

THEOREM — *For Your Notebook*

THEOREM 10.11

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.

Proof: Ex. 27, p. 685

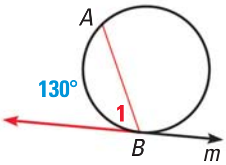


$m\angle 1 = \frac{1}{2} m\overset{\frown}{AB}$ $\quad m\angle 2 = \frac{1}{2} m\overset{\frown}{BCA}$

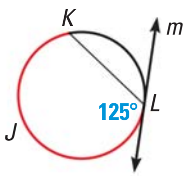
EXAMPLE 1 Find angle and arc measures

Line *m* is tangent to the circle. Find the measure of the red angle or arc.

a.



b.



Solution

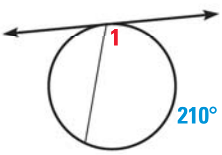
a. $m\angle 1 = \frac{1}{2}(130°) = 65°$

b. $m\overset{\frown}{KJL} = 2(125°) = 250°$

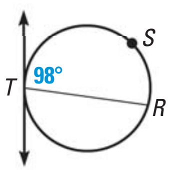
✓ GUIDED PRACTICE for Example 1

Find the indicated measure.

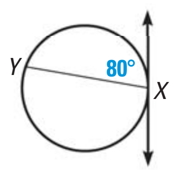
1. $m\angle 1$



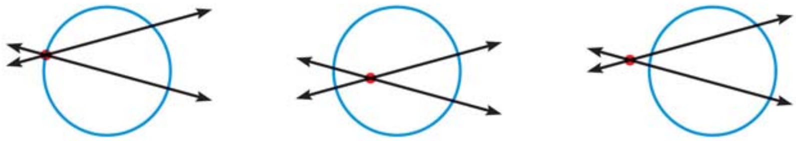
2. $m\overset{\frown}{RST}$



3. $m\overset{\frown}{XY}$



INTERSECTING LINES AND CIRCLES If two lines intersect a circle, there are three places where the lines can intersect.



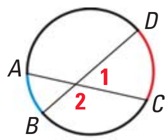
on the circle | inside the circle | outside the circle

You can use Theorems 10.12 and 10.13 to find measures when the lines intersect *inside* or *outside* the circle.

THEOREMS *For Your Notebook*

THEOREM 10.12 Angles Inside the Circle Theorem

If two chords intersect *inside* a circle, then the measure of each angle is one half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.



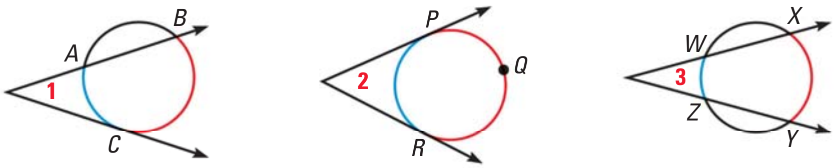
$m\angle 1 = \frac{1}{2}(m\overset{\frown}{DC} + m\overset{\frown}{AB})$,

$m\angle 2 = \frac{1}{2}(m\overset{\frown}{AD} + m\overset{\frown}{BC})$

Proof: Ex. 28, p. 685

THEOREM 10.13 Angles Outside the Circle Theorem

If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is one half the *difference* of the measures of the intercepted arcs.

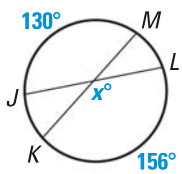


$m\angle 1 = \frac{1}{2}(m\overset{\frown}{BC} - m\overset{\frown}{AC})$ $m\angle 2 = \frac{1}{2}(m\overset{\frown}{PQR} - m\overset{\frown}{PR})$ $m\angle 3 = \frac{1}{2}(m\overset{\frown}{XY} - m\overset{\frown}{WZ})$

Proof: Ex. 29, p. 685

EXAMPLE 2 Find an angle measure inside a circle

Find the value of *x*.



Solution

The chords $\overline{JL}$ and $\overline{KM}$ intersect inside the circle.

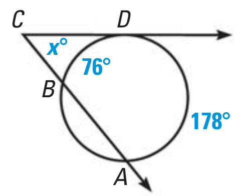
$x° = \frac{1}{2}(m\overset{\frown}{JM} + m\overset{\frown}{LK})$ **Use Theorem 10.12.**

$x° = \frac{1}{2}(130° + 156°)$ **Substitute.**

$x = 143$ **Simplify.**

EXAMPLE 3 Find an angle measure outside a circle

Find the value of x.



Solution

The tangent $\overrightarrow{CD}$ and the secant $\overrightarrow{CB}$ intersect outside the circle.

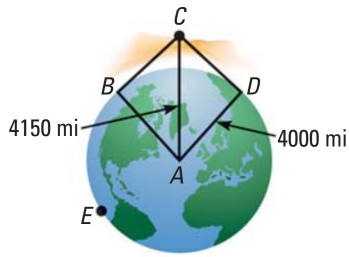
$$m\angle BCD = \frac{1}{2}\left(m\overset{\frown}{AD} - m\overset{\frown}{BD}\right)$$ **Use Theorem 10.13.**

$$x^\circ = \frac{1}{2}(178^\circ - 76^\circ)$$ **Substitute.**

$$x = 51$$ **Simplify.**

EXAMPLE 4 Solve a real-world problem

SCIENCE The Northern Lights are bright flashes of colored light between 50 and 200 miles above Earth. Suppose a flash occurs 150 miles above Earth. What is the measure of arc BD, the portion of Earth from which the flash is visible? (Earth's radius is approximately 4000 miles.)



Not drawn to scale

Solution

Because $\overline{CB}$ and $\overline{CD}$ are tangents, $\overline{CB} \perp \overline{AB}$ and $\overline{CD} \perp \overline{AD}$. Also, $\overline{BC} \cong \overline{DC}$ and $\overline{CA} \cong \overline{CA}$. So, $\triangle ABC \cong \triangle ADC$ by the Hypotenuse-Leg Congruence Theorem, and $\angle BCA \cong \angle DCA$. Solve right $\triangle CBA$ to find that $m\angle BCA \approx 74.5^\circ$. So, $m\angle BCD \approx 2(74.5^\circ) \approx 149^\circ$. Let $m\overset{\frown}{BD} = x^\circ$.

$$m\angle BCD = \frac{1}{2}\left(m\overset{\frown}{DEB} - m\overset{\frown}{BD}\right)$$ **Use Theorem 10.13.**

$$149^\circ \approx \frac{1}{2}[(360^\circ - x^\circ) - x^\circ]$$ **Substitute.**

$$x \approx 31$$ **Solve for x.**

▶ The measure of the arc from which the flash is visible is about 31°.

AVOID ERRORS
Because the value for $m\angle BCD$ is an approximation, use the symbol $\approx$ instead of $=$.

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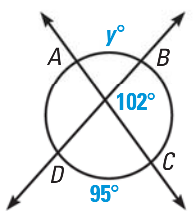
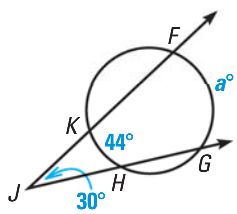
✓ GUIDED PRACTICE for Examples 2, 3, and 4

Find the value of the variable.

4.



5.



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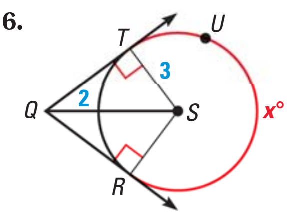


10.5 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 9, and 23

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 13, 15, 19, and 26

SKILL PRACTICE

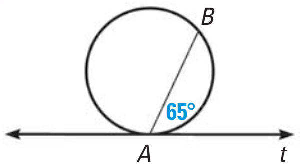
1. **VOCABULARY** Copy and complete: The points A, B, C, and D are on a circle and $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{CD}$ at P. If $m\angle APC = \frac{1}{2}(m\widehat{BD} - m\widehat{AC})$, then P is __?__ (*inside*, *on*, or *outside*) the circle.

2. ★ **WRITING** What does it mean in Theorem 10.12 if $m\widehat{AB} = 0°$? Is this consistent with what you learned in Lesson 10.4? *Explain* your answer.

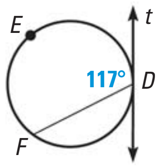
EXAMPLE 1 on p. 680 for Exs. 3–6

FINDING MEASURES **Line t is tangent to the circle. Find the indicated measure.**

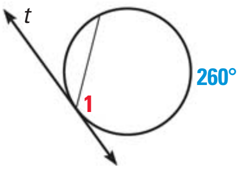
3. $m\widehat{AB}$



4. $m\widehat{DEF}$



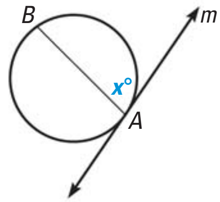
5. $m\angle 1$



6. ★ **MULTIPLE CHOICE** The diagram at the right is not drawn to scale. $\overline{AB}$ is any chord that is not a diameter of the circle. Line m is tangent to the circle at point A. Which statement must be true?

Ⓐ $x \le 90$ Ⓑ $x \ge 90$

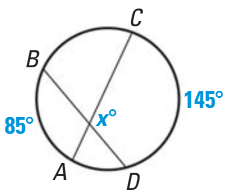
Ⓒ $x = 90$ Ⓓ $x \ne 90$



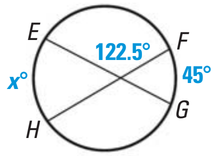
FINDING MEASURES **Find the value of x.**

EXAMPLE 2 on p. 681 for Exs. 7–9

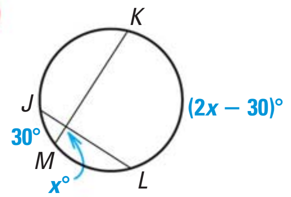
7.



8.

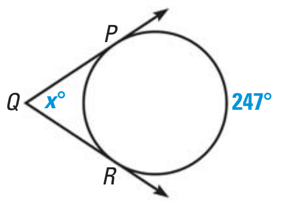


9.

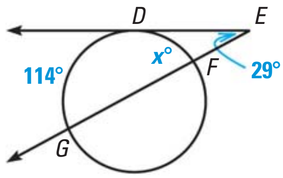


EXAMPLE 3 on p. 682 for Exs. 10–13

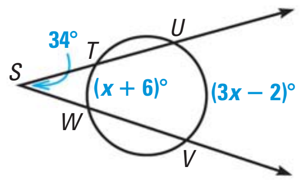
10.



11.



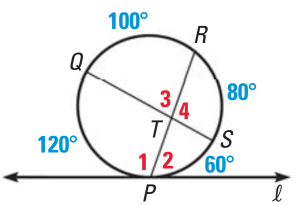
12.



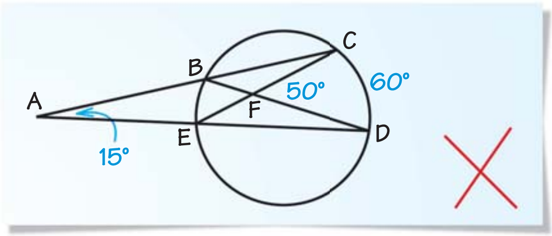
13. ★ **MULTIPLE CHOICE** In the diagram, ℓ is tangent to the circle at P. Which relationship is not true?

Ⓐ $m\angle 1 = 110°$ Ⓑ $m\angle 2 = 70°$

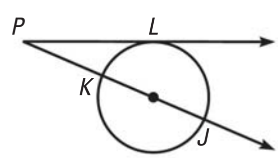
Ⓒ $m\angle 3 = 80°$ Ⓓ $m\angle 4 = 90°$



14. ERROR ANALYSIS *Describe* the error in the diagram below.

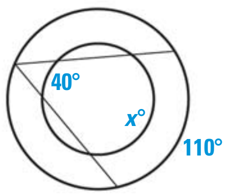


15. ★ SHORT RESPONSE In the diagram at the right, $\overrightarrow{PL}$ is tangent to the circle and $\overline{KJ}$ is a diameter. What is the range of possible angle measures of $\angle LPJ$? *Explain*.

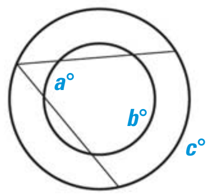


16. CONCENTRIC CIRCLES The circles below are concentric.

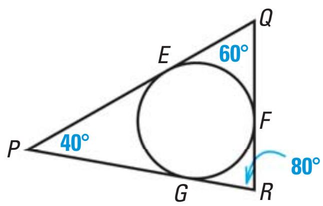
a. Find the value of x.



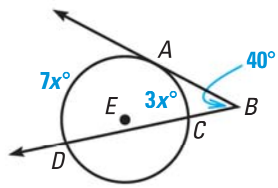
b. Express c in terms of a and b.



17. INSCRIBED CIRCLE In the diagram, the circle is inscribed in $\triangle PQR$. Find $m\overset{\frown}{EF}$, $m\overset{\frown}{FG}$, and $m\overset{\frown}{GE}$.



18. ALGEBRA In the diagram, $\overrightarrow{BA}$ is tangent to $\odot E$. Find $m\overset{\frown}{CD}$.



19. ★ WRITING Points A and B are on a circle and t is a tangent line containing A and another point C.

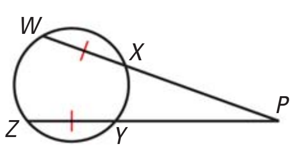
a. Draw two different diagrams that illustrate this situation.

b. Write an equation for $m\overset{\frown}{AB}$ in terms of $m\angle BAC$ for each diagram.

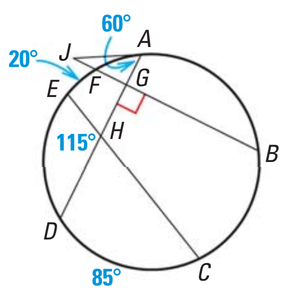
c. When will these equations give the same value for $m\overset{\frown}{AB}$?

CHALLENGE Find the indicated measure(s).

20. Find $m\angle P$ if $m\overset{\frown}{WZY} = 200°$.



21. Find $m\overset{\frown}{AB}$ and $m\overset{\frown}{ED}$.

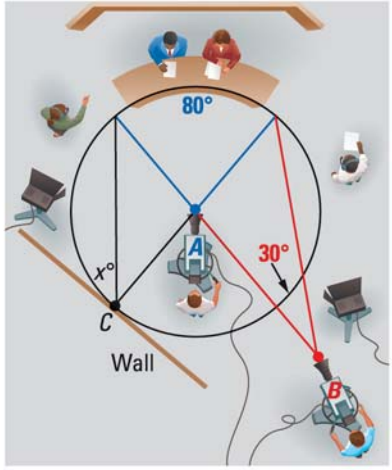


○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

PROBLEM SOLVING

VIDEO RECORDING In the diagram at the right, television cameras are positioned at *A*, *B*, and *C* to record what happens on stage. The stage is an arc of $\odot A$. Use the diagram for Exercises 22–24.



22. Find $m\angle A$, $m\angle B$, and $m\angle C$.

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23. The wall is tangent to the circle. Find x without using the measure of $\angle C$.

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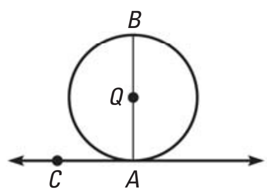
24. You would like Camera *B* to have a 30° view of the stage. Should you move the camera closer or further away from the stage? *Explain.*

EXAMPLE 4 on p. 682 for Ex. 25

25. **HOT AIR BALLOON** You are flying in a hot air balloon about 1.2 miles above the ground. Use the method from Example 4 to find the measure of the arc that represents the part of Earth that you can see. The radius of Earth is about 4000 miles.

26. ★ **EXTENDED RESPONSE** A cart is resting on its handle. The angle between the handle and the ground is 14° and the handle connects to the center of the wheel. What are the measures of the arcs of the wheel between the ground and the cart? *Explain.*

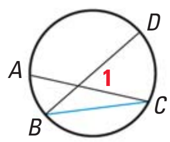
27. **PROVING THEOREM 10.11** The proof of Theorem 10.11 can be split into three cases. The diagram at the right shows the case where $\overline{AB}$ contains the center of the circle. Use Theorem 10.1 to write a paragraph proof for this case. What are the other two cases? (*Hint:* See Exercises 31–33 on page 678.) Draw a diagram and write plans for proof for the other cases.



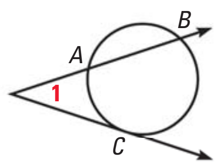
28. **PROVING THEOREM 10.12** Write a proof of Theorem 10.12.

GIVEN ▶ Chords $\overline{AC}$ and $\overline{BD}$ intersect.

PROVE ▶ $m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{DC} + m\overset{\frown}{AB}\right)$



29. **PROVING THEOREM 10.13** Use the diagram at the right to prove Theorem 10.13 for the case of a tangent and a secant. Draw $\overline{BC}$. *Explain* how to use the Exterior Angle Theorem in the proof of this case. Then copy the diagrams for the other two cases from page 681, draw appropriate auxiliary segments, and write plans for proof for these cases.



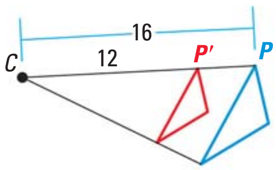
30. **PROOF** Q and R are points on a circle. P is a point outside the circle. $\overline{PQ}$ and $\overline{PR}$ are tangents to the circle. Prove that $\overline{QR}$ is not a diameter.

31. **CHALLENGE** A block and tackle system composed of two pulleys and a rope is shown at the right. The distance between the centers of the pulleys is 113 centimeters and the pulleys each have a radius of 15 centimeters. What percent of the circumference of the bottom pulley is not touching the rope?

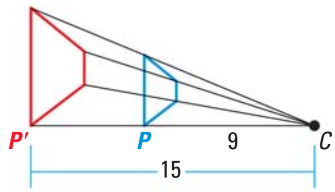
MIXED REVIEW

Classify the dilation and find its scale factor. *(p. 626)*

32.



33.



PREVIEW
Prepare for Lesson 10.6 in Exs. 34–39.

Use the quadratic formula to solve the equation. Round decimal answers to the nearest hundredth. *(pp. 641, 883)*

34. $x^2 + 7x + 6 = 0$

35. $x^2 - x - 12 = 0$

36. $x^2 + 16 = 8x$

37. $x^2 + 6x = 10$

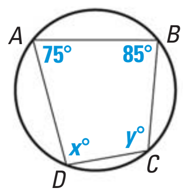
38. $5x + 9 = 2x^2$

39. $4x^2 + 3x - 11 = 0$

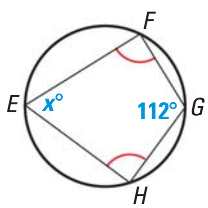
QUIZ *for Lessons 10.4–10.5*

Find the value(s) of the variable(s).

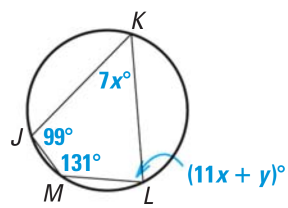
1. $m\widehat{ABC} = z°$ *(p. 672)*



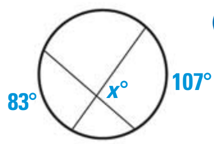
2. $m\widehat{GHE} = z°$ *(p. 672)*



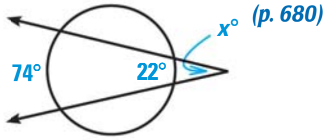
3. $m\widehat{JKL} = z°$ *(p. 672)*



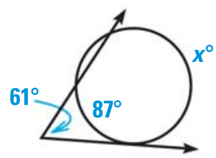
4. *(p. 680)*



5. *(p. 680)*



6. *(p. 680)*



7. **MOUNTAIN** You are on top of a mountain about 1.37 miles above sea level. Find the measure of the arc that represents the part of Earth that you can see. Earth's radius is approximately 4000 miles. *(p. 680)*

MIXED REVIEW of Problem Solving

STATE TEST PRACTICE
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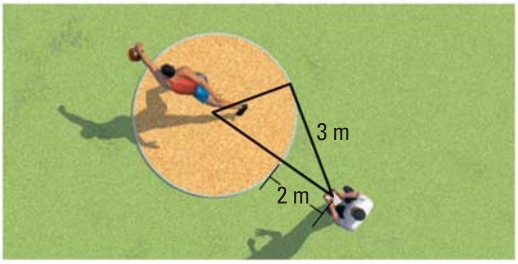
Lessons 10.1–10.5

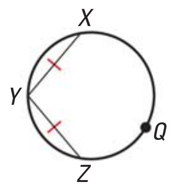
1. MULTI-STEP PROBLEM An official stands 2 meters from the edge of a discus circle and 3 meters from a point of tangency.



a. Find the radius of the discus circle.

b. How far is the official from the center of the discus circle?

2. GRIDDED ANSWER In the diagram, $\overline{XY} \cong \overline{YZ}$ and $m\widehat{XQZ} = 199°$. Find $m\widehat{YZ}$ in degrees.



3. MULTI-STEP PROBLEM A wind turbine has three equally spaced blades that are each 131 feet long.

a. What is the measure of the arc between any two blades?

b. The highest point reached by a blade is 361 feet above the ground. Find the distance x between the lowest point reached by the blades and the ground.

c. What is the distance y from the tip of one blade to the tip of another blade? Round your answer to the nearest tenth.

4. EXTENDED RESPONSE The Navy Pier Ferris Wheel in Chicago is 150 feet tall and has 40 spokes.

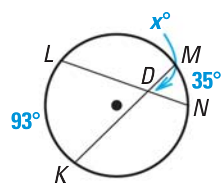
a. Find the measure of the angle between any two spokes.

b. Two spokes form a central angle of 72°. How many spokes are between the two spokes?

c. The bottom of the wheel is 10 feet from the ground. Find the diameter and radius of the wheel. *Explain* your reasoning.

5. OPEN-ENDED Draw a quadrilateral inscribed in a circle. Measure two consecutive angles. Then find the measures of the other two angles algebraically.

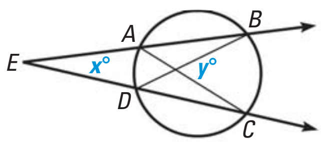
6. MULTI-STEP PROBLEM Use the diagram.



a. Find the value of x.

b. Find the measures of the other three angles formed by the intersecting chords.

7. SHORT RESPONSE Use the diagram to show that $m\widehat{DA} = y° - x°$.



Investigating Geometry ACTIVITY Use before Lesson 10.6

@HomeTutor classzone.com Keystrokes

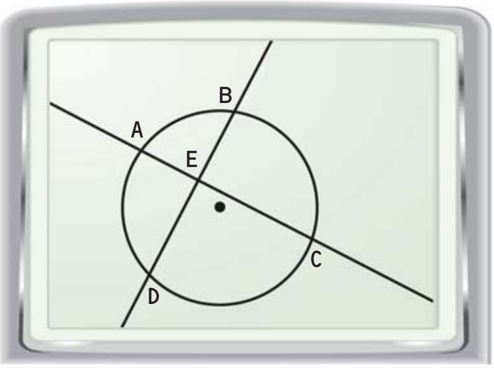
10.6 Investigate Segment Lengths

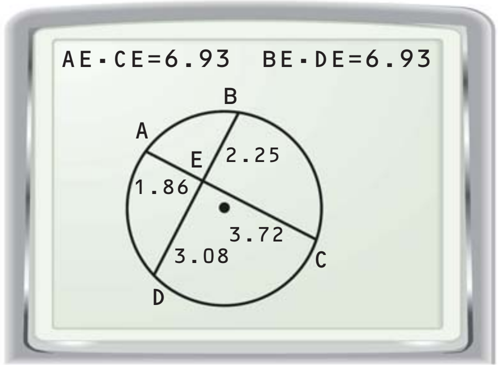
MATERIALS • graphing calculator or computer

QUESTION What is the relationship between the lengths of segments in a circle?

You can use geometry drawing software to find a relationship between the segments formed by two intersecting chords.

EXPLORE Draw a circle with two chords





STEP 1 ***Draw a circle*** Draw a circle and choose four points on the circle. Label them A, B, C, and D.

STEP 2 ***Draw secants*** Draw secants $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ and label the intersection point E.

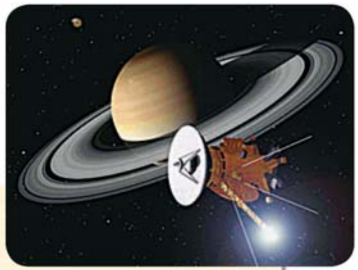
STEP 3 ***Measure segments*** Note that $\overline{AC}$ and $\overline{BD}$ are chords. Measure $\overline{AE}$, $\overline{CE}$, $\overline{BE}$, and $\overline{DE}$ in your diagram.

STEP 4 ***Perform calculations*** Calculate the products $AE \cdot CE$ and $BE \cdot DE$.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. What do you notice about the products you found in Step 4?
2. Drag points A, B, C, and D, keeping point E inside the circle. What do you notice about the new products from Step 4?
3. Make a conjecture about the relationship between the four chord segments.
4. Let $\overline{PQ}$ and $\overline{RS}$ be two chords of a circle that intersect at the point T. If $PT = 9$, $QT = 5$, and $RT = 15$, use your conjecture from Exercise 3 to find ST.

10.6 Find Segment Lengths in Circles



Before You found angle and arc measures in circles.

Now You will find segment lengths in circles.

Why? So you can find distances in astronomy, as in Example 4.

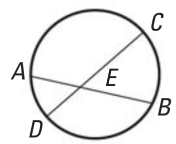
Key Vocabulary
- **segments of a chord**
- **secant segment**
- **external segment**

When two chords intersect in the interior of a circle, each chord is divided into two segments that are called **segments of the chord**.

THEOREM *For Your Notebook*

THEOREM 10.14 Segments of Chords Theorem

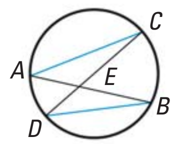
If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



$EA \cdot EB = EC \cdot ED$

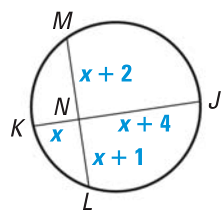
Proof: Ex. 21, p. 694

Plan for Proof To prove Theorem 10.14, construct two similar triangles. The lengths of the corresponding sides are proportional, so $\frac{EA}{ED} = \frac{EC}{EB}$. By the Cross Products Property, $EA \cdot EB = EC \cdot ED$.



EXAMPLE 1 Find lengths using Theorem 10.14

ALGEBRA **Find *ML* and *JK*.**



Solution

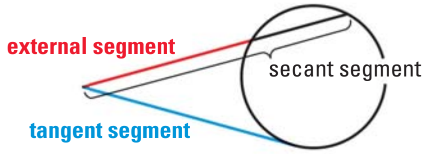
$NK \cdot NJ = NL \cdot NM$	Use Theorem 10.14.
$x \cdot (x + 4) = (x + 1) \cdot (x + 2)$	Substitute.
$x^2 + 4x = x^2 + 3x + 2$	Simplify.
$4x = 3x + 2$	Subtract x^2 from each side.
$x = 2$	Solve for x.

Find *ML* and *JK* by substitution.

$ML = (x + 2) + (x + 1)$
$= 2 + 2 + 2 + 1$
$= 7$

$JK = x + (x + 4)$
$= 2 + 2 + 4$
$= 8$

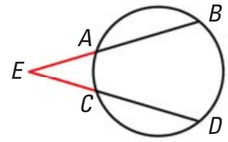
TANGENTS AND SECANTS A **secant segment** is a segment that contains a chord of a circle, and has exactly one endpoint outside the circle. The part of a secant segment that is outside the circle is called an **external segment**.



THEOREM — *For Your Notebook*

THEOREM 10.15 Segments of Secants Theorem

If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.



$EA \cdot EB = EC \cdot ED$

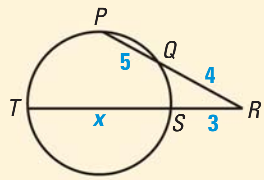
Proof: Ex. 25, p. 694

EXAMPLE 2 Standardized Test Practice

What is the value of x?

Ⓐ 6 Ⓑ $6\frac{2}{3}$

Ⓒ 8 Ⓓ 9



Solution

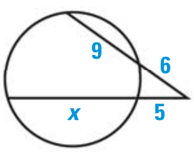
$RQ \cdot RP = RS \cdot RT$	**Use Theorem 10.15.**
$4 \cdot (5 + 4) = 3 \cdot (x + 3)$	**Substitute.**
$36 = 3x + 9$	**Simplify.**
$9 = x$	**Solve for x.**

▶ The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

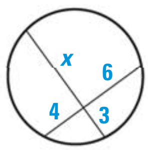
✓ GUIDED PRACTICE for Examples 1 and 2

Find the value(s) of x.

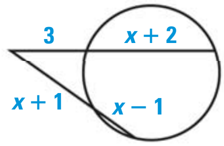
1.



2.



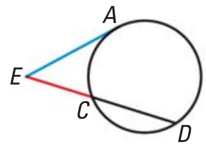
3.



THEOREM *For Your Notebook*

THEOREM 10.16 Segments of Secants and Tangents Theorem

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.

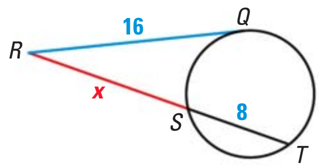


$EA^2 = EC \cdot ED$

Proof: Ex. 26, p. 694

EXAMPLE 3 Find lengths using Theorem 10.16

Use the figure at the right to find *RS*.



Solution

$RQ^2 = RS \cdot RT$	**Use Theorem 10.16.**
$16^2 = x \cdot (x + 8)$	**Substitute.**
$256 = x^2 + 8x$	**Simplify.**
$0 = x^2 + 8x - 256$	**Write in standard form.**
$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-256)}}{2(1)}$	**Use quadratic formula.**
$x = -4 \pm 4\sqrt{17}$	**Simplify.**

Use the positive solution, because lengths cannot be negative.

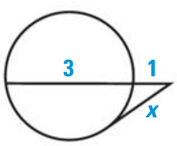
▶ So, $x = -4 + 4\sqrt{17} \approx 12.49$, and $RS \approx 12.49$.

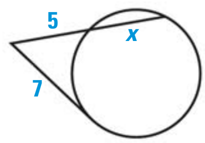
Animated Geometry at classzone.com

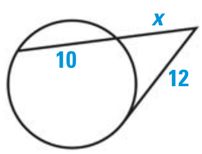
ANOTHER WAY
For an alternative method for solving the problem in Example 3, turn to page 696 for the **Problem Solving Workshop**.

✓ GUIDED PRACTICE for Example 3

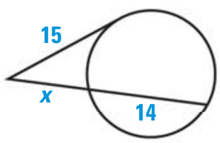
Find the value of *x*.

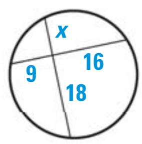
4.


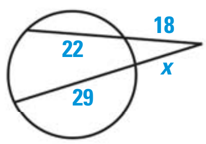
5.


6.


Determine which theorem you would use to find *x*. Then find the value of *x*.

7.


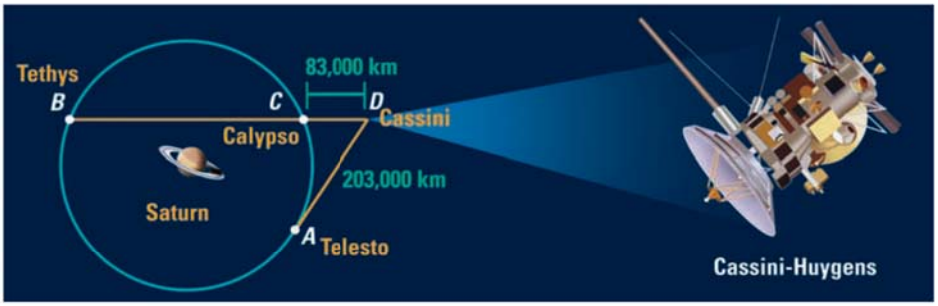
8.


9.


10. In the diagram for Theorem 10.16, what must be true about *EC* compared to *EA*?

EXAMPLE 4 Solve a real-world problem

SCIENCE Tethys, Calypso, and Telesto are three of Saturn's moons. Each has a nearly circular orbit 295,000 kilometers in radius. The Cassini-Huygens spacecraft entered Saturn's orbit in July 2004. Telesto is on a point of tangency. Find the distance DB from Cassini to Tethys.



Solution

$DC \cdot DB = AD^2$	**Use Theorem 10.16.**
$83{,}000 \cdot DB \approx 203{,}000^2$	**Substitute.**
$DB \approx 496{,}494$	**Solve for *DB*.**

▶ Cassini is about 496,494 kilometers from Tethys.

✓ GUIDED PRACTICE for Example 4

11. Why is it appropriate to use the approximation symbol ≈ in the last two steps of the solution to Example 4?

10.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 9, and 21

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 24, and 27

SKILL PRACTICE

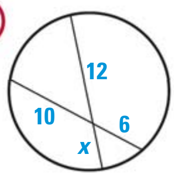
1. VOCABULARY Copy and complete: The part of the secant segment that is outside the circle is called a(n) __?__.

2. ★ WRITING *Explain* the difference between a tangent segment and a secant segment.

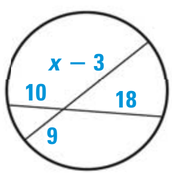
EXAMPLE 1 on p. 689 for Exs. 3–5

FINDING SEGMENT LENGTHS **Find the value of *x*.**

3.



4.



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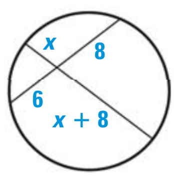
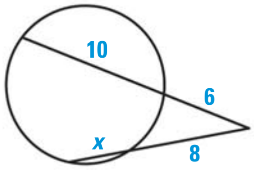


FINDING SEGMENT LENGTHS Find the value of x.

EXAMPLE 2
on p. 690
for Exs. 6–8

6.



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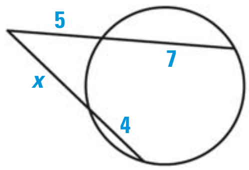

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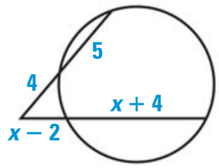
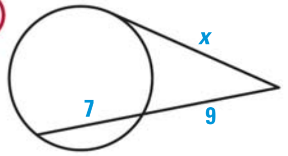
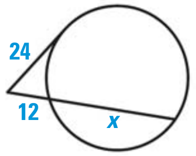


EXAMPLE 3
on p. 691
for Exs. 9–11

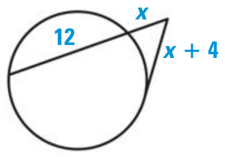
9.



10.

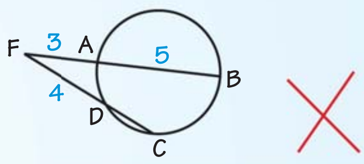


11.



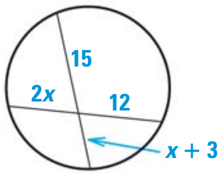
12. ERROR ANALYSIS *Describe* and correct the error in finding CD.

$CD \cdot DF = AB \cdot AF$
$CD \cdot 4 = 5 \cdot 3$
$CD \cdot 4 = 15$
$CD = 3.75$

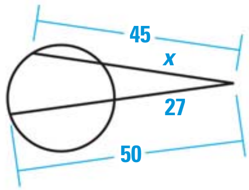


FINDING SEGMENT LENGTHS Find the value of x. Round to the nearest tenth.

13.



14.



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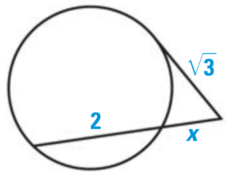
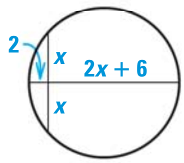


16. ★ **MULTIPLE CHOICE** Which of the following is a possible value of x?

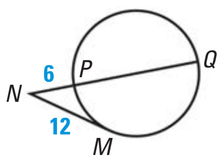
Ⓐ -2 Ⓑ 4

Ⓒ 5 Ⓓ 6

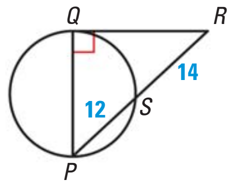


FINDING LENGTHS Find PQ. Round your answers to the nearest tenth.

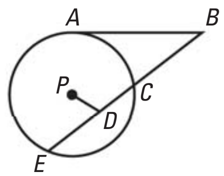
17.



18.



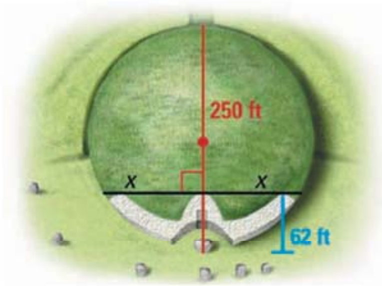
19. CHALLENGE In the figure, $AB = 12$, $BC = 8$, $DE = 6$, $PD = 4$, and A is a point of tangency. Find the radius of $\odot P$.



PROBLEM SOLVING

EXAMPLE 4
on p. 692
for Ex. 20

20. ARCHAEOLOGY The circular stone mound in Ireland called Newgrange has a diameter of 250 feet. A passage 62 feet long leads toward the center of the mound. Find the perpendicular distance x from the end of the passage to either side of the mound.

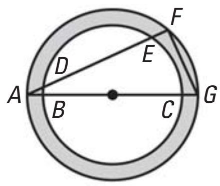


@HomeTutor for problem solving help at classzone.com

21. PROVING THEOREM 10.14 Write a two-column proof of Theorem 10.14. Use similar triangles as outlined in the Plan for Proof on page 689.

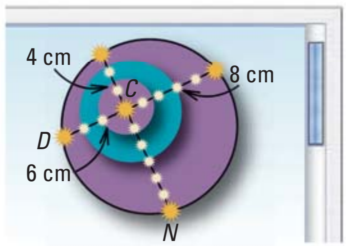
@HomeTutor for problem solving help at classzone.com

22. WELLS In the diagram of the water well, AB, AD, and DE are known. Write an equation for BC using these three measurements.



23. PROOF Use Theorem 10.1 to prove Theorem 10.16 for the special case when the secant segment contains the center of the circle.

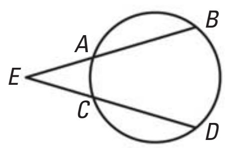
24. ★ SHORT RESPONSE You are designing an animated logo for your website. Sparkles leave point C and move to the circle along the segments shown so that all of the sparkles reach the circle at the same time. Sparkles travel from point C to point D at 2 centimeters per second. How fast should sparkles move from point C to point N? *Explain.*



25. PROVING THEOREM 10.15 Use the plan to prove Theorem 10.15.

GIVEN ▶ $\overline{EB}$ and $\overline{ED}$ are secant segments.

PROVE ▶ $EA \cdot EB = EC \cdot ED$

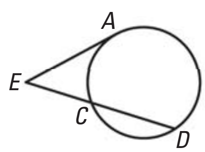


Plan for Proof Draw $\overline{AD}$ and $\overline{BC}$. Show that $\triangle BCE$ and $\triangle DAE$ are similar. Use the fact that corresponding side lengths in similar triangles are proportional.

26. PROVING THEOREM 10.16 Use the plan to prove Theorem 10.16.

GIVEN ▶ $\overline{EA}$ is a tangent segment.
$\overline{ED}$ is a secant segment.

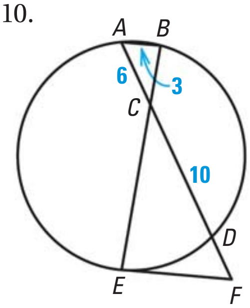
PROVE ▶ $EA^2 = EC \cdot ED$



Plan for Proof Draw $\overline{AD}$ and $\overline{AC}$. Use the fact that corresponding side lengths in similar triangles are proportional.

○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE

27. ★ **EXTENDED RESPONSE** In the diagram, $\overline{EF}$ is a tangent segment, $m\widehat{AD} = 140°$, $m\widehat{AB} = 20°$, $m\angle EFD = 60°$, $AC = 6$, $AB = 3$, and $DC = 10$.



a. Find $m\angle CAB$.

b. Show that $\triangle ABC \sim \triangle FEC$.

c. Let $EF = y$ and $DF = x$. Use the results of part (b) to write a proportion involving x and y. Solve for y.

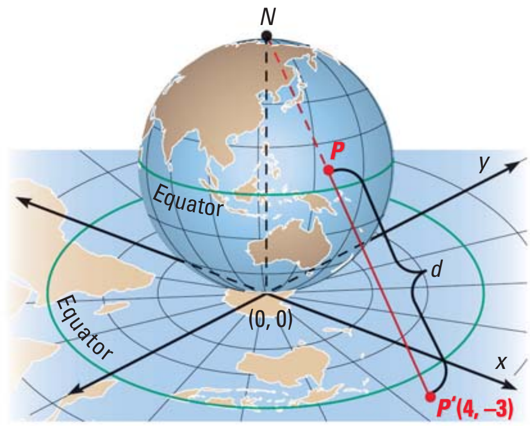
d. Use a theorem from this section to write another equation involving both x and y.

e. Use the results of parts (c) and (d) to solve for x and y.

f. *Explain* how to find CE.

28. **CHALLENGE** Stereographic projection is a map-making technique that takes points on a sphere with radius one unit (Earth) to points on a plane (the map). The plane is tangent to the sphere at the origin.

The map location for each point P on the sphere is found by extending the line that connects N and P. The point's projection is where the line intersects the plane. Find the distance d from the point P to its corresponding point $P'(4, -3)$ on the plane.



Not drawn to scale

MIXED REVIEW

PREVIEW
Prepare for Lesson 10.7 in Exs. 29–32.

Evaluate the expression. *(p. 874)*

29. $\sqrt{(-10)^2 - 8^2}$

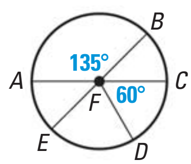
30. $\sqrt{-5 + (-4) + (6 - 1)^2}$

31. $\sqrt{[-2 - (-6)]^2 + (3 - 6)^2}$

32. In right $\triangle PQR$, $PQ = 8$, $m\angle Q = 40°$, and $m\angle R = 50°$. Find QR and PR to the nearest tenth. *(p. 473)*

33. $\overleftrightarrow{EF}$ is tangent to $\odot C$ at E. The radius of $\odot C$ is 5 and $EF = 8$. Find FC. *(p. 651)*

Find the indicated measure. $\overline{AC}$ and $\overline{BE}$ are diameters. *(p. 659)*



34. $m\widehat{AB}$

35. $m\widehat{CD}$

36. $m\widehat{BCA}$

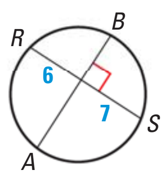
37. $m\widehat{CBD}$

38. $m\widehat{CDA}$

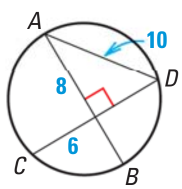
39. $m\widehat{BAE}$

Determine whether $\overline{AB}$ is a diameter of the circle. *Explain.* *(p. 664)*

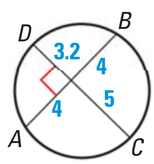
40.



41.



42.



PROBLEM SOLVING WORKSHOP
LESSON 10.6

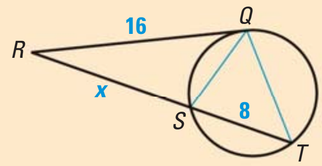
Using ALTERNATIVE METHODS

Another Way to Solve Example 3, page 691

MULTIPLE REPRESENTATIONS You can use similar triangles to find the length of an external secant segment.

PROBLEM

Use the figure at the right to find RS.



METHOD

Using Similar Triangles

STEP 1 **Draw** segments $\overline{QS}$ and $\overline{QT}$, and identify the similar triangles.

Because they both intercept the same arc, $\angle RQS \cong \angle RTQ$.
By the Reflexive Property of Angle Congruence, $\angle QRS \cong \angle TRQ$.
So, $\triangle RSQ \sim \triangle RQT$ by the AA Similarity Postulate.

STEP 2 **Use** a proportion to solve for RS.

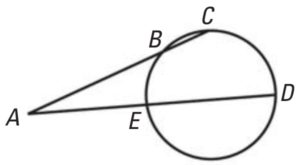
$$\frac{RS}{RQ} = \frac{RQ}{RT} \qquad \Rightarrow \qquad \frac{x}{16} = \frac{16}{x+8}$$

▶ By the Cross Products Property, $x^2 + 8x = 256$. Use the quadratic formula to find that $x = -4 \pm 4\sqrt{17}$. Taking the positive solution, $x = -4 + 4\sqrt{17}$ and $RS = 12.49$.

PRACTICE

1. **WHAT IF?** Find RQ in the problem above if the known lengths are $RS = 4$ and $ST = 9$.

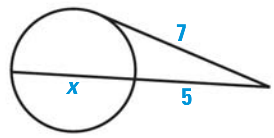
2. **MULTI-STEP PROBLEM** Copy the diagram.



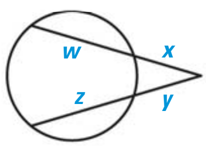
   a. Draw auxiliary segments $\overline{BE}$ and $\overline{CD}$. Name two similar triangles.

   b. If $AB = 15$, $BC = 5$, and $AE = 12$, find DE.

3. **CHORD** Find the value of x.



4. **SEGMENTS OF SECANTS** Use the Segments of Secants Theorem to write an expression for w in terms of x, y, and z.



Extension

Use after Lesson 10.6

Draw a Locus

GOAL Draw the locus of points satisfying certain conditions.

Key Vocabulary
- **locus**

A **locus** in a plane is the set of all points in a plane that satisfy a given condition or a set of given conditions. The word *locus* is derived from the Latin word for "location." The plural of locus is *loci*, pronounced "low-sigh."

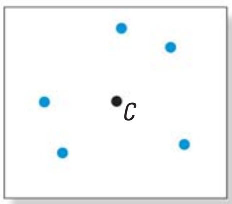
A locus is often described as the path of an object moving in a plane. For example, the reason that many clock faces are circular is that the locus of the end of a clock's minute hand is a circle.

EXAMPLE 1 Find a locus

Draw a point *C* on a piece of paper. Draw and describe the locus of all points on the paper that are 1 centimeter from *C*.

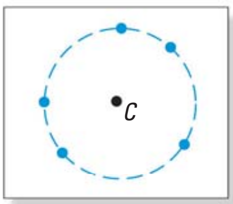
Solution

STEP 1



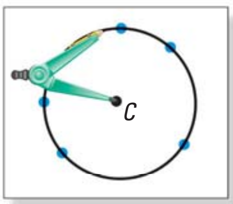
Draw point *C*. Locate several points 1 centimeter from *C*.

STEP 2



Recognize a pattern: the points lie on a circle.

STEP 3



Draw the circle.

▶ The locus of points on the paper that are 1 centimeter from *C* is a circle with center *C* and radius 1 centimeter.

KEY CONCEPT — *For Your Notebook*

How to Find a Locus

To find the locus of points that satisfy a given condition, use the following steps.

STEP 1 **Draw** any figures that are given in the statement of the problem. Locate several points that satisfy the given condition.

STEP 2 **Continue** drawing points until you can recognize the pattern.

STEP 3 **Draw** the locus and describe it in words.

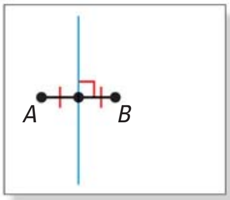
LOCI SATISFYING TWO OR MORE CONDITIONS To find the locus of points that satisfy two or more conditions, first find the locus of points that satisfy each condition alone. Then find the intersection of these loci.

EXAMPLE 2 Draw a locus satisfying two conditions

Points A and B lie in a plane. What is the locus of points in the plane that are equidistant from points A and B and are a distance of AB from B?

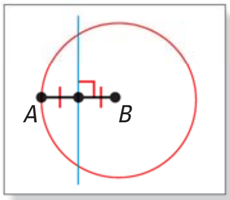
Solution

STEP 1



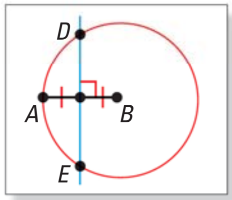
The locus of all points that are equidistant from A and B is the perpendicular bisector of $\overline{AB}$.

STEP 2



The locus of all points that are a distance of AB from B is the circle with center B and radius AB.

STEP 3



These loci intersect at D and E. So D and E form the locus of points that satisfy both conditions.

PRACTICE

EXAMPLE 1
on p. 697
for Exs. 1–4

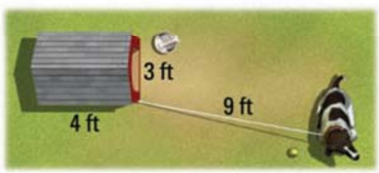
DRAWING A LOCUS **Draw the figure. Then sketch the locus of points on the paper that satisfy the given condition.**

1. Point P, the locus of points that are 1 inch from P
2. Line k, the locus of points that are 1 inch from k
3. Point C, the locus of points that are at least 1 inch from C
4. Line j, the locus of points that are no more than 1 inch from j

EXAMPLE 2
on p. 698
for Exs. 5–9

WRITING **Write a description of the locus. Include a sketch.**

5. Point P lies on line ℓ. What is the locus of points on ℓ and 3 cm from P?
6. Point Q lies on line m. What is the locus of points 5 cm from Q and 3 cm from m?
7. Point R is 10 cm from line k. What is the locus of points that are within 10 cm of R, but further than 10 cm from k?
8. Lines ℓ and m are parallel. Point P is 5 cm from both lines. What is the locus of points between ℓ and m and no more than 8 cm from P?
9. **DOG LEASH** A dog's leash is tied to a stake at the corner of its doghouse, as shown at the right. The leash is 9 feet long. Make a scale drawing of the doghouse and sketch the locus of points that the dog can reach.



10.7 Write and Graph Equations of Circles

Before You wrote equations of lines in the coordinate plane.

Now You will write equations of circles in the coordinate plane.

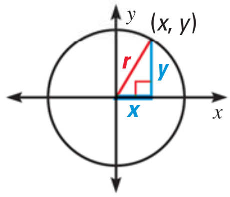
Why? So you can determine zones of a commuter system, as in Ex. 36.

Key Vocabulary
- standard equation of a circle

Let (x, y) represent any point on a circle with center at the origin and radius r. By the Pythagorean Theorem,

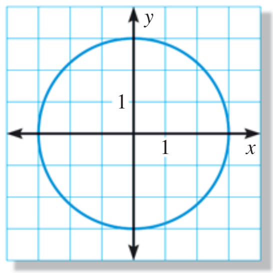
$$x^2 + y^2 = r^2.$$

This is the equation of a circle with radius r and center at the origin.



EXAMPLE 1 Write an equation of a circle

Write the equation of the circle shown.



Solution

The radius is 3 and the center is at the origin.

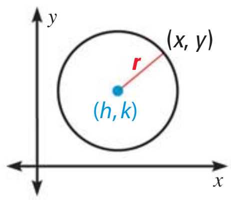
$x^2 + y^2 = r^2$	Equation of circle
$x^2 + y^2 = 3^2$	Substitute.
$x^2 + y^2 = 9$	Simplify.

▶ The equation of the circle is $x^2 + y^2 = 9$.

CIRCLES CENTERED AT (h, k) You can write the equation of *any* circle if you know its radius and the coordinates of its center.

Suppose a circle has radius r and center (h, k). Let (x, y) be a point on the circle. The distance between (x, y) and (h, k) is r, so by the Distance Formula

$$\sqrt{(x - h)^2 + (y - k)^2} = r.$$



Square both sides to find the **standard equation of a circle**.

KEY CONCEPT — *For Your Notebook*

Standard Equation of a Circle

The standard equation of a circle with center (h, k) and radius r is:

$$(x - h)^2 + (y - k)^2 = r^2$$

EXAMPLE 2 Write the standard equation of a circle

Write the standard equation of a circle with center (0, −9) and radius 4.2.

Solution

$(x - h)^2 + (y - k)^2 = r^2$ Standard equation of a circle

$(x - 0)^2 + (y - (-9))^2 = 4.2^2$ Substitute.

$x^2 + (y + 9)^2 = 17.64$ Simplify.

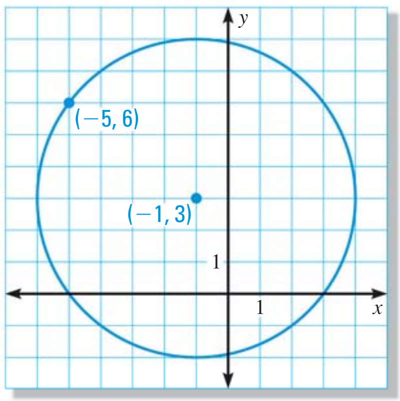
GUIDED PRACTICE for Examples 1 and 2

Write the standard equation of the circle with the given center and radius.

1. Center (0, 0), radius 2.5
2. Center (−2, 5), radius 7

EXAMPLE 3 Write the standard equation of a circle

The point (−5, 6) is on a circle with center (−1, 3). Write the standard equation of the circle.



Solution

To write the standard equation, you need to know the values of h, k, and r. To find r, find the distance between the center and the point (−5, 6) on the circle.

$r = \sqrt{[-5 - (-1)]^2 + (6 - 3)^2}$ Distance Formula

$= \sqrt{(-4)^2 + 3^2}$ Simplify.

$= 5$ Simplify.

Substitute $(h, k) = (-1, 3)$ and $r = 5$ into the standard equation of a circle.

$(x - h)^2 + (y - k)^2 = r^2$ Standard equation of a circle

$[x - (-1)]^2 + (y - 3)^2 = 5^2$ Substitute.

$(x + 1)^2 + (y - 3)^2 = 25$ Simplify.

▶ The standard equation of the circle is $(x + 1)^2 + (y - 3)^2 = 25$.

GUIDED PRACTICE for Example 3

3. The point (3, 4) is on a circle whose center is (1, 4). Write the standard equation of the circle.
4. The point (−1, 2) is on a circle whose center is (2, 6). Write the standard equation of the circle.

EXAMPLE 4 Graph a circle

USE EQUATIONS
If you know the equation of a circle, you can graph the circle by identifying its center and radius.

The equation of a circle is $(x - 4)^2 + (y + 2)^2 = 36$. Graph the circle.

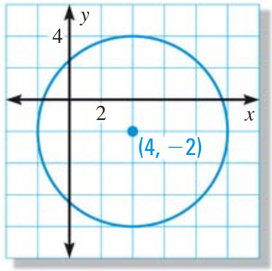
Solution

Rewrite the equation to find the center and radius.

$$(x - 4)^2 + (y + 2)^2 = 36$$

$$(x - 4)^2 + [y - (-2)]^2 = 6^2$$

The center is $(4, -2)$ and the radius is 6. Use a compass to graph the circle.

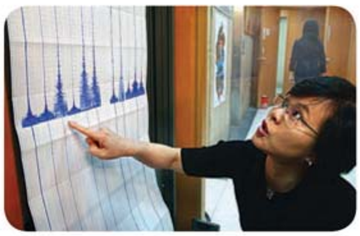


EXAMPLE 5 Use graphs of circles

EARTHQUAKES The epicenter of an earthquake is the point on Earth's surface directly above the earthquake's origin. A seismograph can be used to determine the distance to the epicenter of an earthquake. Seismographs are needed in three different places to locate an earthquake's epicenter.

Use the seismograph readings from locations A, B, and C to find the epicenter of an earthquake.

- The epicenter is 7 miles away from $A(-2, 2.5)$.
- The epicenter is 4 miles away from $B(4, 6)$.
- The epicenter is 5 miles away from $C(3, -2.5)$.



Solution

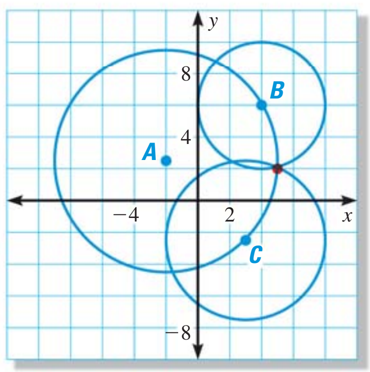
The set of all points equidistant from a given point is a circle, so the epicenter is located on each of the following circles.

$\odot A$ with center $(-2, 2.5)$ and radius 7

$\odot B$ with center $(4, 6)$ and radius 4

$\odot C$ with center $(3, -2.5)$ and radius 5

To find the epicenter, graph the circles on a graph where units are measured in miles. Find the point of intersection of all three circles.



▶ The epicenter is at about $(5, 2)$.

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✓ GUIDED PRACTICE for Examples 4 and 5

5. The equation of a circle is $(x - 4)^2 + (y + 3)^2 = 16$. Graph the circle.

6. The equation of a circle is $(x + 8)^2 + (y + 5)^2 = 121$. Graph the circle.

7. Why are three seismographs needed to locate an earthquake's epicenter?

10.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 17, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 26, and 42

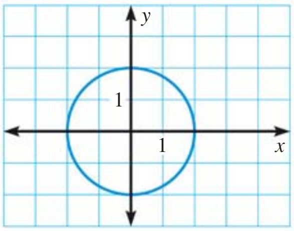
SKILL PRACTICE

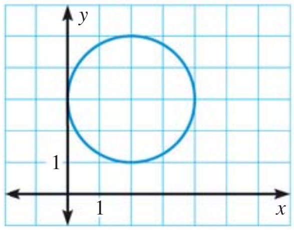
1. **VOCABULARY** Copy and complete: The standard equation of a circle can be written for any circle with known _?_ and _?_.

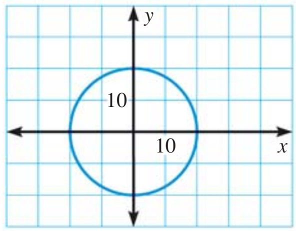
2. ★ **WRITING** *Explain* why the location of the center and one point on a circle is enough information to draw the rest of the circle.

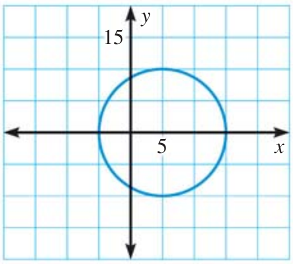
EXAMPLES 1 and 2 on pp. 699–700 for Exs. 3–16

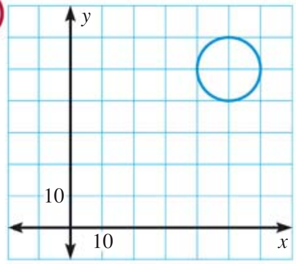
WRITING EQUATIONS **Write the standard equation of the circle.**

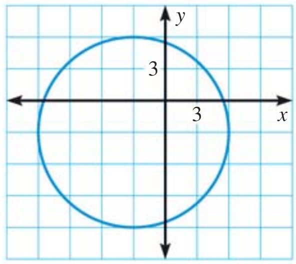
3. 

4. 

5. 

6. 

7. 

8. 

WRITING EQUATIONS **Write the standard equation of the circle with the given center and radius.**

9. Center (0, 0), radius 7
10. Center (−4, 1), radius 1
11. Center (7, −6), radius 8
12. Center (4, 1), radius 5
13. Center (3, −5), radius 7
14. Center (−3, 4), radius 5

15. **ERROR ANALYSIS** *Describe* and correct the error in writing the equation of a circle.

An equation of a circle with center (−3, −5) and radius 3 is $(x - 3)^2 + (y - 5)^2 = 9$.

16. ★ **MULTIPLE CHOICE** The standard equation of a circle is $(x - 2)^2 + (y + 1)^2 = 16$. What is the diameter of the circle?

Ⓐ 2 Ⓑ 4 Ⓒ 8 Ⓓ 16

EXAMPLE 3 on p. 700 for Exs. 17–19

WRITING EQUATIONS **Use the given information to write the standard equation of the circle.**

17. The center is (0, 0), and a point on the circle is (0, 6).
18. The center is (1, 2), and a point on the circle is (4, 2).
19. The center is (−3, 5), and a point on the circle is (1, 8).

EXAMPLE 4
on p. 701
for Exs. 20–25

GRAPHING CIRCLES **Graph the equation.**

20. $x^2 + y^2 = 49$

21. $(x - 3)^2 + y^2 = 16$

22. $x^2 + (y + 2)^2 = 36$

23. $(x - 4)^2 + (y - 1)^2 = 1$

24. $(x + 5)^2 + (y - 3)^2 = 9$

25. $(x + 2)^2 + (y + 6)^2 = 25$

26. ★ **MULTIPLE CHOICE** Which of the points does not lie on the circle described by the equation $(x + 2)^2 + (y - 4)^2 = 25$?

Ⓐ $(-2, -1)$ Ⓑ $(1, 8)$ Ⓒ $(3, 4)$ Ⓓ $(0, 5)$

xy **ALGEBRA** **Determine whether the given equation defines a circle. If the equation defines a circle, rewrite the equation in standard form.**

27. $x^2 + y^2 - 6y + 9 = 4$

28. $x^2 - 8x + 16 + y^2 + 2y + 4 = 25$

29. $x^2 + y^2 + 4y + 3 = 16$

30. $x^2 - 2x + 5 + y^2 = 81$

IDENTIFYING TYPES OF LINES **Use the given equations of a circle and a line to determine whether the line is a *tangent*, *secant*, *secant that contains a diameter*, or none of these.**

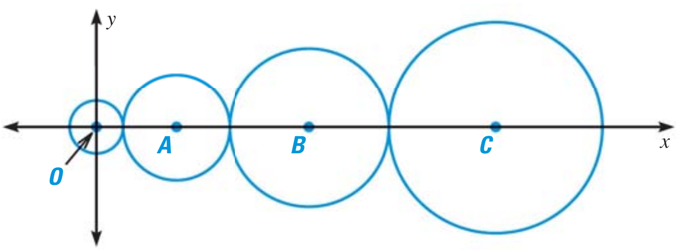
31. **Circle:** $(x - 4)^2 + (y - 3)^2 = 9$
Line: $y = -3x + 6$

32. **Circle:** $(x + 2)^2 + (y - 2)^2 = 16$
Line: $y = 2x - 4$

33. **Circle:** $(x - 5)^2 + (y + 1)^2 = 4$
Line: $y = \frac{1}{5}x - 3$

34. **Circle:** $(x + 3)^2 + (y - 6)^2 = 25$
Line: $y = -\frac{4}{3}x + 2$

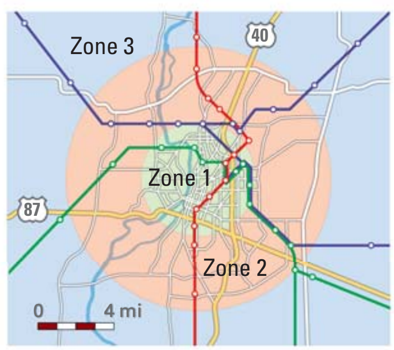
35. **CHALLENGE** Four tangent circles are centered on the x-axis. The radius of $\odot A$ is twice the radius of $\odot O$. The radius of $\odot B$ is three times the radius of $\odot O$. The radius of $\odot C$ is four times the radius of $\odot O$. All circles have integer radii and the point $(63, 16)$ is on $\odot C$. What is the equation of $\odot A$?



PROBLEM SOLVING

EXAMPLE 5
on p. 701
for Ex. 36

36. **COMMUTER TRAINS** A city's commuter system has three zones covering the regions described. Zone 1 covers people living within three miles of the city center. Zone 2 covers those between three and seven miles from the center, and Zone 3 covers those over seven miles from the center.



a. Graph this situation with the city center at the origin, where units are measured in miles.

b. Find which zone covers people living at $(3, 4)$, $(6, 5)$, $(1, 2)$, $(0, 3)$, and $(1, 6)$.

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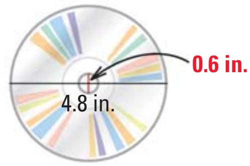
37. **COMPACT DISCS** The diameter of a CD is about 4.8 inches. The diameter of the hole in the center is about 0.6 inches. You place a CD on the coordinate plane with center at (0, 0). Write the equations for the outside edge of the disc and the edge of the hole in the center.



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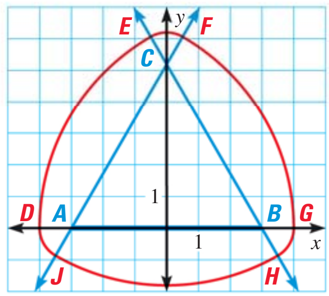
REULEAUX POLYGONS **In Exercises 38–41, use the following information.**

The figure at the right is called a *Reuleaux polygon*. It is not a true polygon because its sides are not straight. $\triangle ABC$ is equilateral.



38. $\overparen{JD}$ lies on a circle with center A and radius AD. Write an equation of this circle.

39. $\overparen{DE}$ lies on a circle with center B and radius BD. Write an equation of this circle.

40. **CONSTRUCTION** The remaining arcs of the polygon are constructed in the same way as $\overparen{JD}$ and $\overparen{DE}$ in Exercises 38 and 39. Construct a Reuleaux polygon on a piece of cardboard.

41. Cut out the Reuleaux polygon from Exercise 40. Roll it on its edge like a wheel and measure its height when it is in different orientations. *Explain* why a Reuleaux polygon is said to have constant width.

42. ★ **EXTENDED RESPONSE** Telecommunication towers can be used to transmit cellular phone calls. Towers have a range of about 3 km. A graph with units measured in kilometers shows towers at points (0, 0), (0, 5), and (6, 3).

a. Draw the graph and locate the towers. Are there any areas that may receive calls from more than one tower?

b. Suppose your home is located at (2, 6) and your school is at (2.5, 3). Can you use your cell phone at either or both of these locations?

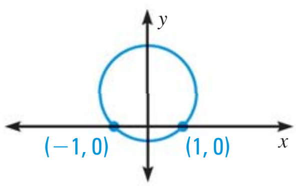
c. City A is located at (−2, 2.5) and City B is at (5, 4). Each city has a radius of 1.5 km. Which city seems to have better cell phone coverage? *Explain*.

43. **REASONING** The lines $y = \frac{3}{4}x + 2$ and $y = -\frac{3}{4}x + 16$ are tangent to $\odot C$ at the points (4, 5) and (4, 13), respectively.

a. Find the coordinates of C and the radius of $\odot C$. *Explain* your steps.

b. Write the standard equation of $\odot C$ and draw its graph.

44. **PROOF** Write a proof.



GIVEN ▶ A circle passing through the points (−1, 0) and (1, 0)

PROVE ▶ The equation of the circle is $x^2 - 2yk + y^2 = 1$ with center at $(0, k)$.

45. **CHALLENGE** The intersecting lines m and n are tangent to $\odot C$ at the points (8, 6) and (10, 8), respectively.

a. What is the intersection point of m and n if the radius r of $\odot C$ is 2? What is their intersection point if r is 10? What do you notice about the two intersection points and the center C?

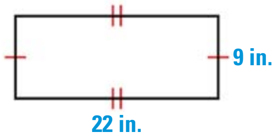
b. Write the equation that describes the locus of intersection points of m and n for all possible values of r.

MIXED REVIEW

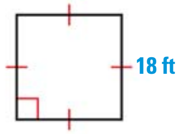
PREVIEW
Prepare for Lesson 11.1 in Exs. 46–48.

Find the perimeter of the figure.

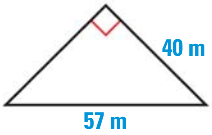
46. *(p. 49)*



47. *(p. 49)*



48. *(p. 433)*



Find the circumference of the circle with given radius r or diameter d. Use $\pi = 3.14$. *(p. 49)*

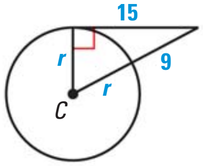
49. $r = 7$ cm

50. $d = 160$ in.

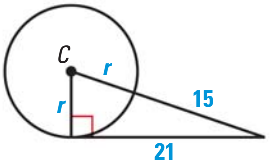
51. $d = 48$ yd

Find the radius r of $\odot C$. *(p. 651)*

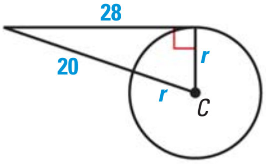
52.



53.



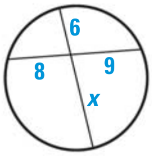
54.



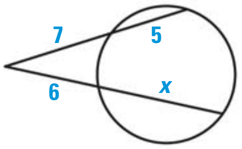
QUIZ *for Lessons 10.6–10.7*

Find the value of x. *(p. 689)*

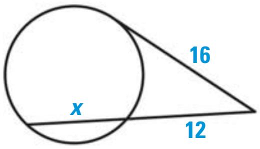
1.



2.



3.



In Exercises 4 and 5, use the given information to write the standard equation of the circle. *(p. 699)*

4. The center is (1, 4), and the radius is 6.

5. The center is (5, −7), and a point on the circle is (5, −3).

6. **TIRES** The diameter of a certain tire is 24.2 inches. The diameter of the rim in the center is 14 inches. Draw the tire in a coordinate plane with center at (−4, 3). Write the equations for the outer edge of the tire and for the rim where units are measured in inches. *(p. 699)*

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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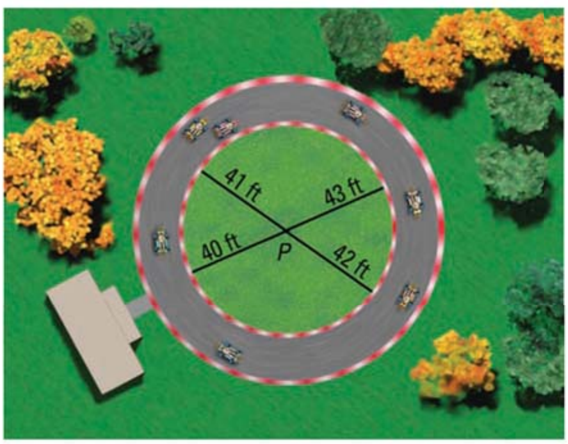
Lessons 10.6–10.7

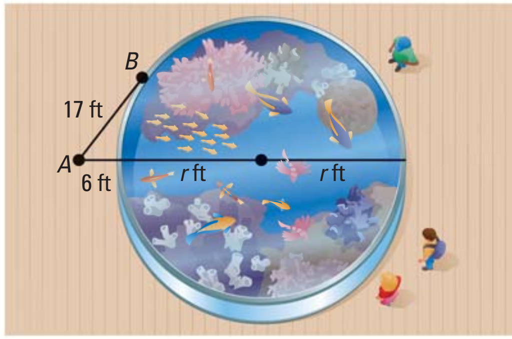
1. **SHORT RESPONSE** A local radio station can broadcast its signal 20 miles. The station is located at the point (20, 30) where units are measured in miles.
   - **a.** Write an inequality that represents the area covered by the radio station.
   - **b.** Determine whether you can receive the radio station's signal when you are located at each of the following points: $E(25, 25)$, $F(10, 10)$, $G(20, 16)$, and $H(35, 30)$.

2. **EXTENDED RESPONSE** Cell phone towers are used to transmit calls. An area has cell phone towers at points (2, 3), (4, 5), and (5, 3) where units are measured in miles. Each tower has a transmission radius of 2 miles.
   - **a.** Draw the area on a graph and locate the three cell phone towers. Are there any areas that can transmit calls using more than one tower?
   - **b.** Suppose you live at (3, 5) and your friend lives at (1, 7). Can you use your cell phone at either or both of your homes?
   - **c.** City A is located at $(-1, 1)$ and City B is located at (4, 7). Each city has a radius of 5 miles. Which city has better coverage from the cell phone towers?

3. **SHORT RESPONSE** You are standing at point P inside a go-kart track. To determine if the track is a circle, you measure the distance to four points on the track, as shown in the diagram. What can you conclude about the shape of the track? *Explain*.



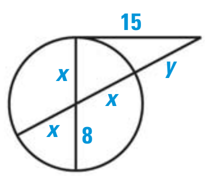
4. **SHORT RESPONSE** You are at point A, about 6 feet from a circular aquarium tank. The distance from you to a point of tangency on the tank is 17 feet.



   - **a.** What is the radius of the tank?
   - **b.** Suppose you are standing 4 feet from another aquarium tank that has a diameter of 12 feet. How far, in feet, are you from a point of tangency?

5. **EXTENDED RESPONSE** You are given seismograph readings from three locations.
   - At $A(-2, 3)$, the epicenter is 4 miles away.
   - At $B(5, -1)$, the epicenter is 5 miles away.
   - At $C(2, 5)$, the epicenter is 2 miles away.
   - **a.** Graph circles centered at A, B, and C with radii of 4, 5, and 2 miles, respectively.
   - **b.** Locate the epicenter.
   - **c.** The earthquake could be felt up to 12 miles away. If you live at (14, 16), could you feel the earthquake? *Explain*.

6. **MULTI-STEP PROBLEM** Use the diagram.



   - **a.** Use Theorem 10.16 and the quadratic formula to write an equation for y in terms of x.
   - **b.** Find the value of x.
   - **c.** Find the value of y.

10 CHAPTER SUMMARY

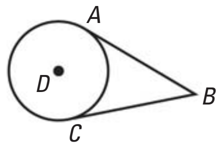
BIG IDEAS — For Your Notebook

Big Idea 1

Using Properties of Segments that Intersect Circles

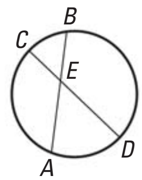
You learned several relationships between tangents, secants, and chords.

Some of these relationships can help you determine that two chords or tangents are congruent. For example, tangent segments from the same exterior point are congruent.



$\overline{AB} \cong \overline{CB}$

Other relationships allow you to find the length of a secant or chord if you know the length of related segments. For example, with the Segments of a Chord Theorem you can find the length of an unknown chord segment.



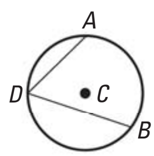
$EA \cdot EB = EC \cdot ED$

Big Idea 2

Applying Angle Relationships in Circles

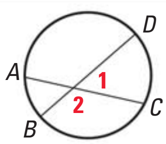
You learned to find the measures of angles formed inside, outside, and on circles.

Angles formed on circles



$m\angle ADB = \frac{1}{2}m\overset{\frown}{AB}$

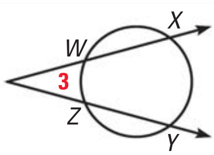
Angles formed inside circles



$m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{AB} + m\overset{\frown}{CD}\right),$

$m\angle 2 = \frac{1}{2}\left(m\overset{\frown}{AD} + m\overset{\frown}{BC}\right)$

Angles formed outside circles



$m\angle 3 = \frac{1}{2}\left(m\overset{\frown}{XY} - m\overset{\frown}{WZ}\right)$

Big Idea 3

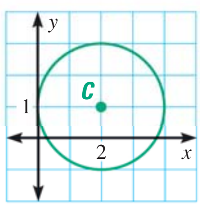
Using Circles in the Coordinate Plane

The standard equation of $\odot C$ is:

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 2)^2 + (y - 1)^2 = 2^2$

$(x - 2)^2 + (y - 1)^2 = 4$



10 CHAPTER REVIEW

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• Multi-Language Glossary
• Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

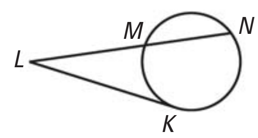
- circle, *p. 651*
  center, radius, diameter
- chord, *p. 651*
- secant, *p. 651*
- tangent, *p. 651*
- central angle, *p. 659*
- minor arc, *p. 659*
- major arc, *p. 659*
- semicircle, *p. 659*
- measure of a minor arc, *p. 659*
- measure of a major arc, *p. 659*
- congruent circles, *p. 660*
- congruent arcs, *p. 660*
- inscribed angle, *p. 672*
- intercepted arc, *p. 672*
- inscribed polygon, *p. 674*
- circumscribed circle, *p. 674*
- segments of a chord, *p. 689*
- secant segment, *p. 690*
- external segment, *p. 690*
- standard equation of a circle, *p. 699*

VOCABULARY EXERCISES

1. Copy and complete: If a chord passes through the center of a circle, then it is called a(n) __?__.
2. Draw and *describe* an inscribed angle and an intercepted arc.
3. **WRITING** *Describe* how the measure of a central angle of a circle relates to the measure of the minor arc and the measure of the major arc created by the angle.

In Exercises 4–6, match the term with the appropriate segment.

4. Tangent segment — **A.** $\overline{LM}$
5. Secant segment — **B.** $\overline{KL}$
6. External segment — **C.** $\overline{LN}$



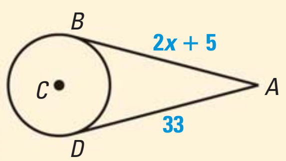
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 10.

10.1 Use Properties of Tangents — *pp. 651–658*

EXAMPLE

In the diagram, *B* and *D* are points of tangency on $\odot C$. Find the value of *x*.



Use Theorem 10.2 to find *x*.

$AB = AD$ — **Tangent segments from the same point are ≅.**

$2x + 5 = 33$ — **Substitute.**

$x = 14$ — **Solve for *x*.**

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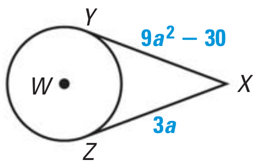
Chapter Review Practice

EXAMPLES
5 and 6
on p. 654
for Exs. 7–9

EXERCISES

Find the value of the variable. *Y* and *Z* are points of tangency on ⊙*W*.

7.



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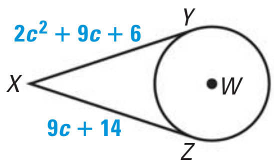
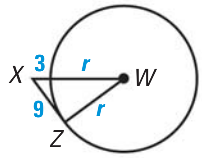


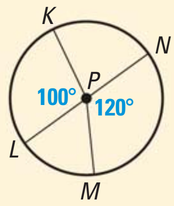
9.



10.2 Find Arc Measures *pp. 659–663*

EXAMPLE

Find the measure of the arc of ⊙*P*. In the diagram, $\overline{LN}$ is a diameter.



a. $\overset{\frown}{MN}$ **b.** $\overset{\frown}{NLM}$ **c.** $\overset{\frown}{NML}$

a. $\overset{\frown}{MN}$ is a minor arc, so $m\overset{\frown}{MN} = m\angle MPN = 120°$.

b. $\overset{\frown}{NLM}$ is a major arc, so $m\overset{\frown}{NLM} = 360° - 120° = 240°$.

c. $\overset{\frown}{NML}$ is a semicircle, so $m\overset{\frown}{NML} = 180°$.

EXAMPLES
1 and 2
on pp. 659–660
for Exs. 10–13

EXERCISES

Use the diagram above to find the measure of the indicated arc.

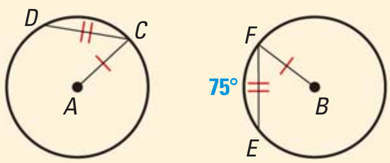
10. $\overset{\frown}{KL}$ **11.** $\overset{\frown}{LM}$ **12.** $\overset{\frown}{KM}$ **13.** $\overset{\frown}{KN}$

10.3 Apply Properties of Chords *pp. 664–670*

EXAMPLE

In the diagram, ⊙*A* ≅ ⊙*B*, $\overline{CD} \cong \overline{FE}$, and $m\overset{\frown}{FE} = 75°$. Find $m\overset{\frown}{CD}$.

By Theorem 10.3, $\overline{CD}$ and $\overline{FE}$ are congruent chords in congruent circles, so the corresponding minor arcs $\overset{\frown}{FE}$ and $\overset{\frown}{CD}$ are congruent. So, $m\overset{\frown}{CD} = m\overset{\frown}{FE} = 75°$.

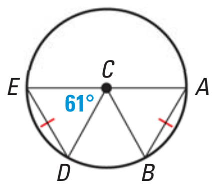


EXAMPLES
1, 3, and 4
on pp. 664, 666
for Exs. 14–16

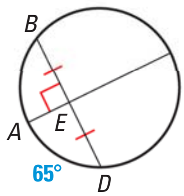
EXERCISES

Find the measure of $\overset{\frown}{AB}$.

14.



15.



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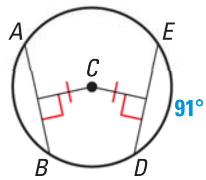


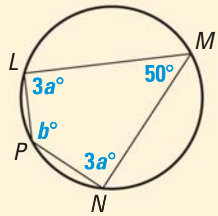
10 CHAPTER REVIEW

10.4 Use Inscribed Angles and Polygons

pp. 672–679

EXAMPLE

Find the value of each variable.



LMNP is inscribed in a circle, so by Theorem 10.10, opposite angles are supplementary.

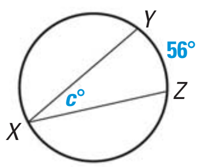
$m\angle L + m\angle N = 180°$	$m\angle P + m\angle M = 180°$
$3a° + 3a° = 180°$	$b° + 50° = 180°$
$6a = 180$	$b = 130$
$a = 30$	

EXERCISES

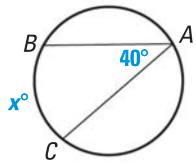
EXAMPLES 1, 2, and 5 on pp. 672–675 for Exs. 17–19

Find the value(s) of the variable(s).

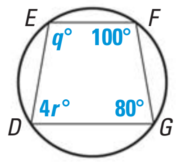
17.



18.



19.

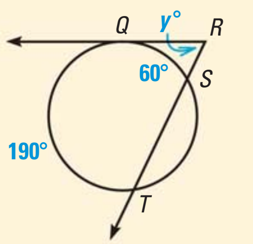


10.5 Apply Other Angle Relationships in Circles

pp. 680–686

EXAMPLE

Find the value of *y*.



The tangent $\overrightarrow{RQ}$ and secant $\overrightarrow{RT}$ intersect outside the circle, so you can use Theorem 10.13 to find the value of *y*.

$y° = \frac{1}{2}(m\overset{\frown}{QT} - m\overset{\frown}{SQ})$ **Use Theorem 10.13.**

$y° = \frac{1}{2}(190° - 60°)$ **Substitute.**

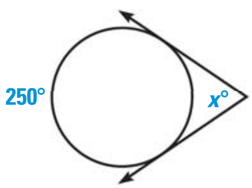
$y = 65$ **Simplify.**

EXERCISES

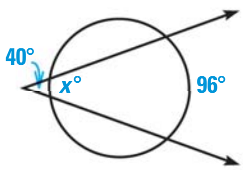
EXAMPLES 2 and 3 on pp. 681–682 for Exs. 20–22

Find the value of *x*.

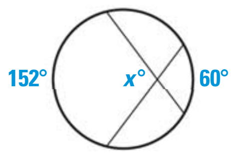
20.



21.



22.

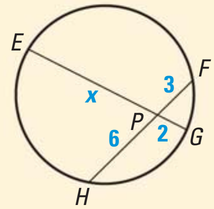


10.6 Find Segment Lengths in Circles *pp. 689–695*

EXAMPLE

Find the value of x.

The chords $\overline{EG}$ and $\overline{FH}$ intersect inside the circle, so you can use Theorem 10.14 to find the value of x.

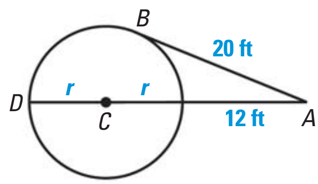


$EP \cdot PG = FP \cdot PH$	**Use Theorem 10.14.**
$x \cdot 2 = 3 \cdot 6$	**Substitute.**
$x = 9$	**Solve for x.**

EXERCISE

EXAMPLE 4 on p. 692 for Ex. 23

23. SKATING RINK A local park has a circular ice skating rink. You are standing at point A, about 12 feet from the edge of the rink. The distance from you to a point of tangency on the rink is about 20 feet. Estimate the radius of the rink.

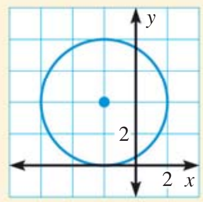


10.7 Write and Graph Equations of Circles *pp. 699–705*

EXAMPLE

Write an equation of the circle shown.

The radius is 2 and the center is at $(-2, 4)$.



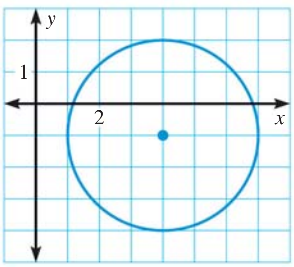
$(x - h)^2 + (y - k)^2 = r^2$	**Standard equation of a circle**
$(x - (-2))^2 + (y - 4)^2 = 4^2$	**Substitute.**
$(x + 2)^2 + (y - 4)^2 = 16$	**Simplify.**

EXERCISES

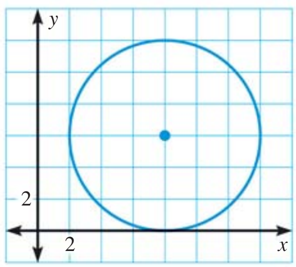
EXAMPLES 1, 2, and 3 on pp. 699–700 for Exs. 24–32

Write an equation of the circle shown.

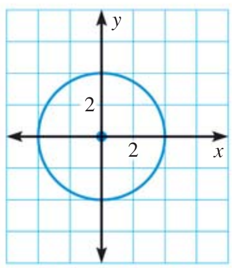
24.



25.



26.



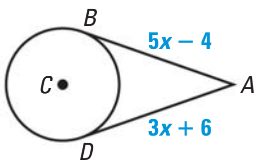
Write the standard equation of the circle with the given center and radius.

27. Center $(0, 0)$, radius 9
28. Center $(-5, 2)$, radius 1.3
29. Center $(6, 21)$, radius 4
30. Center $(-3, 2)$, radius 16
31. Center $(10, 7)$, radius 3.5
32. Center $(0, 0)$, radius 5.2

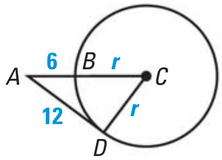
10 CHAPTER TEST

In $\odot C$, B and D are points of tangency. Find the value of the variable.

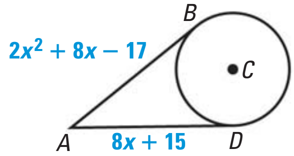
1.



2.

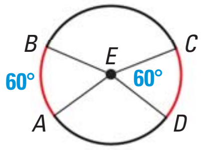


3.

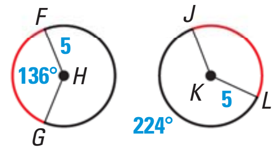


Tell whether the red arcs are congruent. *Explain* why or why not.

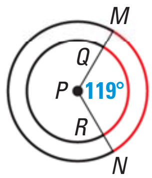
4.



5.

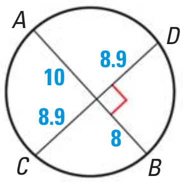


6.

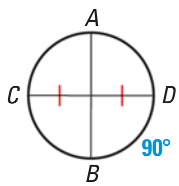


Determine whether $\overline{AB}$ is a diameter of the circle. *Explain* your reasoning.

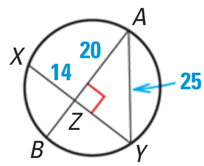
7.



8.

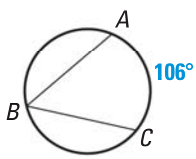


9.

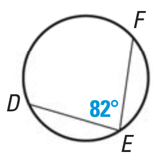


Find the indicated measure.

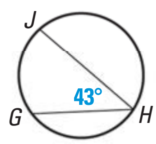
10. $m\angle ABC$



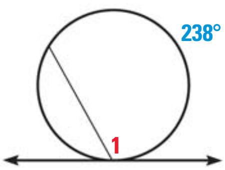
11. $m\overset{\frown}{DF}$



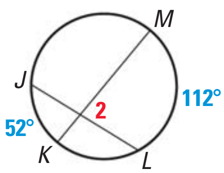
12. $m\overset{\frown}{GHJ}$



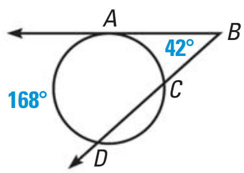
13. $m\angle 1$



14. $m\angle 2$

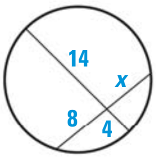


15. $m\overset{\frown}{AC}$

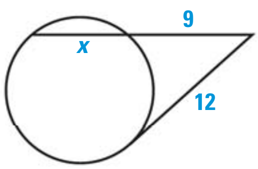


Find the value of x. Round decimal answers to the nearest tenth.

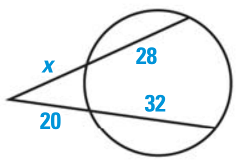
16.



17.



18.



19. Find the center and radius of a circle that has the standard equation $(x + 2)^2 + (y - 5)^2 = 169$.

ALGEBRA REVIEW

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FACTOR BINOMIALS AND TRINOMIALS

EXAMPLE 1 Factor using greatest common factor

Factor $2x^3 + 6x^2$.

Identify the *greatest common factor* of the terms. The greatest common factor (GCF) is the product of all the common factors.

First, factor each term. $2x^3 = 2 \cdot x \cdot x \cdot x$ and $6x^2 = 2 \cdot 3 \cdot x \cdot x$

Then, write the product of the common terms. $\text{GCF} = 2 \cdot x \cdot x = 2x^2$

Finally, use the distributive property with the GCF. $2x^3 + 6x^2 = 2x^2(x + 3)$

EXAMPLE 2 Factor binomials and trinomials

Factor.

a. $2x^2 - 5x + 3$

b. $x^2 - 9$

Solution

a. Make a table of possible factorizations. Because the middle term, $-5x$, is negative, both factors of the third term, 3, must be negative.

Factors of 2	Factors of 3	Possible factorization	Middle term when multiplied	
1, 2	−3, −1	$(x - 3)(2x - 1)$	$-x - 6x = -7x$	✗
1, 2	−1, −3	$(x - 1)(2x - 3)$	$-3x - 2x = -5x$	← Correct

b. Use the special factoring pattern $a^2 - b^2 = (a + b)(a - b)$.

$x^2 - 9 = x^2 - 3^2$ **Write in the form $a^2 - b^2$.**

$= (x + 3)(x - 3)$ **Factor using the pattern.**

EXERCISES

Factor.

EXAMPLE 1 for Exs. 1–9

EXAMPLE 2 for Exs. 10–24

1. $6x^2 + 18x^4$

2. $16a^2 - 24b$

3. $9r^2 - 15rs$

4. $14x^5 + 27x^3$

5. $8t^4 + 6t^2 - 10t$

6. $9z^3 + 3z + 21z^2$

7. $5y^6 - 4y^5 + 2y^3$

8. $30v^7 - 25v^5 - 10v^4$

9. $6x^3y + 15x^2y^3$

10. $x^2 + 6x + 8$

11. $y^2 - y - 6$

12. $a^2 - 64$

13. $z^2 - 8z + 16$

14. $3s^2 + 2s - 1$

15. $5b^2 - 16b + 3$

16. $4x^4 - 49$

17. $25r^2 - 81$

18. $4x^2 + 12x + 9$

19. $x^2 + 10x + 21$

20. $z^2 - 121$

21. $y^2 + y - 6$

22. $z^2 + 12z + 36$

23. $x^2 - 49$

24. $2x^2 - 12x - 14$

10 ★ Standardized TEST PREPARATION

MULTIPLE CHOICE QUESTIONS

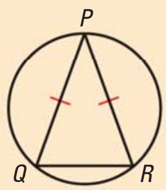
If you have difficulty solving a multiple choice question directly, you may be able to use another approach to eliminate incorrect answer choices and obtain the correct answer.

PROBLEM 1

In the diagram, $\triangle PQR$ is inscribed in a circle. The ratio of the angle measures of $\triangle PQR$ is 4 : 7 : 7. What is $m\overset{\frown}{QR}$?

Ⓐ 20°   Ⓑ 40°

Ⓒ 80°   Ⓓ 140°



METHOD 1

SOLVE DIRECTLY Use the Interior Angles Theorem to find $m\angle QPR$. Then use the fact that $\angle QPR$ intercepts $\overset{\frown}{QR}$ to find $m\overset{\frown}{QR}$.

STEP 1 **Use** the ratio of the angle measures to write an equation. Because $\triangle EFG$ is isosceles, its base angles are congruent. Let $4x° = m\angle QPR$. Then $m\angle Q = m\angle R = 7x°$. You can write:

$$m\angle QPR + m\angle Q + m\angle R = 180°$$

$$4x° + 7x° + 7x° = 180°$$

STEP 2 **Solve** the equation to find the value of x.

$$4x° + 7x° + 7x° = 180°$$

$$18x° = 180°$$

$$x = 10$$

STEP 3 **Find** $m\angle QPR$. From Step 1, $m\angle QPR = 4x°$, so $m\angle QPR = 4 \cdot 10° = 40°$.

STEP 4 **Find** $m\overset{\frown}{QR}$. Because $\angle QPR$ intercepts $\overset{\frown}{QR}$, $m\overset{\frown}{QR} = 2 \cdot m\angle QPR$. So, $m\overset{\frown}{QR} = 2 \cdot 40° = 80°$.

The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

METHOD 2

ELIMINATE CHOICES Because $\angle QPR$ intercepts $\overset{\frown}{QR}$, $m\angle QPR = \frac{1}{2} \cdot m\overset{\frown}{QR}$. Also, because $\triangle PQR$ is isosceles, its base angles, $\angle Q$ and $\angle R$, are congruent. For each choice, find $m\angle QPR$, $m\angle Q$, and $m\angle R$. Determine whether the ratio of the angle measures is 4 : 7 : 7.

Choice A: If $m\overset{\frown}{QR} = 20°$, $m\angle QPR = 10°$. So, $m\angle Q + m\angle R = 180° - 10° = 170°$, and $m\angle Q = m\angle R = \frac{170}{2} = 85°$. The angle measures 10°, 85°, and 85° are not in the ratio 4 : 7 : 7, so Choice A is not correct.

Choice B: If $m\overset{\frown}{QR} = 40°$, $m\angle QPR = 20°$. So, $m\angle Q + m\angle R = 180° - 20° = 160°$, and $m\angle Q = m\angle R = 80°$. The angle measures 20°, 80°, and 80° are not in the ratio 4 : 7 : 7, so Choice B is not correct.

Choice C: If $m\overset{\frown}{QR} = 80°$, $m\angle QPR = 40°$. So, $m\angle Q + m\angle R = 180° - 40° = 140°$, and $m\angle Q = m\angle R = 70°$. The angle measures 40°, 70°, and 70° are in the ratio 4 : 7 : 7. So, $m\overset{\frown}{QR} = 80°$.

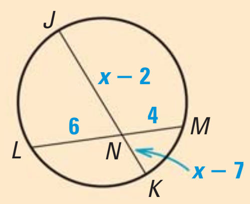
The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

PROBLEM 2

In the circle shown, $\overline{JK}$ intersects $\overline{LM}$ at point N. What is the value of x?

Ⓐ -1 Ⓑ 2

Ⓒ 7 Ⓓ 10



METHOD 1

SOLVE DIRECTLY Write and solve an equation.

STEP 1 **Write** an equation. By the Segments of a Chord Theorem, $NJ \cdot NK = NL \cdot NM$. You can write $(x - 2)(x - 7) = 6 \cdot 4 = 24$.

STEP 2 **Solve** the equation.

$$(x - 2)(x - 7) = 24$$
$$x^2 - 9x + 14 = 24$$
$$x^2 - 9x - 10 = 0$$
$$(x - 10)(x + 1) = 0$$

So, $x = 10$ or $x = -1$.

STEP 3 **Decide** which value makes sense. If $x = -1$, then $NJ = -1 - 2 = -3$. But a distance cannot be negative. If $x = 10$, then $NJ = 10 - 2 = 8$, and $NK = 10 - 7 = 3$. So, $x = 10$.

The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

METHOD 2

ELIMINATE CHOICES Check to see if any choices do not make sense.

STEP 1 **Check** to see if any choices give impossible values for NJ and NK. Use the fact that $NJ = x - 2$ and $NK = x - 7$.

Choice A: If $x = -1$, then $NJ = -3$ and $NK = -8$. A distance cannot be negative, so you can eliminate Choice A.

Choice B: If $x = 2$, then $NJ = 0$ and $NK = -5$. A distance cannot be negative or 0, so you can eliminate Choice B.

Choice C: If $x = 7$, then $NJ = 5$ and $NK = 0$. A distance cannot be 0, so you can eliminate Choice C.

STEP 2 **Verify** that Choice D is correct. By the Segments of a Chord Theorem, $(x - 7)(x - 2) = 6(4)$. This equation is true when $x = 10$.

The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

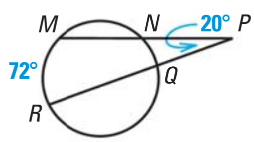
EXERCISES

Explain **why you can eliminate the highlighted answer choice.**

1. In the diagram, what is $m\overset{\frown}{NQ}$?

Ⓐ ~~20°~~ Ⓑ 26°

Ⓒ 40° Ⓓ 52°



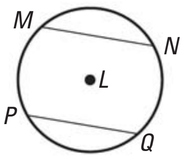
2. Isosceles trapezoid $EFGH$ is inscribed in a circle, $m\angle E = (x + 8)°$, and $m\angle G = (3x + 12)°$. What is the value of x?

Ⓐ Ⓑ 10 Ⓒ 40 Ⓓ 72

10 ★ Standardized TEST PRACTICE

MULTIPLE CHOICE

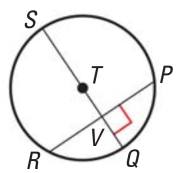
1. In $\odot L$, $\overline{MN} \cong \overline{PQ}$. Which statement is not necessarily true?



Ⓐ $\widehat{MN} \cong \widehat{PQ}$　Ⓑ $\widehat{NQP} \cong \widehat{QNM}$

Ⓒ $\widehat{MP} \cong \widehat{NQ}$　Ⓓ $\widehat{MPQ} \cong \widehat{NMP}$

2. In $\odot T$, $PV = 5x - 2$ and $PR = 4x + 14$. What is the value of x?



Ⓐ -10　Ⓑ 3

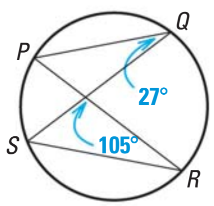
Ⓒ 12　Ⓓ 16

3. What are the coordinates of the center of a circle with equation $(x + 2)^2 + (y - 4)^2 = 9$?

Ⓐ $(-2, -4)$　Ⓑ $(-2, 4)$

Ⓒ $(2, -4)$　Ⓓ $(2, 4)$

4. In the circle shown below, what is $m\widehat{QR}$?



Ⓐ 24°　Ⓑ 27°

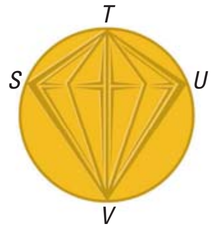
Ⓒ 48°　Ⓓ 96°

5. Regular hexagon *FGHJKL* is inscribed in a circle. What is $m\widehat{KL}$?

Ⓐ 6°　Ⓑ 60°

Ⓒ 120°　Ⓓ 240°

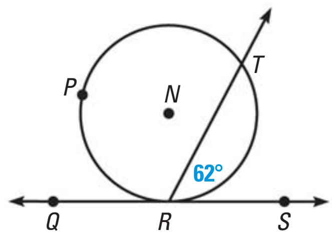
6. In the design for a jewelry store sign, *STUV* is inscribed inside a circle, $ST = TU = 12$ inches, and $SV = UV = 18$ inches. What is the approximate diameter of the circle?



Ⓐ 17 in.　Ⓑ 22 in.

Ⓒ 25 in.　Ⓓ 30 in.

7. In the diagram shown, $\overleftrightarrow{QS}$ is tangent to $\odot N$ at R. What is $m\widehat{RPT}$?



Ⓐ 62°　Ⓑ 118°

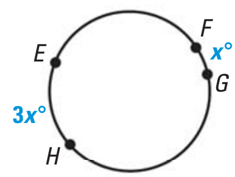
Ⓒ 124°　Ⓓ 236°

8. Two distinct circles intersect. What is the maximum number of common tangents?

Ⓐ 1　Ⓑ 2

Ⓒ 3　Ⓓ 4

9. In the circle shown, $m\widehat{EFG} = 146°$ and $m\widehat{FGH} = 172°$. What is the value of x?

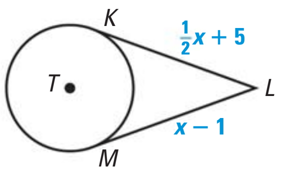


Ⓐ 10.5　Ⓑ 21

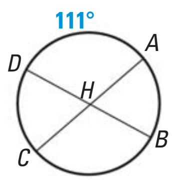
Ⓒ 42　Ⓓ 336

GRIDDED ANSWER

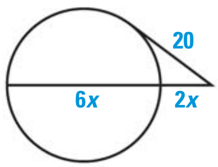
10. $\overline{LK}$ is tangent to $\odot T$ at K. $\overline{LM}$ is tangent to $\odot T$ at M. Find the value of x.



11. In $\odot H$, find $m\angle AHB$ in degrees.

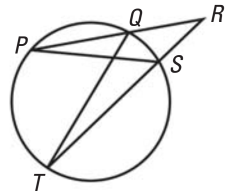


12. Find the value of x.



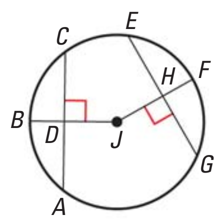
SHORT RESPONSE

13. *Explain* why $\triangle PSR$ is similar to $\triangle TQR$.



14. Let $x°$ be the measure of an inscribed angle, and let $y°$ be the measure of its intercepted arc. Graph y as a function of x for all possible values of x. Give the slope of the graph.

15. In $\odot J$, $\overline{JD} \cong \overline{JH}$. Write two true statements about congruent arcs and two true statements about congruent segments in $\odot J$. *Justify* each statement.



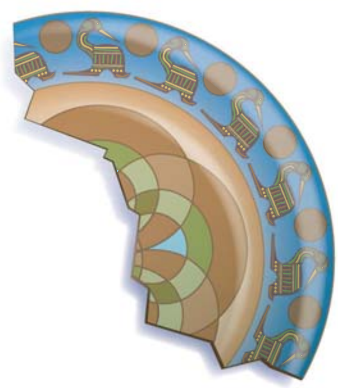
EXTENDED RESPONSE

16. The diagram shows a piece of broken pottery found by an archaeologist. The archaeologist thinks that the pottery is part of a circular plate and wants to estimate the diameter of the plate.

a. Trace the outermost arc of the diagram on a piece of paper. Draw any two chords whose endpoints lie on the arc.

b. Construct the perpendicular bisector of each chord. Mark the point of intersection of the perpendiculars bisectors. How is this point related to the circular plate?

c. Based on your results, *describe* a method the archaeologist could use to estimate the diameter of the actual plate. *Explain* your reasoning.



17. The point $P(3, -8)$ lies on a circle with center $C(-2, 4)$.

a. Write an equation for $\odot C$.

b. Write an equation for the line that contains radius $\overline{CP}$. *Explain*.

c. Write an equation for the line that is tangent to $\odot C$ at point P. *Explain*.