Contents of Student Resources

Operations with Rational Numbers

EXAMPLE **Add or subtract: a.** $-\frac{3}{4} + \frac{5}{8}$ **b.** $8.5 - (-1.4)$

a. Write the fractions with the same denominator, then add.

$$-\frac{3}{4} + \frac{5}{8} = -\frac{6}{8} + \frac{5}{8} = \frac{-6+5}{8} = \frac{-1}{8} = -\frac{1}{8}$$

b. To subtract a rational number, add its opposite.

$8.5 - (-1.4) = 8.5 + 1.4 = 9.9$ **The opposite of -1.4 is 1.4, because $(-1.4) + (1.4) = 0$.**

The product or quotient of two numbers with the *same* sign is **positive**.

The product or quotient of two numbers with *different* signs is **negative**.

EXAMPLE **Multiply: a. $4(5)$ b. $(-4)(-5)$ c. $4(-5)$**

a. $4(5) = 20$ **b.** $(-4)(-5) = 20$ **c.** $4(-5) = -20$

EXAMPLE **Divide $-\frac{1}{4} \div \frac{2}{5}$.**

To divide by a fraction, multiply by its reciprocal.

$$-\frac{1}{4} \div \frac{2}{5} = -\frac{1}{4} \times \frac{5}{2} = -\frac{1 \times 5}{4 \times 2} = -\frac{5}{8}$$

The reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$, because $\frac{2}{5} \times \frac{5}{2} = 1$.

PRACTICE

Add, subtract, multiply, or divide.

1. $4 - (-7)$
2. $-13 + 28$
3. $-5 \cdot 3$
4. $32 \div (-8)$
5. $(-2)(-3)(-4)$
6. $-8.1 + 4.5$
7. $(-2.7) \div (-9)$
8. $0.85 - 0.9$
9. $12.1 + (-0.5)$
10. $(-2.6) \cdot (-8.1)$
11. $-1.5 - 3.4$
12. $-3.6 \div 1.5$
13. $-3.1 \cdot 4.2$
14. $0.48 \div 4$
15. $-5.4 + (-3.8)$
16. $0.6 - 1.8$
17. $-\frac{5}{6} - \frac{1}{4}$
18. $-\frac{3}{4} \cdot \frac{7}{12}$
19. $\frac{4}{7} \div \frac{2}{3}$
20. $-\frac{11}{12} + \frac{7}{9}$
21. $-\frac{2}{3} + \left(-\frac{1}{4}\right)$
22. $\frac{5}{12} \div \frac{3}{8}$
23. $\frac{7}{9} - \left(-\frac{1}{6}\right)$
24. $\frac{5}{8} \cdot \frac{2}{11}$

Simplifying and Evaluating Expressions

To evaluate expressions involving more than one operation, mathematicians have agreed on the following set of rules, called the **order of operations**.

1. **Evaluate** expressions inside grouping symbols.
2. **Evaluate** powers.
3. **Multiply** and **divide** from left to right.
4. **Add** and **subtract** from left to right.

EXAMPLE **Simplify: a.** $10 + (1 - 5)^2 \div (-8)$ **b.** $3|-9 + 2| - 2 \cdot 6$

a. $10 + (1 - 5)^2 \div (-8)$

$= 10 + (-4)^2 \div (-8)$ **Subtract.**

$= 10 + 16 \div (-8)$ **Evaluate powers.**

$= 10 + (-2)$ **Divide.**

$= 8$ **Add.**

b. $3|-9 + 2| - 2 \cdot 6$

$= 3|-7| - 2 \cdot 6$ **Add.**

$= 3(7) - 2 \cdot 6$ **Absolute value**

$= 21 - 12$ **Multiply.**

$= 9$ **Subtract.**

To evaluate an algebraic expression, substitute values for the variables. Evaluate the resulting numerical expression using the order of operations.

EXAMPLE **Evaluate the expression when $x = 4$ and $y = 9$.**

a. $\dfrac{x^2 - 1}{x + 2} = \dfrac{4^2 - 1}{4 + 2} = \dfrac{16 - 1}{4 + 2} = \dfrac{15}{6} = \dfrac{5}{2} = 2\frac{1}{2}$

b. $[(2x + y) - 3x] \div 2 = (-x + y) \div 2 = (-4 + 9) \div 2 = 5 \div 2 = 2.5$

c. $2|x - 3y| = 2|4 - 3(9)| = 2|4 - 27| = 2|-23| = 2(23) = 46$

PRACTICE

Simplify the expression.

1. $5^2 - (-2)^3$ **2.** $-8 \cdot 3 - 12 \div 2$ **3.** $21|-7 + 4| - 4^3$ **4.** $24 \div (8 - |5 - 1|)$

5. $4(2 - 5)^2$ **6.** $4 + 21 \div 7 - 6^2$ **7.** $19.6 \div (2.8 \div 0.4)$ **8.** $20 - 4[2 + (10 - 3^2)]$

9. $\dfrac{6 + 3 \cdot 4}{2^2 - 7}$ **10.** $\dfrac{18 + |-2|}{(4 - 6)^2}$ **11.** $3(6x) + 7x$ **12.** $3|-5y + 4y|$

Evaluate the expression when $x = -3$ and $y = 5$.

13. $-4x^2$ **14.** $(-4x)^2$ **15.** $x(x + 8)$ **16.** $(11 - x) \div 2$

17. $3 \cdot |x - 2|$ **18.** $7x^2 - 2y$ **19.** $5 - |3x + y|$ **20.** $4x^3 + 3y$

21. $\dfrac{y^2 - 1}{5 - y^2}$ **22.** $|6y| - |x|$ **23.** $\dfrac{-6(2x + y)}{5 - x}$ **24.** $\dfrac{x - 7}{x + 7} + 1$

Properties of Exponents

An **exponent** tells you how many times to multiply a **base**. The expression 4^5 is called a **power** with base 4 and exponent 5.

$4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024$

Product of Powers	Power of a Product	Power of a Power	
$a^m \cdot a^n = a^{m+n}$ Add exponents.	$(a \cdot b)^m = a^m \cdot b^m$ Find the power of each factor.	$(a^m)^n = a^{mn}$ Multiply exponents.	
Quotient of Powers	**Power of a Quotient**	**Negative Exponent**	**Zero Exponent**
$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$ Subtract exponents.	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$ Find the power of the numerator and the power of the denominator.	$a^{-n} = \frac{1}{a^n}, a \neq 0$	$a^0 = 1, a \neq 0$

EXAMPLE Simplify the expression. Use positive exponents.

a. $x^2 \cdot x^5 = x^{2+5} = x^7$

b. $(2xy)^3 = 2^3 \cdot x^3 \cdot y^3 = 8x^3y^3$

c. $(y^4)^5 = y^{4 \cdot 5} = y^{20}$

d. $(-35)^0 = 1$

e. $\frac{m^9}{m^6} = m^{9-6} = m^3$

f. $\left(\frac{z}{4}\right)^3 = \frac{z^3}{4^3} = \frac{z^3}{64}$

g. $12^{-4} = \frac{1}{12^4} = \frac{1}{20{,}736}$

h. $\frac{20x^2y^{-4}z^5}{4x^4yz^3} = \frac{20}{4}x^{(2-4)}y^{(-4-1)}z^{(5-3)} = 5x^{-2}y^{-5}z^2 = \frac{5z^2}{x^2y^5}$

PRACTICE

Evaluate the power.

1. 5^2 **2.** $\left(-\frac{1}{2}\right)^3$ **3.** 4^{-2} **4.** 13^0

5. $5^3 \cdot 5^4$ **6.** $\left(\frac{3}{5}\right)^{-2}$ **7.** $(7^8)^4$ **8.** $\frac{4^6}{4^4}$

Simplify the expression. Write your answer using only positive exponents.

9. $a^5 \cdot a \cdot a^{-2}$ **10.** $3x^8 \cdot (2x)^3$ **11.** $5a^5 \cdot b^{-4}$ **12.** $(m^{-2})^{-3}$

13. $\left(\frac{3}{n}\right)^4$ **14.** $\left(\frac{x^5}{x^2}\right)^3$ **15.** $\frac{1}{m^{-2}}$ **16.** $\left(\frac{a^3}{3b}\right)^{-2}$

17. $(4 \cdot x^3 \cdot y)^2$ **18.** $(2n)^4 \cdot (3n)^2$ **19.** $(5a^3b^{-2}c)^{-1}$ **20.** $(r^2st^3)^0$

21. $\frac{16x^2y}{2xy}$ **22.** $\frac{(3r^{-3}s)^2}{10s}$ **23.** $\frac{3a^2b^0c}{21a^{-3}b^4c^2}$ **24.** $\left(\frac{6kn}{9k^2}\right)^2$

25. $6x^2 \cdot 5xy$ **26.** $2(r^{-4}s^2t)^{-3}$ **27.** $(5a^{-3}bc^4)^{-2} \cdot 15a^8$ **28.** $(3x^2y)^2 \cdot (-4xy^3)$

Using the Distributive Property

You can use the **Distributive Property** to simplify some expressions. Here are four forms of the Distributive Property.

$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ **Addition**

$a(b - c) = ab - ac$ and $(b - c)a = ba - ca$ **Subtraction**

EXAMPLE **Write the expression without parentheses.**

a. $x(x - 7) = x(x) - x(7)$
$= x^2 - 7x$

b. $(n + 5)(-3) = n(-3) + (5)(-3)$
$= -3n - 15$

Like terms are terms of an expression that have identical variable parts. You can use the Distributive Property to combine like terms and to simplify expressions that include adding, subtracting, factoring, and dividing polynomials.

EXAMPLE **Simplify the expression.**

a. $-2x^2 + 6x^2 = (-2 + 6)x^2 = 4x^2$

b. $9y - 4y + 8y = (9 - 4 + 8)y = 13y$

c. $5(x^2 - 3x) + (x + 2) = 5x^2 - 15x + x + 2 = 5x^2 + (-15 + 1)x + 2 = 5x^2 - 14x + 2$

d. $(3x^2 - 4x + 1) - (2x^2 - x - 7) = (3 - 2)x^2 + (-4 + 1)x + (1 + 7) = x^2 - 3x + 8$

e. $\frac{2x^2 - 4x}{2x} = \frac{2x(x - 2)}{2x} = \frac{\cancel{2x}(x - 2)}{\cancel{2x}} = x - 2$

PRACTICE

Use the Distributive Property to write an equivalent expression.

1. $3(x + 7)$ **2.** $-2(9a - 5)$ **3.** $(5n - 2)8$ **4.** $x(3x - 4)$

5. $-(x + 6)$ **6.** $(5b + c)(2a)$ **7.** $4(3x^2 - 2x + 4)$ **8.** $-5a(-a + 3b - 1)$

Simplify the expression.

9. $3x^2 - 9x^2 + x^2$ **10.** $4x - 7x + 12x$ **11.** $3n + 5 - n$ **12.** $-6r + 3s - 5r + 8$

13. $12h^2 + 5h^3 - 7h^2$ **14.** $6.5a + 2.4 - 5a$ **15.** $(x + 8) - (x - 2)$ **16.** $4.5(2r - 6) - 3r$

17. $\frac{1}{2}a + \frac{2}{5}a$ **18.** $\frac{1}{4}(x^2 - 4) + x$ **19.** $\frac{15n + 20}{5}$ **20.** $\frac{16r^3 - 12r^2}{2r}$

21. $(a^2 - 81) + (a^2 + 6a + 5)$ **22.** $(5a^2 + 3a - 2) - (2a^2 - a + 6)$

23. $2x + 3x(x - 4) + 5$ **24.** $3r(5r + 2) - 4(2r^2 - r + 3)$

25. $\frac{8a^3b + 4a^2b^2 - 2ab}{2ab}$ **26.** $\frac{7h^2 - 14h - 35 + 21h}{7}$

Binomial Products

To multiply two binomials, you can use the Distributive Property systematically. Multiply the *first* terms, the *outer* terms, the *inner* terms, and the *last* terms of the binomials. This method is called **FOIL** for the words **F**irst, **O**uter, **I**nner, and **L**ast.

For certain binomial products, you can also use a special product pattern.

$(a + b)^2 = a^2 + 2ab + b^2$ $\quad$ $(a - b)^2 = a^2 - 2ab + b^2$ $\quad$ $(a - b)(a + b) = a^2 - b^2$

EXAMPLE **Find the product.**

$$(x + 2)(3x - 4) = \underbrace{x(3x)}_{\text{First}} + \underbrace{x(-4)}_{\text{Outer}} + \underbrace{2(3x)}_{\text{Inner}} + \underbrace{2(-4)}_{\text{Last}}$$
$$= 3x^2 - 4x + 6x - 8$$
$$= 3x^2 + 2x - 8$$

a. $(x + 5)^2$
$= x^2 + 2(x)(5) + 5^2$
$= x^2 + 10x + 25$

b. $(y - 3)^2$
$= y^2 - 2(y)(3) + 3^2$
$= y^2 - 6y + 9$

c. $(z + 4)(z - 4)$
$= z^2 - 4^2$
$= z^2 - 16$

To simplify some expressions, multiply binomials first.

EXAMPLE **Simplify the expression.**

$2(x + 1)(x + 6) - 4(x^2 - 5x + 4) = 2(x^2 + 7x + 6) - 4(x^2 - 5x + 4)$ **Multiply binomials.**
$= 2x^2 + 14x + 12 - 4x^2 + 20x - 16$ **Distributive Property**
$= -2x^2 + 34x - 4$ **Combine like terms.**

PRACTICE

Find the product.

1. $(a - 2)(a - 9)$ **2.** $(y - 4)^2$ **3.** $(t - 5)(t + 8)$ **4.** $(5n + 1)(n - 4)$
5. $(5a + 2)^2$ **6.** $(x - 10)(x + 10)$ **7.** $(c + 4)(4c - 3)$ **8.** $(n + 7)^2$
9. $(8 - z)^2$ **10.** $(a + 1)(a - 1)$ **11.** $(2x + 1)(x + 1)$ **12.** $(-7z + 6)(3z - 4)$
13. $(2x - 3)(2x + 3)$ **14.** $(5 + n)^2$ **15.** $(2d - 1)(3d + 2)$ **16.** $(a + 3)(a + 3)$
17. $(k - 1.2)^2$ **18.** $(6x - 5)(2x - 3)$ **19.** $(6 - z)(6 + z)$ **20.** $(4 - 5g)(3g + 2)$

Simplify the expression.

21. $3(y - 4)(y + 2) + (2y - 1)(y + 8)$ **22.** $4(t^2 + 3t - 4) + 2(t - 1)(t + 5)$
23. $2(x + 2)(x - 2) + (x - 3)(x + 3)$ **24.** $2(2c^2 + 3c - 1) + 7(c + 2)^2$

Radical Expressions

A **square root** of a number n is a number m such that $m^2 = n$. For example, $9^2 = 81$ and $(-9)^2 = 81$, so the square roots of 81 are 9 and -9.

Every positive number has two square roots, one positive and one negative. Negative numbers have no real square roots. The square root of zero is zero.

The radical symbol, $\sqrt{\ }$, represents a nonnegative square root: $\sqrt{81} = 9$. The opposite of a square root is negative: $-\sqrt{81} = -9$.

A **perfect square** is a number that is the square of an integer. So, 81 is a perfect square. A **radicand** is a number or expression inside a radical symbol.

Properties of Radicals	Simplest Form of a Radical Expression
For $a \geq 0$ and $b \geq 0$: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$	• No perfect square factors other than 1 in the radicand • No fractions in the radicand • No radical signs in the denominator of a fraction

EXAMPLE **Simplify the expression.**

a. $\sqrt{9 + 36} = \sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$

b. $\sqrt{50} - \sqrt{32} = \sqrt{25 \cdot 2} - \sqrt{16 \cdot 2} = 5\sqrt{2} - 4\sqrt{2} = (5 - 4)\sqrt{2} = 1\sqrt{2} = \sqrt{2}$

c. $\sqrt{18} \cdot \sqrt{72} = \sqrt{18 \cdot 72} = \sqrt{1296} = 36$

d. $(8\sqrt{3})^2 = 8^2 \cdot (\sqrt{3})^2 = 64 \cdot 3 = 192$

e. $\frac{6}{\sqrt{2}} = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6 \cdot \sqrt{2}}{(\sqrt{2})^2} = \frac{6 \cdot \sqrt{2}}{2} = 3\sqrt{2}$

f. $\frac{\sqrt{20}}{\sqrt{500}} = \sqrt{\frac{20}{500}} = \sqrt{\frac{1}{25}} = \frac{1}{5}$

PRACTICE

Find all square roots of the number or write *no square roots*.

1. 100 **2.** 64 **3.** $\frac{1}{4}$ **4.** $\frac{9}{25}$

5. -16 **6.** 0 **7.** 0.81 **8.** 0.0016

Simplify the expression.

9. $\sqrt{121}$ **10.** $-\sqrt{169}$ **11.** $-\sqrt{99}$ **12.** $\sqrt{48}$

13. $\sqrt{16 + 4}$ **14.** $\sqrt{(-4)^2 + 6^2}$ **15.** $\sqrt{175} - \sqrt{28}$ **16.** $\sqrt{32} + \sqrt{162}$

17. $\sqrt{8} \cdot \sqrt{10}$ **18.** $4\sqrt{6} \cdot 2\sqrt{15}$ **19.** $\sqrt{210 \cdot 420}$ **20.** $(9\sqrt{3})^2$

21. $\sqrt{137} \cdot \sqrt{137}$ **22.** $\sqrt{12} \cdot \sqrt{48}$ **23.** $5\sqrt{18} \cdot \sqrt{2}$ **24.** $3\sqrt{7} \cdot 5\sqrt{11}$

25. $\frac{\sqrt{192}}{\sqrt{3}}$ **26.** $\sqrt{\frac{2}{49}}$ **27.** $\frac{12}{\sqrt{6}}$ **28.** $\frac{2}{\sqrt{5}}$

Solving Linear Equations

To solve a linear equation, you isolate the variable.

Add the same number to each side of the equation.

Subtract the same number from each side of the equation.

Multiply each side of the equation by the same nonzero number.

Divide each side of the equation by the same nonzero number.

EXAMPLE **Solve the equation: a. $3x - 5 = 13$ b. $2(y - 3) = y + 4$**

a. $3x - 5 = 13$

$3x - 5 + 5 = 13 + 5$	Add 5.
$3x = 18$	Simplify.
$\frac{3x}{3} = \frac{18}{3}$	Divide by 3.
$x = 6$	Simplify.

CHECK $3x - 5 = 13$

$3(6) - 5 \stackrel{?}{=} 13$

$13 = 13$ ✓

b. $2(y - 3) = y + 4$

$2y - 6 = y + 4$	Distributive Property
$2y - y - 6 = y - y + 4$	Subtract y.
$y - 6 = 4$	Simplify.
$y - 6 + 6 = 4 + 6$	Add 6.
$y = 10$	Simplify.

CHECK $2(y - 3) = y + 4$

$2(10 - 3) \stackrel{?}{=} 10 + 4$

$14 = 14$ ✓

PRACTICE

Solve the equation.

1. $x - 8 = 23$

2. $n + 12 = 0$

3. $-18 = 3y$

4. $\frac{a}{6} = 7$

5. $\frac{2}{3}r = 26$

6. $-\frac{4}{5}t = -8$

7. $-4.8 = 1.5z$

8. $0 = -3x + 12$

9. $72 = 90 - x$

10. $7(y - 2) = 21$

11. $5 = 4k + 2 - k$

12. $4n + 1 = -2n + 8$

13. $2c + 3 = 4(c - 1)$

14. $9 - (3r - 1) = 12$

15. $12m + 3(2m + 6) = 0$

16. $\frac{6}{5}y - 2 = 10$

17. $\frac{w - 8}{3} = 4$

18. $-\frac{1}{4}(12 + h) = 7$

19. $2c - 8 = 24$

20. $2.8(5 - t) = 7$

21. $2 - c = -3(2c + 1)$

22. $-4k + 8 = 12 - 5k$

23. $3(z - 2) + 8 = 23$

24. $12 = 5(-3r + 2) - (r - 1)$

25. $12(z + 12) = 15^2$

26. $2 \cdot 3.14 \cdot r = 94.2$

27. $3.1(2f + 1.2) = 0.2(f - 6)$

28. $5(3t - 2) = -3(7 - t)$

29. $20a - 12(a - 3) = 4$

30. $5.5(h - 5.5) = 18.18$

31. $\frac{1}{2} \cdot b \cdot 8 = 10$

32. $\frac{4x + 12}{2} = 3x - 5$

33. $\frac{10 + 7y}{4} = \frac{5 - y}{3}$

34. $\frac{9 - 2x}{7} = x$

35. $\frac{23 - 11c}{7} = 5c$

36. $\frac{4n - 28}{3} = 2n$

Solving and Graphing Linear Inequalities

You can graph solutions to equations and inequalities on a number line.

Symbol	Meaning	Equation or Inequality	Graph
=	equals	$x = 3$	
<	is less than	$x < 3$	
≤	is less than or equal to	$x \leq 3$	
>	is greater than	$x > 3$	
≥	is greater than or equal to	$x \geq 3$	

You can use properties of inequalities to solve linear inequalities.

Add the same number to each side of the inequality.

Subtract the same number from each side of the inequality.

Multiply each side of the inequality by the same positive number. If you multiply by a negative number, reverse the direction of the inequality symbol.

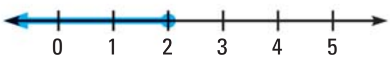
Divide each side of the inequality by the same positive number. If you divide by a negative number, reverse the direction of the inequality symbol.

EXAMPLE **Solve the inequality. Graph the solution.**

a. $2x + 1 \leq 5$

$2x \leq 4$ **Subtract 1 from each side.**

$x \leq 2$ **Divide each side by 2.**



b. $-4y < 18$

$\frac{-4y}{-4} > \frac{18}{-4}$ **Divide by −4 and change < to >.**

$y > -4.5$ **Simplify.**

PRACTICE

Solve the inequality. Graph the solution.

1. $x - 2 < 5$

2. $16 < x + 5$

3. $10 - n \geq 6$

4. $2z \geq -9$

5. $8c + 24 < 0$

6. $6 \geq -3a$

7. $5a - 3 \geq -8$

8. $2n + 7 < 17$

9. $5 > 0.5y + 3$

10. $5 - 3x \leq x + 13$

11. $5r + 2r \leq 6r - 1$

12. $y - 3 \leq 2y + 5$

13. $-2.4m \geq 3.6m - 12$

14. $-2(t - 6) > 7t - 6$

15. $4(8 - z) + 2 > 3z - 8$

16. $-\frac{3}{4}n > 3$

17. $\frac{c}{5} - 8 \leq -6$

18. $\frac{n-5}{2} \geq \frac{2n-6}{3}$

Solving Formulas

A **formula** is an equation that relates two or more real-world quantities. You can rewrite a formula so that any one of the variables is a function of the other variable(s). In each case you isolate a variable on one side of the equation.

EXAMPLE Solve the formula for the indicated variable.

a. Solve $C = 2\pi r$ for r.

$C = 2\pi r$

$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$ **Divide by 2π.**

$\frac{C}{2\pi} = r$ **Simplify.**

$r = \frac{C}{2\pi}$ **Rewrite.**

b. Solve $P = a + b + c$ for a.

$P = a + b + c$

$P - b - c = a + b - b + c - c$ **Subtract.**

$P - b - c = a$ **Simplify.**

$a = P - b - c$ **Rewrite.**

EXAMPLE Rewrite the equation so that *y* is a function of *x*.

a. $2x + y = 3$

$2x - 2x + y = 3 - 2x$ **Subtract 2x.**

$y = 3 - 2x$ **Simplify.**

b. $\frac{1}{4}y = x$

$4 \cdot \frac{1}{4}y = 4 \cdot x$ **Multiply by 4.**

$y = 4x$ **Simplify.**

PRACTICE

Solve the formula for the indicated variable.

1. Solve $P = 4s$ for s.
2. Solve $d = rt$ for r.
3. Solve $V = \ell wh$ for ℓ.
4. Solve $V = \pi r^2 h$ for h.
5. Solve $A = \frac{1}{2}bh$ for b.
6. Solve $d = \frac{m}{v}$ for v.
7. Solve $P = 2(\ell + w)$ for w.
8. Solve $I = prt$ for r.
9. Solve $F = \frac{9}{5}C + 32$ for C.
10. Solve $A = \frac{1}{2}h(b_1 + b_2)$ for h.
11. Solve $S = 2\pi r^2 + 2\pi rh$ for h.
12. Solve $A = P(1 + r)^t$ for P.

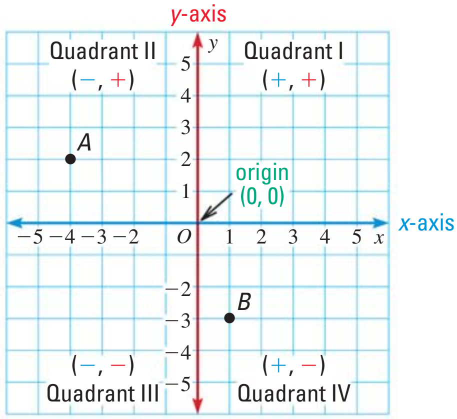
Rewrite the equation so that *y* is a function of *x*.

13. $2x + y = 7$
14. $5x + 3y = 0$
15. $3x - y = -2$
16. $y + 1 = -2(x - 2)$
17. $\frac{4}{5}y = x$
18. $\frac{1}{4}x + 2y = 5$
19. $1.8x - 0.3y = 4.5$
20. $y - 4 = \frac{1}{3}(x + 6)$

Graphing Points and Lines

A **coordinate plane** is formed by the intersection of a horizontal number line called the ***x*-axis** and a vertical number line called the ***y*-axis**. The axes meet at a point called the **origin** and divide the coordinate plane into four **quadrants**, labeled I, II, III, and IV.

Each point in a coordinate plane is represented by an **ordered pair**. The first number is the ***x*-coordinate**, and the second number is the ***y*-coordinate**.



EXAMPLE **Give the coordinates of points *A* and *B* in the graph above.**

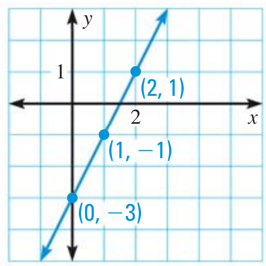
Start at the origin. Count 4 units left and 2 units up. Point *A* is at (−4, 2).
Start at the origin. Count 1 unit right and 3 units down. Point *B* is at (1, −3).

A **solution** of an equation in *x* and *y* is an ordered pair (*x*, *y*) that makes the equation true. The graph of such an equation is the set of points in a coordinate plane that represent all the solutions. A **linear equation** has a line as its graph.

EXAMPLE **Graph the equation $y = 2x - 3$.**

Make a table of values, graph each point, and draw the line.

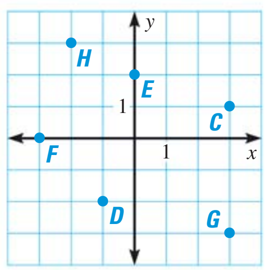
x	$y = 2x - 3$	(x, y)	
0	$y = 2(0) - 3 = -3$	$(0, -3)$	→ 0 units right or left, 3 units down
1	$y = 2(1) - 3 = -1$	$(1, -1)$	→ 1 unit right, 1 unit down
2	$y = 2(2) - 3 = 1$	$(2, 1)$	→ 2 units right, 1 unit up



PRACTICE

Use the graph shown. Give the coordinates of the point.

1. *C*
2. *D*
3. *E*
4. *F*
5. *G*
6. *H*



Plot the point in a coordinate plane.

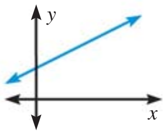
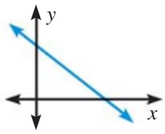
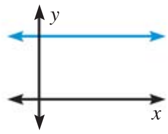
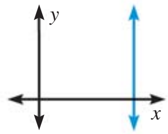
7. $J(-3, 1)$
8. $K(2, -2)$
9. $L(0, -1)$
10. $M\left(\frac{3}{2}, 3\right)$
11. $N\left(-\frac{5}{2}, -\frac{1}{2}\right)$
12. $P(4.5, 0)$

Use a table of values to graph the equation.

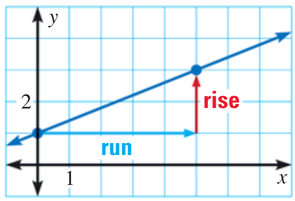
13. $y = 3x - 2$
14. $y = -2x + 1$
15. $y = \frac{2}{3}x - 3$
16. $y = -\frac{1}{2}x$
17. $y = 1.5x - 2.5$
18. $y = 4 - 3x$
19. $4x + 2y = 0$
20. $2x - y = 3$

Slope and Intercepts of a Line

The **slope** of a nonvertical line is the ratio of the vertical change, called the **rise**, to the horizontal change, called the **run**. The table below shows some types of lines and slopes.

Rising Line	Falling Line	Horizontal Line	Vertical Line
			
Positive Slope	Negative Slope	Zero Slope	Undefined Slope

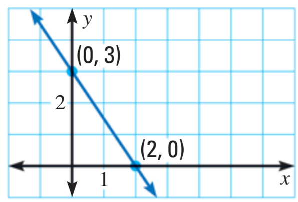
EXAMPLE Find the slope of the line.



Use the graph of the line.

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{2 \text{ units up}}{5 \text{ units right}} = \frac{2}{5}$$

An ***x*-intercept** is the x-coordinate of a point where a graph crosses the x-axis. A ***y*-intercept** is the y-coordinate of a point where a graph crosses the y-axis. The line graphed at the right has x-intercept 2 and y-intercept 3.



EXAMPLE Find the x-intercept and the y-intercept of the graph of $x - 4y = 8$.

To find the x-intercept, let $y = 0$.

$$x - 4(0) = 8$$
$$x = 8$$

The x-intercept is 8.

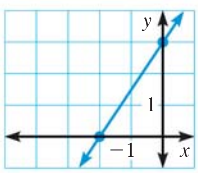
To find the y-intercept, let $x = 0$.

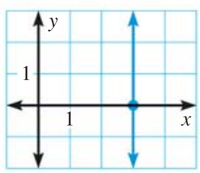
$$0 - 4y = 8$$
$$y = -2$$

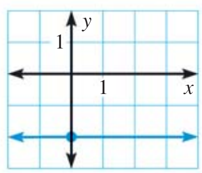
The y-intercept is -2.

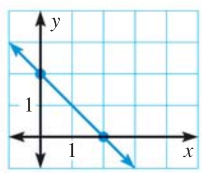
PRACTICE

Find the slope and intercept(s) of the line graphed.

1. 

2. 

3. 

4. 

Find the intercepts of the line with the given equation.

5. $5x - y = 15$
6. $2x + 4y = 12$
7. $y = -x + 3$
8. $y = 3x - 2$
9. $-3x + y = -6$
10. $y = -2x - 7$
11. $y = 5x$
12. $9x - 3y = 15$

Systems of Linear Equations

A **system of linear equations** in two variables is shown at the right. A **solution** of such a system is an ordered pair (x, y) that satisfies both equations. A solution must lie on the graph of both equations.

$x + 2y = 5$	Equation 1
$x - y = -1$	Equation 2

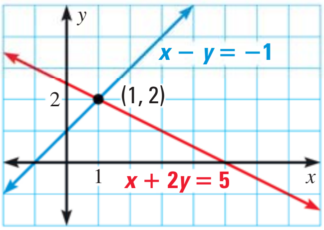
EXAMPLE Use substitution to solve the linear system above.

Solve Equation 2 for x. $\quad x - y = -1$

$x = y - 1 \quad$ Revised Equation 2

In Equation 1, substitute $y - 1$ for x. Solve for y.

$$x + 2y = 5$$
$$(y - 1) + 2y = 5$$
$$3y = 6$$
$$y = 2$$



In Revised Equation 2, substitute 2 for y. $\quad x = y - 1 = 2 - 1 = 1$

Because $x = 1$ and $y = 2$, the solution (x, y) is $(1, 2)$.

The graph verifies that $(1, 2)$ is the point of intersection of the lines.

EXAMPLE Use elimination to solve the linear system above.

Multiply Equation 2 by 2, then add equations.

$$\begin{array}{rcl} x + 2y = 5 & \longrightarrow & x + 2y = 5 \\ x - y = -1 & \longrightarrow & 2x - 2y = -2 \\ \hline & & 3x = 3 \\ & & x = 1 \end{array}$$

Substitute 1 for x in Equation 2 and solve for y. $\quad 1 - y = -1$

$$2 = y$$

Because $x = 1$ and $y = 2$, the solution (x, y) is $(1, 2)$.

Substitute 1 for x and 2 for y in each original equation to check.

PRACTICE

Use substitution to solve the linear system. Check your solution.

1. $3x - 5y = 1$, $y = 2x - 3$

2. $7x + 4y = -13$, $x = -6y + 9$

3. $-4x + 3y = -19$, $2x + y = 7$

4. $x + y = -7$, $2x - 5y = 21$

5. $4x + 9y = -3$, $x + 2y = 0$

6. $0.5x + y = 5$, $1.5x - 2.5y = 4$

7. $2x + 4y = -18$, $3x - y = 1$

8. $4x + 7y = 3$, $6x + y = 14$

Use elimination to solve the linear system. Check your solution.

9. $3x - 6y = -3$, $12x + 6y = 48$

10. $12x + 20y = 56$, $-12x - 7y = -4$

11. $4x - y = 1$, $2x + 3y = -17$

12. $10x + 15y = 90$, $5x - 4y = -1$

13. $18x + 63y = -27$, $3x + 9y = -6$

14. $5x + 7y = 23$, $20x - 30y = 5$

15. $8x - 5y = 14$, $10x - 2y = 9$

16. $-5x + 8y = 4$, $6x - 5y = -14$

Linear Inequalities

A **linear inequality** in x and y can be written in one of the forms shown at the right. A **solution** of a linear inequality is an ordered pair (x, y) that satisfies the inequality. A **graph** of a linear inequality is the graph of all the solutions.

$ax + by < c$	$ax + by > c$
$ax + by \leq c$	$ax + by \geq c$

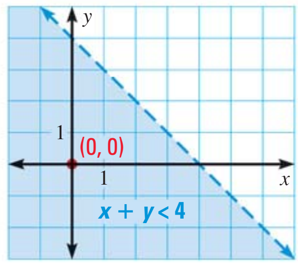
EXAMPLE **Graph the linear inequality $x + y < 4$.**

Graph the corresponding equation $x + y = 4$. Use a dashed line to show that the points on the line are not solutions of the inequality.

Test a point on either side of the line to see if it is a solution.

Test $(3, 2)$ in $x + y < 4$:	Test $(0, 0)$ in $x + y < 4$:
$3 + 2 < 4$ ✗	$0 + 0 < 4$ ✓
So $(3, 2)$ is not a solution.	So $(0, 0)$ is a solution.

Shade the *half-plane* that includes a test point that is a solution.



Two or more linear inequalities form a **system of linear inequalities**. A **solution** of such a system is an ordered pair (x, y) that satisfies all the inequalities in the system. A **graph** of the system shows all the solutions of the system.

EXAMPLE **Graph the system of linear inequalities $x \geq -2$ and $y \leq 3$.**

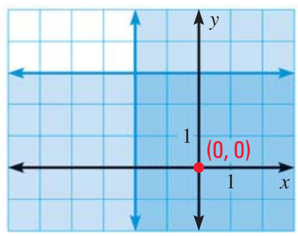
Graph the linear inequality $x \geq -2$. Use a solid line for the graph of $x = 2$ to show that the points on the line are solutions of the inequality. Shade the half-plane to the right of the line.

Graph the linear inequality $y \leq 3$. Use a solid line for the graph of $y = 3$. Shade the half-plane below the line.

The intersection of the shaded half-planes is a graph of the system.

Check solution point $(0, 0)$ in both inequalities $x \geq -2$ and $y \leq 3$.

$0 \geq -2$ ✓ and $0 \leq 3$ ✓



PRACTICE

Graph the linear inequality.

1. $x + y \geq 3$	**2.** $x - y < -2$	**3.** $y \leq -3x$	**4.** $x - 4y > 4$
5. $y > 1$	**6.** $x \leq 2$	**7.** $5x - y \geq 5$	**8.** $2x + 5y < 10$

Graph the system of linear inequalities.

9. $x > 1$ $y > -2$	**10.** $x \leq 4$ $x \geq -2$	**11.** $x - y \leq 1$ $x + y < 5$	**12.** $y < x$ $y \geq 3x$
13. $2x - y \leq 1$ $2x - y \geq -3$	**14.** $x \geq 0$ $y \geq 0$ $4x + 3y < 12$	**15.** $y > -4$ $y < -2$ $x > -3$	**16.** $x + y \geq 0$ $4x - y \geq -5$ $7x + 2y \leq 10$

Quadratic Equations and Functions

A **quadratic equation** is an equation that can be written in the *standard form* $ax^2 + bx + c = 0$, where $a \neq 0$. A quadratic equation can have two solutions, one solution, or no real solutions. When $b = 0$, you can use square roots to solve the quadratic equation.

EXAMPLE Solve the quadratic equation.

a. $x^2 + 5 = 29$
$x^2 = 24$
$x = \pm\sqrt{24}$
$x = \pm 2\sqrt{6} \approx \pm 4.90$

Two solutions

b. $3x^2 - 4 = -4$
$3x^2 = 0$
$x^2 = 0$
$x = 0$

One solution

c. $-6x^2 + 3 = 21$
$-6x^2 = 18$
$x^2 = -3$

No real solution

A **quadratic function** is a function that can be written in the standard form $y = ax^2 + bx + c$, where $a \neq 0$.

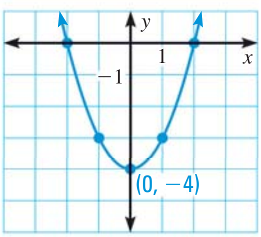
The graph of a quadratic equation is a U-shaped curve called a **parabola**. The **vertex** is the lowest point of a parabola that opens upward ($a > 0$) or the highest point of a parabola that opens downward ($a < 0$). The vertical line passing through the vertex is the **axis of symmetry**.

To graph a quadratic function, you can make a table of values, plot the points, and draw the parabola. The x-intercepts of the graph (if any) are the real solutions of the corresponding quadratic equation.

EXAMPLE Graph the quadratic function. Label the vertex.

a. $y = x^2 - 4$

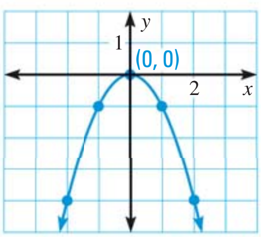
x	y
−2	0
−1	−3
0	−4
1	−3
2	0



Two x-intercepts

b. $y = -x^2$

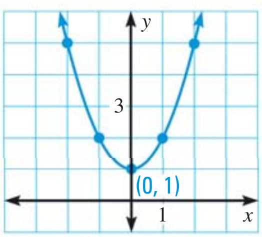
x	y
−2	−4
−1	−1
0	0
1	−1
2	−4



One x-intercept

c. $y = x^2 + 1$

x	y
−2	5
−1	2
0	1
1	2
2	5



No x-intercepts

You can use the **quadratic formula** to solve any quadratic equation.

The solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $a \neq 0$ and $b^2 - 4ac \geq 0$.

EXAMPLE **Use the quadratic formula to solve the equation $8x^2 + 6x = 1$.**

Write the equation in standard form and identify a, b, and c.

The equation $8x^2 + 6x = 1$ is equivalent to $8x^2 + 6x - 1 = 0$. So, $a = 8$, $b = 6$, and $c = -1$.

Use the quadratic formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4(8)(-1)}}{2(8)} = \frac{-6 \pm \sqrt{68}}{16} = \frac{-6 \pm 2\sqrt{17}}{16} = \frac{-3 \pm \sqrt{17}}{8}$$

▶ The solutions of the equation are $\dfrac{-3 + \sqrt{17}}{8} \approx 0.14$ and $\dfrac{-3 - \sqrt{17}}{8} \approx -0.89$.

Check the solutions in the original equation.

$8(0.14)^2 + 6(0.14) \stackrel{?}{=} 1$ $\qquad$ $8(-0.89)^2 + 6(-0.89) \stackrel{?}{=} 1$

$0.9968 \approx 1$ ✓ $\qquad$ $0.9968 \approx 1$ ✓

PRACTICE

Solve the quadratic equation.

1. $x^2 = 144$

2. $x^2 + 7 = -5$

3. $x^2 - (x + 1)^2 = 5$

4. $x^2 - 18 = 0$

5. $8x^2 + 3 = 3$

6. $5x^2 - 2 = -12$

7. $(2x + 3)^2 - 4 = 4x^2 - 7$

8. $3x^2 + 2 = 14$

9. $1 - 4x^2 = 13$

10. $12 - 5x^2 = 12$

11. $15 - 9x^2 = 10$

12. $(x + 2)^2 + 2 = (x - 2)^2 + 8$

Graph the quadratic function. Label the vertex.

13. $y = x^2$

14. $y = x^2 - 3$

15. $y = -x^2 + 4$

16. $y = -2x^2$

17. $y = x^2 + 2$

18. $y = -x^2 - 1$

19. $y = \frac{1}{2}x^2$

20. $y = -\frac{1}{4}x^2$

21. $y = \frac{3}{4}x^2 - 2$

22. $y = 3x^2 + 1$

23. $y = (x - 1)^2$

24. $y = -(x + 2)^2$

Use the quadratic formula to solve the quadratic equation.

25. $x^2 + 6x + 5 = 0$

26. $x^2 - 4x - 2 = 0$

27. $x^2 + 6x = -9$

28. $2x = 8x^2 - 3$

29. $x^2 + 7x + 5 = 1$

30. $x^2 + 2x + 5 = 0$

31. $2x^2 + 8x - 3 = -11$

32. $x^2 + 5x = 6$

33. $5x^2 - 6 = 2x$

34. $3x^2 + 7x - 4 = 0$

35. $2x^2 - 3x = -4$

36. $4x + 4 = 3x^2$

37. $3x^2 - x = 5$

38. $(x + 4)(x - 4) = 8$

39. $(x + 2)(x - 2) = 1$

Functions

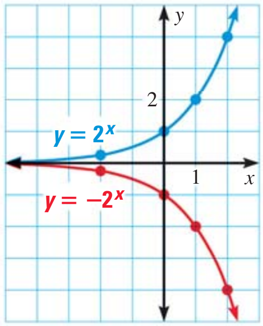
A function can be described by a table of values, a graph, an equation, or words.

EXAMPLE **Graph the exponential functions $y = 2^x$ and $y = -2^x$.**

For each function, make a table of values, plot the points, and draw a curve.

x	$y = 2^x$	(x, y)
−2	$2^{-2} = \frac{1}{4}$	$\left(-2, \frac{1}{4}\right)$
0	$2^0 = 1$	(0, 1)
1	$2^1 = 2$	(1, 2)
2	$2^2 = 4$	(2, 4)

x	$y = -2^x$	(x, y)
−2	$-2^{-2} = -\frac{1}{4}$	$\left(-2, -\frac{1}{4}\right)$
0	$-2^0 = -1$	(0, −1)
1	$-2^1 = -2$	(1, −2)
2	$-2^2 = -4$	(2, −4)



EXAMPLE **The table shows Luke's earnings. Write an equation using his hourly pay rate. How much does Luke earn in 25 hours?**

Hours worked	Earnings (dollars)
8	66
15	123.75
40	330

Use the values in the table to find Luke's hourly pay rate.

$66 \div 8 = 8.25 \qquad 123.75 \div 15 = 8.25 \qquad 330 \div 40 = 8.25$

Write an equation using words. Then use variables.

Earnings = Hourly pay rate • Hours worked

$e = 8.25h$ — **Let e be earnings and h be hours worked.**

$= 8.25(25)$ — **Substitute 25 for h.**

$= 206.25$ — **Multiply.**

▶ Luke earns $206.25 in 25 hours.

PRACTICE

Make a table of values and graph the function.

1. $y = 3^x$
2. $y = -3^x$
3. $y = (0.5)^x$
4. $y = -(0.5)^x$
5. $y = 2x$
6. $y = 2x^2$
7. $y = 2x^3$
8. $y = |2x|$

Write an equation for the function described by the table.

9.

x	1	2	3	4
y	1	4	9	16

10.

x	−2	−1	0	1
y	2	1	0	−1

11. Write an equation using Sue's hourly pay rate of $12. How much does Sue earn in 6 hours? How many hours must Sue work to earn $420?

Problem Solving with Percents

You can use equations to solve problems with percents. Replace words with symbols as shown in the table. To estimate with percents, use compatible numbers.

Words	a is p percent of b.
Symbols	$a = p \cdot b$

EXAMPLE **Use the percent equation to answer the question.**

a. What is 45% of 60?

$a = 0.45 \times 60$

$a = 27$

b. What percent of 28 is 7?

$7 = p \times 28$

$7 \div 28 = p$

$0.25 = p$

$25\% = p$

c. 30% of what number is 12?

$12 = 0.3 \times b$

$12 \div 0.3 = b$

$40 = b$

EXAMPLE **Solve the problem.**

a. Estimate 77% of 80.

$77\% \text{ of } 80 \approx 75\% \times 80$

$= \frac{3}{4} \times 80 = 60$

b. Find the percent of change from \$25 to \$36.

$\frac{\text{new} - \text{old}}{\text{old}} = \frac{36 - 25}{25}$

$= \frac{11}{25}$

$= 0.44 = 44\% \text{ increase}$

PRACTICE

1. A history test has 30 questions. How many questions must you answer correctly to earn a grade of 80%?
2. A class of 27 students has 15 girls. What percent of the class is boys?
3. Jill's goal is to practice her clarinet daily at least 80% of the time. She practiced 25 days in October. Did Jill meet her goal in October?
4. The price of a CD player is \$98. About how much will the CD player cost with a 25% discount?
5. A jacket is on sale for \$48. The original price was \$60. What is the percent of discount?
6. A choir had 38 singers, then 5 more joined. What is the percent of increase?
7. A newspaper conducts a survey and finds that 475 of the residents who were surveyed want a new city park. The newspaper reports that 95% of those surveyed want a new park. How many residents were surveyed?
8. Ron received a raise at work. Instead of earning \$8.75 per hour, he will earn \$9.25. What is the percent of increase in Ron's hourly wage?
9. A school has 515 students. About 260 students ride the school bus. Estimate the percent of the school's students who ride the school bus.

Converting Measurements and Rates

The Table of Measures on page 921 gives many statements of equivalent measures. For each statement, you can write two different conversion factors.

Statement of Equivalent Measures	Conversion Factors
100 cm = 1 m	$\frac{100\text{ cm}}{1\text{ m}} = 1$ and $\frac{1\text{ m}}{100\text{ cm}} = 1$

To convert from one unit of measurement to another, multiply by a conversion factor. Use a conversion factor that allows you to divide out the original unit and keep the desired unit. You can also convert from one rate to another.

EXAMPLE **Copy and complete: a. 5.4 m = _?_ cm   b. 9 ft² = _?_ in.²**

a. $5.4\ \cancel{\text{m}} \times \frac{100\text{ cm}}{1\ \cancel{\text{m}}} = 540\text{ cm}$

b. 1 ft = 12 in., so $1\text{ ft}^2 = 12 \cdot 12 = 144\text{ in.}^2$

Use the conversion factor $\frac{144\text{ in.}^2}{1\text{ ft}^2}$.

$9\ \cancel{\text{ft}^2} \times \frac{144\text{ in.}^2}{1\ \cancel{\text{ft}^2}} = 1296\text{ in.}^2$

EXAMPLE **Copy and complete: $425\frac{\text{ft}}{\text{min}} = \underline{\ ?\ }\frac{\text{mi}}{\text{h}}$.**

Use the conversion factors $\frac{60\text{ min}}{1\text{ h}}$ and $\frac{1\text{ mi}}{5280\text{ ft}}$.

$425\frac{\cancel{\text{ft}}}{\cancel{\text{min}}} \times \frac{60\ \cancel{\text{min}}}{1\text{ h}} \times \frac{1\text{ mi}}{5280\ \cancel{\text{ft}}} \approx 4.8\frac{\text{mi}}{\text{h}}$

PRACTICE

Copy and complete the statement.

1. 500 cm = _?_ m
2. 7 days = _?_ hours
3. 48 oz = _?_ lb
4. 14.8 kg = _?_ g
5. 3200 mL = _?_ L
6. 1200 sec = _?_ min
7. 10 gal = _?_ cups
8. 1 km = _?_ mm
9. 1 mi = _?_ in.
10. $90\text{ ft}^2 = \underline{\ ?\ }\text{ yd}^2$
11. $4\text{ ft}^2 = \underline{\ ?\ }\text{ in.}^2$
12. $12\text{ cm}^2 = \underline{\ ?\ }\text{ mm}^2$
13. $3\text{ m}^3 = \underline{\ ?\ }\text{ cm}^3$
14. $2\text{ yd}^3 = \underline{\ ?\ }\text{ in.}^3$
15. $6500\text{ mm}^3 = \underline{\ ?\ }\text{ cm}^3$
16. $12\frac{\text{mi}}{\text{min}} = \underline{\ ?\ }\frac{\text{mi}}{\text{h}}$
17. $17\frac{\text{km}}{\text{sec}} = \underline{\ ?\ }\frac{\text{km}}{\text{min}}$
18. $0.9\frac{\text{m}}{\text{min}} = \underline{\ ?\ }\frac{\text{mm}}{\text{min}}$
19. $58\frac{\text{mi}}{\text{min}} = \underline{\ ?\ }\frac{\text{ft}}{\text{sec}}$
20. $82\frac{\text{cm}}{\text{min}} = \underline{\ ?\ }\frac{\text{m}}{\text{h}}$
21. $60\frac{\text{mi}}{\text{h}} = \underline{\ ?\ }\frac{\text{ft}}{\text{min}}$
22. $17\frac{\text{km}}{\text{h}} = \underline{\ ?\ }\frac{\text{m}}{\text{sec}}$
23. $0.09\frac{\text{m}^3}{\text{min}} = \underline{\ ?\ }\frac{\text{mm}^3}{\text{min}}$
24. $0.6\frac{\text{km}^2}{\text{year}} = \underline{\ ?\ }\frac{\text{m}^2}{\text{month}}$

Mean, Median, and Mode

Three measures of *central tendency* are mean, median, and mode. One or more of these measures may be more representative of a given set of data than the others.

The **mean** of a data set is the sum of the values divided by the number of values. The mean is also called the *average*.	The **median** of a data set is the middle value when the values are written in numerical order. If a data set has an even number of values, the median is the mean of the two middle values.	The **mode** of a data set is the value that occurs most often. A data set can have no mode, one mode, or more than one mode.

EXAMPLE **The website hits for one week are listed. Which measure of central tendency best represents the data? *Explain.***

Website Hits for One Week

Day	Number of hits
Monday	88
Tuesday	95
Wednesday	87
Thursday	84
Friday	92
Saturday	95
Sunday	11

Mean Add the values. Then divide by the number of values.

$88 + 95 + 87 + 84 + 92 + 95 + 11 = 552$

$\text{Mean} = 552 \div 7 \approx 79$

Median Write the values in order from least to greatest. Then find the middle value(s).

11, 84, 87, **88**, 92, 95, 95

Median = 88

Mode Find the value that occurs most often.
Mode = 95

An *outlier* is a value that is much greater or lower than the other values in a data set. In the data set above, the outlier 11 causes the mean to be lower than the other six data values. So, the **mean** does not represent the data well. The **mode**, 95, does not represent the data well because it is the highest value. The **median**, 88, best represents the data because all but one value lie close to it.

PRACTICE

Tell which measure of central tendency best represents the given data. *Explain*.

1. Daily high temperatures (°F) for a week: 75, 74, 74, 70, 69, 68, 67
2. Movie ticket prices: \$6.75, \$7.50, \$7.25, \$6.75, \$7, \$7.50, \$7.25, \$6.75, \$7
3. Number of eggs bought: 12, 12, 12, 6, 12, 18, 18, 12, 6, 12, 12, 12, 24, 18
4. Number of children in a family: 0, 0, 0, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 3, 3, 4, 4, 5
5. Ages of employees: 36, 22, 30, 27, 41, 58, 33, 27, 62, 39, 21, 24, 22
6. Shoe sizes in a shipment: 5, $5\frac{1}{2}$, 6, $6\frac{1}{2}$, 7, $7\frac{1}{2}$, $7\frac{1}{2}$, 8, 8, 8, $8\frac{1}{2}$, 9, $9\frac{1}{2}$, 10
7. Test scores: 97%, 65%, 68%, 98%, 72%, 60%, 94%, 100%, 99%
8. Favorite of 3 colors: blue, yellow, red, yellow, red, red, blue, red, red, blue

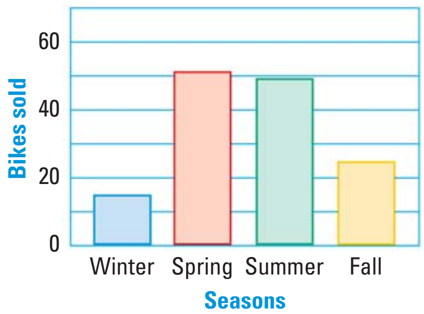
Displaying Data

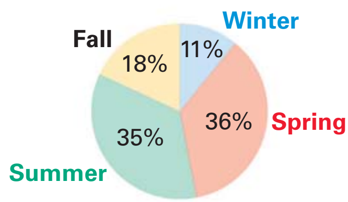
There are many ways to display data. An appropriate data display can help you analyze the data. The table summarizes how data are shown in some data displays.

Circle Graph	Bar Graph	Histogram	Line Graph	Stem-and-Leaf Plot	Box-and-Whisker Plot
Shows data as parts of a whole.	Compares data in distinct categories.	Compares data in intervals.	Shows how data change over time.	Shows data in numerical order.	Shows distribution of data in quartiles.

EXAMPLE **The table shows bike sales at a shop. Display the data in two appropriate ways. *Describe* what each display shows about the data.**

Season	Winter	Spring	Summer	Fall
Bikes sold	15	51	49	25





In the bar graph, the heights of the bars can be used to compare sales for the four seasons. Bikes sales were strongest in the spring and summer.

The circle graph shows the percent of annual sales for each season. Almost $\frac{3}{4}$ of the bikes were sold in the spring and summer.

EXAMPLE **The test scores for a class were 82, 99, 68, 76, 84, 100, 85, 79, 92, 100, 82, 81, 60, 95, 98, 74, 95, 84, 88. Display the distribution of the scores.**

Use a stem-and-leaf plot to organize the data. Identify the *lower* and *upper extremes*, the median, and the *lower* and *upper quartiles* (the medians of the lower and upper half of the ordered data set.)

6	0 8
7	4 6 9
8	1 2 2 4 4 5 8
9	2 5 5 8 9
10	0 0

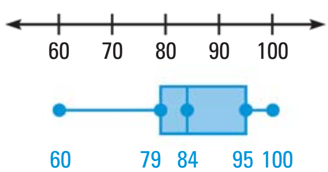
Key: 7 | 4 = 74

Lower and upper extremes: 60 and 100

Median: 84

Lower and upper quartiles: 79 and 95

Then make a box-and-whisker plot. Draw a number line. Below it, plot the lower extreme (60), the lower quartile (79), the median (84), the upper quartile (95), and the upper extreme (100). Draw boxes and "whiskers," as shown.



PRACTICE

Name a data display that would be appropriate for the situation. (There may be more than one choice.) *Explain* your reasoning.

1. A store owner keeps track of how many cell phones are sold each week. The owner wants to see how sales change over a six-month period.
2. You measure the daily high temperature for 31 days in July. You want to see the distribution of the temperatures.
3. The ages of people in a survey are grouped into these intervals: 20–29, 30–39, 40–49, 50–59, 60–69, 70–79. You want to compare the numbers of people in the various groups.

Make a data display that can be used to answer the question. *Explain* why you chose this display. Then answer the question.

4. The table gives the number of gold medals won by U.S. athletes at five Summer Olympic games. *Question:* How has the number of medals won changed over time?

Year	1988	1992	1996	2000	2004
Number of gold medals	36	37	44	40	35

5. Students were surveyed about the amounts they spent at a mall one Saturday. These are the amounts (in dollars): 5, 70, 10, 40, 42, 45, 50, 4, 3, 10, 12, 15, 20, 5, 30, 35, 70, 80. *Question*: If the dollar amounts are grouped into intervals such as 0–9, 10–19, and so on, in which interval do the greatest number of students fall?

Display the data in two appropriate ways. *Describe* what each display shows about the data.

6. During a game, a high school soccer team plays 2 forwards, 4 midfielders, 4 defenders, and 1 goalkeeper.
7. A high school has 131 students taking Geometry. The number of students in each class are: 18, 16, 17, 15, 16, 14, 17 and 18.
8. The table gives the number of calories in 8 different pieces of fresh fruit.

Fruit	Apple	Banana	Mango	Orange	Peach	Pear	Plum	Tangerine
Calories	117	100	85	65	35	60	40	35

The ages of actors in a community theater play are 18, 25, 19, 32, 26, 15, 33, 12, 36, 16, 18, 30, 25, 24, 32, 30, 13, 15, 37, 35, 72, 35. Use these data for Exercises 9–11.

9. Make a stem-and-leaf plot of the data. Identify the lower and upper extremes, the median, and the lower and upper quartiles of the data set.
10. Make a box-and-whisker plot of the data. About what percent of the actors are over 18? How does the box-and-whisker plot help you answer this question?
11. Suppose the two oldest actors drop out of the play. Draw a new box-and-whisker plot without the data values for those actors. How does the distribution of the data change? *Explain*.

Sampling and Surveys

A *survey* is a study of one or more characteristics of a group. A **population** is the group you want information about. A **sample** is part of the population. In a **random sample**, every member of a population has an equal chance of being selected for a survey. A random sample is most likely to represent the population. A sample that is not representative is a *biased sample*.

Using a biased sample may affect the results of a survey. In addition, survey results may be influenced by the use of *biased questions*. A biased question encourages a particular response.

EXAMPLE **Read the description of the survey. Identify any biased samples or questions. *Explain.***

a. A movie theater owner wants to know how often local residents go to the movies each month. The owner asks every tenth ticket buyer.

▸ The sample (every tenth ticket buyer) is unlikely to represent the population (local residents). It is biased because moviegoers are over-represented.

b. The mayor's office asks a random sample of the city's residents the following question: Do you support the necessary budget cuts proposed by the mayor?

▸ The sample is random, so it is not biased. The question is biased because the word *necessary* suggests that people should support the budget cuts.

PRACTICE

Read the description of the survey. Identify any biased samples or questions. *Explain.*

1. The coach of a high school soccer team wants to know whether students are more likely to come watch the team's games on Wednesdays or Thursdays. The team's first game is on a Friday. The coach asks all the students who come to watch which day they prefer.
2. A town's recreation department wants to know whether to build a new skateboard park. The head of the department visits a local park and asks people at the park whether they would like to have a skateboard park built.
3. A television producer wants to know whether people in a city would like to watch a one-hour local news program or a half-hour local news program. A television advertisement is run several times during the day asking viewers to e-mail their preference.
4. The teachers at a music school want to know whether the students at the school practice regularly. Five of the ten teachers at the school ask their students the following question: How many hours do you spend practicing each day?
5. A skating rink owner wants to know the ages of people who use the rink. Over a two-week period, the owner asks every tenth person who uses the rink his or her age.
6. A cello teacher asks some of his students, "Do you practice every day?"

Counting Methods

To count the number of possibilities in a situation, you can make an organized list, draw a tree diagram, make a table, or use the counting principle.

The Counting Principle
If one event can occur in m ways, and for each of these ways a second event can occur in n ways, then the number of ways that the two events can occur together is $m \times n$.

The counting principle can be extended to three or more events.

EXAMPLE **Use four different counting methods to find the number of possible salad specials.**

Salad Special $5.95

Choose 1 salad and 1 dressing

Salad: Lettuce or Spinach

Dressing: Ranch, Blue cheese, or Italian

Method 1 Make an Organized List

Pair each salad with each dressing and list each possible special.

Lettuce salad with ranch

Lettuce salad with blue cheese

Lettuce salad with Italian

Spinach salad with ranch

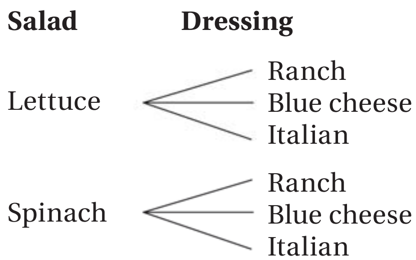
Spinach salad with blue cheese

Spinach salad with Italian

Count the number of specials listed. There are 6 possible salad specials.

Method 2 Draw a Tree Diagram

Arrange the salads and dressings in a tree diagram.



Count the number of branches in the tree diagram. There are 6 possible salad specials.

Method 3 Make a Table

List the salads in the left column. List the dressings in the top row.

	Ranch	Blue cheese	Italian
Lettuce	Lettuce, Ranch	Lettuce, Blue cheese	Lettuce, Italian
Spinach	Spinach, Ranch	Spinach, Blue cheese	Spinach, Italian

Count the number of cells filled. There are 6 possible salad specials.

Method 4 Use the Counting Principle

There are 2 choices of salad, so $m = 2$. There are 3 choices of dressing, so $n = 3$. By the counting principle, the number of ways that the salad and dressing choices can be combined is $m \times n = 2 \times 3 = 6$.

There are 6 possible salad specials.

EXAMPLE **Tyler must choose a 4-digit password for his bank account. Find the number of possible 4-digit passwords using four different digits.**

Because there are many possible passwords, use the counting principle.

For one of the digits in the password, there are 10 choices: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Because one of these digits will be used for the first digit, there are only 9 choices for the next, 8 for the next after that, and so on.

10 choices for first digit × 9 choices for second digit × 8 choices for third digit × 7 choices for fourth digit

$10 \times 9 \times 8 \times 7 = 5040$

▶ There are 5040 possible 4-digit passwords using four different digits.

PRACTICE

Use one of the methods described in the Examples on pages 891 and 892 to solve each problem. *Explain* your reasoning.

1. Ann takes three pairs of shorts (red, blue, and green) and five T-shirts (black, white, yellow, orange, and brown) on a trip. Find the number of different shorts and T-shirt outfits Ann can wear while on the trip.
2. Art students can choose any two pieces of colored paper for a project. There are six colors available and students must choose two different colors. Find the number of different color combinations that can be chosen.
3. Steve must choose four characters for his computer password. Each character can be any letter from A through Z or any digit from 0 through 9. All letters and digits may be used more than once. Find the number of possible passwords.
4. A restaurant offers a pizza special, as shown at the right. Assuming that two different toppings are ordered, find the number of two-topping combinations that can be ordered.
5. Each of the locker combinations at a gym uses three numbers from 0 through 49. Find the number of different locker combinations that are possible.

Large Pizza Special

Any 2 toppings for $12.49

Pepperoni	**Green Olive**
Sausage	**Green Pepper**
Ground Beef	**Red Onion**
Black Olive	**Mushroom**

6. A movie theater sells three sizes of popcorn and six different soft drinks. Each soft drink can be bought in one of three sizes. Find the number of different popcorn and soft drink combinations that can be ordered.
7. A class has 28 students and elects two students to be class officers. One student will be president and one will be vice president. How many different combinations of class officers are possible?
8. Some students are auditioning for parts in the play *Our Town*. Twenty girls try out for the parts listed at the right. In how many different ways can 5 of the 20 girls be assigned these roles?
9. Bill, Allison, James, and Caroline are friends. In how many different ways can they stand in a row for a photo?
10. A cafeteria serves 4 kinds of sandwiches: cheese, veggie, peanut butter, and bologna. Students can choose any two sandwiches for lunch. How many different sandwich combinations are possible?

Parts in *Our Town*

Emily Webb
Mrs. Gibbs
Mrs. Webb
Mrs. Soames
Rebecca Gibbs

Probability

The **probability** of an event is a measure of the likelihood that the event will occur. An event that cannot occur has a probability of 0, and an event that is certain to occur has a probability of 1. Other probabilities lie between 0 and 1. You can write a probability as a decimal, a fraction, or a percent.

Probability of an Event

When all outcomes are equally likely, the probability of an event, P(event), is $\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$.

When you consider the probability of two events occurring, the events are called **compound events**. Compound events can be dependent or independent.

Two events are **independent events** if the occurrence of one event *does not* affect the occurrence of another. For two independent events A and B, $P(A \text{ and } B) = P(A) \cdot P(B)$.	Two events are **dependent events** if the occurrence of one event *does* affect the occurrence of another. For two dependent events A and B, $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$, where $P(B \mid A)$ is the probability of B given that A has occurred.

EXAMPLE **A box holds 12 yellow marbles and 12 orange marbles. Without looking, you take a marble. Then you take another marble without replacing the first. Find the probability that both marbles are yellow.**

There are 24 marbles in the box when you take the first one, and only 23 when you take the second. So, the events are dependent.

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A) = \frac{12}{24} \cdot \frac{11}{23} = \frac{11}{46} \approx 0.24, \text{ or } 24\%$$

PRACTICE

Identify the events as *independent* or *dependent*. Then answer the question.

1. There are 20 socks in your drawer, and 12 of them are white. You grab a sock without looking. Then you grab a second sock without putting the first one back. What is the probability that both socks are white?
2. You flip a coin two times. What is the probability that you get heads each time?
3. Your math, literature, Spanish, history, and science homework assignments are organized in five folders. You randomly choose one folder, finish your assignment, and then choose a new folder. What is the probability that you do your math homework first, and then history?
4. You roll a red number cube and a blue number cube. What is the probability that you roll an even number on the red cube and a number greater than 2 on the blue cube?
5. You flip a coin three times. What is the probability that you do not get heads on any of the flips?

Problem Solving Plan and Strategies

Here is a 4-step **problem solving plan** that you can use to solve problems.

STEP 1	**Read and understand the problem.**	Read the problem carefully. Organize the given information and decide what you need to find. Check for unnecessary or missing information. Supply missing facts, if needed.
STEP 2	**Make a plan to solve the problem.**	Choose a problem solving strategy. Choose the correct operations to use. Decide if you will use a tool such as a calculator, graph, or spreadsheet.
STEP 3	**Carry out the plan to solve the problem.**	Use the problem solving strategy and any tools you have chosen. Estimate before you calculate, if possible. Do any calculations that are needed. Answer the question that the problem asks.
STEP 4	**Check to see if your answer is reasonable.**	Reread the problem. See if your answer agrees with the given information and with any estimate you have made.

Here are some **problem solving strategies** that you can use to solve problems.

Strategy	When to use	How to use
Guess, check, and revise	Guess, check, and revise when you need a place to start or you want to see how the problem works.	Make a reasonable guess. Check to see if your guess solves the problem. If it does not, revise your guess and check again.
Draw a diagram or a graph	Draw a diagram or a graph when a problem involves any relationships that you can represent visually.	Draw a diagram or a graph that shows given information. See what your diagram reveals that can help you solve the problem.
Make a table or an organized list	Make a table or list when a problem requires you to record, generate, or organize information.	Make a table with columns, rows, and any given information. Generate a systematic list that can help you solve the problem.
Use an equation or a formula	Use an equation or a formula when you know a relationship between quantities.	Write an equation or formula that shows the relationship between known quantities. Solve the equation to solve the problem.
Use a proportion	Use a proportion when you know that two ratios are equal.	Write a proportion using the two equal ratios. Solve the proportion to solve the problem.
Look for a pattern	Look for a pattern when a problem includes numbers or diagrams that you need to analyze.	Look for a pattern in any given information. Organize, extend, or generalize the pattern to help you solve the problem.
Break a problem into parts	Break a problem into parts when a problem cannot be solved in one step but can be solved in parts.	Break the problem into parts and solve each part. Put the answers together to help you solve the original problem.
Solve a simpler or related problem	Solve a simpler or related problem when a problem seems difficult and can be made easier by using simpler numbers or conditions.	Think of a way to make the problem easier. Solve the simpler or related problem. Use what you learned to help you solve the original problem.
Work backward	Work backward when a problem gives you an end result and you need to find beginning conditions.	Work backward from the given information until you solve the problem. Work forward through the problem to check your answer.

EXAMPLE **A marching band receives a $2800 donation to buy new drums and piccolos. Each drum costs $350 and each piccolo costs $400. How many of each type of instrument can the band buy?**

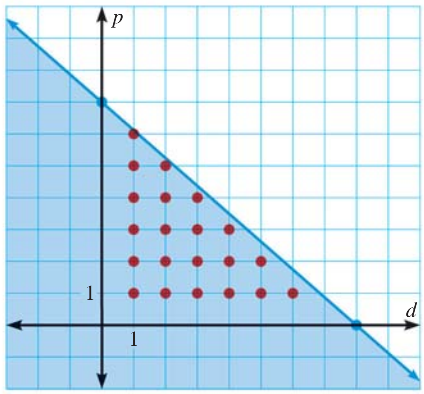
STEP 1 **Choose** two strategies, *Use an Equation* and *Draw a Graph*.

STEP 2 **Write** an inequality. Let d = the number of drums and p = the number of piccolos.

Cost of drums	•	Number of drums	+	Cost of piccolos	•	Number of piccolos	≤	$2800

$350d + 400p \leq 2800$

STEP 3 **Graph** and shade the solution region of the inequality.



The band can buy only whole numbers of instruments. Also, you can assume that the band will buy at least one of each type of instrument. Mark each point in the solution region that has whole number coordinates greater than or equal to 1.

▸ The red points on the graph show 21 different ways that the band can buy drums and piccolos without spending more than $2800.

PRACTICE

1. A cell phone company offers a plan with an initial registration fee of $25 and a monthly fee of $15. How much will the plan cost for one year?
2. Rita wants to attend a swim camp that costs $220. She has $56 in a bank account. She also earns $25 each week walking dogs. Will Rita be able to make a full payment for the camp in 5 weeks? *Explain* your reasoning.
3. What is the 97th number in the pattern 4, 3, 2, 1, 4, 3, 2, 1, 4, 3, 2, 1, . . .?
4. Sam makes a down payment of $120 on a $360 bike. He will pay $30 each month until the balance is paid. How many monthly payments will he make?
5. Marie is buying tree seedlings for the school. She can spend no more than $310 on aspen and birch trees. She wants at least 20 trees in all and twice as many aspen trees as birch trees. Find three possible ways that Marie can buy the trees.

Tree Seedlings

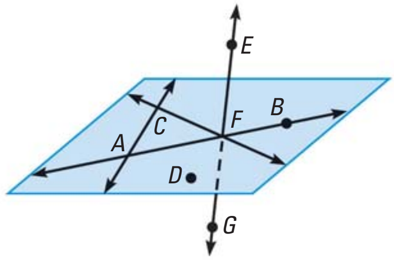
Aspen	$10 each
Birch	$12 each

6. In how many different ways can you make 75¢ in change using quarters, dimes, and nickels?
7. Charlie is cutting a rectangular cake that is 9 inches by 13 inches into equal-sized rectangular pieces. Each piece of cake should be at least 2 inches on each side. What is the greatest number of pieces Charlie can cut?
8. Streamers cost $1.70 per roll and balloons cost $1.50 per bag. If the student council has $40 to spend for parent night and buys 10 rolls of streamers, how many bags of balloons can the student council buy?

Extra Practice

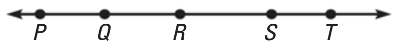
Chapter 1

1.1 In Exercises 1–5, use the diagram.



1. Name three points that are collinear. Then give a name for the line that contains the points.
2. Name the intersection of plane *ABC* and $\overleftrightarrow{EG}$.
3. Name two pairs of opposite rays.
4. Are points *A*, *C*, and *G* coplanar? *Explain*.
5. Name a line that intersects plane *AFD* at more than one point.

1.2 In the diagram, *P*, *Q*, *R*, *S*, and *T* are collinear, *PT* = 54, *QT* = 42, *QS* = 31, and *RS* = 17. Find the indicated length.



6. *PQ*
7. *PS*
8. *QR*
9. *PR*
10. *ST*
11. *RT*

1.2 Point *B* is between *A* and *C* on $\overline{AC}$. Use the given information to write an equation in terms of *x*. Solve the equation. Then find *AB* and *BC*, and determine whether $\overline{AB}$ and $\overline{BC}$ are congruent.

12. $AB = x + 3$
 $BC = 2x + 1$
 $AC = 10$
13. $AB = 3x - 7$
 $BC = 3x - 1$
 $AC = 16$
14. $AB = 11x - 16$
 $BC = 8x - 1$
 $AC = 78$
15. $AB = 4x - 5$
 $BC = 2x - 7$
 $AC = 54$
16. $AB = 14x + 5$
 $BC = 10x + 15$
 $AC = 80$
17. $AB = 3x - 7$
 $BC = 2x + 5$
 $AC = 108$

1.3 Find the coordinates of the midpoint of the segment with the given endpoints.

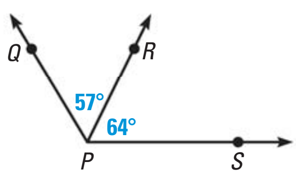
18. $A(2, -4)$, $B(7, 1)$
19. $C(-3, -2)$, $D(-8, 4)$
20. $E(-2.3, -1.9)$, $F(3.1, -9.7)$
21. $G(3, -7)$, $H(-1, 9)$
22. $I(4, 3)$, $J(2, 2)$
23. $K(1.7, -7.9)$, $L(8.5, -8.2)$

1.3 Find the length of the segment with given endpoint and midpoint *M*.

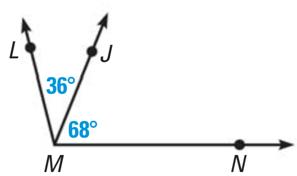
24. $Z(0, 1)$ and $M(7, 1)$
25. $Y(4, 3)$ and $M(1, 7)$
26. $X(0, -1)$ and $M(12, 4)$
27. $W(5, 3)$ and $M(-10, -5)$
28. $V(-3, -4)$ and $M(9, 5)$
29. $U(3, 2)$ and $M(11, -4)$

1.4 Use the given information to find the indicated angle measure.

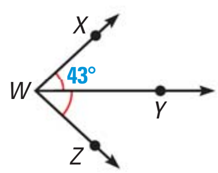
30. $m\angle QPS =$ ___?___



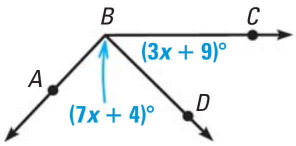
31. $m\angle LMN =$ ___?___



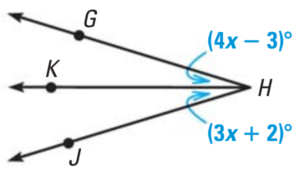
32. $m\angle XWZ =$ ___?___



1.4 **33.** Given $m\angle ABC = 133°$, find $m\angle ABD$.

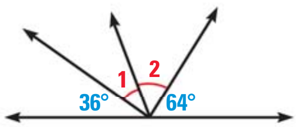


34. Given $m\angle GHK = 17°$, find $m\angle KHJ$.

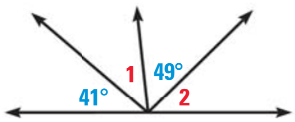


1.5 **Tell whether ∠1 and ∠2 are *vertical angles*, *adjacent angles*, a *linear pair*, *complementary*, or *supplementary*. There may be more than one answer.**

35.



36.



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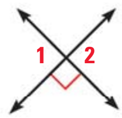
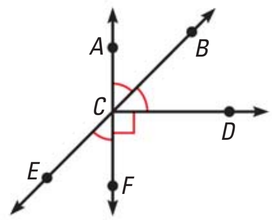


1.5 **Use the diagram.**



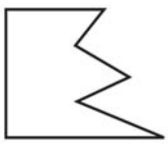
38. Name two supplementary angles that are not a linear pair.

39. Name two vertical angles that are not complementary.

40. Name three pairs of complementary angles. Tell whether each pair contains vertical angles, adjacent angles, or neither.

1.6 **Tell whether the figure is a polygon. If it is not, *explain* why. If it is, tell whether it is *convex* or *concave*.**

41.



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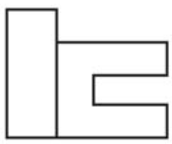

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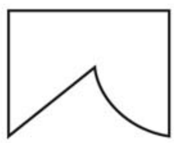

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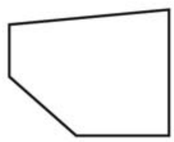
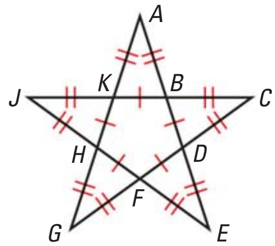


1.6 **In Exercises 45 and 46, use the diagram.**

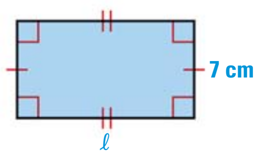


45. Identify two different equilateral polygons in the diagram. Classify each by the number of sides.

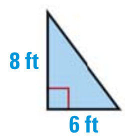
46. Name one of each of the following figures as it appears in the five-pointed star diagram: triangle, quadrilateral, pentagon, hexagon, heptagon.

1.7 **Use the information about the figure to find the indicated measure.**

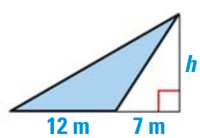
47. Area = 91 cm^2
Find the length ℓ.



48. Find the area of the triangle.



49. Area = 66 m^2
Find the height h.



1.7 **Find the perimeter and area of the triangle with the given vertices. Round to the nearest tenth.**

50. $A(2, 1)$, $B(3, 6)$, $C(6, 1)$

51. $D(1, 1)$, $E(3, 1)$, $F(6, 5)$

Chapter 2

2.1 ***Describe*** **the pattern in the numbers. Write the next number in the pattern.**

1. 17, 23, 15, 21, 13, 19,...

2. 1, 0.5, 0.25, 0.125, 0.0625,...

3. 2, 3, 5, 7, 11, 13,...

4. 7.0, 7.5, 8.0, 8.5,...

5. $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$

6. 2, 2, 4, 6, 10, 16, 26,...

2.1 **Show the conjecture is false by finding a counterexample.**

7. The difference of any two numbers is a value that lies between those two numbers.

8. The value of $2x$ is always greater than the value of x.

9. If an angle A can be bisected, then angle A must be obtuse.

2.2 **For the given statement, write the if-then form, the converse, the inverse, and the contrapositive.**

10. Two lines that intersect form two pairs of vertical angles.

11. All squares are four-sided regular polygons.

2.2 **Decide whether the statement is *true* or *false*. If false, provide a counterexample.**

12. If a figure is a hexagon, then it is a regular polygon.

13. If two angles are complementary, then the sum of their measures is 90°.

2.3 **Write the statement that follows from the pair of statements that are given.**

14. If a triangle is equilateral, then it has congruent angles.
If a triangle has congruent angles, then it is regular.

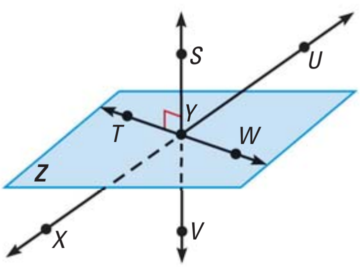
15. If two coplanar lines are not parallel, then they intersect.
If two lines intersect, then they form congruent vertical angles.

2.3 **Select the word(s) that make(s) the conclusion true.**

16. John only does his math homework when he is in study hall. John is doing his math homework. So, John (*is*, *may be*, *is not*) in study hall.

17. May sometimes buys pretzels when she goes to the supermarket. May is at the supermarket. So, she (*will*, *might*, *will not*) buy pretzels.

2.4 **Use the diagram to determine if the statement is *true* or *false*.**



18. $\overleftrightarrow{SV} \perp$ plane Z

19. $\overleftrightarrow{XU}$ intersects plane Z at point Y.

20. $\overleftrightarrow{TW}$ lies in plane Z.

21. $\angle SYT$ and $\angle WYS$ are vertical angles.

22. $\angle SYT$ and $\angle TYV$ are complementary angles.

23. $\angle TYU$ and $\angle UYW$ are a linear pair.

24. $\angle UYV$ is acute.

EXTRA PRACTICE

2.5 Solve the equation. Write a reason for each step.

25. $4x + 15 = 39$ **26.** $6x + 47 = 10x - 9$ **27.** $2(-7x + 3) = -50$

28. $54 + 9x = 3(7x + 6)$ **29.** $13(2x - 3) - 20x = 3$ **30.** $31 + 25x = 7x - 14 + 3x$

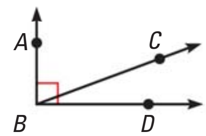
2.6 Copy and complete the statement. Name the property illustrated.

31. If $m\angle JKL = m\angle GHI$ and $m\angle GHI = m\angle ABC$, then $\underline{\ ?\ } = \underline{\ ?\ }$.

32. If $m\angle MNO = m\angle PQR$, then $m\angle PQR = \underline{\ ?\ }$

33. $m\angle XYZ = \underline{\ ?\ }$

2.6 **34.** Copy and complete the proof.



GIVEN ▶ Point C is in the interior of $\angle ABD$. $\angle ABD$ is a right angle.

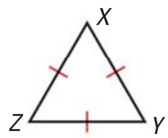
PROVE ▶ $\angle ABC$ and $\angle CBD$ are complementary.

STATEMENTS	REASONS
1. $\angle ABD$ is a right angle.	**1.** Given
2. $m\angle ABD = 90°$	**2.** $\underline{\ ?\ }$
3. $\underline{\ ?\ }$	**3.** Given
4. $m\angle ABD = m\angle ABC + m\angle CBD$	**4.** $\underline{\ ?\ }$
5. $\underline{\ ?\ } = m\angle ABC + m\angle CBD$	**5.** Substitution Property of Equality
6. $\underline{\ ?\ }$	**6.** Definition of complementary angles

2.6 **35.** Use the given information and the diagram to prove the statement.

GIVEN ▶ $\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$

PROVE ▶ The perimeter of $\triangle XYZ$ is $3 \cdot XY$.



2.7 Copy and complete the statement. $\angle AGD$ is a right angle and $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{EF}$ intersect at point G.

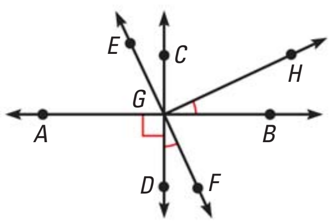
36. If $m\angle CGF = 158°$, then $m\angle EGD = \underline{\ ?\ }$.

37. If $m\angle EGA = 67°$, then $m\angle FGD = \underline{\ ?\ }$.

38. If $m\angle FGC = 149°$, then $m\angle EGA = \underline{\ ?\ }$.

39. $m\angle DGB = \underline{\ ?\ }$

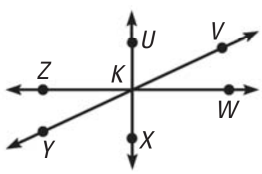
40. $m\angle FGH = \underline{\ ?\ }$



2.7 **41.** Write a two-column proof.

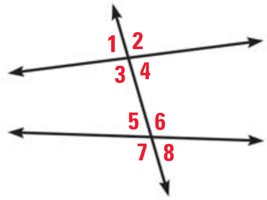
GIVEN ▶ $\angle UKV$ and $\angle VKW$ are complements.

PROVE ▶ $\angle YKZ$ and $\angle XKY$ are complements.



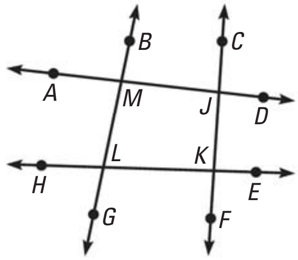
Chapter 3

3.1 Classify the angle pair as *corresponding, alternate interior, alternate exterior,* or *consecutive interior* angles.



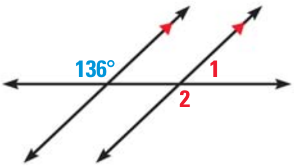
1. $\angle 6$ and $\angle 2$
2. $\angle 7$ and $\angle 2$
3. $\angle 5$ and $\angle 3$
4. $\angle 4$ and $\angle 5$
5. $\angle 1$ and $\angle 5$
6. $\angle 3$ and $\angle 6$

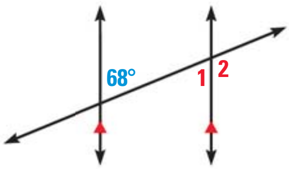
3.1 Copy and complete the statement. List all possible correct answers.



7. $\angle AMB$ and __?__ are corresponding angles.
8. $\angle AML$ and __?__ are alternate interior angles.
9. $\angle CJD$ and __?__ are alternate exterior angles.
10. $\angle LMJ$ and __?__ are consecutive interior angles.
11. __?__ is a transversal of $\overleftrightarrow{AD}$ and $\overleftrightarrow{HE}$.

3.2 Find $m\angle 1$ and $m\angle 2$. *Explain* your reasoning.

12.


13.


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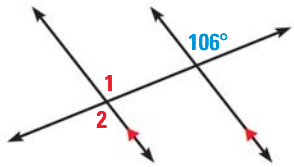
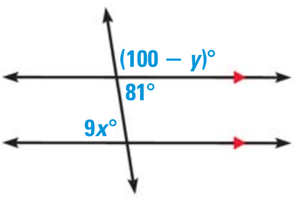


3.2 Find the values of x and y.

15.


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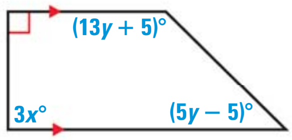

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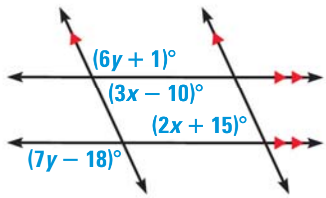
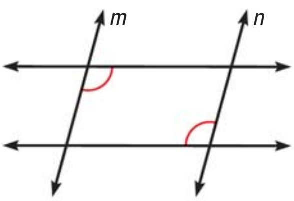


3.3 Is there enough information to prove $m \parallel n$? If so, state the postulate or theorem you would use.

18.


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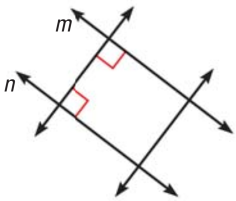

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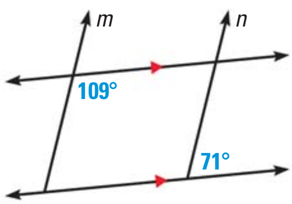

3.3 Can you prove that lines a and b are parallel? If so, *explain* how.

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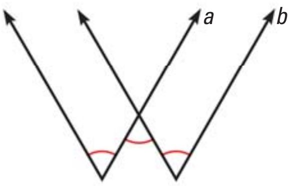

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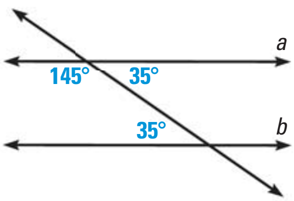

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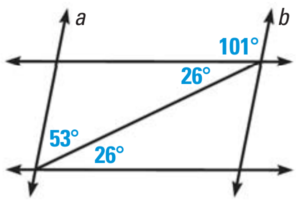


3.4 Tell whether the lines through the given points are *parallel*, *perpendicular*, or *neither*. *Justify* your answer.

24. **Line 1:** (7, 4), (10, 5)
Line 2: (2, 3), (8, 5)

25. **Line 1:** (−3, 1), (−2, 5)
Line 2: (−1, −3), (5, −2)

26. **Line 1:** (−6, 0), (8, 7)
Line 2: (1, 4), (2, 2)

3.4 Tell which line through the given points is steeper.

27. **Line 1:** (0, −6), (−4, −9)
Line 2: (−2, 5), (1, 9)

28. **Line 1:** (−1, −5), (−1, 3)
Line 2: (−3, 4), (−5, 4)

29. **Line 1:** (1, 1), (2, 6)
Line 2: (1, 1), (3, 10)

3.5 Write an equation of the line that passes through the given point *P* and has the given slope *m*.

30. $P(4, 7), m = 2$

31. $P(-3, 0), m = \frac{2}{3}$

32. $P(9, 4), m = -\frac{1}{3}$

3.5 Write an equation of the line that passes through point *P* and is parallel to the line with the given equation.

33. $P(1, -2), y = -2x - 6$

34. $P(6, 3), y = -\frac{1}{3}x + 12$

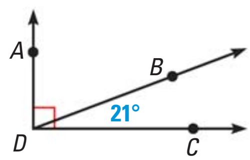
35. $P(-7, 3), y = x + 3$

36. $P(0, 3), y = 4x - 2$

37. $P(-9, 4), y = \frac{2}{5}x + 1$

38. $P(8, -3), y = x - 5$

3.6 Find $m\angle ADB$.

39.


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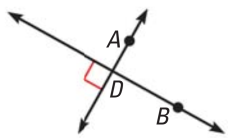

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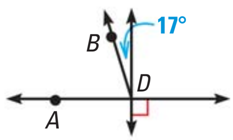

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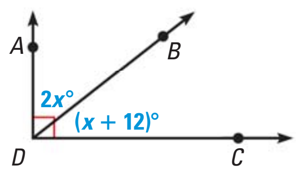

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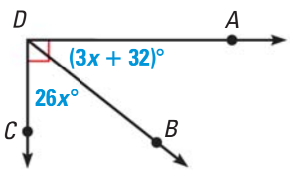

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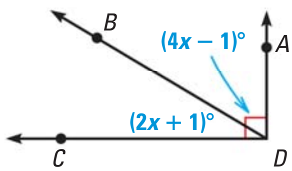
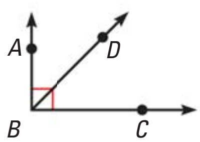


3.6 **45.** Copy and complete the proof.



GIVEN ▶ $\overrightarrow{BA} \perp \overrightarrow{BC}$,
$\overrightarrow{BD}$ bisects $\angle ABC$.

PROVE ▶ $m\angle ABD = 45°$

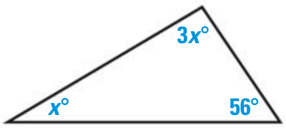
STATEMENTS	REASONS
1. $\overrightarrow{BA} \perp \overrightarrow{BC}$	1. ?
2. ?	2. Definition of perpendicular lines
3. $m\angle ABC = 90°$	3. ?
4. ?	4. Given
5. $m\angle ABD = m\angle DBC$	5. ?
6. $m\angle ABC =$? + ?	6. Angle Addition Postulate
7. $m\angle ABD + m\angle DBC = 90°$	7. ?
8. $m\angle ABD +$? $= 90°$	8. Substitution Property of Equality
9. $2(m\angle ABD) = 90°$	9. ?
10. $m\angle ABD = 45°$	10. ?

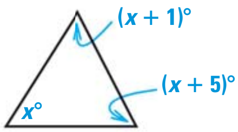
Chapter 4

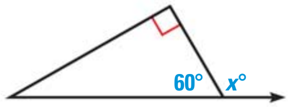
4.1 A triangle has the given vertices. Graph the triangle and classify it by its sides. Then determine if it is a right triangle.

1. $A(-1, -2)$, $B(-1, 2)$, $C(4, 2)$
2. $A(-1, -1)$, $B(3, 1)$, $C(2, -2)$
3. $A(-3, 4)$, $B(2, 4)$, $C(5, -2)$

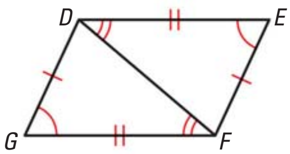
4.1 Find the value of x. Then classify the triangle by its angles.

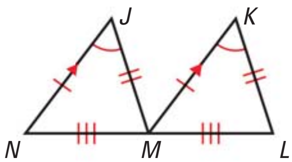
4. 

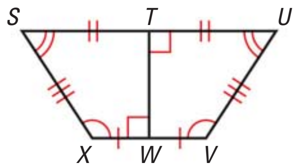
5. 

6. 

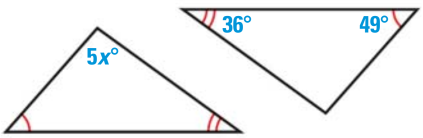
4.2 Write a congruence statement for any figures that can be proved congruent. *Explain* your reasoning.

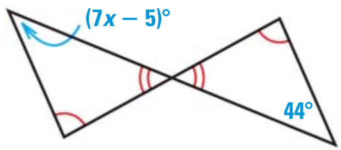
7. 

8. 

9. 

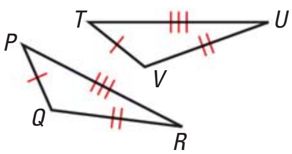
4.2 Find the value of x.

10. 

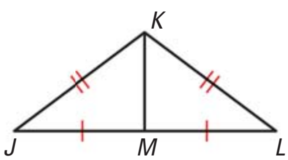
11. 

4.3 Decide whether the congruence statement is true. *Explain* your reasoning.

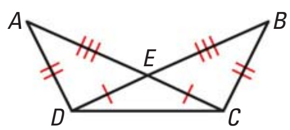
12. $\triangle PQR \cong \triangle TUV$



13. $\triangle JKM \cong \triangle LMK$



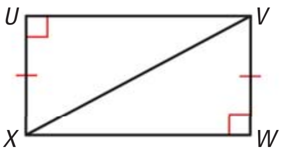
14. $\triangle ACD \cong \triangle BDC$

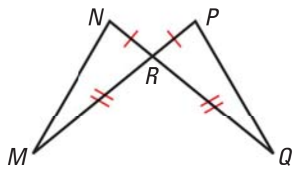


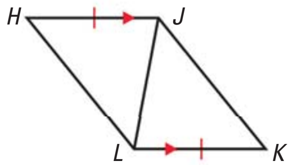
4.3 Use the given coordinates to determine if $\triangle ABC \cong \triangle PQR$.

15. $A(-2, 1)$, $B(2, 6)$, $C(6, 2)$, $P(-1, -2)$, $Q(3, 3)$, $R(7, -1)$
16. $A(-4, 5)$, $B(2, 6)$, $C(-2, 3)$, $P(2, 1)$, $Q(8, 2)$, $R(5, -1)$

4.4 Name the congruent triangles in the diagram. *Explain*.

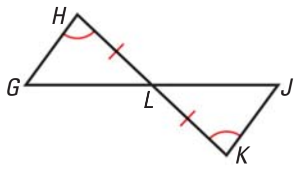
17. 

18. 

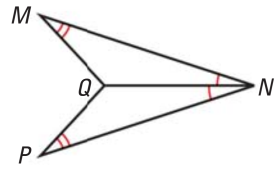
19. 

4.5 Is it possible to prove that the triangles are congruent? If so, state the postulate or theorem you would use.

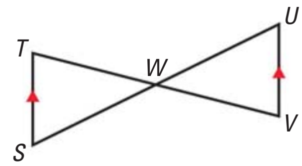
20. $\triangle GHL$, $\triangle JKL$



21. $\triangle MNQ$, $\triangle PNQ$



22. $\triangle STW$, $\triangle UVW$



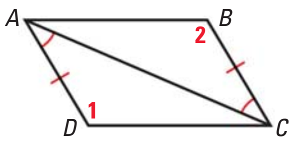
4.5 Tell whether you can use the given information to determine whether $\triangle ABC \cong \triangle DEF$. *Explain* your reasoning.

23. $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DE}$, $\angle B \cong \angle E$

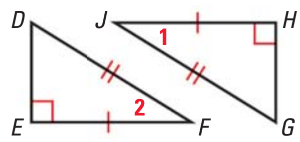
24. $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\angle A \cong \angle D$

4.6 Use the information in the diagram to write a plan for proving that $\angle 1 \cong \angle 2$.

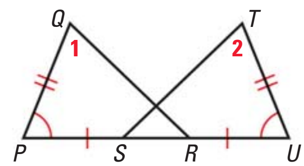
25.



26.



27.



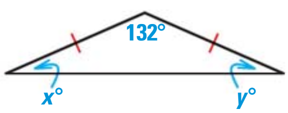
4.6 Use the vertices of $\triangle ABC$ and $\triangle DEF$ to show that $\angle A \cong \angle D$. *Explain.*

28. $A(0, 8)$, $B(6, 0)$, $C(0, 0)$, $D(3, 10)$, $E(9, 2)$, $F(3, 2)$

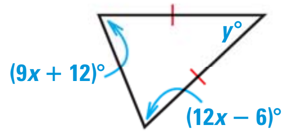
29. $A(-3, -2)$, $B(-2, 3)$, $C(2, 2)$, $D(5, 1)$, $E(6, 6)$, $F(10, 5)$

4.7 Find the value(s) of the variable(s).

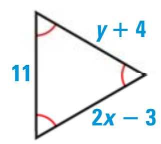
30.



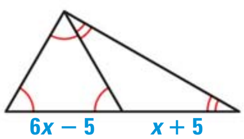
31.



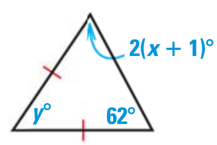
32.



33.



34.



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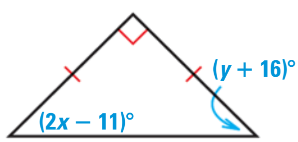
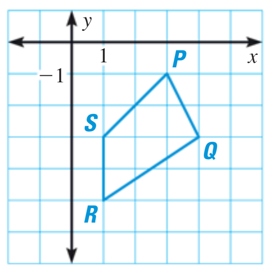


4.8 Copy the figure and draw its image after the transformation.

36. Reflection: in the y-axis

37. Reflection: in the x-axis

38. Translation: $(x, y) \rightarrow (x - 3, y + 7)$



4.8 Use the coordinates to graph $\overline{AB}$ and $\overline{CD}$. Tell whether $\overline{CD}$ is a rotation of $\overline{AB}$ about the origin. If so, give the angle and direction of rotation.

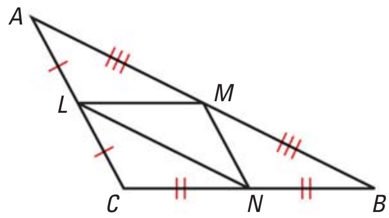
39. $A(4, 2)$, $B(1, 1)$, $C(-4, -2)$, $D(-1, -1)$

40. $A(-1, 3)$, $B(0, 2)$, $C(-1, 2)$, $D(-3, 1)$

Chapter 5

5.1 Copy and complete the statement.

1. $\overline{LN} \parallel$ __?__
2. $\overline{CB} \parallel$ __?__
3. $\overline{MN} \parallel$ __?__
4. $AM =$ __?__ $=$ __?__
5. $MN =$ __?__ $=$ __?__

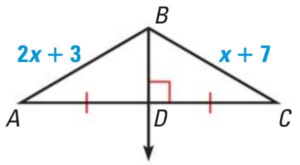


5.1 Place the figure in a coordinate plane in a convenient way. Assign coordinates to each vertex.

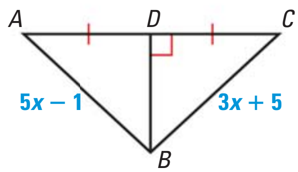
6. Isosceles right triangle: leg length is 4 units
7. Scalene triangle: one side length is 6 units
8. Square: side length is 5 units
9. Right triangle: leg lengths are s and t

5.2 Find the length of $\overline{AB}$.

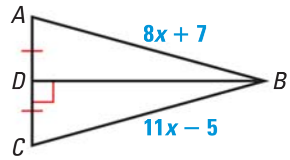
10.



11.

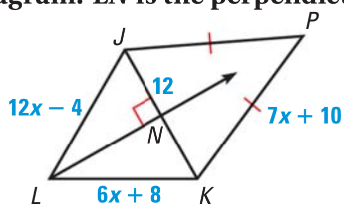


12.



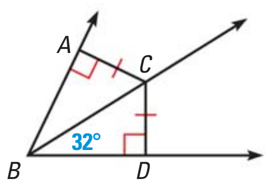
5.2 In Exercises 13–17, use the diagram. $\overrightarrow{LN}$ is the perpendicular bisector of $\overline{JK}$.

13. Find KN.
14. Find LJ.
15. Find KP.
16. Find JP.
17. Is P on LN?

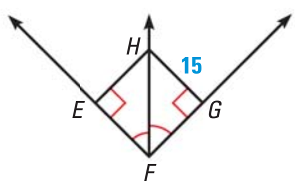


5.3 Use the information in the diagram to find the measure.

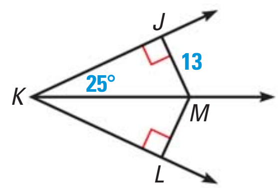
18. Find $m\angle ABC$.



19. Find EH.

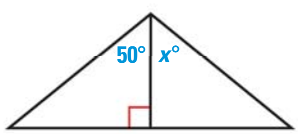


20. $m\angle JKL = 50°$. Find LM.

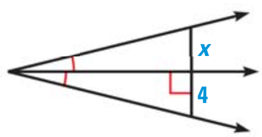


5.3 Can you find the value of x? *Explain*.

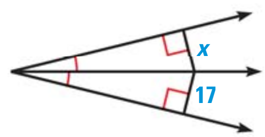
21.



22.

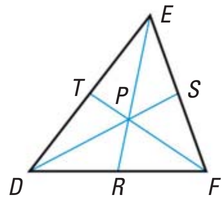


23.



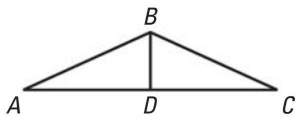
5.4 ***P*** **is the centroid of $\triangle DEF$, $FP = 14$, $RE = 24$, and $PS = 8.5$. Find the length of the segment.**

24. $\overline{TF}$
25. $\overline{DP}$
26. $\overline{DS}$
27. $\overline{PR}$



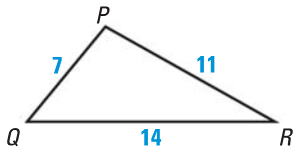
5.4 **Use the diagram shown and the given information to decide whether $\overline{BD}$ is a *perpendicular bisector*, an *angle bisector*, a *median*, or an *altitude* of $\triangle ABC$.**

28. $\overline{BD} \perp \overline{AC}$
29. $\angle ABD \cong \angle CBD$
30. $\overline{AD} \cong \overline{CD}$
31. $\overline{BD} \perp \overline{AC}$ and $\overline{AD} \cong \overline{CD}$
32. $\triangle ABD \cong \triangle CBD$
33. $\overline{BD} \perp \overline{AC}$ and $\overline{AB} \cong \overline{CB}$

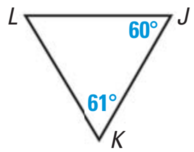


5.5 **List the sides and angles in order from smallest to largest.**

34.



35.



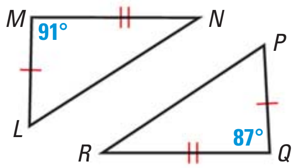
36.

5.5 ***Describe*** **the possible lengths of the third side of the triangle given the lengths of the other two sides.**

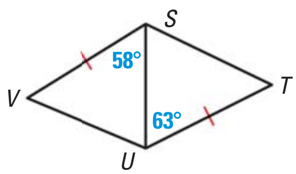
37. 9 inches, 8 inches
38. 24 feet, 13 feet
39. 3 inches, 9 inches
40. 1 foot, 17 inches
41. 4 feet, 2 yards
42. 2 yards, 6 feet

5.6 **Copy and complete with $>$, $<$ or $=$. *Explain*.**

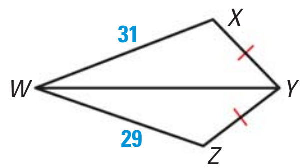
43. LN __?__ PR



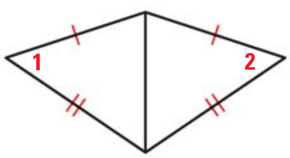
44. VU __?__ ST



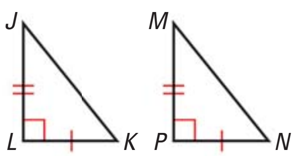
45. $m\angle WYX$ __?__ $m\angle WYZ$



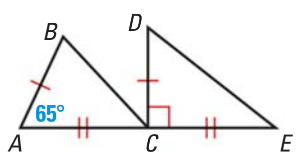
46. $m\angle 1$ __?__ $m\angle 2$



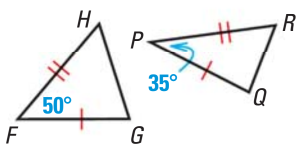
47. JK __?__ MN



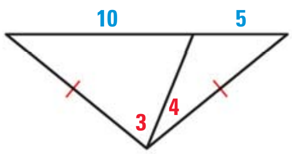
48. BC __?__ DE



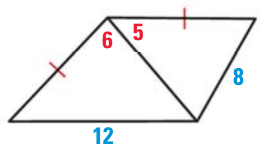
49. GH __?__ QR



50. $m\angle 3$ __?__ $m\angle 4$



51. $m\angle 5$ __?__ $m\angle 6$



EXTRA PRACTICE

Chapter 6

6.1 The measures of the angles of a triangle are in the extended ratio given. Find the measures of the angles of the triangle.

1. 1 : 3 : 5 **2.** 1 : 5 : 6 **3.** 2 : 3 : 5 **4.** 5 : 6 : 9

6.1 Solve the proportion.

5. $\frac{x}{14} = \frac{6}{21}$ **6.** $\frac{15}{y} = \frac{20}{4}$ **7.** $\frac{3}{2z+1} = \frac{1}{7}$ **8.** $\frac{a-3}{2} = \frac{2a-1}{6}$

9. $\frac{6}{3} = \frac{x+8}{-1}$ **10.** $\frac{x+6}{3} = \frac{x-5}{2}$ **11.** $\frac{x-2}{4} = \frac{x+10}{10}$ **12.** $\frac{12}{8} = \frac{5+t}{t-3}$

6.1 Find the geometric mean of the two numbers.

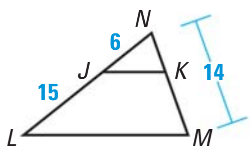
13. 4 and 9 **14.** 3 and 48 **15.** 9 and 16 **16.** 7 and 11

6.2 Copy and complete the statement.

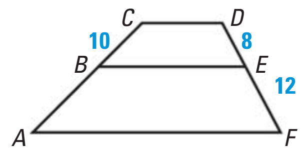
17. If $\frac{7}{x} = \frac{9}{y}$, then $\frac{x}{7} = \frac{?}{?}$. **18.** If $\frac{2}{8} = \frac{1}{x}$, then $\frac{8+2}{2} = \frac{?}{?}$.

6.2 Use the diagram and the given information to find the unknown length.

19. Given $\frac{NJ}{NK} = \frac{NL}{NM}$, find NK.

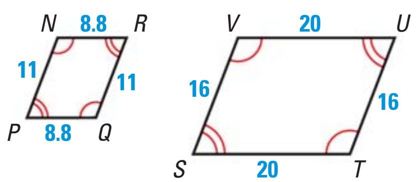


20. Given $\frac{CB}{DE} = \frac{BA}{EF}$, find CA.

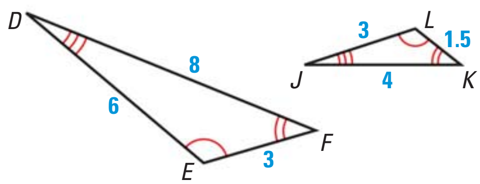


6.3 Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor.

21.



22.

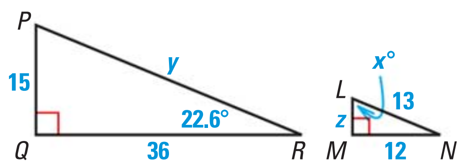


6.3 In the diagram, $\triangle PQR \sim \triangle LMN$.

23. Find the scale factor of $\triangle PQR$ to $\triangle LMN$.

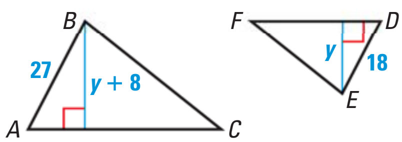
24. Find the values of x, y, and z.

25. Find the perimeter of each triangle.

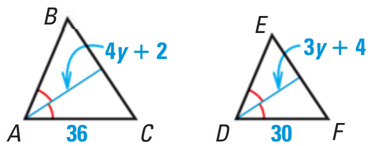


6.3 $\triangle ABC \sim \triangle DEF$. Identify the blue special segment and find the value of y.

26.

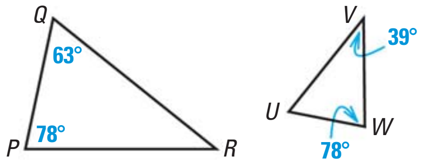


27.

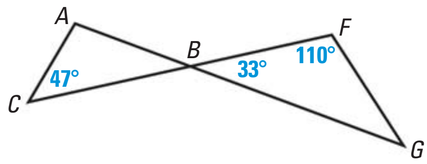


6.4 **In Exercises 28–31, determine whether the triangles are similar. If they are, write a similarity statement. *Explain* your reasoning.**

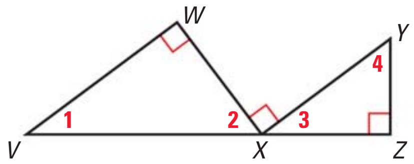
28.



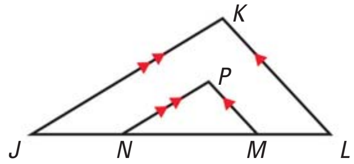
29.



30.

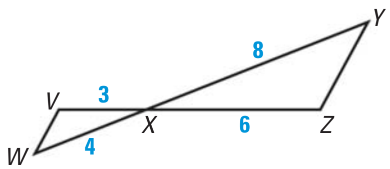


31.



6.5 **Show that the triangles are similar and write a similarity statement. *Explain* your reasoning.**

32.



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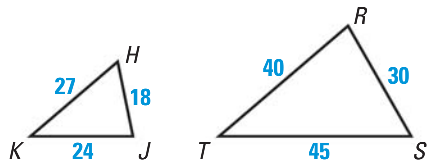
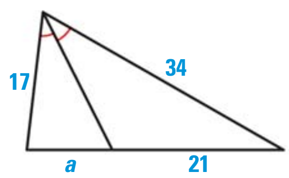


6.6 **Use the diagram to find the value of each variable.**

34.



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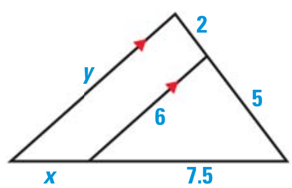

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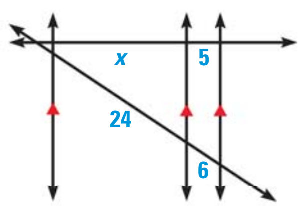


6.7 **Draw a dilation of the polygon with the given vertices using the given scale factor of *k*.**

37. $A(1, 1)$, $B(4, 1)$, $C(1, 2)$; $k = 3$

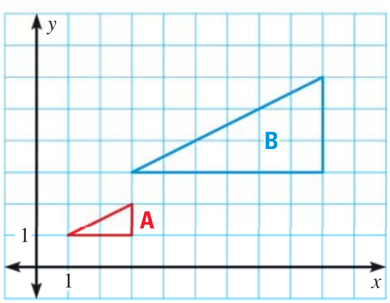
38. $A(2, 2)$, $B(-2, 2)$, $C(-1, -1)$, $D(2, -1)$; $k = 5$

39. $A(2, 2)$, $B(8, 2)$, $C(2, 6)$; $k = \frac{1}{2}$

40. $A(3, -6)$, $B(6, -6)$, $C(6, 9)$, $D(-3, 9)$; $k = \frac{1}{3}$

6.7 **Determine whether the dilation from Figure A to Figure B is a *reduction* or an *enlargement*. Then find its scale factor.**

41.



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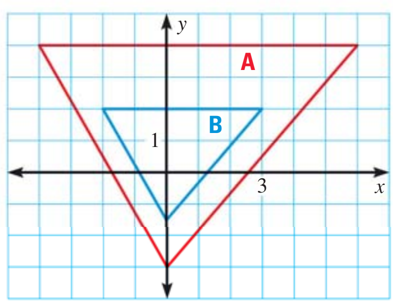
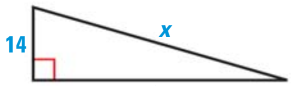
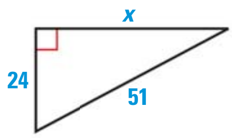
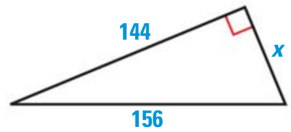


Chapter 7

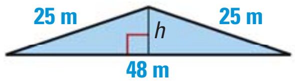
7.1 Find the unknown side length of the right triangle using the Pythagorean Theorem or a Pythagorean triple.

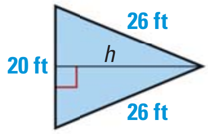
1. 

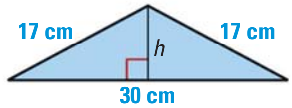
2. 

3. 

7.1 Find the area of the isosceles triangle.

4. 

5. 

6. 

7.2 Tell whether the given side lengths of a triangle can represent a right triangle.

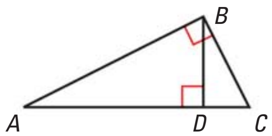
7. 24, 32, and 40
8. 21, 72, and 75
9. 11, 25, and 27
10. 7, 11, and 13
11. 17, 19, and $5\sqrt{26}$
12. 9, 10, and $\sqrt{181}$

7.2 Decide if the segment lengths form a triangle. If so, would the triangle be *acute*, *right*, or *obtuse*?

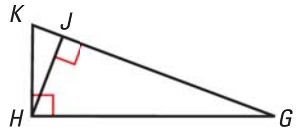
13. 14, 21, and 25
14. 32, 60, and 68
15. 11, 19, and 32
16. 3, 9, and $3\sqrt{11}$
17. 12, 15, and $3\sqrt{40}$
18. $4\sqrt{21}$, 25, and 31

7.3 Write a similarity statement for the three similar triangles in the diagram. Then complete the proportion.

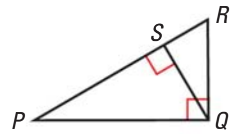
19. $\frac{AB}{AD} = \frac{BC}{?}$



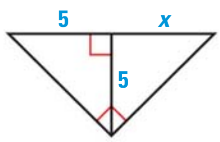
20. $\frac{KJ}{HJ} = \frac{?}{JG}$

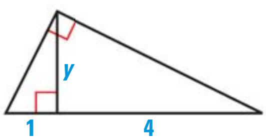


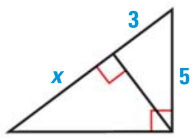
21. $\frac{SR}{RQ} = \frac{RQ}{?}$

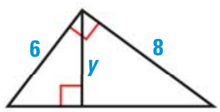


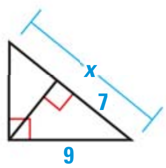
7.3 Find the value of the variable. Round decimal answers to the nearest tenth.

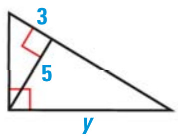
22. 

23. 

24. 

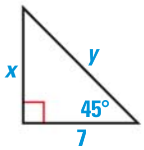
25. 

26. 

27. 

7.4 **Find the value of each variable. Write your answers in simplest radical form.**

28.



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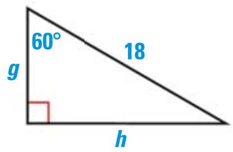

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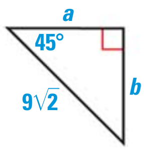

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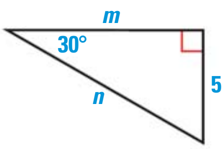

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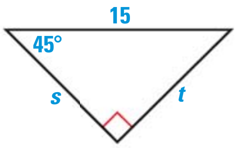

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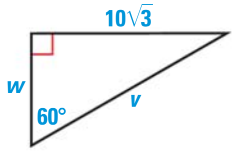
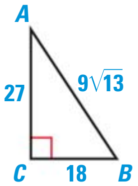


7.5 **Find tan *A* and tan *B*. Write each answer as a fraction and as a decimal rounded to four places.**

34.



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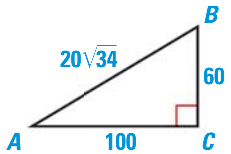

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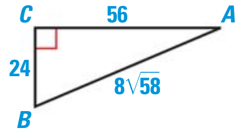
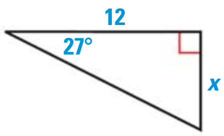


7.5 **Use a tangent ratio to find the value of *x*. Round to the nearest tenth. Check your solution using the tangent of the other acute angle.**

37.



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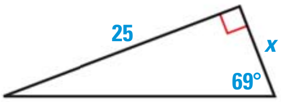

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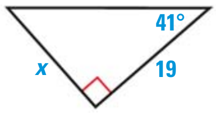
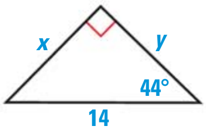


7.6 **Use a sine or cosine ratio to find the value of each variable. Round decimals to the nearest tenth.**

40.



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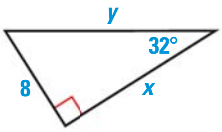

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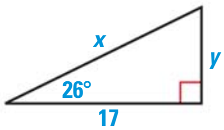

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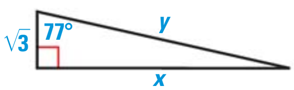

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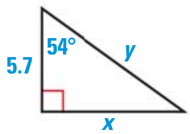

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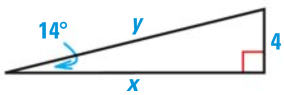
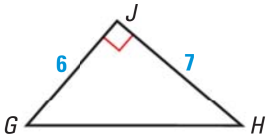


7.7 **Solve the right triangle. Round decimal answers to the nearest tenth.**

46.

47.



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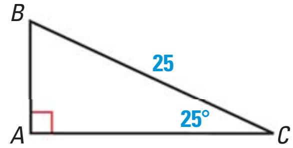

Chapter 8

8.1 Find the value of x.

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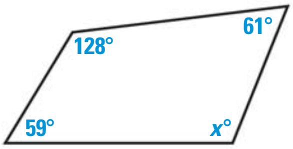

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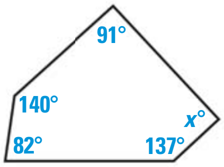

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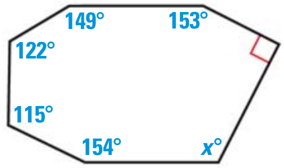

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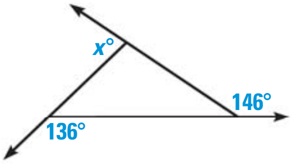

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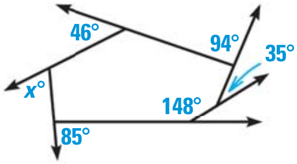

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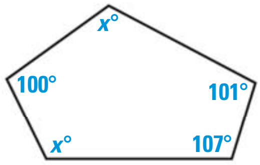
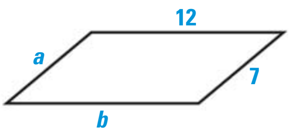


8.1 Find the measure of an interior angle and an exterior angle of the indicated regular polygon.

7. Regular hexagon
8. Regular 9-gon
9. Regular 17-gon

8.2 Find the value of each variable in the parallelogram.

10.


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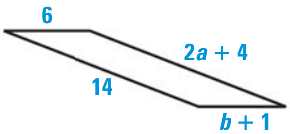

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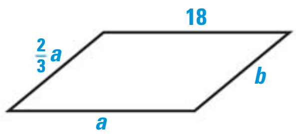
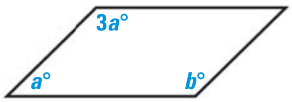


13.


14.


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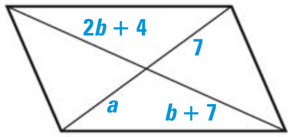
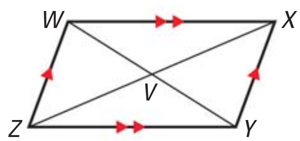


8.2 Use the diagram to copy and complete the statement.

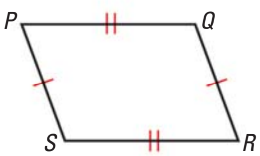
16. $\angle WXV \cong$ ___?___
17. $\angle ZWV \cong$ ___?___
18. $\angle WVX \cong$ ___?___
19. $WV =$ ___?___
20. $WZ =$ ___?___
21. $2 \cdot ZV =$ ___?___



8.3 The vertices of quadrilateral *ABCD* are given. Draw *ABCD* in a coordinate plane and show that it is a parallelogram.

22. $A(5, 6)$, $B(7, 3)$, $C(5, -2)$, $D(3, 1)$
23. $A(-8, 2)$, $B(-6, 3)$, $C(-1, 2)$, $D(-3, 1)$
24. $A(-1, 11)$, $B(2, 14)$, $C(6, 11)$, $D(3, 8)$
25. $A(-1, -5)$, $B(4, -4)$, $C(6, -9)$, $D(1, -10)$

8.3 *Describe* how to prove that quadrilateral *PQRS* is a parallelogram.

26.


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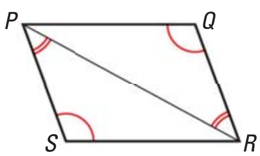

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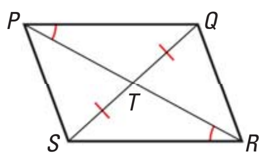
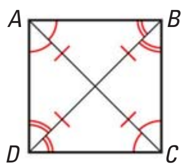


8.4 **Classify the special quadrilateral. *Explain* your reasoning.**

29.


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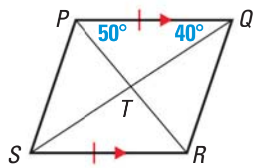

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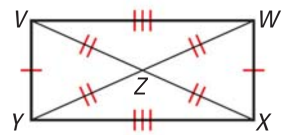
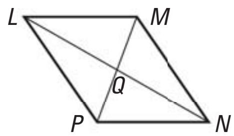


8.4 **The diagonals of rhombus *LMNP* intersect at *Q*. Given that *LM* = 5 and $m\angle QLM = 30°$, find the indicated measure.**

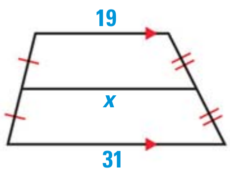
32. $m\angle LMQ$

33. $m\angle LQM$

34. MN



8.5 **Find the value of *x*.**

35.


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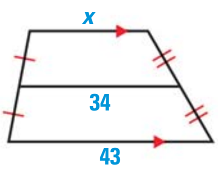

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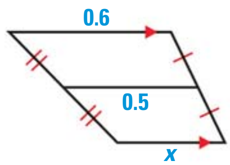
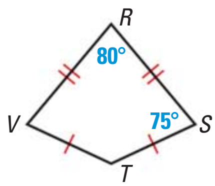


8.5 ***RSTV* is a kite. Find $m\angle V$.**

38.


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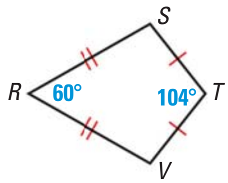

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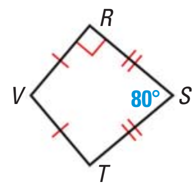
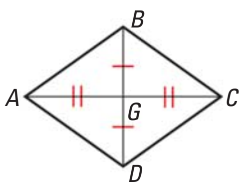


8.6 **Give the most specific name for the quadrilateral. *Explain* your reasoning.**

41.


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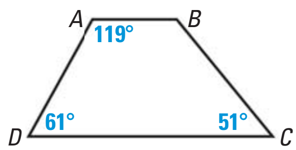

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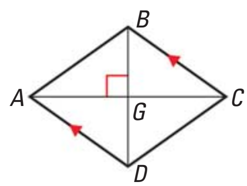

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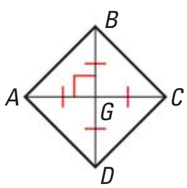

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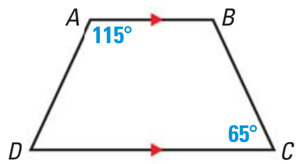
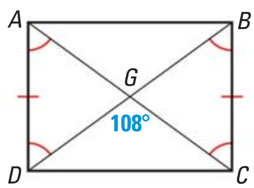


46.


8.6 **The vertices of quadrilateral *DEFG* are given. Give the most specific name for *DEFG*. *Justify* your answer.**

47. $D(6, 8)$, $E(9, 12)$, $F(12, 8)$, $G(9, 6)$

48. $D(1, 2)$, $E(4, 1)$, $F(3, -2)$, $G(0, -1)$

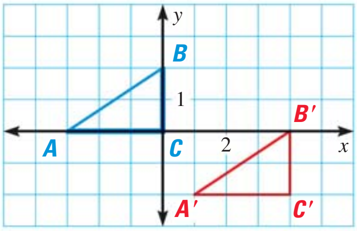
49. $D(10, 3)$, $E(14, 4)$, $F(20, 2)$, $G(12, 0)$

50. $D(-2, 10)$, $E(1, 13)$, $F(5, 13)$, $G(-2, 6)$

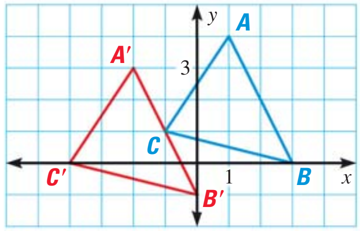
Chapter 9

9.1 **$\triangle A'B'C'$ is the image of $\triangle ABC$ after a translation. Write a rule for the translation. Then *verify* that the translation is an isometry.**

1.



2.



9.1 **Use the point $P(7, -3)$. Find the component form of the vector that describes the translation to P'.**

3. $P'(-3, 4)$ **4.** $P'(1, -1)$ **5.** $P'(3, 2)$ **6.** $P'(-8, -11)$

9.2 **Add, subtract, or multiply.**

7. $\begin{bmatrix} 2 \\ 7 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

8. $\begin{bmatrix} 5 & -3 \\ -9 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 4 & -1 \end{bmatrix}$

9. $\begin{bmatrix} 7 & -3 \\ 5 & 9 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 6 & 8 \end{bmatrix}$

9.2 **Find the image matrix that represents the translation of the polygon. Then graph the polygon and its image.**

10. $\begin{bmatrix} 3 & -5 & 7 \\ -2 & -2 & 1 \end{bmatrix}$; 6 units left

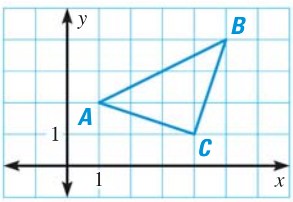
11. $\begin{bmatrix} 1 & 9 & 4 & 3 \\ 5 & 6 & 5 & 2 \end{bmatrix}$; 1 unit right and 7 units down

12. $\begin{bmatrix} 7 & -3 & 0 \\ 6 & 8 & -4 \end{bmatrix}$; 3 units right and 4 units up

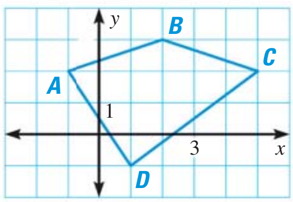
13. $\begin{bmatrix} 9 & 6 & 4 & 2 & 3 \\ -1 & -4 & -4 & -4 & 2 \end{bmatrix}$; 4 units left and 5 units up

9.3 **Graph the reflection of the polygon in the given line.**

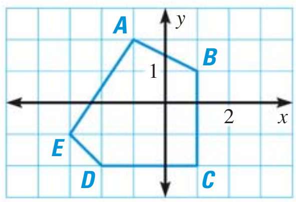
14. y-axis



15. $x = 1$

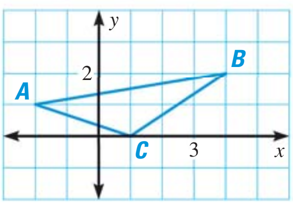


16. $y = x$

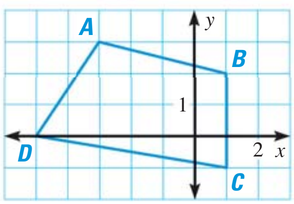


9.4 **Rotate the figure the given number of degrees about the origin. List the coordinates of the vertices of the image.**

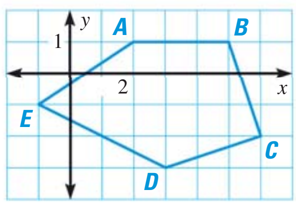
17. 270°



18. 180°



19. 90°



9.4 **Find the image matrix that represents the rotation of the polygon about the origin. Then graph the polygon and its image.**

20. $\begin{array}{c} \begin{matrix} P & Q & R \end{matrix} \\ \begin{bmatrix} 1 & 2 & 4 \\ 4 & 1 & 3 \end{bmatrix} \end{array}$; 180°

21. $\begin{array}{c} \begin{matrix} S & T & V \end{matrix} \\ \begin{bmatrix} 4 & 2 & 1 \\ 2 & -3 & 0 \end{bmatrix} \end{array}$; 90°

22. $\begin{array}{c} \begin{matrix} A & B & C & D \end{matrix} \\ \begin{bmatrix} 4 & -1 & -2 & 1 \\ 0 & -1 & -2 & -3 \end{bmatrix} \end{array}$; 270°

9.5 **The vertices of $\triangle ABC$ are $A(1, 1)$, $B(4, 1)$, and $C(2, 4)$. Graph the image of $\triangle ABC$ after a composition of the transformations in the order they are listed.**

23. **Translation:** $(x, y) \rightarrow (x - 2, y + 3)$
Rotation: 270° about the origin

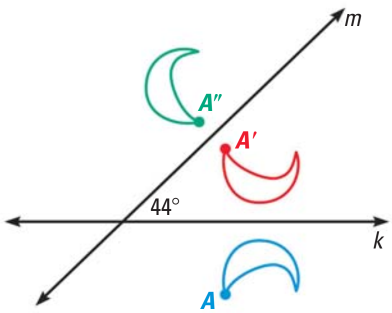
24. **Reflection:** in the line $x = 2$
Translation: $(x, y) \rightarrow (x + 3, y)$

25. **Rotation:** 180° about the origin
Reflection: in the line $y = -2$

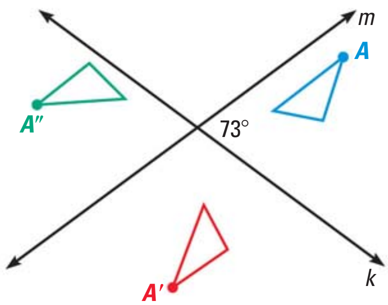
26. **Translation:** $(x, y) \rightarrow (x - 4, y - 4)$
Reflection: in the line $y = x$

9.5 **Find the angle of rotation that maps A onto A''.**

27.



28.



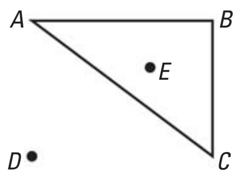
9.6 **Determine whether the flag has *line symmetry* and whether it has *rotational symmetry*. Identify all lines of symmetry and angles of rotation that map the figure onto itself.**

29.

30.

31.

9.7 **Copy the diagram. Then draw the given dilation.**



32. Center B; $k = 2$

33. Center E; $k = 3$

34. Center D; $k = \frac{1}{2}$

35. Center A; $k = \frac{2}{3}$

36. Center C; $k = \frac{3}{2}$

37. Center E; $k = \frac{1}{3}$

9.7 **Find the image matrix that represents a dilation of a polygon centered at the origin with a given scale factor. Then graph the polygon and its image.**

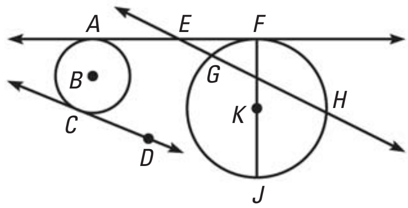
38. $\begin{array}{c} \begin{matrix} G & H & J \end{matrix} \\ \begin{bmatrix} 1 & 3 & 4 \\ 4 & 2 & 4 \end{bmatrix} \end{array}$; $k = 3$

39. $\begin{array}{c} \begin{matrix} K & L & M & N \end{matrix} \\ \begin{bmatrix} 2 & 4 & 5 & 6 \\ -2 & -2 & 4 & 0 \end{bmatrix} \end{array}$; $k = \frac{1}{2}$

40. $\begin{array}{c} \begin{matrix} P & Q & R \end{matrix} \\ \begin{bmatrix} -3 & -3 & -1 \\ -1 & -3 & -3 \end{bmatrix} \end{array}$; $k = 4$

Chapter 10

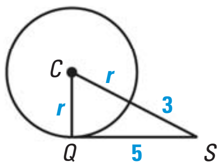
10.1 Use the diagram to give an example of the term.



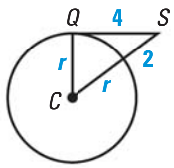
1. Radius
2. Common tangent
3. Tangent
4. Secant
5. Center
6. Point of tangency
7. Chord
8. Diameter

10.1 Find the value(s) of the variable. *P*, *Q*, and *R* are points of tangency.

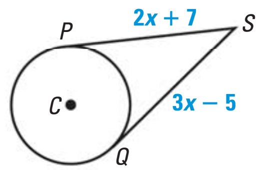
9.



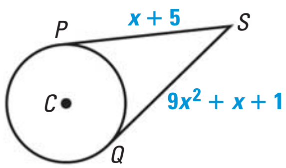
10.



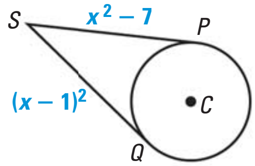
11.



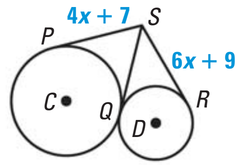
12.



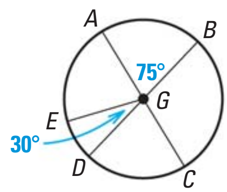
13.



14.



10.2 $\overline{AC}$ and $\overline{BD}$ are diameters of $\odot G$. Determine whether the arc is a *minor arc*, a *major arc*, or a *semicircle* of $\odot G$. Then find the measure of the arc.



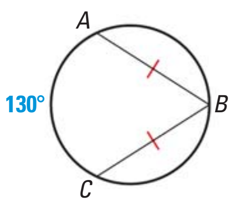
15. $\overset{\frown}{ED}$
16. $\overset{\frown}{EB}$
17. $\overset{\frown}{EC}$
18. $\overset{\frown}{BEC}$
19. $\overset{\frown}{BC}$
20. $\overset{\frown}{BCD}$

10.2 In $\odot C$, $m\overset{\frown}{AD} = 50°$, *B* bisects $\overset{\frown}{AD}$, and $\overline{AE}$ is a diameter. Find the measure of the arc.

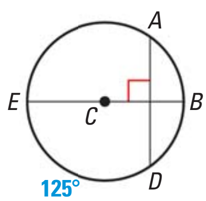
21. $\overset{\frown}{AED}$
22. $\overset{\frown}{BD}$
23. $\overset{\frown}{DE}$
24. $\overset{\frown}{BAE}$

10.3 Find the measure of $\overset{\frown}{AB}$.

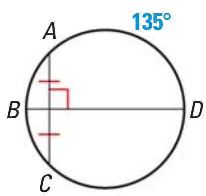
25.



26.

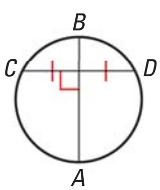


27.



10.3 In Exercises 28–30, what can you conclude about the diagram shown? State theorems to justify your answer.

28.



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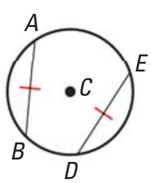
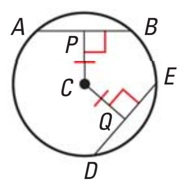
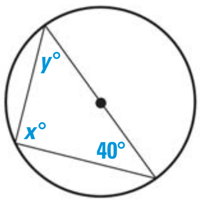


30.



10.4 Find the values of the variables.

31.


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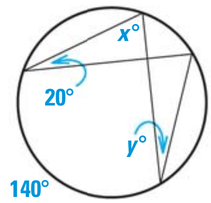

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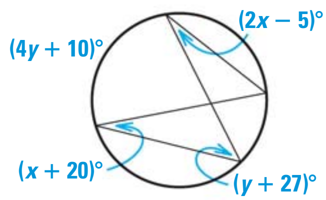

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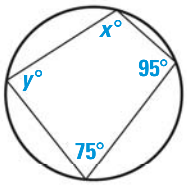

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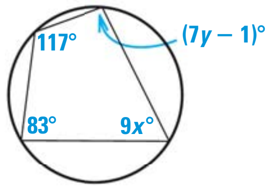

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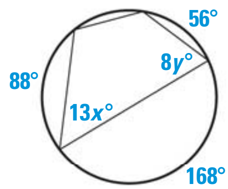
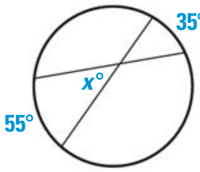


10.5 Find the value of *x*.

37.


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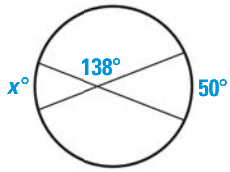

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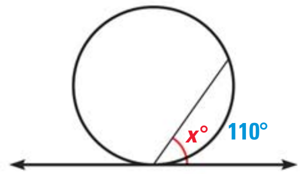

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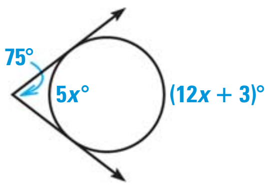

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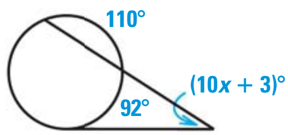

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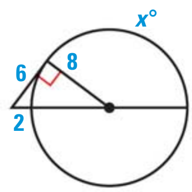
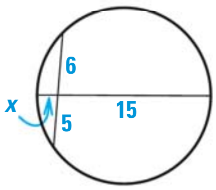


10.6 Find the value of *x*.

43.


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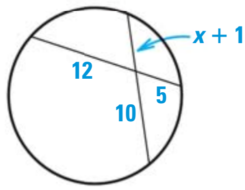

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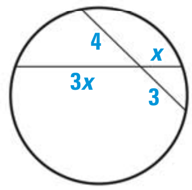

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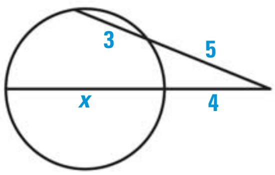

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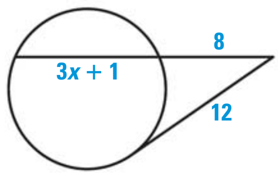

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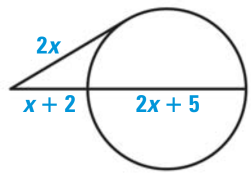


10.7 Use the given information to write the standard equation for the circle.

49. The center is $(0, -2)$, and the radius is 4 units.

50. The center is $(2, -3)$, and a point on the circle is $(7, -8)$.

51. The center is (m, n), and a point on the circle is $(m + h, n + k)$.

10.7 Graph the equation.

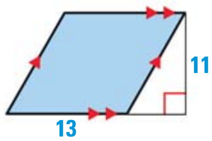
52. $x^2 + y^2 = 25$

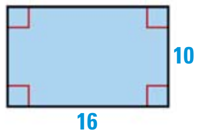
53. $x^2 + (y - 5)^2 = 121$

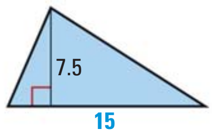
54. $(x + 4)^2 + (y - 1)^2 = 49$

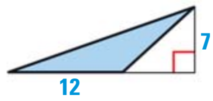
Chapter 11

11.1 Find the area of the polygon.

1. 

2. 

3. 

4. 

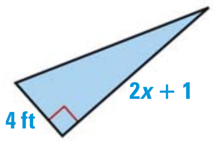
11.1 The lengths of the hypotenuse and one leg of a right triangle are given. Find the perimeter and area of the triangle.

5. Hypotenuse: 25 cm; leg: 20 cm

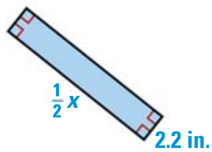
6. Hypotenuse: 51 ft; leg: 24 ft

11.1 Find the value of x.

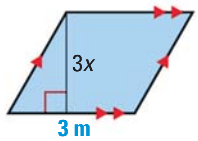
7. $A = 22\text{ ft}^2$



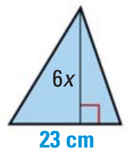
8. $A = 14.3\text{ in.}^2$



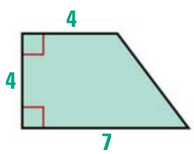
9. $A = 7.2\text{ m}^2$

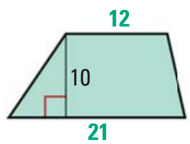


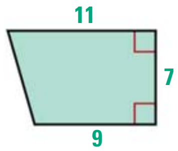
10. $A = 276\text{ cm}^2$

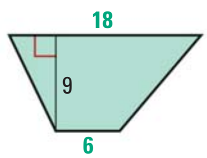


11.2 Find the area of the trapezoid.

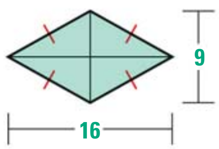
11. 

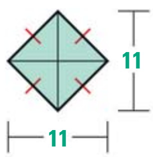
12. 

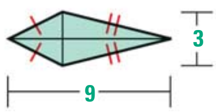
13. 

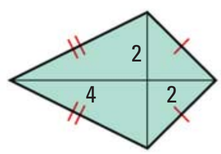
14. 

11.2 Find the area of the rhombus or kite.

15. 

16. 

17. 

18. 

11.3 The ratio of the areas of two similar figures is given. Write the ratio of the lengths of the corresponding sides.

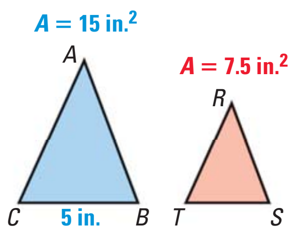
19. Ratio of areas $= 100:81$

20. Ratio of areas $= 25:100$

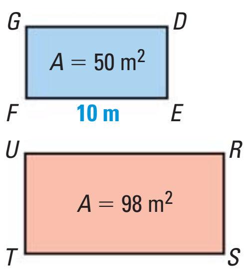
21. Ratio of areas $= 8:1$

11.3 Use the given area to find ST.

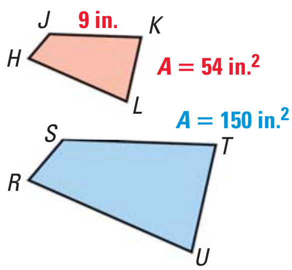
22. $\triangle ABC \sim \triangle RST$



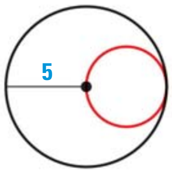
23. $DEFG \sim RSTU$

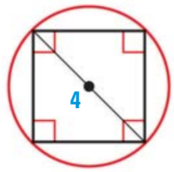


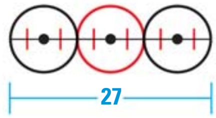
24. $HJKL \sim RSTU$

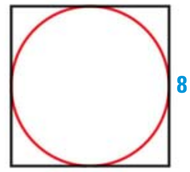


11.4 **Find the circumference of the red circle.**

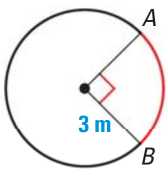
25. 

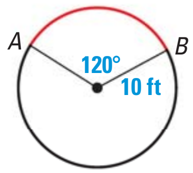
26. 

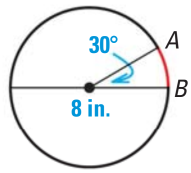
27. 

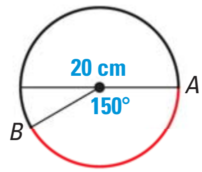
28. 

11.4 **Find the length of $\widehat{AB}$.**

29. 

30. 

31. 

32. 

11.5 **Find the exact area of a circle with the given radius r or diameter d. Then find the area to the nearest hundredth.**

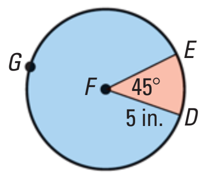
33. $r = 3$ in.

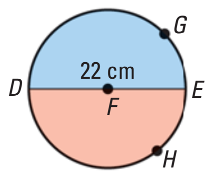
34. $r = 2.5$ cm

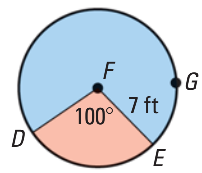
35. $d = 20$ ft

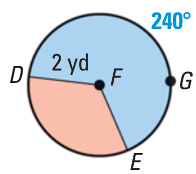
36. $d = 13$ m

11.5 **Find the areas of the sectors formed by $\angle DFE$.**

37. 

38. 

39. 

40. 

11.6 **Find the measure of a central angle of a regular polygon with the given number of sides.**

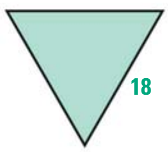
41. 8 sides

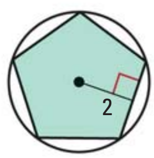
42. 12 sides

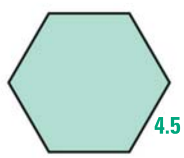
43. 20 sides

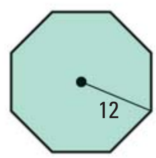
44. 25 sides

11.6 **Find the perimeter and area of the regular polygon.**

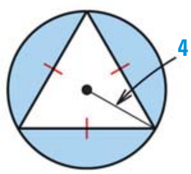
45. 

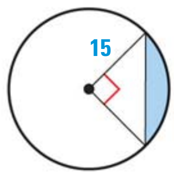
46. 

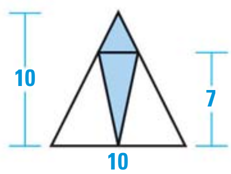
47. 

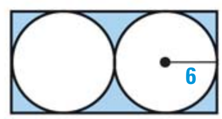
48. 

11.7 **Find the probability that a randomly chosen point in the figure lies in the shaded region.**

49. 

50. 

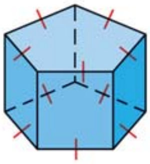
51. 

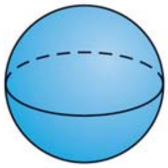
52. 

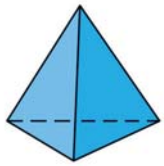
11.7 **53.** A local radio station plays your favorite song once every two hours. Your favorite song is 4.5 minutes long. If you randomly turn on the radio, what is the probability that your favorite song will be playing?

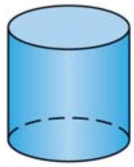
Chapter 12

12.1 **Determine whether the solid is a polyhedron. If it is, name the polyhedron. *Explain* your reasoning.**

1. 

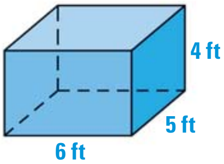
2. 

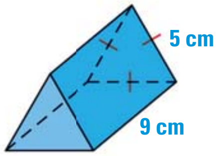
3. 

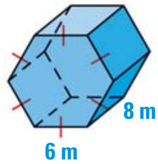
4. 

12.1 **5.** Determine the number of faces on a solid with six vertices and ten edges.

12.2 **Find the surface area of the right prism. Round to two decimal places.**

6. 

7. 

8. 

12.2 **Find the surface area of the right cylinder with the given radius r and height h. Round to two decimal places.**

9. $r = 2$ cm
$h = 11$ cm

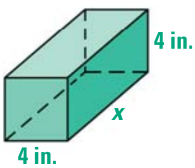
10. $r = 1$ m
$h = 1$ m

11. $r = 22$ in.
$h = 9$ in.

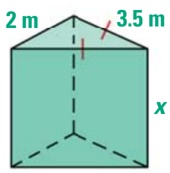
12. $r = 17$ mm
$h = 5$ mm

12.2 **Solve for x given the surface area S of the right prism or right cylinder. Round to two decimal places.**

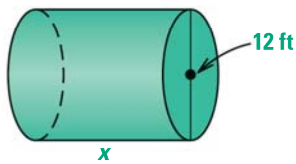
13. $S = 192$ in.^{2}



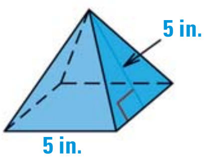
14. $S = 33.7$ m^2

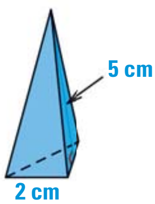


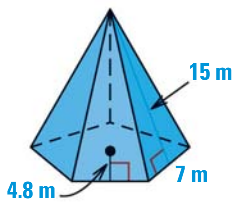
15. $S = 754$ ft^2



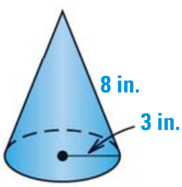
12.3 **Find the surface area of the regular pyramid. Round to two decimal places.**

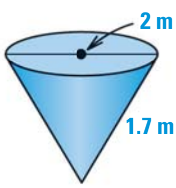
16. 

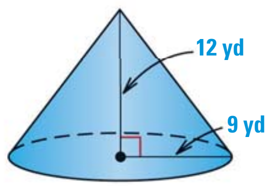
17. 

18. 

12.3 **Find the surface area of the right cone. Round to two decimal places.**

19. 

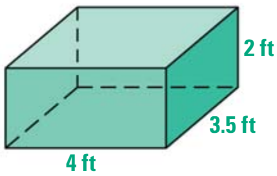
20. 

21. 

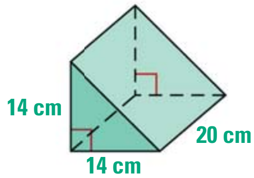
EXTRA PRACTICE

12.4 **Find the volume of the right prism or right cylinder. Round to two decimal places.**

22.



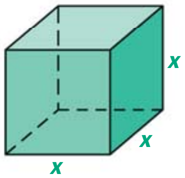
23.



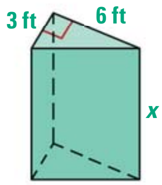
24.

12.4 **Find the value of x. Round to two decimal places, if necessary.**

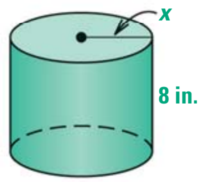
25. $V = 8\text{ cm}^3$



26. $V = 72\text{ ft}^3$

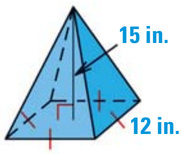


27. $V = 628\text{ in.}^3$



12.5 **Find the volume of the solid. Round to two decimal places.**

28.



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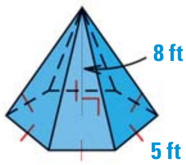

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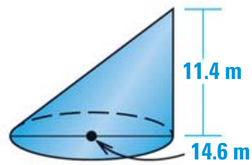
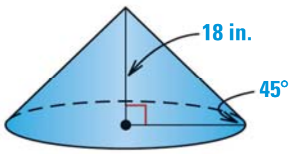


12.5 **Find the volume of the right cone. Round to two decimal places.**

31.



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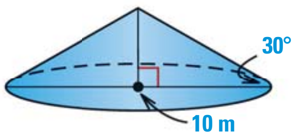
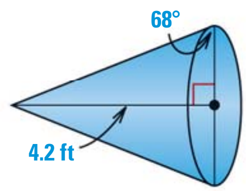


33.



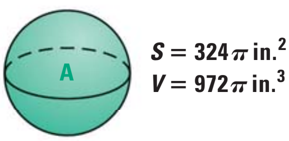
12.6 **Find the surface area and volume of a sphere with the given radius r or diameter d. Round to two decimal places.**

34. $r = 13$ m　**35.** $r = 1.8$ in.　**36.** $d = 28$ yd　**37.** $d = 13.7$ cm

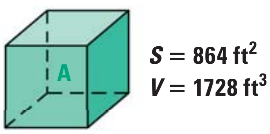
38. $r = 20$ in.　**39.** $r = 17.5$ mm　**40.** $d = 15.2$ m　**41.** $d = 23$ ft

12.7 **Solid A (shown) is similar to Solid B (not shown) with the given scale factor of A to B. Find the surface area and volume of Solid B.**

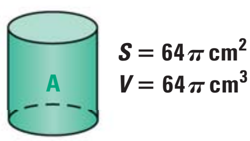
42. Scale factor of 3 : 2



43. Scale factor of 2 : 1



44. Scale factor of 4 : 7



12.7 **45.** Two similar cylinders have volumes 12π cubic units and 324π cubic units. Find the scale factor of the smaller cylinder to the larger cylinder.

Symbols

Symbol	Meaning	Page
$-a$	opposite of a	xxii
$\overleftrightarrow{AB}$	line AB	2
$\overline{AB}$	segment AB	3
$\overrightarrow{AB}$	ray AB	3
$\cdot$	multiplication, times	8
AB	the length of AB	9
$\lvert x \rvert$	absolute value of x	9
x_1	x sub one	9
(x, y)	ordered pair	11
$=$	is equal to	11
$\cong$	is congruent to	11
$\sqrt{a}$	square root of a	14
$\angle ABC$	angle ABC	24
$m\angle A$	measure of angle A	24
$^\circ$	degree(s)	24
$\urcorner$	right angle symbol	25
n-gon	polygon with n sides	43
π	pi; irrational number ≈ 3.14	49
$\approx$	is approximately equal to	50
$\ldots$	and so on	72
$\perp$	is perpendicular to	81
$\rightarrow$	implies	94
$\leftrightarrow$	if and only if	94
$\sim p$	negation of statement p	94
$\parallel$	is parallel to	147
m	slope	171
$\triangle ABC$	triangle ABC	217

Symbol	Meaning	Page
$\triangle$s	triangles	227
$\angle$s	angles	250
$\rightarrow$	maps to	272
$<$	is less than	328
$>$	is greater than	328
$\neq$	is not equal to	337
$\frac{a}{b}$, $a:b$	ratio of a to b	356
$\sim$	is similar to	372
$\stackrel{?}{=}$	is this statement true?	389
$\nparallel$	is not parallel to	398
tan	tangent	466
sin	sine	473
cos	cosine	473
$\sin^{-1}$	inverse sine	483
$\cos^{-1}$	inverse cosine	483
$\tan^{-1}$	inverse tangent	483
$\square ABCD$	parallelogram $ABCD$	515
$\ncong$	is not congruent to	531
A'	A prime	572
$\overrightarrow{AB}$	vector AB	574
$\langle a, b\rangle$	component form of a vector	574
A''	A double prime	608
$\odot P$	circle with center P	651
$m\overset{\frown}{AB}$	measure of minor arc AB	659
$m\overset{\frown}{ABC}$	measure of major arc ABC	659
$P(A)$	probability of event A	771

Measures

Time	
60 seconds (sec) = 1 minute (min) 60 minutes = 1 hour (h) 24 hours = 1 day 7 days = 1 week 4 weeks (approx.) = 1 month	365 days, 52 weeks (approx.), 12 months = 1 year 10 years = 1 decade 100 years = 1 century

Metric	United States Customary
Length 10 millimeters (mm) = 1 centimeter (cm) 100 cm, 1000 mm = 1 meter (m) 1000 m = 1 kilometer (km)	**Length** 12 inches (in.) = 1 foot (ft) 36 in., 3 ft = 1 yard (yd) 5280 ft, 1760 yd = 1 mile (mi)
Area 100 square millimeters (mm^2) = 1 square centimeter (cm^2) 10,000 cm^2 = 1 square meter (m^2) 10,000 m^2 = 1 hectare (ha)	**Area** 144 square inches ($in.^2$) = 1 square foot (ft^2) 9 ft^2 = 1 square yard (yd^2) 43,560 ft^2, 4840 yd^2 = 1 acre (A)
Volume 1000 cubic millimeters (mm^3) = 1 cubic centimeter (cm^3) 1,000,000 cm^3 = 1 cubic meter (m^3)	**Volume** 1728 cubic inches ($in.^3$) = 1 cubic foot (ft^3) 27 ft^3 = 1 cubic yard (yd^3)
Liquid Capacity 1000 milliliters (mL), 1000 cubic centimeters (cm^3) = 1 liter (L) 1000 L = 1 kiloliter (kL)	**Liquid Capacity** 8 fluid ounces (fl oz) = 1 cup (c) 2 c = 1 pint (pt) 2 pt = 1 quart (qt) 4 qt = 1 gallon (gal)
Mass 1000 milligrams (mg) = 1 gram (g) 1000 g = 1 kilogram (kg) 1000 kg = 1 metric ton (t)	**Weight** 16 ounces (oz) = 1 pound (lb) 2000 lb = 1 ton
Temperature **Degrees Celsius (°C)** 0°C = freezing point of water 37°C = normal body temperature 100°C = boiling point of water	**Temperature** **Degrees Fahrenheit (°F)** 32°F = freezing point of water 98.6°F = normal body temperature 212°F = boiling point of water

Formulas

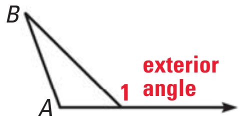
Angles

Sum of the measures of the interior angles of a triangle: 180° *(p. 218)*

Sum of the measures of the interior angles of a convex n-gon: $(n - 2) \cdot 180°$ *(p. 507)*

Exterior angle of a triangle:

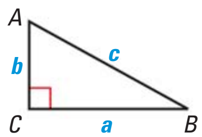
$m\angle 1 = m\angle A + m\angle B$ *(p. 219)*



Sum of the measures of the exterior angles of a convex polygon: 360° *(p. 509)*

Right Triangles

Pythagorean Theorem:
$c^2 = a^2 + b^2$ *(p. 433)*



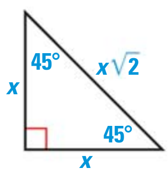
Trigonometric ratios:

$\sin A = \frac{BC}{AB}$ *(p. 473)* $\quad \sin^{-1}\frac{BC}{AB} = m\angle A$ *(p. 483)*

$\cos A = \frac{AC}{AB}$ *(p. 473)* $\quad \cos^{-1}\frac{AC}{AB} = m\angle A$ *(p. 483)*

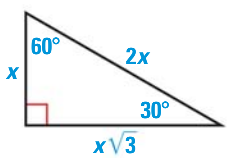
$\tan A = \frac{BC}{AC}$ *(p. 466)* $\quad \tan^{-1}\frac{BC}{AC} = m\angle A$ *(p. 483)*

45°-45°-90° triangle *(p. 457)*



Ratio of sides: $1:1:\sqrt{2}$

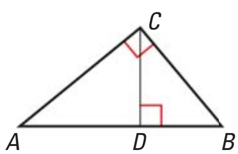
30°-60°-90° triangle *(p. 459)*



Ratio of sides: $1:\sqrt{3}:2$

$\triangle ABC \sim \triangle ACD \sim \triangle CBD$
(p. 449)

$\frac{BD}{CD} = \frac{CD}{AD}, \frac{AB}{CB} = \frac{CB}{DB}, \frac{AB}{AC} = \frac{AC}{AD}$
(p. 451)



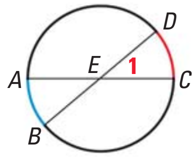
$\frac{BD}{CD} = \frac{CD}{AD}$, and $CD = \sqrt{AD \cdot DB}$ *(pp. 359, 452)*

Circles

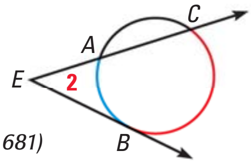
Angle and segments formed by two chords:

$m\angle 1 = \frac{1}{2}(m\overset{\frown}{CD} + m\overset{\frown}{AB})$ *(p. 681)*

$EA \cdot EC = EB \cdot ED$ *(p. 689)*



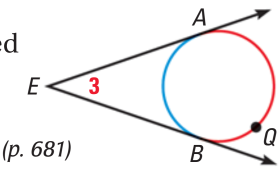
Angle and segments formed by a tangent and a secant:



$m\angle 2 = \frac{1}{2}(m\overset{\frown}{BC} - m\overset{\frown}{AB})$ *(p. 681)*

$EB^2 = EA \cdot EC$ *(p. 691)*

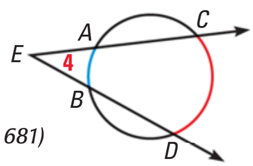
Angle and segments formed by two tangents:



$m\angle 3 = \frac{1}{2}(m\overset{\frown}{AQB} - m\overset{\frown}{AB})$ *(p. 681)*

$EA = EB$ *(p. 654)*

Angle and segments formed by two secants:



$m\angle 4 = \frac{1}{2}(m\overset{\frown}{CD} - m\overset{\frown}{AB})$ *(p. 681)*

$EA \cdot EC = EB \cdot ED$ *(p. 690)*

Coordinate Geometry

Given: points $A(x_1, y_1)$ and $B(x_2, y_2)$

Midpoint of $\overline{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ *(p. 16)*

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ *(p. 17)*

Slope of $\overleftrightarrow{AB} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$ *(p. 171)*

Slope-intercept form of a linear equation with slope m and y-intercept b: $y = mx + b$ *(p. 180)*

Standard equation of a circle with center (h, k) and radius r: $(x - h)^2 + (y - k)^2 = r^2$ *(p. 699)*

Taxicab distance $AB = |x_2 - x_1| + |y_2 - y_1|$
(p. 198)

Perimeter

P = perimeter, C = circumference, s = side, ℓ = length, w = width, a, b, c = lengths of the sides of a triangle, r = radius

Polygon:	P = sum of side lengths	*(p. 49)*
Square:	$P = 4s$	*(p. 49)*
Rectangle:	$P = 2\ell + 2w$	*(p. 49)*
Triangle:	$P = a + b + c$	*(p. 49)*
Regular n-gon:	$P = ns$	*(pp. 49, 765)*
Circle:	$C = 2\pi r$	*(p. 49)*
Arc length of $\overset{\frown}{AB}$	$= \frac{m\overset{\frown}{AB}}{360°} \cdot 2\pi r$	*(p. 747)*

Area

A = area, s = side, b = base, h = height, ℓ = length, w = width, d = diagonal, a = apothem, P = perimeter, r = radius

Square:	$A = s^2$	*(pp. 49, 720)*
Rectangle:	$A = lw$	*(pp. 49, 720)*
Triangle:	$A = \frac{1}{2}bh$	*(pp. 49, 721)*
Parallelogram:	$A = bh$	*(p. 721)*
Trapezoid:	$A = \frac{1}{2}h(b_1 + b_2)$	*(p. 730)*
Rhombus:	$A = \frac{1}{2}d_1d_2$	*(p. 731)*
Kite:	$A = \frac{1}{2}d_1d_2$	*(p. 731)*
Equilateral triangle:	$A = \frac{1}{4}\sqrt{3}s^2$	*(pp. 726, 766)*
Regular polygon:	$A = \frac{1}{2}aP$	*(p. 763)*
Circle:	$A = \pi r^2$	*(pp. 49, 755)*
Area of a sector:	$A = \frac{m\overset{\frown}{AB}}{360°} \cdot \pi r^2$	*(p. 756)*

Surface Area

B = area of a base, P = perimeter, C = circumference, h = height, r = radius, ℓ = slant height

Right prism:	$S = 2B + Ph$	*(p. 804)*
Right cylinder:	$S = 2B + Ch = 2\pi r^2 + 2\pi rh$	*(p. 805)*
Regular pyramid:	$S = B + \frac{1}{2}P\ell$	*(p. 811)*
Right cone:	$S = B + \frac{1}{2}C\ell = \pi r^2 + \pi r\ell$	*(p. 812)*
Sphere:	$S = 4\pi r^2$	*(p. 838)*

Volume

V = volume, B = area of a base, h = height, r = radius, s = side length

Cube:	$V = s^3$	*(p. 819)*
Prism:	$V = Bh$	*(p. 820)*
Cylinder:	$V = Bh = \pi r^2 h$	*(p. 820)*
Pyramid:	$V = \frac{1}{3}Bh$	*(p. 829)*
Cone:	$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$	*(p. 829)*
Sphere:	$V = \frac{4}{3}\pi r^3$	*(p. 840)*

Miscellaneous

Geometric mean of a and b: $\sqrt{a \cdot b}$ *(p. 359)*

Euler's Theorem for Polyhedra, F = faces, V = vertices, E = edges: $F + V = E + 2$ *(p. 795)*

Given: similar polygons or similar solids with a scale factor of $a:b$

Ratio of perimeters $= a:b$ *(p. 374)*

Ratio of areas $= a^2:b^2$ *(p. 737)*

Ratio of volumes $= a^3:b^3$ *(p. 848)*

Given a quadratic equation $ax^2 + bx + c = 0$, the solutions are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ *(pp. 641, 883)*

TABLES

Squares and Square Roots

No.	Square	Sq. Root
1	1	1.000
2	4	1.414
3	9	1.732
4	16	2.000
5	25	2.236
6	36	2.449
7	49	2.646
8	64	2.828
9	81	3.000
10	100	3.162
11	121	3.317
12	144	3.464
13	169	3.606
14	196	3.742
15	225	3.873
16	256	4.000
17	289	4.123
18	324	4.243
19	361	4.359
20	400	4.472
21	441	4.583
22	484	4.690
23	529	4.796
24	576	4.899
25	625	5.000
26	676	5.099
27	729	5.196
28	784	5.292
29	841	5.385
30	900	5.477
31	961	5.568
32	1024	5.657
33	1089	5.745
34	1156	5.831
35	1225	5.916
36	1296	6.000
37	1369	6.083
38	1444	6.164
39	1521	6.245
40	1600	6.325
41	1681	6.403
42	1764	6.481
43	1849	6.557
44	1936	6.633
45	2025	6.708
46	2116	6.782
47	2209	6.856
48	2304	6.928
49	2401	7.000
50	2500	7.071

No.	Square	Sq. Root
51	2601	7.141
52	2704	7.211
53	2809	7.280
54	2916	7.348
55	3025	7.416
56	3136	7.483
57	3249	7.550
58	3364	7.616
59	3481	7.681
60	3600	7.746
61	3721	7.810
62	3844	7.874
63	3969	7.937
64	4096	8.000
65	4225	8.062
66	4356	8.124
67	4489	8.185
68	4624	8.246
69	4761	8.307
70	4900	8.367
71	5041	8.426
72	5184	8.485
73	5329	8.544
74	5476	8.602
75	5625	8.660
76	5776	8.718
77	5929	8.775
78	6084	8.832
79	6241	8.888
80	6400	8.944
81	6561	9.000
82	6724	9.055
83	6889	9.110
84	7056	9.165
85	7225	9.220
86	7396	9.274
87	7569	9.327
88	7744	9.381
89	7921	9.434
90	8100	9.487
91	8281	9.539
92	8464	9.592
93	8649	9.644
94	8836	9.695
95	9025	9.747
96	9216	9.798
97	9409	9.849
98	9604	9.899
99	9801	9.950
100	10,000	10.000

No.	Square	Sq. Root
101	10,201	10.050
102	10,404	10.100
103	10,609	10.149
104	10,816	10.198
105	11,025	10.247
106	11,236	10.296
107	11,449	10.344
108	11,664	10.392
109	11,881	10.440
110	12,100	10.488
111	12,321	10.536
112	12,544	10.583
113	12,769	10.630
114	12,996	10.677
115	13,225	10.724
116	13,456	10.770
117	13,689	10.817
118	13,924	10.863
119	14,161	10.909
120	14,400	10.954
121	14,641	11.000
122	14,884	11.045
123	15,129	11.091
124	15,376	11.136
125	15,625	11.180
126	15,876	11.225
127	16,129	11.269
128	16,384	11.314
129	16,641	11.358
130	16,900	11.402
131	17,161	11.446
132	17,424	11.489
133	17,689	11.533
134	17,956	11.576
135	18,225	11.619
136	18,496	11.662
137	18,769	11.705
138	19,044	11.747
139	19,321	11.790
140	19,600	11.832
141	19,881	11.874
142	20,164	11.916
143	20,449	11.958
144	20,736	12.000
145	21,025	12.042
146	21,316	12.083
147	21,609	12.124
148	21,904	12.166
149	22,201	12.207
150	22,500	12.247

Trigonometric Ratios

Angle	Sine	Cosine	Tangent
1°	.0175	.9998	.0175
2°	.0349	.9994	.0349
3°	.0523	.9986	.0524
4°	.0698	.9976	.0699
5°	.0872	.9962	.0875
6°	.1045	.9945	.1051
7°	.1219	.9925	.1228
8°	.1392	.9903	.1405
9°	.1564	.9877	.1584
10°	.1736	.9848	.1763
11°	.1908	.9816	.1944
12°	.2079	.9781	.2126
13°	.2250	.9744	.2309
14°	.2419	.9703	.2493
15°	.2588	.9659	.2679
16°	.2756	.6313	.2867
17°	.2924	.9563	.3057
18°	.3090	.9511	.3249
19°	.3256	.9455	.3443
20°	.3420	.9397	.3640
21°	.3584	.9336	.3839
22°	.3746	.9272	.4040
23°	.3907	.9205	.4245
24°	.4067	.9135	.4452
25°	.4226	.9063	.4663
26°	.4384	.8988	.4877
27°	.4540	.8910	.5095
28°	.4695	.8829	.5317
29°	.4848	.8746	.5543
30°	.5000	.8660	.5774
31°	.5150	.8572	.6009
32°	.5299	.8480	.6249
33°	.5446	.8387	.6494
34°	.5592	.8290	.6745
35°	.5736	.8192	.7002
36°	.5878	.8090	.7265
37°	.6018	.7986	.7536
38°	.6157	.7880	.7813
39°	.6293	.7771	.8098
40°	.6428	.7660	.8391
41°	.6561	.7547	.8693
42°	.6691	.7431	.9004
43°	.6820	.7314	.9325
44°	.6947	.7193	.9657
45°	.7071	.7071	1.0000

Angle	Sine	Cosine	Tangent
46°	.7193	.6947	1.0355
47°	.7314	.6820	1.0724
48°	.7431	.6991	1.1106
49°	.7547	.6561	1.1504
50°	.7660	.6428	1.1918
51°	.7771	.6293	1.2349
52°	.7880	.6157	1.2799
53°	.7986	.6018	1.3270
54°	.8090	.5878	1.3764
55°	.8192	.5736	1.4281
56°	.8290	.5592	1.4826
57°	.8387	.5446	1.5399
58°	.8480	.5299	1.6003
59°	.8572	.5150	1.6643
60°	.8660	.5000	1.7321
61°	.8746	.4848	1.8040
62°	.8829	.4695	1.8807
63°	.8910	.4540	1.9626
64°	.8988	.4384	2.0503
65°	.9063	.4226	2.1445
66°	.9135	.4067	2.2460
67°	.9205	.3907	2.3559
68°	.9272	.3746	2.4751
69°	.9336	.3584	2.6051
70°	.9397	.3420	2.7475
71°	.9455	.3256	2.9042
72°	.9511	.3090	0.0777
73°	.9563	.3746	3.2709
74°	.9613	.3584	3.4874
75°	.9659	.3420	3.7321
76°	.9703	.2419	4.0108
77°	.9744	.2250	4.3315
78°	.9781	.2079	4.7046
79°	.9816	.1908	5.1446
80°	.9848	.1736	5.6713
81°	.9877	.1564	6.3138
82°	.9903	.1392	7.1154
83°	.9925	.1219	8.1443
84°	.9945	.1045	9.5144
85°	.9962	.0872	11.4301
86°	.9976	.0698	14.3007
87°	.9986	.0523	19.0811
88°	.9994	.0349	28.6363
89°	.9998	.0175	52.2900

Postulates

1 **Ruler Postulate** The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the coordinate of the point. The distance between points A and B, written as AB, is the absolute value of the difference between the coordinates of A and B. *(p. 9)*

2 **Segment Addition Postulate** If B is between A and C, then $AB + BC = AC$. If $AB + BC = AC$, then B is between A and C. *(p. 10)*

3 **Protractor Postulate** Consider $\overrightarrow{OB}$ and a point A on one side of $\overrightarrow{OB}$. The rays of the form $\overrightarrow{OA}$ can be matched one to one with the real numbers from 0 to 180. The measure of $\angle AOB$ is equal to the absolute value of the difference between the real numbers for $\overrightarrow{OA}$ and $\overrightarrow{OB}$. *(p. 24)*

4 **Angle Addition Postulate** If P is in the interior of $\angle RST$, then $m\angle RST = m\angle RSP + m\angle PST$. *(p. 25)*

5 Through any two points there exists exactly one line. *(p. 96)*

6 A line contains at least two points. *(p. 96)*

7 If two lines intersect, then their intersection is exactly one point. *(p. 96)*

8 Through any three noncollinear points there exists exactly one plane. *(p. 96)*

9 A plane contains at least three noncollinear points. *(p. 96)*

10 If two points lie in a plane, then the line containing them lies in the plane. *(p. 96)*

11 If two planes intersect, then their intersection is a line. *(p. 96)*

12 **Linear Pair Postulate** If two angles form a linear pair, then they are supplementary. *(p. 126)*

13 **Parallel Postulate** If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line. *(p. 148)*

14 **Perpendicular Postulate** If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line. *(p. 148)*

15 **Corresponding Angles Postulate** If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent. *(p. 154)*

16 **Corresponding Angles Converse** If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel. *(p. 161)*

17 **Slopes of Parallel Lines** In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope. Any two vertical lines are parallel. *(p. 172)*

18 **Slopes of Perpendicular Lines** In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1. Horizontal lines are perpendicular to vertical lines. *(p. 172)*

19 **Side-Side-Side (SSS) Congruence Postulate** If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent. *(p. 234)*

20 **Side-Angle-Side (SAS) Congruence Postulate** If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent. *(p. 240)*

21 **Angle-Side-Angle (ASA) Congruence Postulate** If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent. *(p. 249)*

22 **Angle-Angle (AA) Similarity Postulate** If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar. *(p. 381)*

23 **Arc Addition Postulate** The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. *(p. 660)*

24 **Area of a Square Postulate** The area of a square is the square of the length of its side, or $A = s^2$. *(p. 720)*

25 **Area Congruence Postulate** If two polygons are congruent, then they have the same area. *(p. 720)*

26 **Area Addition Postulate** The area of a region is the sum of the areas of its nonoverlapping parts. *(p. 720)*

27 **Volume of a Cube** The volume of a cube is the cube of the length of its side, or $V = s^3$. *(p. 819)*

28 **Volume Congruence Postulate** If two polyhedra are congruent, then they have the same volume. *(p. 819)*

29 **Volume Addition Postulate** The volume of a solid is the sum of the volumes of all its nonoverlapping parts. *(p. 819)*

Theorems

2.1 Properties of Segment Congruence Segment congruence is reflexive, symmetric, and transitive.

Reflexive: For any segment AB, $\overline{AB} \cong \overline{AB}$.

Symmetric: If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive: If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$. *(p. 113)*

2.2 Properties of Angles Congruence Angle congruence is reflexive, symmetric, and transitive.

Reflexive: For any angle A, $\angle A \cong \angle A$.

Symmetric: If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive: If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$. *(p. 113)*

2.3 Right Angles Congruence Theorem All right angles are congruent. *(p. 124)*

2.4 Congruent Supplements Theorem If two angles are supplementary to the same angle (or to congruent angles), then the two angles are congruent. *(p. 125)*

2.5 Congruent Complements Theorem If two angles are complementary to the same angle (or to congruent angles), then the two angles are congruent. *(p. 125)*

2.6 Vertical Angles Congruence Theorem Vertical angles are congruent. *(p. 126)*

3.1 Alternate Interior Angles Theorem If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent. *(p. 155)*

3.2 Alternate Exterior Angles Theorem If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent. *(p. 155)*

3.3 Consecutive Interior Angles Theorem If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary. *(p. 155)*

3.4 Alternate Interior Angles Converse If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel. *(p. 162)*

3.5 Alternate Exterior Angles Converse If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel. *(p. 162)*

3.6 Consecutive Interior Angles Converse If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel. *(p. 162)*

3.7 Transitive Property of Parallel Lines If two lines are parallel to the same line, then they are parallel to each other. *(p. 164)*

3.8 If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular. *(p. 190)*

3.9 If two lines are perpendicular, then they intersect to form four right angles. *(p. 190)*

3.10 If two sides of two adjacent acute angles are perpendicular, then the angles are complementary. *(p. 191)*

3.11 Perpendicular Transversal Theorem If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other. *(p. 192)*

3.12 Lines Perpendicular to a Transversal Theorem In a plane, if two lines are perpendicular to the same line, then they are parallel to each other. *(p. 192)*

4.1 Triangle Sum Theorem The sum of the measures of the interior angles of a triangle is 180°. *(p. 218)*

Corollary The acute angles of a right triangle are complementary. *(p. 220)*

4.2 Exterior Angle Theorem The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles. *(p. 219)*

4.3 Third Angles Theorem If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent. *(p. 227)*

4.4 Properties of Triangle Congruence Triangle congruence is reflexive, symmetric, and transitive.

Reflexive: For any $\triangle ABC$, $\triangle ABC \cong \triangle ABC$.

Symmetric: If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.

Transitive: If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$. *(p. 228)*

4.5 Hypotenuse-Leg (HL) Congruence Theorem If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent. ***(p. 241)***

4.6 Angle-Angle-Side (AAS) Congruence Theorem If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent. ***(p. 249)***

4.7 Base Angles Theorem If two sides of a triangle are congruent, then the angles opposite them are congruent. ***(p. 264)***

Corollary If a triangle is equilateral, then it is equiangular. ***(p. 265)***

4.8 Converse of the Base Angles Theorem If two angles of a triangle are congruent, then the sides opposite them are congruent. ***(p. 264)***

Corollary If a triangle is equiangular, then it is equilateral. ***(p. 265)***

5.1 Midsegment Theorem The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long as that side. ***(p. 295)***

5.2 Perpendicular Bisector Theorem If a point is on a perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment. ***(p. 303)***

5.3 Converse of the Perpendicular Bisector Theorem If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment. ***(p. 303)***

5.4 Concurrency of Perpendicular Bisectors Theorem The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle. ***(p. 305)***

5.5 Angle Bisector Theorem If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle. ***(p. 310)***

5.6 Converse of the Angle Bisector Theorem If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle. ***(p. 310)***

5.7 Concurrency of Angle Bisectors of a Triangle The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle. ***(p. 312)***

5.8 Concurrency of Medians of a Triangle The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side. ***(p. 319)***

5.9 Concurrency of Altitudes of a Triangle The lines containing the altitudes of a triangle are concurrent. ***(p. 320)***

5.10 If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side. ***(p. 328)***

5.11 If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle. ***(p. 328)***

5.12 Triangle Inequality Theorem The sum of the lengths of any two sides of a triangle is greater than the length of the third side. ***(p. 330)***

5.13 Hinge Theorem If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second. ***(p. 335)***

5.14 Converse of the Hinge Theorem If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second. ***(p. 335)***

6.1 If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths. ***(p. 374)***

6.2 Side-Side-Side (SSS) Similarity Theorem If the corresponding side lengths of two triangles are proportional, then the triangles are similar. ***(p. 388)***

6.3 Side-Angle-Side (SAS) Similarity Theorem If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar. ***(p. 390)***

6.4 Triangle Proportionality Theorem If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally. ***(p. 397)***

6.5 Converse of the Triangle Proportionality Theorem If a line divides two sides of a triangle proportionally, then it is parallel to the third side. ***(p. 397)***

6.6 If three parallel lines intersect two transversals, then they divide the transversals proportionally. ***(p. 398)***

6.7 If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides. ***(p. 398)***

7.1 **Pythagorean Theorem** In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. ***(p. 433)***

7.2 **Converse of the Pythagorean Theorem** If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle. ***(p. 441)***

7.3 If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is an acute triangle. ***(p. 442)***

7.4 If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is an obtuse triangle. ***(p. 442)***

7.5 If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other. ***(p. 449)***

7.6 **Geometric Mean (Altitude) Theorem** In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments. The length of the altitude is the geometric mean of the lengths of the two segments. ***(p. 452)***

7.7 **Geometric Mean (Leg) Theorem** In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments. The length of each leg of the right triangle is the geometric mean of the lengths of hypotenuse and the segment of the hypotenuse that is adjacent to the leg. ***(p. 452)***

7.8 **45°-45°-90° Triangle Theorem** In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg. ***(p. 457)***

7.9 **30°-60°-90° Triangle Theorem** In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg. ***(p. 459)***

8.1 **Polygon Interior Angles Theorem** The sum of the measures of the interior angles of a convex n-gon is $(n - 2) \cdot 180°$. ***(p. 507)***

Corollary The sum of the measures of the interior angles of a quadrilateral is 360°. ***(p. 507)***

8.2 **Polygon Exterior Angles Theorem** The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360°. ***(p. 509)***

8.3 If a quadrilateral is a parallelogram, then its opposite sides are congruent. ***(p. 515)***

8.4 If a quadrilateral is a parallelogram, then its opposite angles are congruent. ***(p. 515)***

8.5 If a quadrilateral is a parallelogram, then its consecutive angles are supplementary. ***(p. 516)***

8.6 If a quadrilateral is a parallelogram, then its diagonals bisect each other. ***(p. 517)***

8.7 If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. ***(p. 522)***

8.8 If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. ***(p. 522)***

8.9 If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram. ***(p. 523)***

8.10 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. ***(p. 523)***

Rhombus Corollary A quadrilateral is a rhombus if and only if it has four congruent sides. ***(p. 533)***

Rectangle Corollary A quadrilateral is a rectangle if and only if it has four right angles. ***(p. 533)***

Square Corollary A quadrilateral is a square if and only if it is a rhombus and a rectangle. ***(p. 533)***

8.11 A parallelogram is a rhombus if and only if its diagonals are perpendicular. ***(p. 535)***

8.12 A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles. ***(p. 535)***

8.13 A parallelogram is a rectangle if and only if its diagonals are congruent. ***(p. 535)***

8.14 If a trapezoid is isosceles, then both pairs of base angles are congruent. ***(p. 543)***

8.15 If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid. ***(p. 543)***

8.16 A trapezoid is isosceles if and only if its diagonals are congruent. ***(p. 543)***

8.17 **Midsegment Theorem for Trapezoids** The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases. ***(p. 544)***

8.18 If a quadrilateral is a kite, then its diagonals are perpendicular. ***(p. 545)***

8.19 If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent. ***(p. 545)***

9.1 **Translation Theorem** A translation is an isometry. ***(p. 573)***

9.2 **Reflection Theorem** A reflection is an isometry. ***(p. 591)***

9.3 **Rotation Theorem** A rotation is an isometry. ***(p. 601)***

9.4 **Composition Theorem** The composition of two (or more) isometries is an isometry. ***(p. 609)***

9.5 **Reflections in Parallel Lines** If lines k and m are parallel, then a reflection in line k followed by a reflection in line m is the same as a translation. If P'' is the image of P, then:

(1) $\overline{PP'}$ is perpendicular to k and m, and
(2) $PP'' = 2d$, where d is the distance between k and m. ***(p. 609)***

9.6 **Reflections in Intersecting Lines** If lines k and m intersect at point P, then a reflection in k followed by a reflection in m is the same as a rotation about point P. The angle of rotation is $2x°$, where $x°$ is the measure of the acute or right angle formed by k and m. ***(p. 610)***

10.1 In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle. ***(p. 653)***

10.2 Tangent segments from a common external point are congruent. ***(p. 654)***

10.3 In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent. ***(p. 664)***

10.4 If one chord is a perpendicular bisector of another chord, then the first chord is a diameter. ***(p. 665)***

10.5 If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc. ***(p. 665)***

10.6 In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center. ***(p. 666)***

10.7 **Measure of an Inscribed Angle Theorem** The measure of an inscribed angle is one half the measure of its intercepted arc. ***(p. 672)***

10.8 If two inscribed angles of a circle intercept the same arc, then the angles are congruent. ***(p. 673)***

10.9 If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle. ***(p. 674)***

10.10 A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary. ***(p. 675)***

10.11 If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc. ***(p. 680)***

10.12 **Angles Inside the Circle** If two chords intersect inside a circle, then the measure of each angle is one half the sum of the measures of the arcs intercepted by the angle and its vertical angle. ***(p. 681)***

10.13 **Angles Outside the Circle** If a tangent and a secant, two tangents, or two secants intersect outside a circle, then the measure of the angle formed is one half the difference of the measures of the intercepted arcs. ***(p. 681)***

10.14 **Segments of Chords Theorem** If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord. ***(p. 689)***

10.15 **Segments of Secants Theorem** If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment. ***(p. 690)***

10.16 **Segments of Secants and Tangents Theorem** If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment. ***(p. 691)***

11.1 **Area of a Rectangle** The area of a rectangle is the product of its base and height. $A = bh$ ***(p. 720)***

POSTULATES AND THEOREMS

11.2 Area of a Parallelogram The area of a parallelogram is the product of a base and its corresponding height. $A = bh$ *(p. 721)*

11.3 Area of a Triangle The area of a triangle is one half the product of a base and its corresponding height. $A = \frac{1}{2}bh$ *(p. 721)*

11.4 Area of a Trapezoid The area of a trapezoid is one half the product of the height and the sum of the lengths of the bases.
$A = \frac{1}{2}h(b_1 + b_2)$ *(p. 730)*

11.5 Area of a Rhombus The area of a rhombus is one half the product of the lengths of its diagonals. $A = \frac{1}{2}d_1d_2$ *(p. 731)*

11.6 Area of a Kite The area of a kite is one half the product of the lengths of its diagonals.
$A = \frac{1}{2}d_1d_2$ *(p. 731)*

11.7 Areas of Similar Polygons If two polygons are similar with the lengths of corresponding sides in the ratio of $a : b$, then the ratio of their areas is $a^2 : b^2$. *(p. 737)*

11.8 Circumference of a Circle The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$, where d is the diameter of the circle and r is the radius of the circle. *(p. 746)*

Arc Length Corollary In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.

$\frac{\text{Arc length of } \overset{\frown}{AB}}{2\pi r} = \frac{m\overset{\frown}{AB}}{360^\circ}$, or

Arc length of $\overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360^\circ} \cdot 2\pi r$ *(p. 747)*

11.9 Area of a Circle The area of a circle is π times the square of the radius. $A = \pi r^2$ *(p. 755)*

11.10 Area of a Sector The ratio of the area A of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc to 360°.

$\frac{A}{\pi r^2} = \frac{m\overset{\frown}{AB}}{360^\circ}$, or $A = \frac{m\overset{\frown}{AB}}{360^\circ} \cdot \pi r^2$ *(p. 756)*

11.11 Area of a Regular Polygon The area of a regular n-gon with side length s is half the product of the apothem a and the perimeter P, so $A = \frac{1}{2}aP$, or $A = \frac{1}{2}a \cdot ns$. *(p. 763)*

12.1 Euler's Theorem The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula $F + V = E + 2$. *(p. 795)*

12.2 Surface Area of a Right Prism The surface area S of a right prism is $S = 2B + Ph = aP + Ph$, where a is the apothem of the base, B is the area of a base, P is the perimeter of a base, and h is the height. *(p. 804)*

12.3 Surface Area of a Right Cylinder The surface area S of a right cylinder is $S = 2B + Ch = 2\pi r^2 + 2\pi rh$, where B is the area of a base, C is the circumference of a base, r is the radius of a base, and h is the height. *(p. 805)*

12.4 Surface Area of a Regular Pyramid The surface area S of a regular pyramid is $S = B + \frac{1}{2}P\ell$, where B is the area of the base, P is the perimeter of the base, and ℓ is the slant height. *(p. 811)*

12.5 Surface Area of a Right Cone The surface area S of a right cone is $S = B + \frac{1}{2}C\ell = \pi r^2 + \pi r\ell$, where B is the area of the base, C is the circumference of the base, r is the radius of the base, and ℓ is the slant height. *(p. 812)*

12.6 Volume of a Prism The volume V of a prism is $V = Bh$, where B is the area of a base and h is the height. *(p. 820)*

12.7 Volume of a Cylinder The volume V of a cylinder is $V = Bh = \pi r^2h$, where B is the area of a base, h is the height, and r is the radius of a base. *(p. 820)*

12.8 Cavalieri's Principle If two solids have the same height and the same cross-sectional area at every level, then they have the same volume. *(p. 821)*

12.9 Volume of a Pyramid The volume V of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height. *(p. 829)*

12.10 Volume of a Cone The volume V of a cone is $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2h$, where B is the area of the base, h is the height, and r is the radius of the base. *(p. 829)*

12.11 Surface Area of a Sphere The surface area S of a sphere with radius r is $S = 4\pi r^2$. *(p. 838)*

12.12 Volume of a Sphere The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$. *(p. 840)*

12.13 Similar Solids Theorem If two similar solids have a scale factor of $a : b$, then corresponding areas have a ratio of $a^2 : b^2$, and corresponding volumes have a ratio of $a^3 : b^3$. *(p. 848)*

Additional Proofs

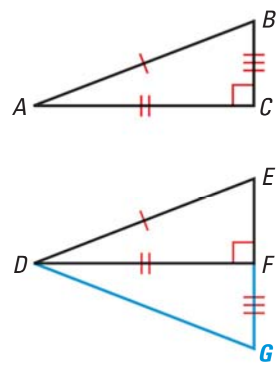
Proof of Theorem 4.5
Hypotenuse-Leg (HL) Congruence Theorem

THEOREM 4.5
PAGE 241

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

GIVEN ▶ In $\triangle ABC$, $\angle C$ is a right angle.
In $\triangle DEF$, $\angle F$ is a right angle.
$\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$

PROVE ▶ $\triangle ABC \cong \triangle DEF$



Plan for Proof Construct $\triangle DGF$ with $\overline{GF} \cong \overline{BC}$, as shown. Prove that $\triangle ABC \cong \triangle DGF$. Then use the fact that corresponding parts of congruent triangles are congruent to show that $\triangle DGF \cong \triangle DEF$. By the Transitive Property of Congruence, you can show that $\triangle ABC \cong \triangle DEF$.

STATEMENTS	REASONS
1. $\angle C$ is a right angle. $\angle DFE$ is a right angle.	**1.** Given
2. $\overline{DF} \perp \overline{EG}$	**2.** Definition of perpendicular lines
3. $\angle DFG$ is a right angle.	**3.** If 2 lines are $\perp$, then they form 4 rt. $\angle$s.
4. $\angle C \cong \angle DFG$	**4.** Right Angles Congruence Theorem
5. $\overline{AC} \cong \overline{DF}$	**5.** Given
6. $\overline{BC} \cong \overline{GF}$	**6.** Given by construction
7. $\triangle ABC \cong \triangle DGF$	**7.** SAS Congruence Postulate
8. $\overline{DG} \cong \overline{AB}$	**8.** Corresp. parts of $\cong$ $\triangle$s are $\cong$.
9. $\overline{AB} \cong \overline{DE}$	**9.** Given
10. $\overline{DG} \cong \overline{DE}$	**10.** Transitive Property of Congruence
11. $\angle E \cong \angle G$	**11.** If 2 sides of a $\triangle$ are $\cong$, then the $\angle$s opposite them are $\cong$.
12. $\angle DFG \cong \angle DFE$	**12.** Right Angles Congruence Theorem
13. $\triangle DGF \cong \triangle DEF$	**13.** AAS Congruence Theorem
14. $\triangle ABC \cong \triangle DEF$	**14.** Transitive Property of $\cong$ $\triangle$s

ADDITIONAL PROOFS

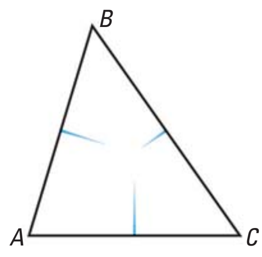
Proof of Theorem 5.4
Concurrency of Perpendicular Bisectors of a Triangle

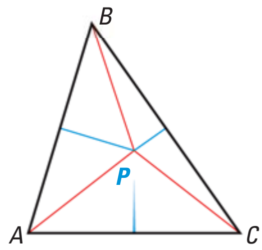
THEOREM 5.4
PAGE 305
The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.

GIVEN ▶ $\triangle ABC$; the $\perp$ bisectors of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$

PROVE ▶ The $\perp$ bisectors intersect in a point; that point is equidistant from A, B, and C.

Plan for Proof Show that P, the point of intersection of the perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$, also lies on the perpendicular bisector of $\overline{AC}$. Then show that P is equidistant from the vertices of the triangle, A, B, and C.





STATEMENTS	REASONS
1. $\triangle ABC$; the $\perp$ bisectors of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$	**1.** Given
2. The perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$ intersect at some point P.	**2.** ABC is a triangle, so its sides $\overline{AB}$ and $\overline{BC}$ cannot be parallel; therefore, segments perpendicular to those sides cannot be parallel. So, the perpendicular bisectors must intersect in some point. Call it P.
3. Draw $\overline{PA}$, $\overline{PB}$, and $\overline{PC}$.	**3.** Through any two points there is exactly one line.
4. $PA = PB$, $PB = PC$	**4.** In a plane, if a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment. (Theorem 5.2)
5. $PA = PC$	**5.** Substitution Property of Equality
6. P is on the perpendicular bisector of $\overline{AC}$.	**6.** In a plane, if a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment. (Theorem 5.3)
7. $PA = PB = PC$, so P is equidistant from the vertices of the triangle.	**7.** From the results of Steps 4 and 5 and the definition of equidistant

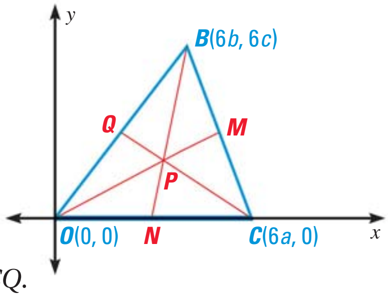
Proof of Theorem 5.8
Concurrency of Medians of a Triangle

THEOREM 5.8
PAGE 319

The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

GIVEN ▶ $\triangle OBC$; medians $\overline{OM}$, $\overline{BN}$, and $\overline{CQ}$

PROVE ▶ The medians intersect in a point P; that point is two thirds of the distance from vertices O, B, and C to midpoints M, N, and Q.



Plan for Proof The medians $\overline{OM}$ and $\overline{BN}$ intersect at some point P. Show that point P lies on $\overleftrightarrow{CQ}$. Then show that $OP = \frac{2}{3}OM$, $BP = \frac{2}{3}BN$, and $CP = \frac{2}{3}CQ$.

WRITE PROOFS

Because you want to prove something involving the fraction $\frac{2}{3}$, it is convenient to position the vertices at points whose coordinates are multiples of both 2 and 3.

STEP 1 **Find** the equations of the lines containing the medians $\overline{OM}$, $\overline{BN}$, and $\overline{CQ}$.

By the *Midpoint Formula*,

the coordinates of M are $\left(\frac{6b + 6a}{2}, \frac{6c + 0}{2}\right) = (3b + 3a, 3c)$;

the coordinates of N are $\left(\frac{0 + 6a}{2}, \frac{0 + 0}{2}\right) = (3a, 0)$;

the coordinates of Q are $\left(\frac{6b + 0}{2}, \frac{6c + 0}{2}\right) = (3b, 3c)$.

By the *slope formula*,

slope of $\overline{OM} = \frac{3c - 0}{(3b + 3a) - 0} = \frac{3c}{3b + 3a} = \frac{c}{b + a}$;

slope of $\overline{BN} = \frac{6c - 0}{6b - 3a} = \frac{6c}{6b - 3a} = \frac{2c}{2b - a}$;

slope of $\overline{CQ} = \frac{0 - 3c}{6a - 3b} = \frac{-3c}{6a - 3b} = \frac{-c}{2a - b} = \frac{c}{b - 2a}$.

Using the *point-slope form of an equation of a line*,

the equation of $\overleftrightarrow{OM}$ is $y - 0 = \frac{c}{b + a}(x - 0)$, or $y = \frac{c}{b + a}x$;

the equation of $\overleftrightarrow{BN}$ is $y - 0 = \frac{2c}{2b - a}(x - 3a)$, or $y = \frac{2c}{2b - a}(x - 3a)$;

the equation of $\overleftrightarrow{CQ}$ is $y - 0 = \frac{c}{b - 2a}(x - 6a)$, or $y = \frac{c}{b - 2a}(x - 6a)$.

STEP 2 **Find** the coordinates of the point P where two medians (say, $\overline{OM}$ and $\overline{BN}$) intersect. Using the substitution method, set the values of y in the equations of $\overleftrightarrow{OM}$ and $\overleftrightarrow{BN}$ equal to each other:

$$\frac{c}{b + a}x = \frac{2c}{2b - a}(x - 3a)$$

$$cx(2b - a) = 2c(x - 3a)(b + a)$$

$$2cxb - cxa = 2cxb + 2cxa - 6cab - 6ca^2$$

$$-3cxa = -6cab - 6ca^2$$

$$x = 2b + 2a$$

Substituting to find y, $y = \frac{c}{b + a}x = \frac{c}{b + a}(2b + 2a) = 2c$.

So, the coordinates of P are $(2b + 2a, 2c)$.

ADDITIONAL PROOFS

STEP 3 **Show** that P is on $\overleftrightarrow{CQ}$.

Substituting the x-coordinate for P into the equation of $\overleftrightarrow{CQ}$, $y = \frac{c}{b - 2a}([2b + 2a] - 6a) = \frac{c}{b - 2a}(2b - 4a) = 2c$.

So, $P(2b + 2a, 2c)$ is on $\overleftrightarrow{CQ}$ and the three medians intersect at the same point.

STEP 4 **Find** the distances OM, OP, BN, BP, CQ, and CP.
Use the *Distance Formula*.

$$OM = \sqrt{((3b + 3a) - 0)^2 + (3c - 0)^2} = \sqrt{(3(b + a))^2 + (3c)^2} = \sqrt{9\left((b + a)^2 + c^2\right)} = 3\sqrt{(b + a)^2 + c^2}$$

$$OP = \sqrt{((2b + 2a) - 0)^2 + (2c - 0)^2} = \sqrt{(2(b + a))^2 + (2c)^2} = \sqrt{4\left((b + a)^2 + c^2\right)} = 2\sqrt{(b + a)^2 + c^2}$$

$$BN = \sqrt{(3a - 6b)^2 + (0 - 6c)^2} = \sqrt{(3a - 6b)^2 + (-6c)^2} = \sqrt{(3(a - 2b))^2 + (3(-2c))^2} = \sqrt{9(a - 2b)^2 + 9\left(4c^2\right)} = \sqrt{9\left((a - 2b)^2 + 4c^2\right)} = 3\sqrt{(a - 2b)^2 + 4c^2}$$

$$BP = \sqrt{((2b + 2a) - 6b)^2 + (2c - 6c)^2} = \sqrt{(2a - 4b)^2 + (-4c)^2} = \sqrt{(2(a - 2b))^2 + (2(-2c))^2} = \sqrt{4(a - 2b)^2 + 4\left(4c^2\right)} = \sqrt{4\left((a - 2b)^2 + 4c^2\right)} = 2\sqrt{(a - 2b)^2 + 4c^2}$$

$$CQ = \sqrt{(6a - 3b)^2 + (0 - 3c)^2} = \sqrt{(3(2a - b))^2 + (-3c)^2} = \sqrt{9\left((2a - b)^2 + c^2\right)} = 3\sqrt{(2a - b)^2 + c^2}$$

$$CP = \sqrt{(6a - (2b + 2a))^2 + (0 - 2c)^2} = \sqrt{(4a - 2b)^2 + (-2c)^2} = \sqrt{(2(2a - b))^2 + 4c^2} = \sqrt{4\left((2a - b)^2 + c^2\right)} = 2\sqrt{(2a - b)^2 + c^2}$$

STEP 5 **Multiply** OM, BN, and CQ by $\frac{2}{3}$.

$$\frac{2}{3}OM = \frac{2}{3}\left(3\sqrt{(b + a)^2 + c^2}\right) = 2\sqrt{(b + a)^2 + c^2}$$

$$\frac{2}{3}BN = \frac{2}{3}\left(3\sqrt{(a - 2b)^2 + 4c^2}\right) = 2\sqrt{(a - 2b)^2 + 4c^2}$$

$$\frac{2}{3}CQ = \frac{2}{3}\left(3\sqrt{(2a - b)^2 + c^2}\right) = 2\sqrt{(2a - b)^2 + c^2}$$

Thus, $OP = \frac{2}{3}OM$, $BP = \frac{2}{3}BN$, and $CP = \frac{2}{3}CQ$.

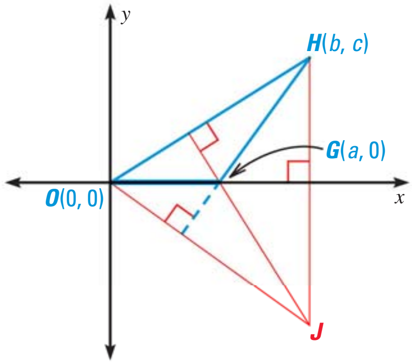
ADDITIONAL PROOFS

Proof of Theorem 5.9
Concurrency of Altitudes of a Triangle

THEOREM 5.9
PAGE 320
The lines containing the altitudes of a triangle are concurrent.

GIVEN ▶ $\triangle OGH$

PROVE ▶ The altitudes to the sides of $\triangle OGH$ all intersect at J.



Plan for Proof Find the equations of the lines containing the altitudes of $\triangle OGH$. Find the intersection point of two of these lines. Show that the intersection point is also on the line containing the third altitude.

STEP 1 **Find** the slopes of the lines containing the sides $\overline{OH}$, $\overline{GH}$, and $\overline{OG}$.

Slope of $\overleftrightarrow{OH} = \frac{c}{b}$ Slope of $\overleftrightarrow{GH} = \frac{c}{b-a}$ Slope of $\overleftrightarrow{OG} = 0$

WRITE PROOFS
Choose a general triangle, with one vertex at the origin and one side along an axis. In the proof shown, the triangle is obtuse.

STEP 2 **Use** the *Slopes of Perpendicular Lines Postulate* to find the slopes of the lines containing the altitudes.

Slope of line containing altitude to $\overline{OH} = \frac{-b}{c}$

Slope of line containing altitude to $\overline{GH} = \frac{-(b-a)}{c} = \frac{a-b}{c}$

The line containing the altitude to $\overline{OG}$ has an undefined slope.

STEP 3 **Use** the *point-slope form of an equation of a line* to write equations for the lines containing the altitudes.

An equation of the line containing the altitude to $\overline{OH}$ is

$$y - 0 = \frac{-b}{c}(x - a), \text{ or } y = \frac{-b}{c}x + \frac{ab}{c}.$$

An equation of the line containing the altitude to $\overline{GH}$ is

$$y - 0 = \frac{a-b}{c}(x - 0), \text{ or } y = \frac{a-b}{c}x.$$

An equation of the vertical line containing the altitude to $\overline{OG}$ is $x = b$.

STEP 4 **Find** the coordinates of the point J where the lines containing two of the altitudes intersect. Using substitution, set the values of y in two of the above equations equal to each other, then solve for x:

$$\frac{-b}{c}x + \frac{ab}{c} = \frac{a-b}{c}x$$

$$\frac{ab}{c} = \frac{a-b}{c}x + \frac{b}{c}x$$

$$\frac{ab}{c} = \frac{a}{c}x$$

$$x = b$$

Next, substitute to find y: $y = \frac{-b}{c}x + \frac{ab}{c} = \frac{-b}{c}(b) + \frac{ab}{c} = \frac{ab - b^2}{c}$.

So, the coordinates of J are $\left(b, \frac{ab - b^2}{c}\right)$.

STEP 5 **Show** that J is on the line that contains the altitude to side $\overline{OG}$. J is on the vertical line with equation $x = b$ because its x-coordinate is b. Thus, the lines containing the altitudes of $\triangle OGH$ are concurrent.

Proof of Theorem 8.17
Midsegment Theorem for Trapezoids

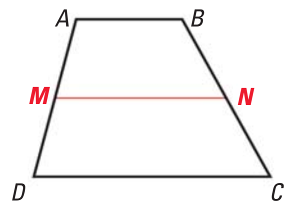
THEOREM 8.17
PAGE 544

The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

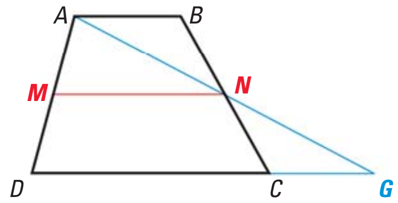
GIVEN ▶ Trapezoid $ABCD$ with midsegment $\overline{MN}$

PROVE ▶ $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$,

$MN = \frac{1}{2}(AB + DC)$



Plan for Proof Draw $\overline{AN}$, then extend $\overline{AN}$ and $\overline{DC}$ so that they intersect at point G. Then prove that $\triangle ANB \cong \triangle GNC$, and use the fact that $\overline{MN}$ is a midsegment of $\triangle ADG$ to prove that $MN = \frac{1}{2}(AB + DC)$.



STATEMENTS	REASONS
1. $ABCD$ is a trapezoid with midsegment $\overline{MN}$.	**1.** Given
2. Draw $\overline{AN}$, then extend $\overline{AN}$ and $\overline{DC}$ so that they intersect at point G.	**2.** Through any two points there is exactly one line.
3. N is the midpoint of $\overline{BC}$.	**3.** Definition of midsegment of a trapezoid
4. $\overline{BN} \cong \overline{NC}$	**4.** Definition of midpoint
5. $\overline{AB} \parallel \overline{DC}$	**5.** Definition of trapezoid
6. $\angle ABN \cong \angle GCN$	**6.** Alternate Interior ⦞ Theorem
7. $\angle ANB \cong \angle GNC$	**7.** Vertical angles are congruent.
8. $\triangle ANB \cong \triangle GNC$	**8.** ASA Congruence Postulate
9. $\overline{AN} \cong \overline{GN}$	**9.** Corresp. parts of $\cong$ ⧍ are $\cong$.
10. N is the midpoint of $\overline{AG}$.	**10.** Definition of midpoint
11. $\overline{MN}$ is the midsegment of $\triangle AGD$.	**11.** Definition of midsegment of a $\triangle$
12. $\overline{MN} \parallel \overline{DG}$ (so $\overline{MN} \parallel \overline{DC}$)	**12.** Midsegment of a $\triangle$ Theorem
13. $\overline{MN} \parallel \overline{AB}$	**13.** Two lines $\parallel$ to the same line are $\parallel$.
14. $MN = \frac{1}{2}DG$	**14.** Midsegment of a $\triangle$ Theorem
15. $DG = DC + CG$	**15.** Segment Addition Postulate
16. $\overline{CG} \cong \overline{AB}$	**16.** Corresp. parts of $\cong$ ⧍ are $\cong$.
17. $CG = AB$	**17.** Definition of congruent segments
18. $DG = DC + AB$	**18.** Substitution Property of Equality
19. $MN = \frac{1}{2}(DC + AB)$	**19.** Substitution Property of Equality

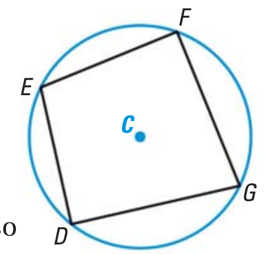
ADDITIONAL PROOFS

Proof of Theorem 10.10
A Theorem about Inscribed Quadrilaterals

THEOREM 10.10
PAGE 675

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

STEP 1 **Prove** that if a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.



GIVEN ▶ $DEFG$ is inscribed in $\odot C$.

PROVE ▶ $\angle D$ and $\angle F$ are supplementary,
$\angle E$ and $\angle G$ are supplementary.

Paragraph Proof Arcs $\widehat{EFG}$ and $\widehat{GDE}$ together make a circle, so $m\widehat{EFG} + m\widehat{GDE} = 360°$ by the Arc Addition Postulate. $\angle D$ is inscribed in $\widehat{EFG}$ and $\angle F$ is inscribed in $\widehat{GDE}$, so the angle measures are half the arc measures. Using the Substitution and Distributive Properties, the sum of the measures of the opposite angles is

$$m\angle D + m\angle F = \frac{1}{2}m\widehat{EFG} + \frac{1}{2}m\widehat{GDE} = \frac{1}{2}(m\widehat{EFG} + m\widehat{GDE}) = \frac{1}{2}(360°) = 180°.$$

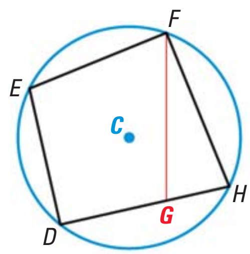
So, $\angle D$ and $\angle F$ are supplementary by definition. Similarly, $\angle E$ and $\angle G$ are inscribed in $\widehat{FGD}$ and $\widehat{DEF}$ and $m\angle E + m\angle G = 180°$. Then $\angle E$ and $\angle G$ are supplementary by definition.

STEP 2 **Prove** that if the opposite angles of a quadrilateral are supplementary, then the quadrilateral can be inscribed in a circle.

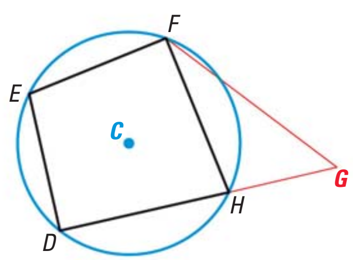
GIVEN ▶ $\angle E$ and $\angle G$ are supplementary (or $\angle D$ and $\angle F$ are supplementary).

PROVE ▶ $DEFG$ is inscribed in $\odot C$.

Plan for Proof Draw the circle that passes through D, E, and F. Use an *indirect proof* to show that the circle also passes through G. Begin by assuming that G does not lie on $\odot C$.



Case 1 *G lies inside $\odot C$.* Let H be the intersection of $\overrightarrow{DG}$ and $\odot C$. Then $DEFH$ is inscribed in $\odot C$ and $\angle E$ is supplementary to $\angle DHF$ (by proof above). Then $\angle DGF \cong \angle DHF$ by the given information and the Congruent Supplements Theorem. This implies that $\overline{FG} \parallel \overline{FH}$, which is a contradiction.



Case 2 *G lies outside $\odot C$.* Let H be the intersection of $\overrightarrow{DG}$ and $\odot C$. Then $DEFH$ is inscribed in $\odot C$ and $\angle E$ is supplementary to $\angle DHF$ (by proof above). Then $\angle DGF \cong \angle DHF$ by the given information and the Congruent Supplements Theorem. This implies that $\overline{FG} \parallel \overline{FH}$, which is a contradiction.

Because the original assumption leads to a contradiction in both cases, G lies on $\odot C$ and $DEFG$ is inscribed in $\odot C$.

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Photo Credit: Bridgeman Art Library; **823** *bottom left* C-Squared Studios/Getty Images; *bottom center* Eising Food Photography/Stockfood America; *bottom right* PhotoObjects; **824** *center right* Jay Penni Photography/McDougal Littell; *bottom right* Schafer & Hill/Getty Images; **825** Claire Rydell/Index Stock Imagery; **829** Amr Nabil/AP/Wide World Photos; **830** Sylvain Grandadam/Getty Images; **834** Thinkstock/PictureQuest; **835** Mystic Seaport Museum - Mystic, CT; **838** *bottom* Royalty-Free/Corbis; *top* Bob Daemmrich/PhotoEdit; **840** Jump Photography Archive; **841** Corbis; **842** *bottom left* Royalty-Free/Getty Images; *bottom center* Michel Tcherevkoff/Getty Images; *bottom right* Bonnie Jacobs/Alamy; **844** Richard Price/Getty Images; **847** © Claes Oldenburg, Coosje van Bruggen, *Binoculars*,1991 Steel frame. Exterior: concrete and cement plaster painted with elastomeric paint. Interior: gypsum plaster 45 x 44 x 18 ft. (13.7 x 13.4 x 5.5 m) Central component of a building designed by Frank O. Gehry and Associates, 340 Main St., Venice, California. Photo Credit: Nik Wheeler/Corbis; **852** PhotoObjects/PictureQuest; **855** *top right* Royalty-Free/Corbis; **861** Royalty-Free/Corbis; **864** Eric Kamp/Index Stock Imagery; **867** James Morrell/Getty Images.

Illustrations and Maps

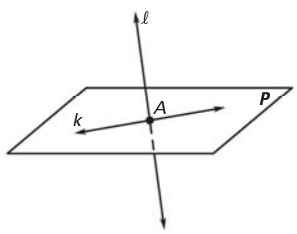
Argosy **1, 71, 145, 199, 215, 290** *top right*, **290** *top left*, **293, 355, 386** *top center*, **426, 431, 463** *top left*, **503** *bottom right*, **503** *center right*, **505, 523, 532** *center left*, **566, 567, 571, 578** *center*, **579, 613, 620, 624** *top right*, **640, 644, 647** *bottom*, **649, 682, 685** *center right*, **694** *center right*, **716, 717, 719, 730, 732, 735, 742, 761, 788** *center right*, **791, 800** *center*, **808, 816, 818** *left*, **831, 834** *top*, **838, 844** *center right*, **855** *right*; Kenneth Batelman **226, 300, 315** *bottom right*, **386** *center right*, **394** *center right*, **417, 424, 475** *center*, **685** *top right*, **706** *right*, **749**; Steve Cowden **6, 21, 36, 39, 41, 58** *bottom*, **77, 101, 115, 156**; Stephen Durke **187, 223, 308** *top center*, **308** *bottom right*, **317** *top right*, **319, 320, 335, 335, 340, 342** *center right*, **383, 390, 416** *left*, **439, 450, 479** *bottom right*, **479** *center right*, **485, 488** *top*, **575, 578** *center right*, **596** *center*, **657** *center*, **674, 687** *top left*, **774, 834** *center right*; John Francis **336, 393, 475** *top*, **520** *center*, **539, 614, 623, 624** *center right*, **631, 760, 778** *bottom left*, **778** *top right*, **822, 826**; Patrick Gnan/Deborah Wolfe, Ltd. **10** *bottom right*, **31, 110, 119, 167, 238** *bottom right*, **248** *bottom right*, **254, 254, 305, 334, 339, 352** *center right*, **403** *both*, **422, 445, 471, 474, 488** *center*, **536, 663** *center right*, **665, 670, 686, 698, 745** *left*, **751, 763, 776, 796, 825**; Chris Lyons **238** *top*, **333** *center*, **512** *center*, **526, 543, 591, 595, 597**; Steve McEntee **159** *top*, **196, 261, 262, 311, 317, 452, 482, 596** *center right*, **752, 752** *top right*; Sarah Buss/McDougal Littell **578** *top right*; Karen Minot **91, 168, 333** *center right*, **369, 703, 738, 754, 844** *bottom right*; Paul Mirocha **298, 834** *bottom right*; Laurie O'Keefe **363, 694** *top right*, **773, 778** *bottom right*, **817**; Jun Park **848**; David Puckett **329, 329**; Tony Randazzo/American Artists **706** *left*, **726, 767**; Mark Schroeder **365, 520** *top*, **528, 548, 663** *top right*, **669** *center*, **669** *bottom*, **692, 827, 853**; Dan Stuckenschneider **151, 162, 170** *top left*, **170** *center right*, **176, 177, 278** *center*, **315** *center right*, **316, 324** *top right*, **438, 455, 463** *top right*; Robert Ulrich **510**; Carol Zuber-Mallison **32** *top*, **159** *bottom*, **213, 695.** All other illustrations © McDougal Littell/Houghton Mifflin Company.

Selected Answers

Chapter 1

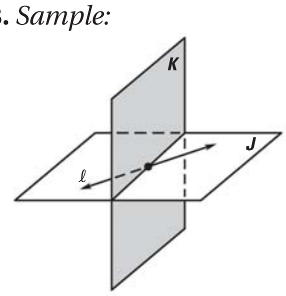
1.1 Skill Practice (pp. 5–7) **1. a.** point Q **b.** line segment MN **c.** ray ST **d.** line FG **3.** $\overleftrightarrow{QW}$, line g **5.** *Sample answer:* points R, Q, S; point T **7.** Yes; through any three points not on the same line, there is exactly one plane. **9.** $\overrightarrow{VY}$, $\overrightarrow{VX}$, $\overrightarrow{VZ}$, $\overrightarrow{VW}$ **11.** $\overrightarrow{WX}$

15. *Sample:*

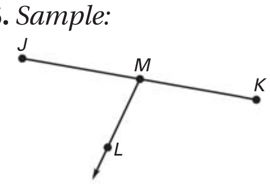


17. point R
19. $\overleftrightarrow{RS}$
21. yes; yes

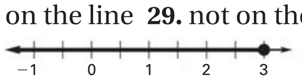
23. *Sample:*



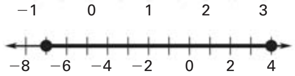
25. *Sample:*



27. on the line **29.** not on the line **31.** on the line

33. 

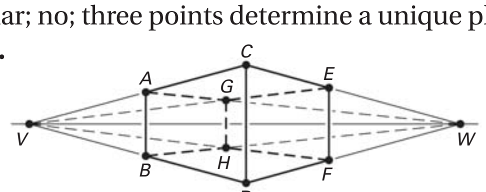
ray

35. 

segment

1.1 Problem Solving (pp. 7–8) **41.** intersection of a line and a plane **43.** Four points are not necessarily coplanar; no; three points determine a unique plane.

45. a–c.

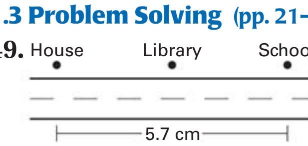


1.2 Skill Practice (pp. 12–13) **1.** $\overline{MN}$ means segment MN while MN is the length of $\overline{MN}$. **3.** 2.1 cm **5.** 3.5 cm **7.** 44 **9.** 23 **11.** 13 **13.** congruent **15.** not congruent **17.** 7 **19.** 9 **21.** 10 **23.** 20 **25.** 30 **29.** $(3x - 16) + (4x - 8) = 60$; 12; 20, 40

1.2 Problem Solving (pp. 13–14)
33. a. 1883 mi **b.** about 50 mi/h
35. a. *Sample:*

b. 21 ft

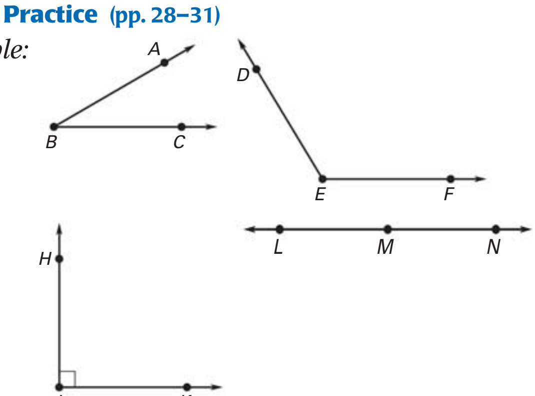
1.3 Skill Practice (pp. 19–20) **1.** Distance Formula **3.** $10\frac{1}{4}$ in. **5.** 26 cm **7.** $4\frac{3}{4}$ in. **9.** $2\frac{3}{8}$ in. **11.** 10 **13.** 1 **15.** 70 **17.** (5, 5) **19.** (1, 4) **21.** $\left(1\frac{1}{2}, -1\right)$ **23.** $\left(\frac{m}{2}, \frac{n}{2}\right)$; when x_2 and y_2 are replaced by zero in the Midpoint Formula and x_1 and y_1 are replaced by m and n the result is $\left(\frac{m}{2}, \frac{n}{2}\right)$. **25.** $(-3, 10)$ **27.** (4, 8) **29.** $(-18, 22)$ **31.** 4.5 **33.** 5.7 **35.** 7; $-\frac{1}{2}$ **37.** 40; 5 **39.** 9; $-3\frac{1}{2}$ **43.** $AB = 3\sqrt{5}$, $CD = 2\sqrt{10}$; not congruent **45.** $JK = 8\sqrt{2}$, $LM = \sqrt{130}$; not congruent

1.3 Problem Solving (pp. 21–22)
49. 

2.85 km

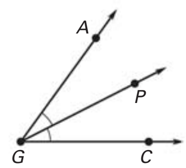
51. objects B and D; objects A and C **53. a.** 191 yd **b.** 40 yd **c.** About 1.5 min; find the total distance, about 230 yards, and divide by 150 yards per minute.

1.4 Skill Practice (pp. 28–31)
1. *Sample:*

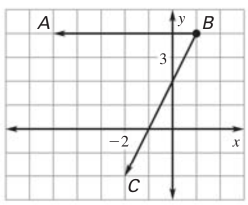


3. $\angle ABC$, $\angle B$, $\angle CBA$; B, $\overrightarrow{BA}$, $\overrightarrow{BC}$ **5.** $\angle MTP$, $\angle T$, $\angle PTM$; T, $\overrightarrow{TM}$, $\overrightarrow{TP}$ **7.** straight **9.** right **11.** 90°; right **13.** 135°; obtuse **15–19.** Sample answers are given. **15.** $\angle BCA$; right **17.** $\angle DFB$; straight **19.** $\angle CDB$; acute **23.** 65° **25.** 55° **29.** $m\angle XWY = 104°$, $m\angle ZWY = 52°$

31. $m\angle XWZ = 35.5°$, $m\angle YWZ = 35.5°$ **33.** 38°
35. 142° **37.** 53°
39. If a ray bisects $\angle AGC$ its vertex must be at point G. *Sample:*

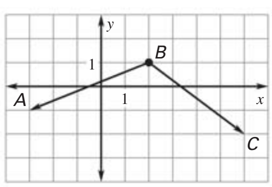


41. 80° **43.** 75°; both angle measures are 5° less.
45.



Acute.
Sample answer: (−2, 0)

47.

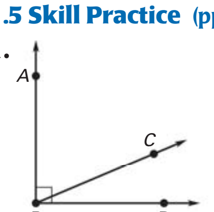


Obtuse.
Sample answer: (2, 0)

1.4 Problem Solving (pp. 31–32) **51.** 34° **53. a.** 112° **b.** 56° **c.** 56° **d.** 56° **55.** *Sample answer:* acute: $\angle ABG$, obtuse: $\angle ABC$, right: $\angle DGE$, straight: $\angle DGF$ **57.** about 140° **59.** about 62° **61.** about 107°

1.5 Skill Practice (pp. 38–40)

1.

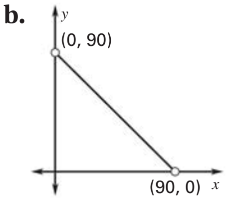


No. *Sample answer:* Any two angles whose angle measures add up to 90° are complementary, but they do not have to have a common vertex and side.

3. adjacent **5.** adjacent **7.** $\angle GLH$ and $\angle HLJ$, $\angle GLJ$ and $\angle JLK$ **9.** 69° **11.** 85° **13.** 25° **15.** 153° **17.** 135°, 45° **19.** 54°, 36° **21.** linear pair **23.** vertical angles **25.** linear pair **27.** neither **29.** The angles are complementary so they should be equal to 90°; $x + 3x = 90°$, $4x = 90$, $x = 22.5$. **31.** 10, 35 **33.** 55, 30 **35.** Never; a straight angle is 180°, and it is not possible to have a supplement of an angle that is 180°. **37.** Always; the sum of complementary angles is 90°, so each angle must be less than 90°, making them acute. **39.** 71°, 19° **41.** 68°, 22° **43.** 58°, 122°

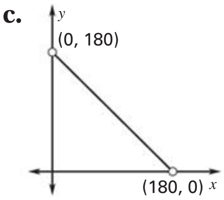
1.5 Problem Solving (pp. 40–41) **47.** neither **49–51.** Sample answers are given. **49.** $\angle FGB$, $\angle BGC$ **51.** $\angle AGE$, $\angle EGD$ **53.** *Sample answer:* Subtract 90° from $m\angle FGB$. **55. a.** $y_1 = 90 - x$, $0 < x < 90$; $y_2 = 180 - x$, $0 < x < 180$; the measure of the complement must be less than 90° and the measure of its supplement must be less than 180°.

55. b.



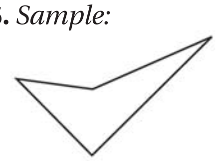
$0 < y_1 < 90$

c.

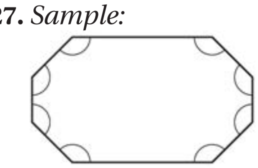


$0 < y_2 < 180$

1.6 Skill Practice (pp. 44–46) **1.** n is the number of sides of a polygon. **3.** polygon; concave **5.** polygon; convex **9.** Pentagon; regular; it has 5 congruent sides and angles. **11.** Triangle; neither; the sides and/or the angles are not all congruent. **13.** Quadrilateral; equiangular; it has 4 sides and 4 congruent angles. **15.** 8 in. **17.** 3 ft **19.** sometimes **21.** never **23.** never **25.** *Sample:*



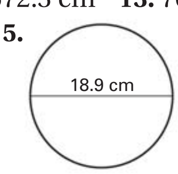
27. *Sample:*



29. 1

1.6 Problem Solving (pp. 46–47) **33.** triangle; regular **35.** octagon; regular **39.** 105 mm; each side of the button is 15 millimeters long, so the perimeter of the button is 15(7) = 105 millimeters. **41. a.** 3 **b.** 5 **c.** 6 **d.** 8

1.7 Skill Practice (pp. 52–54) **1.** *Sample answer:* The diameter is twice the radius. **3.** (52)(9) must be divided by 2; $\frac{52(9)}{2} = 234\text{ ft}^2$. **5.** 22.4 m, 29.4 m^2 **7.** 180 yd, 1080 yd^2 **9.** 36 cm, 36 cm^2 **11.** 84.8 cm, 572.3 cm^2 **13.** 76.0 cm, 459.7 cm^2
15.



59.3 cm, 280.4 cm^2

17. 12.4 **21.** 1.44 **23.** 8,000,000 **25.** 3,456 **27.** 14.5 m **29.** 4.5 in. **31.** 6 in., 3 in. **33.** Octagon; dodecagon; the square has 4 sides, so a polygon with the same side length and twice the perimeter would have to have 2(4) = 8 sides, an octagon; a polygon with the same side length and three times the perimeter would have to have 4(3) = 12 sides, a dodecagon. **35.** $\sqrt{346}$ in. **37.** $5\sqrt{42}$ km

1.7 Problem Solving (pp. 54–56) **41.** 1350 yd^2; 450 ft **43. a.** 15 in. **b.** 6 in.; the spoke is 21 inches long from the center to the tip, and it is 15 inches from the center to the outer edge, so 21 − 15 = 6 inches is the length of the handle.

45. a. 106.4 m^2 **b.** 380 rows, 175 columns. *Sample answer:* The panel is 1520 centimeters high and each module is 4 centimeters so there are $1520 \div 4 = 380$ rows; the panel is 700 centimeters wide and each module is 4 centimeters therefore there are $700 \div 4 = 175$ columns.

1.7 Problem Solving Workshop (p. 57)
1. 2.4 h **3.** $26,730

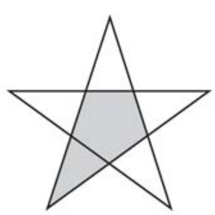
Chapter Review (pp. 60–63) **1.** endpoints **3.** midpoint **5.** *Sample answer:* points P, Y, Z **7.** $\overrightarrow{YZ}$, $\overrightarrow{YX}$ **9.** 1.2 **11.** 7 **13.** 16 **15.** 8.6; (3.5, 3.5) **17.** 16.4; (5, −0.5) **19.** 5 **21.** 162°; obtuse **23.** 7° **25.** 88° **27.** 124° **29.** 168° **31.** 92°, 88°; obtuse **33.** Quadrilateral; equiangular; it has four congruent angles but its four sides are not all congruent. **35.** 21 **37.** 14 in., 11.3 in.^{2} **39.** 5 m

Algebra Review (p. 65) **1.** 6 **3.** −2 **5.** $1\frac{1}{2}$ **7.** 4 **9.** −11 **11.** 17 people

Chapter 2

2.1 Skill Practice (pp. 75–76) **1.** *Sample answer:* A guess based on observation

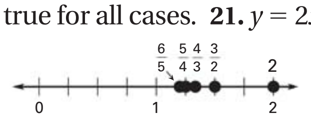
3.



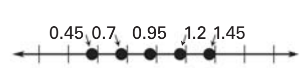
7. The numbers are 4 times the previous number; 768. **9.** The rate of decrease is increasing by 1; −6. **11.** The numbers are increasing by successive multiples of 3; 25. **13.** even

15. *Sample answer:* $(3 + 4)^2 = 7^2 = 49 \neq 3^2 + 4^2 = 9 + 16 = 25$ **17.** *Sample answer:* $3 \cdot 6 = 18$ **19.** To be true, a conjecture must be true for all cases. **21.** $y = 2x$

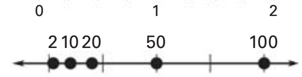
23. Previous numerator becomes the next denominator while the numerator is one more than the denominator; $\frac{6}{5}$.



25. 0.25 is being added to each number; 1.45.



27. Multiply the first number by 10 to get the second number, take half of the second number to get the third number, and repeat the pattern; 500.



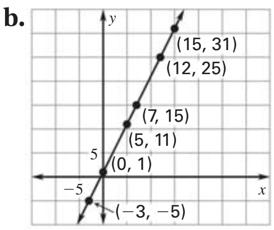
29. $r > 1$; $0 < r < 1$; raising numbers greater than one by successive natural numbers increases the result while raising a number between 0 and 1 by successive natural numbers decreases the result.

2.1 Problem Solving (pp. 77–78) **33.** *Sample answer:* The number of e-mail messages will increase in 2004; the number of e-mail messages has increased for the past 7 years.

35. a.

x	y
−3	−5
0	1
5	11
7	15
12	25
15	31

b.



c. Double the value of x and add 1 to the result, $y = 2x + 1$. **37. a.** sum, two **b.** 144, 233, 377 **c.** *Sample answer:* spiral patterns on the head of a sunflower

2.2 Skill Practice (pp. 82–84) **1.** converse **3.** If $x = 6$, then $x^2 = 36$. **5.** If a person is registered to vote, then they are allowed to vote. **7.** If an angle is a right angle, then its angle measure is 90°; if an angle measures 90°, then it is a right angle; if an angle is not a right angle, then it does not measure 90°; if an angle does not measure 90°, then it is not a right angle. **9.** If $3x + 10 = 16$, then $x = 2$; if $x = 2$, then $3x + 10 = 16$; if $3x + 10 \neq 16$, then $x \neq 2$; if $x \neq 2$, then $3x + 10 \neq 16$.

11. False. *Sample:*

13. False. *Sample answer:* $m\angle ABC = 60°$, $m\angle GEF = 120°$ **15.** False. *Sample answer:* 2 **17.** False; there is no indication of a right angle in the diagram. **19.** An angle is obtuse if and only if its measure is between 90° and 180°. **21.** Points are coplanar if and only if they lie on the same plane. **23.** good definition **27.** If $-x > -6$, then $x < 6$; true. **29.** *Sample answer:* If the dog sits, she gets a treat.

2.2 Problem Solving (pp. 84–85) **31.** true **33.** Find a counterexample. *Sample answer:* Tennis is a sport but the participants do not wear helmets. **35.** *Sample answer:* If a student is a member of the Jazz band, then the student is a member of the Band but not the Chorus. **37.** no

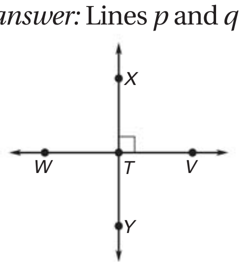
2.3 Skill Practice (pp. 90–91) **1.** Detachment **3.** *Sample answer:* The door to this room is closed. **5.** $-15 < -12$ **7.** If a rectangle has four equal side lengths, then it is a regular polygon. **9.** If you play the clarinet, then you are a musician. **11.** The sum is even; the sum of two even integers is even; $2n$ and $2m$ are even, $2n + 2m = 2(m + n)$, $2(n + m)$ is even.

13. Linear pairs are not the only pairs of angles that are supplementary; angles C and D are supplementary, the sum of their measures is 180°.

2.3 Problem Solving (pp. 91–93) **17.** You will get a raise if the revenue is greater then its cost. **19.** is **21.** Deductive; laws of logic were used to reach the conclusion. **23.** $2n + (2n + 1) = (2n + 2n) + 1 = 4n + 1$, which is odd. **25.** True; since the game is not sold out, Arlo goes and buys a hot dog. **27.** False; Mia will buy popcorn.

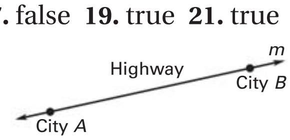
Extension (p. 95) **1.** $\sim q \to \sim p$ **3.** Polygon *ABCDE* is not equiangular and not equilateral. **5.** Polygon *ABCDE* is equiangular and equilateral if and only if it is a regular polygon. **7.** No; it is false when the hypothesis is true while the conclusion is false.

2.4 Skill Practice (pp. 99–100) **1.** line perpendicular to a plane **3.** Postulate 5 **5. a.** If three points are not collinear, then there exists exactly one plane that contains all three points. **b.** If there is a plane, then three noncollinear points exist on the plane; if three points are collinear, then there does not exist exactly one plane that contains all three; if there is not exactly one plane containing three points then the three points are collinear. **c.** contrapositive
7. *Sample answer:* Lines p and q intersecting in point H
9. *Sample:*



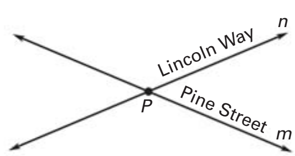
no; $\overline{XY}$ does not necessarily bisect $\overline{WV}$.

11. False. *Sample answer:* Consider a highway with two houses on the right side and one house on the left. **13.** False. *Sample answer:* Consider any pair of opposite sides of a rectangular prism.
15. false **17.** false **19.** true **21.** true **23.** false
25. *Sample:*



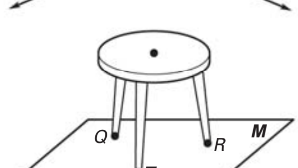
Postulate 5

Sample:



Postulate 7

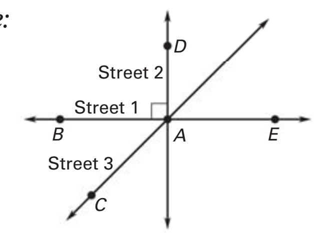
Sample:



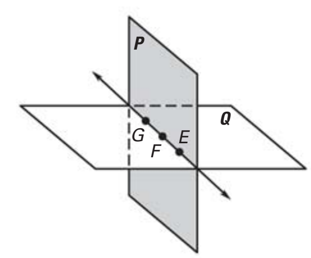
Postulate 8

27. *Sample answer:* Postulate 9 guarantees three noncollinear points on a plane while Postulate 5 guarantees that through any two there exist exactly one line therefore there exists at least one line in the plane.

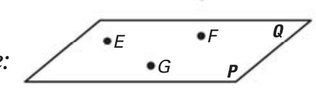
2.4 Problem Solving (pp. 101–102) **31.** Postulate 7 **33.** *Sample answer:* A stoplight with a red, yellow, and green light. **35.** *Sample answer:* A line passing through the second row of the pyramid. **37.** *Sample answer:* The person at the top and the two people at each end of the bottom row.
39. a. *Sample:*



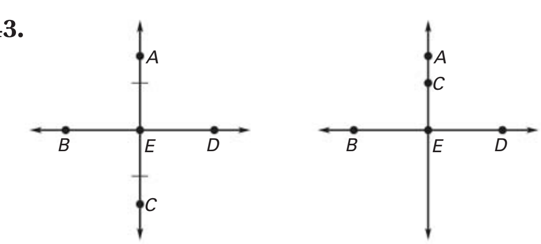
b. Building A **c.** right angle **d.** No; since $\angle CAE$ is obtuse, Building E must be on the east side of Building A. **e.** Street 1
41. They must be collinear. *Sample:*



They must be noncollinear. *Sample:*



43.



2.5 Skill Practice (pp. 108–109) **1.** Reflexive Property of Equality for Angle Measure **3.** Subtraction Property of Equality, Addition Property of Equality, Division Property of Equality

7. $4x + 9 = 16 - 3x$	Given
$7x + 9 = 16$	Addition Property of Equality
$7x = 7$	Subtraction Property of Equality
$x = 1$	Division Property of Equality

9. $3(2x + 11) = 9$	Given
$6x + 33 = 9$	Distributive Property
$6x = -24$	Subtraction Property of Equality
$x = -4$	Division Property of Equality

11. $44 - 2(3x + 4) = -18x$	Given
$44 - 6x - 8 = -18x$	Distributive Property
$36 - 6x = -18x$	Simplify.
$36 = -12x$	Addition Property of Equality
$-3 = x$	Division Property of Equality

13. $2x - 15 - x = 21 + 10x$	Given
$x - 15 = 21 + 10x$	Simplify.
$-15 = 21 + 9x$	Subtraction Property of Equality
$-36 = 9x$	Subtraction Property of Equality
$-4 = x$	Division Property of Equality

15. $5x + y = 18$	Given
$y = 18 - 5x$	Subtraction Property of Equality

17. $12 - 3y = 30x$	Given
$-3y = 30x - 12$	Subtraction Property of Equality
$y = \frac{30x - 12}{-3}$	Division Property of Equality
$y = -10x + 4$	Simplify.

19. $2y + 0.5x = 16$	Given
$2y = -0.5x + 16$	Subtraction Property of Equality
$y = \frac{-0.5x + 16}{2}$	Division Property of Equality
$y = -0.25x + 8$	Simplify.

21. $20 + CD$ **23.** AB, CD **25.** $m\angle 1 = m\angle 3$ **27.** *Sample answer:* Look in the mirror and see your reflection; 12 in. = 1 ft, so 1 ft = 12 in.; 10 pennies = 1 dime and 1 dime = 2 nickels, so 10 pennies = 2 nickels.

29. $AD = CB$	Given
$DC = BA$	Given
$AC = AC$	Reflexive Property of Equality
$AD + DC = CB + DC$	Addition Property of Equality
$AD + DC = CB + BA$	Substitution
$AD + DC + AC = CB + BA + AC$	Addition Property of Equality

2.5 Problem Solving (pp. 110–111)

31. $P = 2\ell + 2w$	Given
$P - 2w = 2\ell$	Subtraction Property of Equality
$\frac{P - 2w}{2} = \ell$	Division Property of Equality

length: 16.5 m

33. Row 1: Marked in diagram; Row 2: Substitute $m\angle GHF$ for 90°; Row 3: Angle Addition Postulate; Row 4: Substitution Property of Equality; Row 5: $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$; Substitution Property of Equality; Row 6: Subtract $m\angle 1$ from both sides. **35.** 116°

2.6 Skill Practice (pp. 116–117) **1.** A theorem is a statement that can be proven; a postulate is a rule that is accepted without proof. **3.** 3. Substitution; 4. $AC = 11$ **5.** $\overline{SE}$ **7.** $\angle J$, $\angle L$ **9.** Reflexive Property of Congruence **11.** Reflexive Property of Equality **13.** The reason is the Transitive Property of Congruence not the Reflexive Property of Congruence.

15. Cottage — Snack Shop — Bike Rental — Arcade — Kite Shop

17. Equation	**Explanation**	**Reason**
$\overline{QR} \cong \overline{PQ}$, $\overline{RS} \cong \overline{PQ}$	Write original statement.	Given
$2x + 5 = 10 - 3x$	Marked in diagram.	Transitive Property of Congruent Segments
$5x + 5 = 10$	Add $3x$ to each side.	Addition Property of Equality
$5x = 5$	Subtract 5 from each side.	Subtraction Property of Equality
$x = 1$	Divide each side by 5.	Division Property of Equality

19. A proof is deductive reasoning because it uses facts, definitions, accepted properties, and laws of logic.

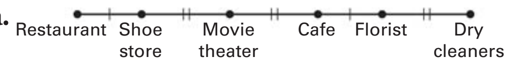
2.6 Problem Solving (pp. 118–119) **21.** 2. Definition of angle bisector; 4. Transitive Property of Congruence

23. Statements	Reasons
1. $2AB = AC$	1. Given
2. $AC = AB + BC$	2. Segment Addition Postulate
3. $2AB = AB + BC$	3. Transitive Property of Segment Equality
4. $AB = BC$	4. Subtraction Property of Equality

25. Statements	Reasons
1. A is an angle.	1. Given
2. $m\angle A = m\angle A$	2. Reflexive Property of Equality
3. $\angle A \cong \angle A$	3. Definition of congruent angles

27. Equiangular; the Transitive Property of Congruent Angles implies $m\angle 1 = m\angle 3$, so all angle measures are the same.

29. a.



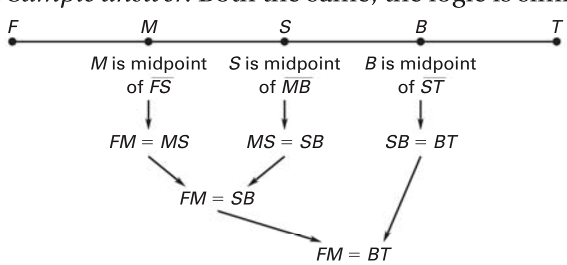
b. Given: $\overline{RS} \cong \overline{CF}$, $\overline{SM} \cong \overline{MC} \cong \overline{FD}$, Prove: $\overline{RM} \cong \overline{CD}$

c.

Statements	Reasons
1. $\overline{RS} \cong \overline{CF}$, $\overline{SM} \cong \overline{MC} \cong \overline{FD}$	1. Given
2. $RS + SM = RM$	2. Segment Addition Postulate
3. $CF + FD = CD$	3. Segment Addition Postulate
4. $CF + FD = RM$	4. Substitution Property of Equality
5. $FM = BT$	5. Transitive Property of Segment Congruence
6. $RM = CD$	6. Definition of congruent segments

2.6 Problem Solving Workshop (p. 121) **1. a.** *Sample answer:* The logic used is similar; one uses segment length and the other uses segment congruence. **b.** *Sample answer:* Both the same; the logic is similar.

3.



Statements	Reasons
1. M is halfway between F and S; S is halfway between M and B; B is halfway between S and T.	1. Given
2. M is the midpoint of $\overline{FS}$; S is the midpoint of $\overline{MB}$; B is the midpoint of $\overline{ST}$.	2. Definition of midpoint
3. $FM = MS$, $MS = SB$, $SB = BT$	3. Definition of midpoint
4. $FM = SB$	4. Transitive Property of Equality
5. $FM = BT$	5. Transitive Property of Equality

5. a. *Sample answer:* The proof on page 114 is angle congruence while this one is segment congruence. **b.** *Sample answer:* If $\overline{FG} \cong \overline{DE}$ is the second statement, the reason would have to be Symmetric Property of Segment Congruence and that is what is being proven and you cannot use a property that you are proving as a reason in the proof.

2.7 Skill Practice (pp. 127–129) **1.** vertical **3.** $\angle MSN$ and $\angle PSQ$, $\angle NSP$ and $\angle QSR$; indicated in diagram, Congruent Complements Theorem **5.** $\angle FGH$ and $\angle WXZ$; Right Angles Congruence Theorem **7.** Yes; perpendicular lines form right angles. **9.** 168°, 12°, 12° **11.** 118°, 118°, 62° **13.** $x = 13$, $y = 20$ **15.** *Sample answer:* It was assumed that $\angle 1$ and $\angle 3$, and $\angle 2$ and $\angle 4$ are linear pairs, but they are not; $\angle 1$ and $\angle 4$, and $\angle 2$ and $\angle 3$ are not vertical angles and are not congruent. **17.** 30° **19.** 27° **21.** 58° **23.** true **25.** false **27.** true **29.** 140°, 40°, 140°, 40° **31.** $\angle FGH$ and $\angle EGH$; Definition of angle bisector **33.** *Sample answer:* $\angle CEB$ and $\angle DEB$; Right Angle Congruence Theorem

2.7 Problem Solving (pp. 129–131)

37. 1. Given; 2. Definition of complementary angles; 3. $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 3$; 4. $m\angle 2 = m\angle 3$; 5. Definition of congruent angles

39.

Statements	Reasons
1. $\overline{JK} \perp \overline{JM}$, $\overline{KL} \perp \overline{ML}$, $\angle J \cong \angle M$, $\angle K \cong \angle L$	1. Given
2. $\angle J$ and $\angle L$ are right angles.	2. Definition of perpendicular lines
3. $\angle M$ and $\angle K$ are right angles.	3. Right Angle Congruence Theorem
4. $\overline{JM} \perp \overline{ML}$ and $\overline{JK} \perp \overline{KL}$	4. Definition of perpendicular lines

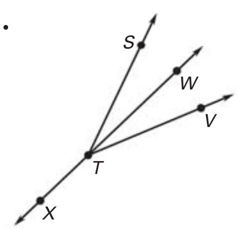
41.

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are complementary; $\angle 3$ and $\angle 2$ are complementary.	1. Given
2. $m\angle 1 + m\angle 2 = 90°$, $m\angle 3 + m\angle 2 = 90°$	2. Definition of complementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	3. Transitive Property of Equality
4. $m\angle 1 = m\angle 3$	4. Subtraction Property of Equality
5. $\angle 1 \cong \angle 3$	5. Definition of congruent angles

43.

Statements	Reasons
1. $\angle QRS$ and $\angle PSR$ are supplementary.	1. Given
2. $\angle QRS$ and $\angle QRL$ are a linear pair.	2. Definition of linear pair
3. $\angle QRS$ and $\angle QRL$ are supplementary.	3. Definition of linear pair
4. $\angle QRL$ and $\angle PSR$ are supplementary.	4. Congruent Supplements Theorem

45. a.



b. $\angle STV$ is bisected by $\overline{TW}$, and $\overline{TX}$ and $\overline{TW}$ are opposite rays, $\angle STX \cong \angle VTX$

c.

Statements	Reasons
1. $\angle STV$ is bisected by $\overline{TW}$; $\overline{TX}$ and $\overline{TW}$ are opposite rays.	1. Given
2. $\angle STW \cong \angle VTW$	2. Definition of angle bisector
3. $\angle VTW$ and $\angle VTX$ are a linear pair; $\angle STW$ and $\angle STX$ are a linear pair.	3. Definition of linear pair
4. $\angle VTW$ and $\angle VTX$ are supplementary; $\angle STW$ and $\angle STX$ are supplementary.	4. Definition of linear pair
5. $\angle STW$ and $\angle VTX$ are supplementary.	5. Substitution
6. $\angle STX \cong \angle VTX$	6. Congruent Supplements Theorem

Chapter Review (pp. 134–137) **1.** theorem **3.** $m\angle A = m\angle C$ **5.** *Sample answer:* $\frac{-10}{-2} = 5$ **7.** Yes. *Sample answer:* This is the definition for complementary angles. **9.** $\angle B$ measures 90°. **11.** The sum of two odd integers is even. *Sample answer:* $7 + 1 = 8$; $2n + 1$ and $2m + 1$ are odd, but their sum $(2n + 1) + (2m + 1) = 2m + 2n + 2 = 2(m + n + 1)$ is even.

15.

$15x + 22 = 7x + 62$	Given
$8x + 22 = 62$	Subtraction Property of Equality
$8x = 40$	Subtraction Property of Equality
$x = 5$	Division Property of Equality

17.

$5x + 2(2x - 23) = -154$	Given
$5x + 4x - 46 = -154$	Distributive Property
$9x - 46 = -154$	Simplify.
$9x = -108$	Addition Property of Equality
$x = -12$	Division Property of Equality

19. Reflexive Property of Congruence

21.

$\angle A \cong \angle B, \angle B \cong \angle C$	Given
$m\angle A = m\angle B$, $m\angle B = m\angle C$	Definition of angle congruence
$m\angle A = m\angle C$	Transitive Property of Equality
$\angle A \cong \angle C$	Definition of angle congruence

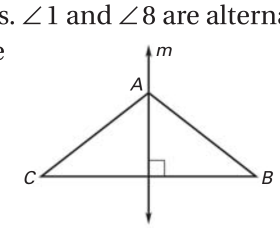
23. 123°, 57°, 123°

Algebra Review (p. 139) **1.** $\frac{x^2}{4}$ **3.** $m + 7$ **5.** $\frac{k+3}{-2k+3}$ **7.** 2 **9.** $\frac{x-2}{2x-1}$ **11.** $-6\sqrt{5}$ **13.** $\pm 8\sqrt{2}$ **15.** $12\sqrt{6}$ **17.** $20\sqrt{2}$ **19.** $100\sqrt{2}$ **21.** 25 **23.** $\sqrt{13}$

Chapter 3

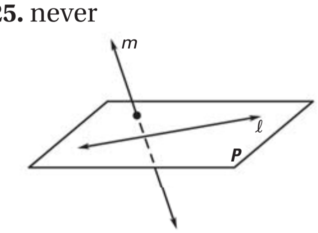
3.1 Skill Practice (pp. 150–151) **1.** transversal **3.** $\overleftrightarrow{AB}$ **5.** $\overleftrightarrow{BF}$ **7.** $\overleftrightarrow{MK}$, $\overleftrightarrow{LS}$ **9.** No. *Sample answer:* There is no arrow indicating they are parallel. **11.** $\angle 1$ and $\angle 5$, $\angle 3$ and $\angle 7$, $\angle 2$ and $\angle 6$, $\angle 4$ and $\angle 8$ **13.** $\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$ **15.** $\angle 1$ and $\angle 8$ are not in corresponding positions. $\angle 1$ and $\angle 8$ are alternate exterior angles.

17. 1 line

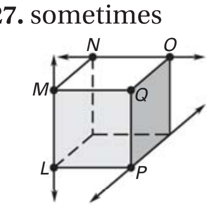


19. consecutive interior **21.** alternate exterior **23.** corresponding

25. never



27. sometimes

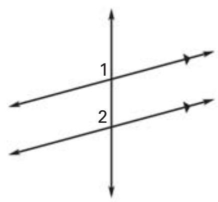


29. $\angle CFJ$, $\angle HJG$ **31.** $\angle DFC$, $\angle CJH$

3.1 Problem Solving (pp. 151–152) **35.** skew **39.** The adjacent interior angles are supplementary thus the measure of the other two angles must be 90°. **41.** false

3.2 Skill Practice (pp. 157–158)

1. *Sample:*



5. 110°; Alternate Exterior Angles Theorem **7.** 63°; Consecutive Interior Angles Theorem

9. Corresponding Angles Postulate **11.** Alternate Interior Angles Theorem **13.** Alternate Exterior Angles Theorem **15.** Alternate Exterior Angles Theorem **17.** $m\angle 1 = 150°$, Corresponding Angles Postulate; $m\angle 2 = 150°$, Vertical Angles Congruence Theorem **19.** $m\angle 1 = 122°$, $m\angle 2 = 58°$; Alternate Interior Angles Theorem, Consecutive Interior Angles Theorem **21.** *Sample answer:* $\angle 1 \cong \angle 4$ by the Alternate Exterior Angles Theorem; $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$ by Vertical Angles Congruence Theorem, Alternate Interior Angles Theorem, and the Transitive Property of Angle Congruence. **23.** $m\angle 1 = 90°$, supplementary to the right angle by the Consecutive Interior Angles Theorem; $m\angle 3 = 65°$, it forms a linear pair with the angle measuring 115°; $m\angle 2 = 115°$, supplementary to $\angle 3$ by the Consecutive Interior Angles Theorem **25.** *Sample answer:* $\angle BAC$ and $\angle DCA$, $\angle DAC$ and $\angle BCA$ **27.** 45, 85 **29.** 65, 60 **31.** 13, 12

3.2 Problem Solving (pp. 159–160)

37.

Statements	Reasons
1. $p \parallel q$	1. Given
2. $\angle 1 \cong \angle 3$	2. Corresponding Angles Postulate
3. $\angle 3 \cong \angle 2$	3. Vertical Angles Congruence Theorem
4. $\angle 1 \cong \angle 4$	4. Transitive Property of Angle Congruence

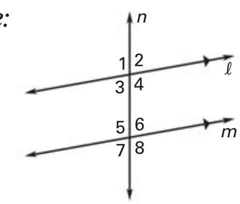
39. a. yes; $\angle 1$ and $\angle 4$, $\angle 1$ and $\angle 5$, $\angle 1$ and $\angle 8$, $\angle 4$ and $\angle 5$, $\angle 4$ and $\angle 8$, $\angle 5$ and $\angle 8$, $\angle 3$ and $\angle 2$, $\angle 3$ and $\angle 7$, $\angle 3$ and $\angle 6$, $\angle 2$ and $\angle 7$, $\angle 2$ and $\angle 6$, $\angle 7$ and $\angle 6$; yes; $\angle 1$ and $\angle 3$, $\angle 1$ and $\angle 2$, $\angle 1$ and $\angle 6$, $\angle 1$ and $\angle 7$, $\angle 2$ and $\angle 4$, $\angle 2$ and $\angle 5$, $\angle 2$ and $\angle 8$, $\angle 3$ and $\angle 4$, $\angle 3$ and $\angle 8$, $\angle 3$ and $\angle 5$, $\angle 5$ and $\angle 6$, $\angle 5$ and $\angle 7$, $\angle 6$ and $\angle 8$, $\angle 7$ and $\angle 8$. **b.** *Sample answer:* The transversal stays parallel to the floor.

41.

Statements	Reasons
1. $n \parallel p$	1. Given
2. $\angle 1 \cong \angle 3$	2. Corresponding Angles Postulate
3. $\angle 3$ and $\angle 2$ are supplementary.	3. Definition of linear pair
4. $\angle 1$ and $\angle 2$ are supplementary.	4. Substitution

3.3 Skill Practice (pp. 165–167)

1. *Sample:*



3. 40 **5.** 15 **7.** 60 **9.** The student believes that $x = y$ but there is no indication that they are equal.

11. yes; Alternate Exterior Angles Converse **13.** yes; Corresponding Angles Converse **15.** yes; Vertical Angles Congruence Theorem, Corresponding Angles Converse **17. a.** $m\angle DCG = 115°$, $m\angle CGH = 65°$ **b.** They are consecutive interior angles. **c.** yes; Consecutive Interior Angles Converse **19.** yes; Consecutive Interior Angles Converse **21.** no **25.** *Sample answer:* $\angle 1 \cong \angle 4$ therefore $\angle 4$ and $\angle 7$ are supplementary. Lines *j* and *k* are parallel by the Consecutive Interior Angles Converse. **27. a.** 1 line **b.** infinite number of lines **c.** 1 plane

3.3 Problem Solving (pp. 167–169) **29.** Alternate Interior Angles Converse Theorem **31.** substitution, Definition of supplementary angles, Consecutive Interior Angles Theorem **33.** Yes. *Sample answer:* 1st is parallel to 2nd by the Corresponding Angles Converse Postulate. 2nd is parallel to 3rd by the Alternate Exterior Angles Converse Theorem. 3rd is parallel to 4th by the Alternate Interior Angles Converse Theorem. They are all parallel by the Transitive Property of Parallel Lines.

35.

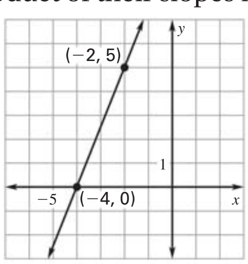
Statements	Reasons
1. $a \parallel b$, $\angle 2 \cong \angle 3$	1. Given
2. $\angle 2$ and $\angle 4$ are supplementary.	2. Consecutive Interior Angles Theorem
3. $\angle 3$ and $\angle 4$ are supplementary.	3. Substitution
4. $c \parallel d$	4. Consecutive Interior Angles Converse Theorem

37. You are given that $\angle 3$ and $\angle 5$ are supplementary. By the Linear Pair Postulate, $\angle 5$ and $\angle 6$ are also supplementary. So $\angle 3 \cong \angle 6$ by the Congruent Supplements Theorem. By the converse of the Alternate Interior Angles Theorem, $m \parallel n$.

39. a. *Sample answer:* Corresponding Angles Converse Theorem **b.** Slide the triangle along a fixed horizontal line and use the edge that forms the 90° angle to draw vertical lines. **40–44.** Sample answers are given. **41.** Vertical Angles Congruence Theorem followed by the Consecutive Interior Angles Converse Theorem **43.** Vertical Angles Congruence Theorem followed by the Corresponding Angles Converse Postulate

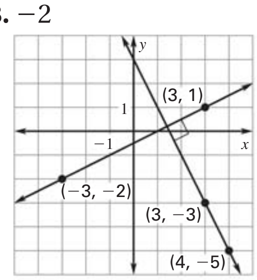
3.4 Skill Practice (pp. 175–176) **1.** The slope of a nonvertical line is the ratio of vertical change (rise) to horizontal change (run) between any two points on the line. **7.** $\frac{1}{2}$ **9.** 0 **11.** Slope was computed using $\frac{\text{run}}{\text{rise}}$, it should be $\frac{\text{rise}}{\text{run}}$; $m = \frac{3}{4}$. **13.** Perpendicular; the product of their slopes is -1. **15.** Perpendicular; the product of their slopes is -1.

17.

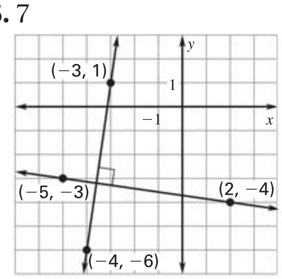


19. line 2 **21.** line 1

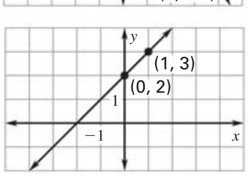
23. -2



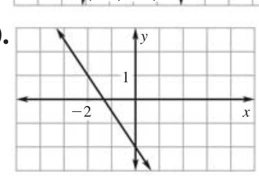
25. 7



27.

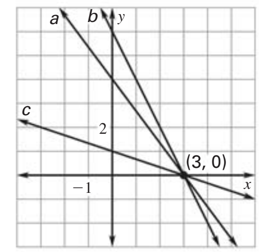


29.



3.4 Problem Solving (pp. 176–178) **33.** $\frac{2}{3}$

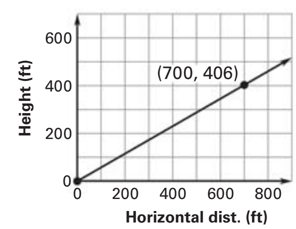
35. line b; line c. *Sample:*



37. a. 406 ft

Horizontal Distance	50	100	150	200	250	300	350
Height	29	58	87	116	145	174	203
Horizontal Distance	**400**	**450**	**500**	**550**	**600**	**650**	**700**
Height	232	261	290	319	348	377	406

b. $\frac{29}{50}$



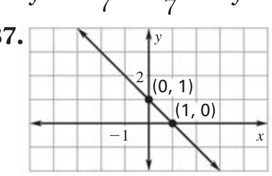
c. $\frac{144}{271}$; Duquesne

39. \$1150 per year **41. a.** 1985 to 1990. *Sample answer:* about 2 million people per year **b.** 1995 to 2000. *Sample answer:* about 3 million people per year **c.** *Sample answer:* There was moderate but steady increase in attendance for the NFL over the time period of 1985–2000.

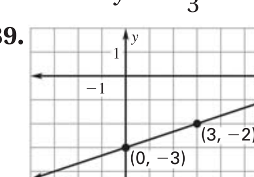
3.5 Skill Practice (pp. 184–186) **1.** The point of intersection on the y-axis when graphing a line.

3. $y = \frac{4}{3}x - 4$ **5.** $y = -\frac{3}{2}x - \frac{1}{2}$ **7.** $y = \frac{3}{2}x - \frac{3}{2}$ **11.** $y = 3x + 2$ **13.** $y = -\frac{5}{2}x$ **15.** $y = -\frac{11}{5}x - 12$ **17.** $y = 4x - 16$ **19.** $y = -\frac{2}{3}x - \frac{22}{3}$ **21.** $y = 7$ **23.** $y = -2x - 1$ **25.** $y = \frac{1}{5}x + \frac{37}{5}$ **27.** $y = -\frac{5}{2}x - 4$ **31.** $y = -\frac{3}{7}x + \frac{4}{7}$ **33.** $y = \frac{1}{2}x + 2$ **35.** $y = -\frac{5}{3}x - \frac{40}{3}$

37.

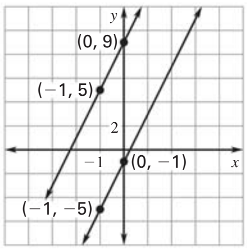


39.

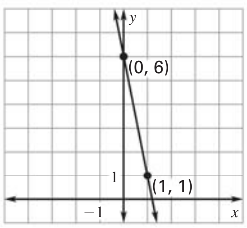


45. To find the x-intercept, let $y = 0$, $5x - 3(0) = -15$, $x = -3$, $(-3, 0)$. To find the y-intercept, let $x = 0$, $5(0) - 3y = -15$, $y = 5$, $(0, 5)$. **47.** $y = 0.5x + 7$ and $-x + 2y = -5$ **49.** 4, 4; $y = -x + 4$ **51.** -20, 10; $y = \frac{1}{2}x + 10$

SELECTED ANSWERS

53. 

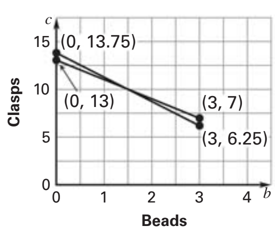
none

55. 

infinitely many

57. 4

3.5 Problem Solving (pp. 186–187) **61.** $y = 2.1x + 2000$; slope: gain in weight per day, y-intercept: starting weight before the growth spurt **63.** $2x + 3y = 24$; A: cost of a small slice, B: cost of a large slice, C: amount of money you can spend **65. a.** $2b + c = 13$, $5b + 2c = 27.50$

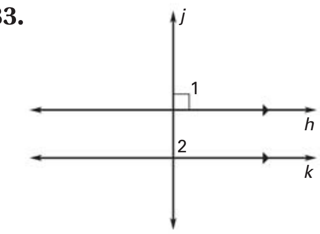
b. 

c. *Sample answer:* It's where the number of packages of beads and the number of packages of clasps would be the same for both girls.

3.5 Problem Solving Workshop (p. 189) **1.** 27 h **3.** 115 buttons **5.** *Sample answer:* In each case an equation modeling the situation was solved.

3.6 Skill Practice (pp. 194–195) **1.** $\overline{AB}$; it's $\perp$ to the parallel lines. **3.** If two sides of two adjacent acute angles are perpendicular, then the angles are complementary. **5.** 25° **7.** 52° **9.** Since the two angles labeled $x°$ form a linear pair of congruent angles, $t \perp n$; since the two lines are perpendicular to the same line, they are parallel to each other. **11.** *Sample answer:* Draw a line. Construct a second line perpendicular to the first line. Construct a third line perpendicular to the second line. **13.** There is no information to indicate that $y \parallel z$ or $y \perp x$. **15.** 13 **17.** 33 **19.** Lines f and g; they are perpendicular to line d. **23.** 4.1 **27.** 2.5

3.6 Problem Solving (pp. 196–197) **29.** Point C; the shortest distance is the length of the perpendicular segment. **31.** Definition of linear pair; $m\angle 1 + m\angle 2 = 180°$; Definition of angle congruence; Division Property of Equality; $\angle 1$ is a right angle; Definition of perpendicular.

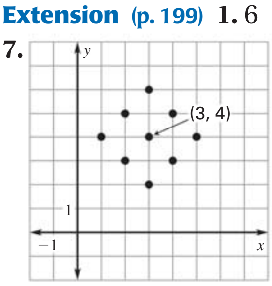
33. 

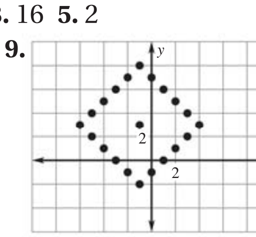
Given $h \parallel k$, $j \perp h$

Prove $j \perp k$

Statements	Reasons
1. $h \parallel k$, $j \perp h$	1. Given
2. $\angle 1 \cong \angle 2$	2. Corresponding Angles Postulate
3. $\angle 1$ is a right angle.	3. $\perp$ lines intersect to form 4 right angles
4. $m\angle 1 = 90°$	4. Definition of right angle
5. $m\angle 2 = 90°$	5. Definition of angle congruence
6. $\angle 2$ is a right angle.	6. Definition of right angle
7. $j \perp k$	7. Definition of perpendicular lines

Extension (p. 199) **1.** 6 **3.** 16 **5.** 2

7. 

9. 

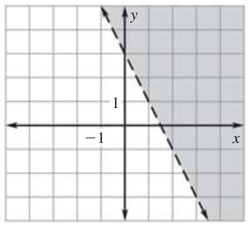
11. (1, 0) **13.** (10, 4)

Chapter Review (pp. 202–205) **1.** skew lines **3.** $\angle 5$ **5.** $\angle 6$ **7.** standard form **9.** $\overleftrightarrow{NR}$ **11.** $\overleftrightarrow{JN}$ **13.** $m\angle 1 = 54°$, vertical angles; $m\angle 2 = 54°$, corresponding angles **15.** $m\angle 1 = 135°$, corresponding angles; $m\angle 2 = 45°$, supplementary angles **17.** 13, 132 **19.** 35°. *Sample answer:* $\angle 2$ and $\angle 3$ are complementary, $\angle 1$ and $\angle 2$ are corresponding angles for two parallel lines cut by a transversal. **21.** 133 **23.** perpendicular **25. a.** $y = 6x - 19$ **b.** $y = -\frac{1}{6}x - \frac{1}{2}$ **27.** 3.2

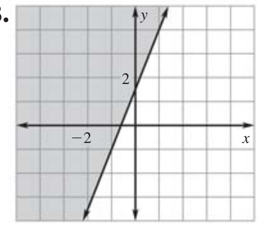
SELECTED ANSWERS

Algebra Review (p. 207)

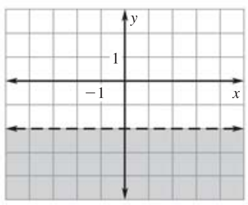
1.



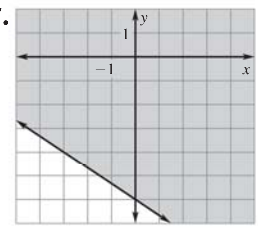
3.



5.



7.



9. 6 mo **11.** after 100 min

Cumulative Review (pp. 212–213) **1.** 28, 56 **3.** acute **5.** acute **7.** 40 in., 84 in.^{2} **9.** 15.2 yd, 14.44 yd^2

11. Each number is being multiplied by $\frac{1}{4}$; $\frac{1}{2}$. **13.** $x = 4$

15. The musician is playing a string instrument.

17.

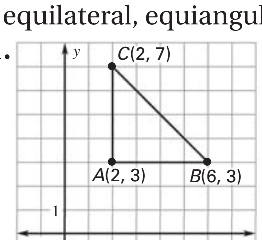
Equation	Reason
$-4(x + 3) = -28$	Given
$x + 3 = 7$	Division Property of Equality
$x = 4$	Subtraction Property of Equality

19. 29 **21.** $x = 9, y = 31$ **23.** $x = 101, y = 79$ **25.** 0 **27.** 2 **29. a.** $y = -x + 10$ **b.** $y = x + 14$ **31.** Yes; if two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular. **33.** *Sample answer:* parallel and perpendicular lines **35.** 89 mi **37.** If you want the lowest television prices, then come see Matt's TV Warehouse; if you want the lowest television prices; come see Matt's TV Warehouse. **39.** Yes. *Sample answer:* Transitive Property of Congruence of Segments

Chapter 4

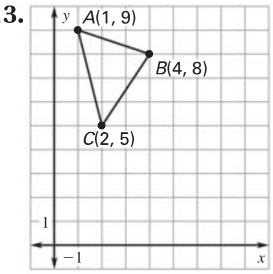
4.1 Skill Practice (pp. 221–222) **1.** C **3.** F **5.** B **7.** No; in a right triangle, the other two angles are complementary so they are both less than 90°. **9.** equilateral, equiangular

11.



isosceles; right triangle

13.



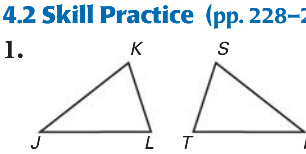
scalene; not a right triangle

15. 30; right **17.** 92° **19.** 158° **21.** 50° **23.** 50° **25.** 40° **27.** $m\angle P = 45°$, $m\angle Q = 90°$, $m\angle R = 45°$ **29.** Isosceles does not guarantee the third side is congruent to the two congruent sides; so if $\triangle ABC$ is equilateral, then it is isosceles as well. **33.** 118, 96 **35.** 26, 64 **37.** 35, 37

4.1 Problem Solving (pp. 223–224) **41.** 2 in.; 60°; in an equilateral triangle all sides have the same length $\left(\frac{6}{3}\right)$. In an equiangular triangle the angles always measure 60°. **45.** 115° **47.** 65° **49. a.** $2\sqrt{2x} + 5\sqrt{2x} + 2\sqrt{2x} = 180$ **b.** 40°, 100°, 40° **c.** obtuse **51.** *Sample answer:* They both reasoned correctly but their initial plan was incorrect. The measure of the exterior angle should be 150°.

4.2 Skill Practice (pp. 228–229)

1.

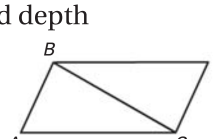


$\overline{JK} \cong \overline{RS}$, $\overline{KL} \cong \overline{ST}$, $\overline{JL} \cong \overline{RT}$, $\angle J \cong \angle R$, $\angle K \cong \angle S$, $\angle L \cong \angle T$

3. $\angle A$ and $\angle D$, $\angle C$ and $\angle F$, $\angle B$ and $\angle E$, $\overline{AB}$ and $\overline{DE}$, $\overline{AC}$ and $\overline{DF}$, $\overline{BC}$ and $\overline{EF}$. *Sample answer:* $\triangle CAB \cong \triangle FDE$. **5.** 124° **7.** 8 **9.** $\triangle ZYX$ **11.** $\triangle XYZ \cong \triangle ZWX$; all corresponding sides and angles are congruent. **13.** $\triangle BAG \cong \triangle CDF$; all corresponding sides and angles are congruent. **15.** 20 **17.** Student still needs to show that corresponding sides are congruent. **19.** 3, 1

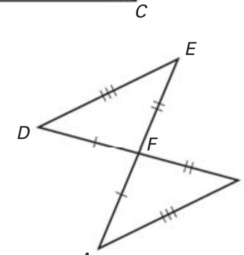
4.2 Problem Solving (pp. 230–231) **23.** Reflexive Property of Congruent Triangles **25.** length, width, and depth

27.



Yes; alternate interior angles are congruent.

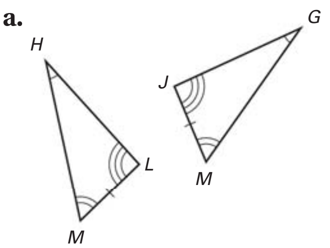
29. no



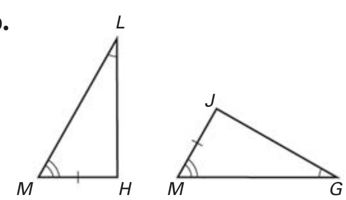
31. a. Corresponding parts of congruent figures are congruent. **b.** They are supplementary to two congruent angles and therefore are congruent. **c.** *Sample answer:* All right angles are congruent. **d.** Yes; all corresponding parts of both triangles are congruent.

4.2 Problem Solving Workshop (p. 232)

1. a.



b.



4.3 Skill Practice (pp. 236–237) **1.** corresponding angles **3.** corresponding sides **5.** not true; $\triangle RST \cong \triangle PQT$ **7.** true; SSS **9.** congruent **11.** congruent **13.** Stable; the figure has diagonal support with fixed side lengths. **15.** Stable; the figure has diagonal support with fixed side lengths. **19.** Not congruent; the congruence statement should read $\triangle ABC \cong \triangle FED$.

4.3 Problem Solving (pp. 238–239) **23.** Gate 1. *Sample answer:* Gate 1 has a diagonal support that forms two triangles with fixed side lengths, and these triangles cannot change shape. Gate 2 is not stable because that gate is a quadrilateral which can take many different shapes.

25.

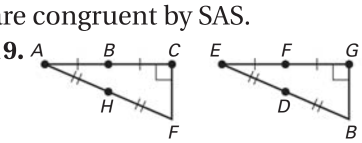
Statements	Reasons
1. $\overline{WX} \cong \overline{VZ}$, $\overline{WY} \cong \overline{VY}$, $\overline{YZ} \cong \overline{YX}$	1. Given
2. $\overline{WV} \cong \overline{VW}$	2. Reflexive Property of Congruence
3. $WY = VY$, $YZ = YX$	3. Definition of segment congruence
4. $WY + YZ = VY + YZ$	4. Addition Property of Equality
5. $WY + YZ = VY + YX$	5. Substitution Property of Equality
6. $WZ = VX$	6. Segment Addition Postulate
7. $\overline{WZ} \cong \overline{VX}$	7. Definition of segment congruence
8. $\triangle VWX \cong \triangle WVZ$	8. SSS

27.

Statements	Reasons
1. $\overline{FM} \cong \overline{FN}$, $\overline{DM} \cong \overline{HN}$, $\overline{EF} \cong \overline{GF}$, $\overline{DE} \cong \overline{HG}$	1. Given
2. $MN = NM$	2. Reflexive Property of Equality
3. $FM = FN$, $DM = HN$, $EF = GF$	3. Definition of segment congruence
4. $EF + FN = GF + FN$, $DM + MN = HN + MN$	4. Addition Property of Equality
5. $EF + FN = GF + FM$, $DM + MN = HN + NM$	5. Substitution Property of Equality
6. $EN = GM$, $DN = HM$	6. Segment Addition Postulate
7. $\overline{EN} \cong \overline{GM}$, $\overline{DN} \cong \overline{HM}$	7. Definition of segment congruence
8. $\triangle DEN \cong \triangle HGM$	8. SSS

29. Only one triangle can be created from three fixed sides.

4.4 Skill Practice (pp. 243–244) **1.** included **3.** $\angle XYW$ **5.** $\angle ZWY$ **7.** $\angle XYZ$ **9.** not enough **11.** not enough **13.** enough **17.** *Sample answer:* $\triangle STU$, $\triangle RVU$; they are congruent by SAS.

19. 

HL

21. SAS **23.** Yes; they are congruent by the SAS Congruence Postulate. **25.** $\overline{AC} \cong \overline{DF}$ **27.** $\overline{BC} \cong \overline{EF}$ **29.** Because $\overline{RM} \perp \overline{PQ}$, $\angle RMQ$ and $\angle RMP$ are right angles and thus are congruent. $\overline{QM} \cong \overline{MP}$ and $\overline{MR} \cong \overline{MR}$. It follows that $\triangle RMP \cong \triangle RMQ$ by SAS.

4.4 Problem Solving (pp. 245–246) **31.** SAS **33.** Two sides and the included angle of one sail need to be congruent to the corresponding sides and angle of the second sail; the two sails need to be right triangles with congruent hypotenuses and one pair of congruent corresponding legs.

35.

Statements	Reasons
1. $\overline{PQ}$ bisects $\angle SPT$, $\overline{ST} \cong \overline{TP}$	1. Given
2. $\angle SPQ \cong \angle TPQ$	2. Definition of angle bisector
3. $\overline{PQ} \cong \overline{PQ}$	3. Reflexive Property of Congruence
4. $\triangle SPQ \cong \triangle TPQ$	4. SAS

SELECTED ANSWERS

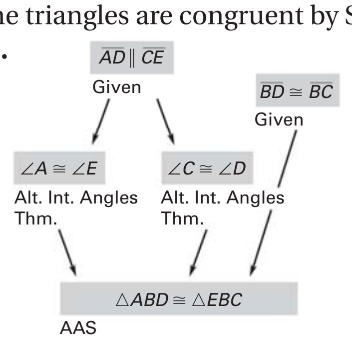
37.

Statements	Reasons
1. $\overline{JM} \cong \overline{LM}$	1. Given
2. $\angle KJM$ and $\angle KLM$ are right angles.	2. Given
3. $\triangle JKM$ and $\triangle LKM$ are right triangles.	3. Definition of right triangle
4. $\overline{KM} \cong \overline{KM}$	4. Reflexive Property of Congruence
5. $\triangle JKM \cong \triangle LKM$	5. HL

4.5 Skill Practice (pp. 252–253) **1.** *Sample answer:* A flow proof shows the flow of a logical argument. **3.** yes; AAS **5.** yes; ASA **9.** $\angle F$, $\angle L$ **11.** $\angle AFE \cong \angle DFB$ by the Vertical Angles Theorem. **13.** $\angle EDA \cong \angle DCB$ by the Corresponding Angles Postulate. **15.** No; there is no AAA postulate or theorem. **17.** No; the segments that are congruent are not corresponding sides. **19.** yes; the SAS Congruence Postulate **21. a.** $\overline{BC}$ and $\overline{AD}$ are parallel with $\overline{AC}$ being a transversal. The Alternate Interior Angles Theorem applies. **b.** $\overline{AB}$ and $\overline{CD}$ are parallel with $\overline{AC}$ being a transversal. The Alternate Interior Angles Theorem applies. **c.** Using parts 21a, 21b, and the fact that $\overline{AC} \cong \overline{CA}$, it can be shown they are congruent by ASA.

4.5 Problem Solving (pp. 254–255) **23.** Two pairs of angles and an included pair of sides are congruent. The triangles are congruent by SAS.

25.

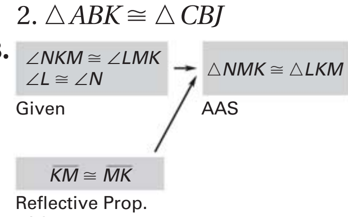


27. AAS **29.** Since all right angles are congruent the two triangles are congruent by either AAS, if the side is not included, or ASA if it is the included side.

31.

Statements	Reasons
1. $\overline{AK} \cong \overline{CJ}$, $\angle BJK \cong \angle BKJ$, $\angle A \cong \angle C$	1. Given
2. $\triangle ABK \cong \triangle CBJ$	2. ASA

33.



4.6 Skill Practice (pp. 259–260) **1.** congruent **3.** $\triangle CBA$, $\triangle CBD$; SSS **5.** $\triangle JKM$, $\triangle LKM$; HL **7.** $\triangle JNH$, $\triangle KLG$; AAS **9.** The angle is not the included angle; the triangles cannot be said to be congruent. **11.** Show $\triangle NML \cong \triangle PQL$ by AAS since $\angle NLM \cong \angle PLQ$ by the Vertical Angles Congruence Theorem. Then use the Corresponding Parts of Congruent Triangles Theorem. **13.** 20, 120, ± 6 **15.** Show $\triangle KFG \cong \triangle HGF$ by AAS, which gives you $\overline{HG} \cong \overline{KF}$. This along with $\angle FJK \cong \angle GJH$ by vertical angles gives you $\triangle FJK \cong \triangle GJH$, therefore $\angle 1 \cong \angle 2$. **17.** Show $\triangle STR \cong \triangle QTP$ by ASA using the givens and vertical angles *STR* and *QTP*. Since $\overline{QP} \cong \overline{SR}$ you now have $\triangle QSP \cong \triangle SQR$, which gives you $\angle PST \cong \angle RQT$. This along with vertical angles *PTS* and *RTQ* gives you $\triangle PTS \cong \triangle RTQ$ which gives you $\angle 1 \cong \angle 2$. **19.** Show $\triangle KNP \cong \triangle MNP$ by SSS. Now $\angle KPL \cong \angle MPL$ and $\overline{PL} \cong \overline{PL}$ leads to $\triangle LKP \cong \triangle LMP$ which gives you $\angle 1 \cong \angle 2$. **21.** The triangles are congruent by SSS.

23.

Statements	Reasons
1. $\angle T \cong \angle U$, $\angle Z \cong \angle X$, $\overline{YZ} \cong \overline{YX}$	1. Given
2. $\triangle TYZ \cong \triangle UYX$	2. AAS
3. $\angle TYZ \cong \angle UYX$	3. Corr. parts of $\cong$ △s are $\cong$.
4. $m\angle TYZ = m\angle UYX$	4. Definition of angle congruence
5. $m\angle TYW + m\angle WYZ = m\angle TYZ$, $m\angle TYW + m\angle VYX = m\angle UYX$	5. Angle Addition Postulate
6. $m\angle TYW + m\angle WYZ = m\angle TYW + m\angle VYX$	6. Transitive Property of Equality
7. $m\angle WYZ = m\angle VYX$	7. Subtraction Property of Equality
8. $\angle WYZ \cong \angle VYX$	8. Definition of angle congruence

4.6 Problem Solving (pp. 261–263)

29.

Statements	Reasons
1. $\overline{PQ} \parallel \overline{VS}$, $\overline{QU} \parallel \overline{ST}$, $\overline{PQ} \cong \overline{VS}$	1. Given
2. $\angle QPU \cong \angle SVT$, $\angle QUP \cong \angle STV$	2. Corresponding Angles Postulate
3. $\triangle PQU \cong \triangle VST$	3. AAS
4. $\angle Q \cong \angle S$	4. Corr. parts of $\cong$ △s are $\cong$.

33. No; the given angle is not an included angle. **35.** Yes; $\angle BDA \cong \angle BDC$, $\overline{AD} \cong \overline{CD}$ and $\overline{BD} \cong \overline{BD}$. By SAS, $\triangle ABD \cong \triangle CBD$. By Corr. parts of $\cong$ △s are $\cong$, $\overline{AB} \cong \overline{BC}$.

SELECTED ANSWERS

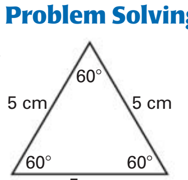
37.

Statements	Reasons
1. $\overline{MN} \cong \overline{KN}$, $\angle PMN \cong \angle NKL$	1. Given
2. $\angle MNP \cong \angle KNL$	2. Vertical Angles Congruence Theorem
3. $\triangle PMN \cong \triangle LKN$	3. ASA
4. $\overline{MP} \cong \overline{KL}$, $\angle MPJ \cong \angle KLQ$	4. Corr. parts of $\cong$ $\triangle$s are $\cong$.
5. $\overline{MJ} \cong \overline{PN}$, $\overline{KQ} \cong \overline{LN}$	5. Given in diagram
6. $\angle KQL$ and $\angle MJP$ are right angles.	6. Theorem 3.9
7. $\angle KQL \cong \angle MJP$	7. Right Angles Congruence Theorem
8. $\triangle MJP \cong \triangle KQL$	8. AAS
9. $\angle 1 \cong \angle 2$	9. Corr. parts of $\cong$ $\triangle$s are $\cong$.

4.7 Skill Practice (pp. 267–268) **1.** The angle formed by the legs is the vertex angle. **3.** A, D; Base Angles Theorem **5.** $\overline{CD}$, $\overline{CE}$; Converse of Base Angles Theorem **7.** 12 **9.** 60° **11.** 20 **13.** 8 **15.** 39, 39 **17.** 45, 5 **21.** There is not enough information to find x or y. We need to know the measure of one of the vertex angles. **23.** 16 ft **25.** 39 in. **27.** possible **29.** possible **31.** $\triangle ABD \cong \triangle CDB$ by SAS making $\overline{BA} \cong \overline{BC}$ by Corresponding parts of congruent triangles are congruent. **33.** 60, 120; solve the system $x + y = 180$ and $180 + 2x - y = 180$. **35.** 50°, 50°, 80°; 65°, 65°, 50°; there are two distinct exterior angles. If the angle is supplementary to the base angle, the base angle measures 50°. If the angle is supplementary to the vertex angle, then the base angle measures 65°.

4.7 Problem Solving (pp. 269–270)

39.



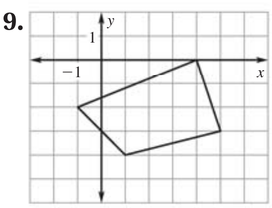
41. a. $\angle A$, $\angle ACB$, $\angle CBD$, and $\angle CDB$ are congruent and $\overline{BC} \cong \overline{CB}$ making $\triangle ABC \cong \triangle BCD$ by AAS. **b.** $\triangle ABC$, $\triangle BCD$, $\triangle CDE$, $\triangle DEF$, $\triangle EFG$ **c.** $\angle BCD$, $\angle CDE$, $\angle DEF$, $\angle EFG$

43. If a triangle is equilateral it is also isosceles, using these two facts it can be shown that the triangle is equiangular.

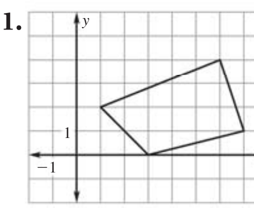
47. Yes; $m\angle ABC = 50°$ and $m\angle BAC = 50°$. The Converse of Base Angles Theorem guarantees that $\overline{AC} \cong \overline{BC}$ making $\triangle ABC$ isosceles. **49.** *Sample answer:* Choose point $P(x, y) \neq (2, 2)$ and set $PT = PU$. Solve the equation $\sqrt{x^2 + (y - 4)^2} = \sqrt{(x - 4)^2 + y^2}$ and get $y = x$. The point (2, 2) is excluded because it is a point on $\overleftrightarrow{TU}$.

4.8 Skill Practice (pp. 276–277) **1.** Subtract one from each x-coordinate and add 4 to each y-coordinate. **3.** translation **5.** reflection **7.** no

9.

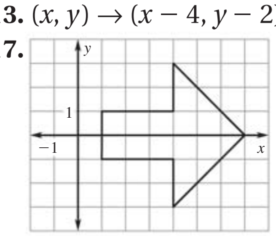


11.



13. $(x, y) \to (x - 4, y - 2)$ **15.** $(x, y) \to (x + 2, y - 1)$

17.



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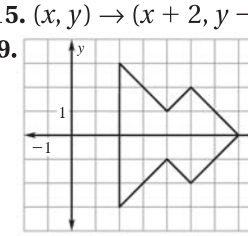

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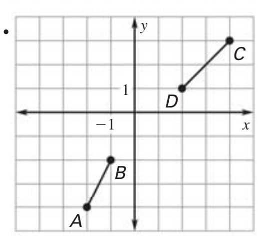
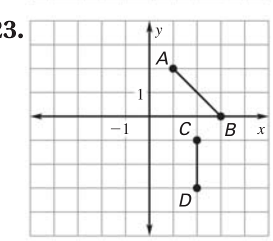


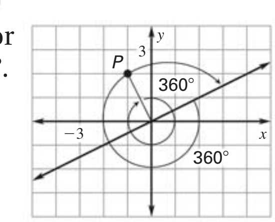
not a rotation

23.



not a rotation

25. Yes; take any point or any line and rotate 360°.

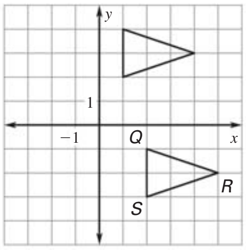


27. (3, 4) **29.** (2, 3) **31.** (13, −5) **33.** $\overline{UV}$ **35.** $\triangle DST$

SELECTED ANSWERS

4.8 Problem Solving (pp. 278–279) **39.** 90° clockwise, 90° counterclockwise **41. a.** $(x, y) \rightarrow (x - 1, y + 2)$ **b.** $(x, y) \rightarrow (x + 2, y - 1)$ c. No; the translation needed does not match a knight's move.

Chapter Review (pp. 282–285) **1.** equiangular **3.** An isosceles triangle has at least two congruent sides while a scalene triangle has no congruent sides. **5.** $\angle P$ and $\angle L$, $\angle Q$ and $\angle M$, $\angle R$ and $\angle N$; $\overline{PQ}$ and $\overline{LM}$, $\overline{QR}$ and $\overline{MN}$, $\overline{RP}$ and $\overline{NL}$ **7.** 120° **9.** 60° **11.** 60° **13.** 18 **15.** true; SSS **17.** true; SAS **19.** $\angle F$, $\angle J$ **21.** Show $\triangle ACD$ and $\triangle BED$ are congruent by AAS, which makes $\overline{AD}$ congruent to $\overline{BD}$. $\triangle ABD$ is then an isosceles triangle, which makes $\angle 1$ and $\angle 2$ congruent. **23.** Show $\triangle QVS$ congruent to $\triangle QVT$ by SSS, which gives us $\angle QSV$ congruent to $\angle QTV$. Using vertical angles and the Transitive Property you get $\angle 1$ congruent to $\angle 2$. **25.** 20

27.


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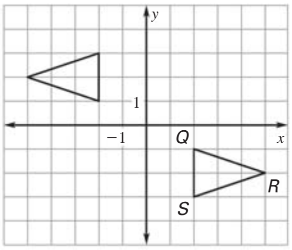
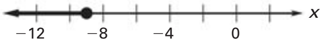
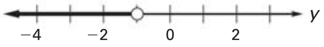
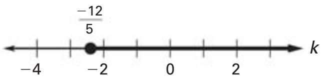


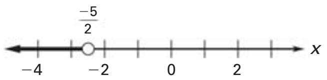
Algebra Review (p. 287)

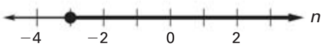
1. $x > 2$


3. $x \leq -9$


5. $y < -1$


7. $k \geq -\frac{12}{5}$


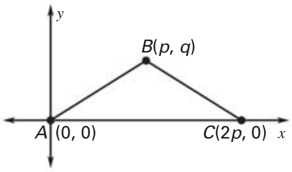
9. $x < -\frac{5}{2}$


11. $n \geq -3$


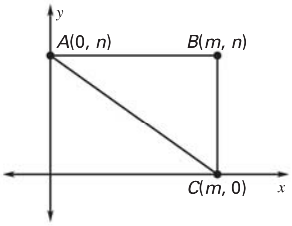
13. 2, 8 **15.** 0, 8 **17.** $-\frac{7}{3}$, 3 **19.** −0.8, 3.4 **21.** $-\frac{1}{3}$, 1 **23.** −5, 14 **25.** $-\frac{6}{5}$, 2 **27.** $\frac{7}{3}$, 5

Chapter 5

5.1 Skill Practice (pp. 298–299) **1.** midsegment **3.** 13 **5.** 6 **7.** $\overline{XZ}$ **9.** $\overline{JX}$, $\overline{KL}$ **11.** $\overline{YL}$, $\overline{LZ}$ **13.** (0, 0), (7, 0), (0, 7) **15.** *Sample answer:* (0, 0), (2*m*, 0), (*a*, *b*) **17.** (0, 0), (*s*, 0), (*s*, *s*), (0, *s*) **19.** *Sample answer:* (0, 0), (*r*, 0), (0, *s*)

21.


$AB = \sqrt{p^2 + q^2}$, $\frac{q}{p}$, $\left(\frac{p}{2}, \frac{p}{2}\right)$; $BC = \sqrt{p^2 + q^2}$, $-\frac{q}{p}$, $\left(\frac{3p}{2}, \frac{q}{2}\right)$; $CA = 2p$, 0, $(p, 0)$; no; yes; it's not a right triangle because none of the slopes are negative reciprocals and it is isosceles because two of the sides have the same measure.

23.


$AB = m$, 0, $\left(\frac{m}{2}, \frac{n}{2}\right)$, $BC = n$, 0, $\left(m, \frac{n}{2}\right)$, $CA = \sqrt{m^2 + n^2}$, $-\frac{n}{m}$, $\left(\frac{m}{2}, \frac{n}{2}\right)$; yes; no; one side is vertical and one side is horizontal thus the triangle is a right triangle. It is not isosceles since none of the sides have the same measure.

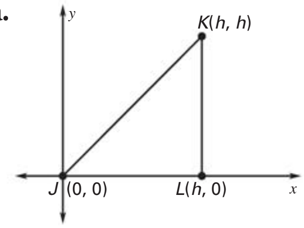
25. 13 **27.** You don't know that $\overline{DE}$ and $\overline{BC}$ are parallel. **29.** (0, *k*). *Sample answer:* Since $\triangle OPQ$ and $\triangle RSQ$ are right triangles with $\overline{OP} \cong \overline{RS}$ and $\overline{PQ} \cong \overline{SQ}$, the triangles are congruent by SAS. **33.** $GE = \frac{1}{2}DB$, $EF = \frac{1}{2}BC$, area of $\triangle EFG = \frac{1}{2}\left[\frac{1}{2}DB\left(\frac{1}{2}BC\right)\right] = \frac{1}{8}(DB)(BC)$, area of $\triangle BCD = \frac{1}{2}(DB)(BC)$.

5.1 Problem Solving (pp. 300–301) **35.** 10 ft **37.** The coordinates of *W* are (3, 3) and the coordinates of *V* are (7, 3). The slope of $\overline{WV}$ is 0 and the slope of $\overline{OH}$ is 0 making $\overline{WV} \parallel \overline{OH}$. $WV = 4$ and $OH = 8$ thus $WV = \frac{1}{2}OH$. **39.** 16. *Sample answer: DE* is half the length of $\overline{FG}$ which makes $FG = 8$. *FG* is half the length of $\overline{AC}$ which makes $AC = 16$. **41.** *Sample answer:* You already know the coordinates of *D* are (*q*, *r*) and can show the coordinates of *F* are (*p*, 0) since $\left(\frac{2p + 0}{2}, \frac{0 + 0}{2}\right) = (p, 0)$. The slope of $\overline{DF}$ is $\frac{r - 0}{q - p} = \frac{r}{q - p}$ and the slope of $\overline{BC}$ is $\frac{2r - 0}{2q - 2p} = \frac{r}{q - p}$ making them parallel. $DF = \sqrt{(q - p)^2 + r^2}$ and $BC = \sqrt{(2q - 2p)^2 + (2r)^2} = 2\sqrt{(q - p)^2 + r^2}$ making $DF = \frac{1}{2}BC$. **43. a.** $\frac{1}{2}$ **b.** $\frac{5}{4}$ **c.** $\frac{19}{8}$ **45.** *Sample answer:* $\triangle ABD$ and $\triangle CBD$ are congruent right isosceles triangles with $A(0, p)$, $B(0, 0)$, $C(p, 0)$ and $D\left(\frac{p}{2}, \frac{p}{2}\right)$. $AB = p$, $BC = p$, and $\overline{AB}$ is a vertical line and $\overline{BC}$ is a horizontal line, so $\overline{AB} \perp \overline{BC}$. By definition, $\triangle ABC$ is a right isosceles triangle.

SELECTED ANSWERS

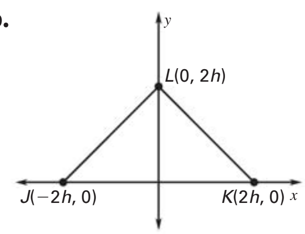
5.1 Problem Solving Workshop (p. 302) **1.** The slopes of $\overline{AC}$ and $\overline{BC}$ are negative reciprocals of each other, so $\overline{AC} \perp \overline{BC}$ making $\angle C$ a right angle; $AC = h\sqrt{2}$ and $BC = h\sqrt{2}$ making $\triangle ABC$ isosceles.

3. a.



$JL = LK = h$ and $\overline{JL}$ is a horizontal line and $\overline{LK}$ is a vertical line, so $\overline{JL} \perp \overline{LK}$; $h\sqrt{2}$, $\left(\frac{h}{2}, \frac{h}{2}\right)$.

b.

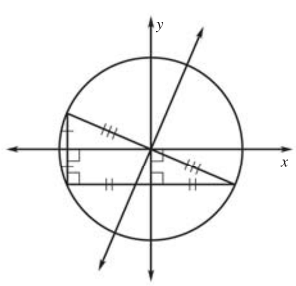


$JL = LK = 2h\sqrt{2}$ and the slope of $\overline{JL} = 1$ and the slope of $\overline{LK} = -1$, so $\overline{JL} \perp \overline{LK}$; $4h$, $(0, 0)$.

5. *Sample answer:* $PQRS$ with $P(0, 0)$, $Q(0, m)$, $R(n, m)$, and $S(n, 0)$. $PR = QS = \sqrt{m^2 + n^2}$ making $\overline{PR} \cong \overline{QS}$.

5.2 Skill Practice (pp. 306–307) **1.** circumcenter **3.** 15 **5.** 55 **7.** yes **11.** 35 **13.** 50 **15.** Yes; the Converse of the Perpendicular Bisector Theorem guarantees L is on $\overleftrightarrow{JP}$. **17.** 11

19. *Sample:*



21. Always; congruent sides are created.

5.2 Problem Solving (pp. 308–309) **25.** Theorem 5.4 shows you that you can find a point equidistant from three points by using the perpendicular bisectors of the sides of the triangle formed by the three points.

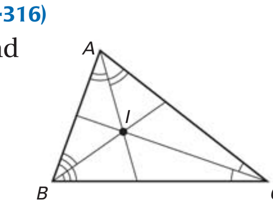
27.

Statements	Reasons
1. $CA = CB$	1. Given
2. Draw $\overleftrightarrow{PC} \perp \overline{AB}$ through point C.	2. Perpendicular Postulate
3. $\overline{CA} \cong \overline{CB}$	3. Definition of segment congruence
4. $\overline{CP} \cong \overline{CP}$	4. Reflexive Property of Segment Congruence
5. $\angle CPA$ and $\angle CPB$ are right angles.	5. Definition of $\perp$ lines
6. $\triangle CPA$ and $\triangle CPB$ right triangles.	6. Definition of right triangle
7. $\triangle CPA \cong \triangle CPB$	7. HL
8. $\overline{PA} \cong \overline{PB}$	8. Corr. parts of $\cong$ ⚠ are $\cong$.
9. C is on the perpendicular bisector of $\overline{AB}$.	9. Definition of perpendicular bisector

5.3 Skill Practice (pp. 313–314) **1.** bisector **3.** 20° **5.** 9 **7.** No; you don't know that $\angle BAD \cong \angle CAD$. **9.** No; you don't know that $\overline{HG} \cong \overline{HF}$, $\overline{HF} \perp \overrightarrow{EF}$, or $\overline{HG} \perp \overrightarrow{EG}$. **11.** No; you don't know that $\overline{HF} \perp \overrightarrow{EF}$, or $\overline{HG} \perp \overrightarrow{EG}$. **13.** 4 **15.** No; the segments with length x and 3 are not perpendicular to their respective rays. **17.** Yes; $x = 7$ using the Angle Bisector Theorem. **19.** 9 **21.** GD is not the perpendicular distance from G to $\overline{CE}$. The same is true about GF; the distance from G to each side of the triangle is the same. **25.** 0.5

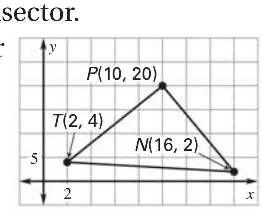
5.3 Problem Solving (pp. 315–316)

29. at the incenter of the pond



31. a. Equilateral; 3; the angle bisector would also be the perpendicular bisector. **b.** Scalene; 6; each angle bisector would be different than the corresponding perpendicular bisector.

33. perpendicular bisectors;

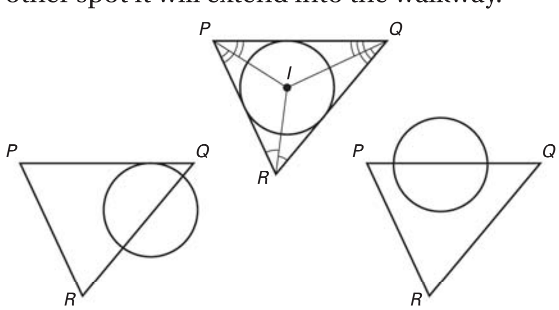


(10, 10); 100 yd; about 628 yd

35.

Statements	Reasons
1. $\angle BAC$ with D interior, $\overline{DB} \perp \overrightarrow{AC}$, $\overline{DC} \perp \overrightarrow{AC}$, $DB = DC$	1. Given
2. $\angle ABD$ and $\angle ACD$ are right angles.	2. Definition of perpendicular
3. $\triangle ABD$ and $\triangle ACD$ are right triangles.	3. Definition of right triangle
4. $\overline{DB} \cong \overline{DC}$	4. Definition of segment congruence
5. $\overline{AD} \cong \overline{AD}$	5. Reflexive Property of Segment Congruence
6. $\triangle ABD \cong \triangle ACD$	6. HL
7. $\angle BAD \cong \angle CAD$	7. Corr. parts of $\cong$ $\triangle$s are $\cong$.
8. $\overrightarrow{AD}$ bisects $\angle BAC$.	8. Definition of angle bisector

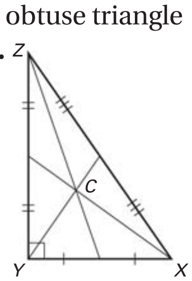
37. a. Use the Concurrency of Angle Bisectors of Triangle Theorem; if you move the circle to any other spot it will extend into the walkway.



b. Yes; the incenter will allow the largest tent possible.

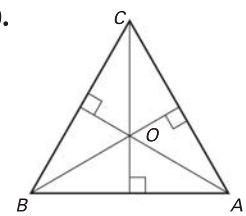
5.4 Skill Practice (pp. 322–323) **1.** circumcenter: when it is an acute triangle, when it is a right triangle, when it is an obtuse triangle; incenter: always, never, never; centroid: always, never, never; orthocenter: when it is an acute triangle, when it is a right triangle, when it is an obtuse triangle **3.** 12 **5.** 10 **9.** (3, 2)

11.

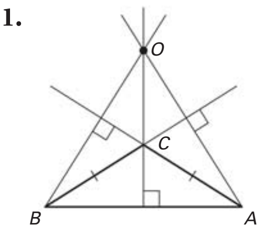


13. no; no; yes **15.** no; yes; no **17.** altitude **19.** median **21.** perpendicular bisector, angle bisector, median, altitude **23.** 6, 22°; $\triangle ABD \cong \triangle CBD$ by HL, use Corr. parts of $\cong$ $\triangle$s are $\cong$. **25.** 3 **27.** $\frac{3}{2}$

29.



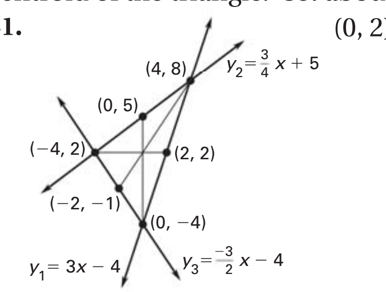
31.



33. $\frac{5}{2}$ **35.** 4

5.4 Problem Solving (pp. 324–325) **37.** B; it is the centroid of the triangle. **39.** about 12.3 in.^{2}; median

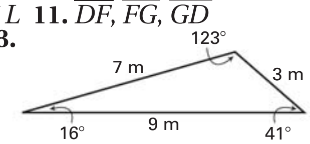
41.



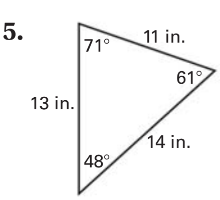
43. b. Their areas are the same. **c.** They weigh the same; it means the weight of $\triangle ABC$ is evenly distributed around its centroid.

5.5 Skill Practice (pp. 331–332) **1.** $\angle A$, $\overline{BC}$; $\angle B$, $\overline{CA}$; $\angle C$, $\overline{AB}$ **3.** *Sample answer:* The longest side is opposite the largest angle. The shortest side is opposite the smallest angle. **5.** *Sample answer:* The longest side is opposite the obtuse angle and the two angles with the same measure are opposite the sides with the same length. **7.** $\overline{XY}$, $\overline{YZ}$, $\overline{ZX}$ **9.** $\angle J$, $\angle K$, $\angle L$ **11.** $\overline{DF}$, $\overline{FG}$, $\overline{GD}$

13.

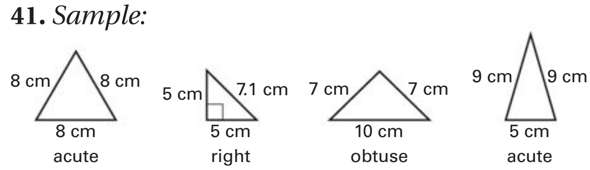


15.



17. No; 3 + 6 is not greater than 9. **19.** yes **21.** 7 in. $< x <$ 17 in. **23.** 6 ft $< x <$ 30 ft **25.** 16 in. $< x <$ 64 in. **27.** $\angle A$ and $\angle B$ are the nonadjacent interior angles to $\angle 1$ thus by the Exterior Angle Inequality Theorem $m\angle 1 = m\angle A + m\angle B$, which guarantees $m\angle 1 > m\angle A$ and $m\angle 1 > m\angle B$. **29.** The longest side is not opposite the largest angle. **31.** yes; $\angle Q$, $\angle P$, $\angle R$ **33.** $2 < x < 15$ **35.** $\angle WXY$, $\angle Z$, $\angle ZXY$, $\angle WYX$ and $\angle ZYX$, $\angle W$; $\angle ZYX$ is the largest angle in $\triangle ZYX$ and $\angle WYX$ is the middle sized angle in $\triangle WXY$ making $\angle W$ the largest angle. $m\angle WXY + m\angle W = m\angle Z + m\angle ZXY$ making $\angle WXY$ the smallest.

5.5 Problem Solving (pp. 333–334) **37.** $m\angle P < m\angle Q$, $m\angle P < m\angle R$; $m\angle Q = m\angle R$ **39. a.** The sum of the other two side lengths is less than 1080. **b.** No; the sum of the distance from Granite Peak to Fort Peck Lake and Granite Peak to Glacier National Park must be more than 565. **c.** $d > 76$ km, $d < 1054$ km **d.** The distance is less than 489 kilometers.
41. *Sample:*



43. *Sample answer:* 3, 4, 17; 2, 5, 17; 4, 4, 16
45. $1\frac{1}{4}$ mi $\le d \le 2\frac{3}{4}$ mi; if the locations are collinear then the distance could be $1\frac{1}{4}$ miles or $2\frac{3}{4}$ miles. If the locations are not collinear then the distance must be between $1\frac{1}{4}$ miles and $2\frac{3}{4}$ miles because of the Triangle Inequality Theorem.

5.6 Skill Practice (pp. 338–339) **1.** You temporarily assume that the desired conclusion is false and this leads to a logical contradiction. **3.** > **5.** < **7.** = **11.** Suppose xy is even. **13.** $\angle A$ could be a right angle. **15.** The Hinge Theorem is about triangles not quadrilaterals. **17.** $x > \frac{1}{2}$ **19.** Using the Converse of the Hinge Theorem $\angle NRQ > \angle NRP$. Since $\angle NRQ$ and $\angle NRP$ are a linear pair $\angle NRQ$ must be obtuse and $\angle NRP$ must be acute.

5.6 Problem Solving (pp. 340–341) **23.** E, A, D, B, C **25. a.** It gets larger; it gets smaller. **b.** KM **c.** *Sample answer:* Since $NL = NK = NM$ and as $m\angle LNK$ increases KL increases and $m\angle KNM$ decreases as KM decreases, you have two pair of congruent sides with $m\angle LNK$ eventually larger than $m\angle KNM$. The Hinge Theorem guarantees KL will eventually be larger than KM. **27.** Prove: If x is divisible by 4, then x is even. Proof: Since x is divisible by 4, $x = 4a$. When you factor out a 2, you get $x = 2(2a)$ which is in the form $2n$, which implies x is an even number; you start the same way by assuming what you are to prove is false, then proceed to show this leads to a contradiction.

Chapter Review (pp. 344–347) **1.** midpoint **3.** B **5.** C **7.** 45 **9.** BA and BC, DA and DC **11.** 25 **13.** 15 **15.** $(-2, 4)$ **17.** 3.5 **19.** 4 in. $< \ell <$ 12 in. **21.** 8 ft $< \ell <$ 32 ft **23.** $\overline{LM}, \overline{MN}, \overline{LN}$; $\angle N, \angle L, \angle M$ **25.** > **27.** C, B, A, D

Algebra Review (p. 349) **1. a.** $\frac{3}{1}$ **b.** $\frac{1}{4}$ **3.** $\frac{5}{4}$ **5.** 9% decrease **7.** about 12.5% increase **9.** 0.25% decrease **11.** 84%; 37.8 h **13.** 107.5%; 86 people

Chapter 6

6.1 Skill Practice (pp. 360–361) **1.** means: n and p, extremes: m and q **3.** 4 : 1 **5.** 600 : 1 **7.** $\frac{7}{1}$ **9.** $\frac{24}{5}$ **11.** $\frac{5 \text{ in.}}{15 \text{ in.}}; \frac{1}{3}$ **13.** $\frac{320 \text{ cm}}{1000 \text{ cm}}; \frac{8}{25}$ **15.** $\frac{5}{2}$ **17.** $\frac{4}{3}$ **19.** 8, 28 **21.** 20°, 70°, 90° **23.** 4 **25.** 42 **27.** 3 **29.** 3 **31.** 6 **33.** 16 **35.** $5\sqrt{2}$ **37.** The unit conversion should be $\frac{1 \text{ ft}}{12 \text{ in.}}$; $\frac{8 \text{ in.}}{3 \text{ ft}} \cdot \frac{1 \text{ ft}}{12 \text{ in.}} = \frac{8}{36} = \frac{2}{9}$. **39.** $\frac{12}{5}$ **41.** $\frac{4}{3}$ **43.** $\frac{7}{11}$ **45.** ± 6 **47.** Obtuse; since the angles are supplementary, $x + 4x = 180$. Find $x = 36$, so the measure of the interior angle is 144°. **49.** 9 **51.** 5 **53.** 72 in., 60 in. **55.** 45, 30

6.1 Problem Solving (pp. 362–363) **57.** 18 ft, 15 ft, 270 ft^2; 270 tiles; \$534.60 **59.** 9 cups, 1.8 cups, 7.2 cups **61.** about 189 hits **63.** All three ratios reduce to 4 : 3. **65.** 600 Canadian dollars **67.** $\frac{a}{b} = \frac{c}{d}$, $b \ne 0$, $d \ne 0$; $\frac{a}{b} \cdot bd = \frac{c}{d} \cdot bd$; $ad = cb$; $ad = bc$

6.2 Skill Practice (pp. 367–368) **1.** scale drawing **3.** $\frac{x}{y}$ **5.** $\frac{y + 15}{15}$ **7.** true **9.** true **11.** 10.5 **13.** about 100 yd **15.** 4 should have been added to the second fraction instead of 3; $\frac{a+3}{3} = \frac{c+4}{4}$. **17.** $\frac{49}{3}$

6.2 Problem Solving (pp. 368–370) **23.** 1 in. : $\frac{1}{3}$ mi **25.** about 8 mi **27.** about 0.0022 mm **29.** 48 ft

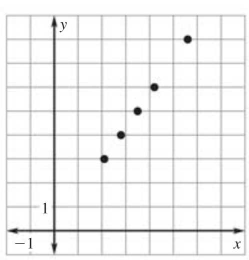
31.
$$\begin{aligned} \frac{a}{b} &= \frac{c}{d} \\ \frac{a}{b} \cdot bd &= \frac{c}{d} \cdot bd \\ ad &= cb \\ ad \cdot \frac{1}{ac} &= cb \cdot \frac{1}{ac} \\ \frac{d}{c} &= \frac{b}{a} \end{aligned}$$

33.
$$\begin{aligned} \frac{a}{b} &= \frac{c}{d} \\ \frac{a}{b} + 1 &= \frac{c}{d} + 1 \\ \frac{a}{b} + \frac{b}{b} &= \frac{c}{d} + \frac{d}{d} \\ \frac{a+b}{b} &= \frac{c+d}{d} \end{aligned}$$

35.
$$\begin{aligned} \frac{a+c}{b+d} &= \frac{a-c}{b-d} \\ (a+c)(b-d) &= (a-c)(b+d) \\ ab - ad + bc - cd &= ab + ad - bc - cd \\ -ad + bc &= ad - bc \\ -2ad &= -2bc \\ ad &= bc \\ \frac{a}{b} &= \frac{c}{d} \end{aligned}$$

6.3 Skill Practice (pp. 376–377) **1.** congruent, proportional **3.** $\angle A \cong \angle L$, $\angle B \cong \angle M$, $\angle C \cong \angle N$; $\frac{AB}{LM} = \frac{BC}{MN} = \frac{CA}{NL}$ **5.** $\angle H \cong \angle W$, $\angle J \cong \angle X$, $\angle K \cong \angle Y$, $\angle L \cong \angle Z$; $\frac{HJ}{WX} = \frac{JK}{XY} = \frac{KL}{YZ} = \frac{LH}{ZW}$ **7.** similar; $RSTU \sim WXYZ$, $\frac{2}{1}$ **9.** $\frac{5}{2}$ **11.** 85, 34 **13.** The larger triangle's perimeter was doubled but should have been halved; perimeter of B = 14. **15.** always **17.** never **19.** altitude, 24 **21.** $10\frac{2}{3}$ in., $13\frac{1}{3}$ in. **23.** $\frac{11}{5}$ **25.** $17\frac{3}{5}$ **27.** No; in similar triangles corresponding angles are congruent.

6.3 Problem Solving (pp. 378–379) **31.** No; the lengths are not proportional. **33. a.** 2.8, 4.2, 5.6, 2.1 **b.**

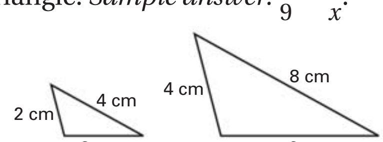


yes **c.** $y = \frac{10}{7}x$; $\frac{10}{7}$; they are the same.

35. Yes; if $\ell = w$ then the larger and smaller image would be similar. *Sample answer:* Let $\ell = 8$, $w = 8$, and $a = 4$; $\frac{w}{w + a} = \frac{8}{12} = \frac{2}{3}$, $\frac{\ell}{\ell + a} = \frac{8}{12} = \frac{2}{3}$. **37. a.** They have the same slope. **b.** $\angle BOA \cong \angle DOC$ by the Vertical Angles Theorem. $\angle OBA \cong \angle ODC$ by the Alternate Interior Angles Theorem. $\angle BAO \cong \angle DCO$ by the Alternate Interior Angles Theorem. **c.** $(-3, 0)$, $(0, 4)$, $(6, 0)$, $(0, -8)$; $AO = 3$, $OB = 4$, $BA = 5$, $CO = 6$, $OD = 8$, $DC = 10$ **d.** Since corresponding angles are congruent and the ratios of corresponding sides are all the same the triangles are similar.

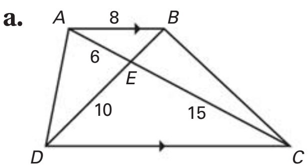
6.4 Skill Practice (pp. 384–385) **1.** similar **3.** $\triangle FED$ **5.** 15, y **7.** 20 **9.** similar; $\triangle FGH \sim \triangle DKLJ$ **11.** not similar **13.** similar; $\triangle YZX \sim \triangle YWU$ **15.** The AA Similarity Postulate is for triangles, not quadrilaterals. **17.** 5 should be replaced by 9, which is the length of the corresponding side of the larger triangle. *Sample answer:* $\frac{4}{9} = \frac{6}{x}$.

19. *Sample:*



21. (10, 0) **23.** (24, 0)

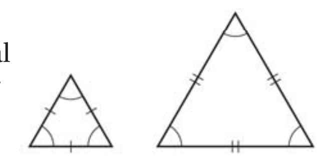
25. a.



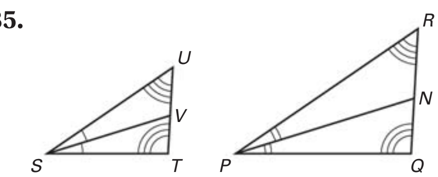
b. *Sample answer:* $\angle ABE$ and $\angle CDE$, $\angle BAE$ and $\angle DCE$ **c.** $\triangle ABE$ and $\triangle CDE$, $\triangle ABE \sim \triangle CDE$ **d.** 4, 20

27. Yes; either $m\angle X$ or $m\angle Y$ could be 90°, and the other angles could be the same. **29.** No; since $m\angle J + m\angle K = 85°$ then $m\angle L = 95°$. Since $m\angle X + m\angle Z = 80°$ then $m\angle X = 100°$ and thus neither $\angle X$ nor $\angle Z$ can measure 95°.

6.4 Problem Solving (pp. 386–387) **31.** about 30.8 in. **33.** The measure of all angles in an equilateral triangle is 60°. *Sample:*

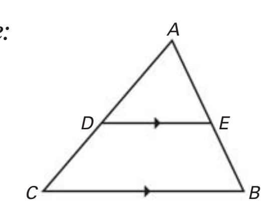


35.



Since $\triangle STU \sim \triangle PQR$ you know that $\angle T \cong \angle Q$ and $\angle UST \cong \angle RPQ$. Since $\overline{SV}$ bisects $\angle TSU$ and $\overline{PN}$ bisects $\angle QPR$ you know that $\angle USV \cong \angle VST$ and $\angle RPN \cong \angle NPQ$ by definition of angle bisector. You know that $m\angle USV + m\angle VST = m\angle UST$ and $m\angle RPN + m\angle NPQ = m\angle RPQ$, therefore, $2m\angle VST = 2m\angle NPQ$ using the Substitution Property of Equality. You now have $\angle VST \cong \angle NPQ$, which makes $\triangle VST \sim \triangle NPQ$ using the AA Similarity Postulate. From this you know that $\frac{SV}{PN} = \frac{ST}{PQ}$.

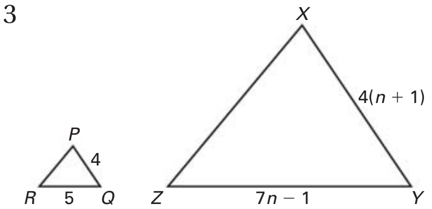
37. a. *Sample:*



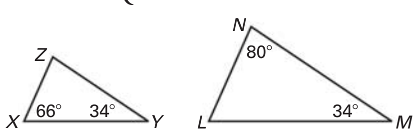
b. $m\angle ADE = m\angle ACB$ and $m\angle AED = m\angle ABC$ **c.** $\triangle ADE \sim \triangle ACB$ **d.** *Sample answer:* $\frac{AD}{AC} = \frac{AE}{AB} = \frac{DE}{CB} = \frac{1}{2}$ **e.** The measures of the angles change, but the equalities remain the same. The lengths of the sides change, but they remain proportional; yes; the triangles remain similar by the AA Similarity Postulate.

SELECTED ANSWERS

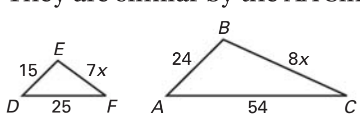
6.5 Skill Practice (pp. 391–393) **1.** $\frac{AC}{PX} = \frac{CB}{XQ} = \frac{AB}{PQ}$
3. $\frac{18}{12} = \frac{15}{10} = \frac{12}{8}; \frac{3}{2}$ **5.** $\triangle RST$ **7.** similar; $\triangle FDE \sim \triangle XWY$; 2 : 3
9. 3



11. $\triangle ABC \sim \triangle DEC$; $\angle ACB \cong \angle DCE$ by the Vertical Angles Congruence Theorem and $\frac{AC}{DC} = \frac{BC}{EC} = \frac{3}{2}$. The triangles are similar using the SAS Similarity Theorem. **13.** *Sample answer:* The triangle correspondence is not listed in the correct order; $\triangle ABC \sim \triangle RQP$.
15.



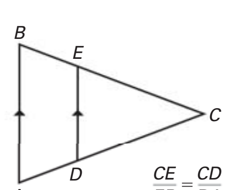
They are similar by the AA Similarity Postulate.
17.



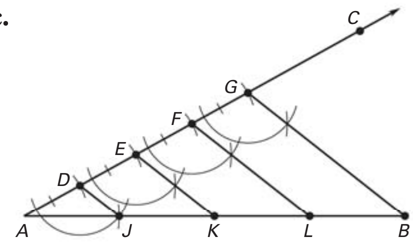
They are not similar since the ratio of corresponding sides is not constant.
19. 45° **21.** 24 **23.** $16\sqrt{2}$

6.5 Problem Solving (pp. 393–395) **29.** The triangle whose sides measure 4 inches, 4 inches, and 7 inches is similar to the triangle whose sides measure 3 inches, 3 inches, and 5.25 inches. **31.** $\angle CBD \cong \angle CAE$
33. a. AA Similarity Postulate **b.** 75 ft **c.** 66 ft
35. *Sample answer:* Given that D and E are midpoints of $\overline{AB}$ and $\overline{BC}$ respectively the Midsegment Theorem guarantees that $\overline{AC} \parallel \overline{DE}$. By the Corresponding Angles Postulate $\angle A \cong \angle BDE$ and so $\angle BDE$ is a right angle. Reasoning similarly $\overline{AB} \parallel \overline{EF}$. By the Alternate Interior Angles Congruence Theorem $\angle BDE \cong \angle DEF$. This makes $\angle DEF$ a right angle that measures 90°.

6.6 Skill Practice (pp. 400–401)
1. If a line parallel to one side of a triangle intersects the other two sides then it divides the two sides proportionally.



3. 9 **5.** Parallel; $\frac{8}{5} = \frac{12}{7.5}$ so the Converse of the Triangle Proportionality Theorem applies. **7.** Parallel; $\frac{20}{18} = \frac{25}{22.5}$ so the Converse of the Triangle Proportionality Theorem applies. **9.** 10 **11.** 1 **15.** 9 **17.** $a = 9$, $b = 4$, $c = 3$, $d = 2$
19. a–b. See figure in part (c).
c.



Theorem 6.6 guarantees that parallel lines divide transversals proportionally. Since $\frac{AD}{DE} = \frac{DE}{EF} = \frac{EF}{FG} = 1$ implies $\frac{AJ}{JK} = \frac{JK}{KL} = \frac{KL}{LB} = 1$ which means $AJ = JK = KL = LB$.

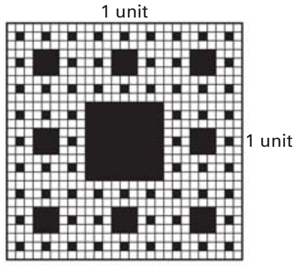
6.6 Problem Solving (pp. 402–403) **21.** 350 yd
23. Since $k_1 \parallel k_2 \parallel k_3$, $\angle FDA \cong \angle CAD$ and $\angle CDA \cong \angle FAD$ by the Alternate Interior Angles Congruence Theorem. $\triangle ACD \sim \triangle DFA$ by the AA Similarity Postulate. Let point G be at the intersection of $\overline{AD}$ and $\overline{BE}$. Using the Triangle Proportionality Theorem $\frac{CB}{BA} = \frac{DG}{GA}$ and $\frac{DE}{EF} = \frac{DG}{GA}$. Using the Transitive Property of Equality $\frac{CB}{BA} = \frac{DE}{EF}$.
25.

The ratio of the lengths of the other two sides is 1 : 1 since in an isosceles triangle these two sides are congruent.

27. Since $\overline{XW} \parallel \overline{AZ}$, $\angle XZA \cong \angle WXZ$ using the Alternate Interior Angles Congruence Theorem. This makes $\triangle AXZ$ isosceles because it is shown that $\angle A \cong \angle WXZ$ and by the Converse of the Base Angles Theorem, $AX = XZ$. Since $\overline{XW} \parallel \overline{AZ}$ using the Triangle Proportionality Theorem you get $\frac{YW}{WZ} = \frac{XY}{AX}$. Substituting you get $\frac{YW}{WX} = \frac{XY}{XZ}$.

6.6 Problem Solving Workshop (p. 405)
1. a. 270 yd **b.** 67.5 yd **3.** 4.5 mi/h **5.** 5.25, 7.5

Extension (p. 407) **1.** 3 : 1. *Sample answer:* It's one unit longer; each of the three edges went from measuring one unit to four edges each measuring $\frac{1}{3}$ of a unit.

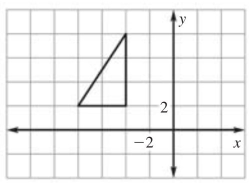
3. a. 

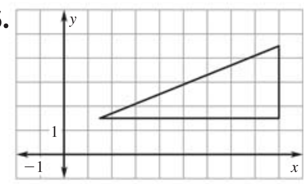
b. *Sample answer:* The upper left square is simply a smaller version of the whole square.

c.

Stage	Number of colored squares	Area of 1 colored square	Total Area
0	0	0	0
1	1	$\frac{1}{9}$	$\frac{1}{9}$
2	8	$\frac{1}{81}$	$\frac{17}{81}$
3	64	$\frac{1}{729}$	$\frac{217}{729}$

6.7 Skill Practice (pp. 412–413) **1.** similar

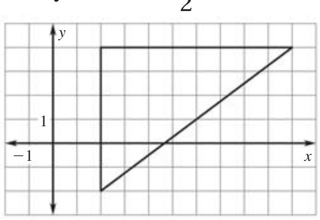
3. 

5. 

9. reduction; $\frac{1}{2}$ **11.** enlargement; 3 **15.** The figures are not similar. **17.** reflection **19.** 2; $m = 4$, $n = 5$

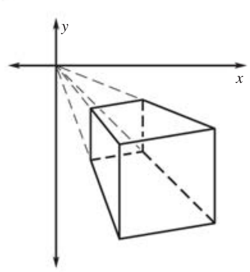
6.7 Problem Solving (pp. 414–415)

25. 24 ft by 12 ft **27.** $\frac{5}{2}$

29. a. 

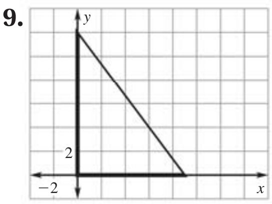
b. $\frac{2}{3}$; they are the same. **c.** $\frac{4}{9}$; it's the square of the scale factor.

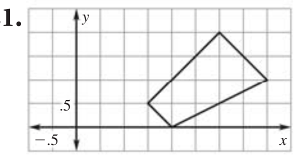
31. Perspective drawings use converging lines to give the illusion that an object is three dimensional. Since the back of the drawing is similar to the front, a dilation can be used to create this illusion with the vanishing point as the center of dilation.



33. The slope of $\overline{PQ}$ is $\frac{d-b}{c-a}$ and the slope of $\overline{XY}$ is $\frac{kd-kb}{kc-ka} = \frac{k(d-b)}{k(c-a)} = \frac{d-b}{c-a}$. Since the slopes are the same, the lines are parallel.

Chapter Review (pp. 418–421) **1.** dilation **3.** In a ratio two numbers are compared. In a proportion two ratios are set equal to one another. *Sample answer:* $\frac{2}{4}, \frac{6}{10} = \frac{3}{5}$ **5.** 45°, 45°, 90° **7.** $\frac{20}{3}$ **9.** similar; $ABCD \sim EFGH$, $\frac{4}{3}$ **11.** 68 in. **13.** The Triangle Sum Theorem tells you that $m\angle D = 60°$ so $\angle A \cong \angle D$ and it was given that $\angle C \cong \angle F$ which gives you $\triangle ABC \sim \triangle DEF$ using the AA Similarity Postulate. **15.** Since $\frac{4}{8} = \frac{3.5}{7}$ and the included angle, $\angle C$, is congruent to itself, $\triangle BCD \sim \triangle ACE$ by the SAS Similarity Theorem. **17.** not parallel

19. 

21. 

Algebra Review (p. 423) **1.** ± 10 **3.** $\pm\sqrt{17}$ **5.** $\pm\sqrt{10}$ **7.** $\pm 2\sqrt{5}$ **9.** $\pm 3\sqrt{2}$ **11.** $\frac{\sqrt{15}}{5}$ **13.** $\frac{\sqrt{21}}{2}$ **15.** $\frac{1}{10}$ **17.** $\frac{\sqrt{2}}{2}$

Cumulative Review (pp. 428–429) **1. a.** 33° **b.** 123° **3. a.** 2° **b.** 92°

5.

$3x - 19 = 47$	Given
$3x = 66$	Addition Property of Equality
$x = 22$	Division Property of Equality

7.

$-5(x + 2) = 25$	Given
$x + 2 = -5$	Division Property of Equality
$x = -7$	Subtraction Property of Equality

9. Alternate Interior Angles Theorem **11.** Corresponding Angles Postulate **13.** Linear Pair Postulate **15.** 78°, 78°, 24°; acute **17.** congruent; $\triangle ABC \cong \triangle CDA$, SSS Congruence Theorem **19.** not congruent **21.** 8 **23.** similar; $\triangle FCD \sim \triangle FHG$, SAS Similarity Theorem **25.** not similar **27. a.** $y = 59x + 250$ **b.** The slope is the monthly membership and the y-intercept is the initial cost to join the club. **c.** \$958 **29.** *Sample answer:* Since $\overline{BC} \parallel \overline{AD}$, you know that $\angle CBD \cong \angle ADB$ by the Alternate Interior Angles Theorem. $\overline{BD} \cong \overline{BD}$ by the Reflexive Property of Segment Congruence and with $\overline{BC} \cong \overline{AD}$ given, then $\triangle BCD \cong \triangle DAB$ by the SAS Congruence Theorem. **31.** 43 mi $< d <$ 397 mi

Chapter 7

7.1 Skill Practice (pp. 436–438) **1.** Pythagorean triple **3.** 130 **5.** 58 **7.** In Step 2, the Distributive Property was used incorrectly; $x^2 = 49 + 576$, $x^2 = 625$, $x = 25$.

9. about 9.1 in. **11.** 120 m^2 **13.** 48 cm^2 **15.** 40 **19.** 15, leg **21.** 52, hypotenuse **23.** 21, leg **25.** $11\sqrt{2}$

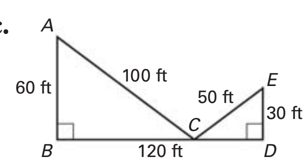
7.1 Problem Solving (pp. 438–439) **31.** about 127.3 ft **33.** *Sample answer:* The longest side of the triangle is opposite the largest angle, which in a right triangle is the right angle.

35. a–b.

BC	AC	CE	AC + CE
10	60.8	114.0	174.8
20	63.2	104.4	167.6
30	67.1	94.9	162
40	72.1	85.4	157.6
50	78.1	76.2	154.3
60	84.9	67.1	152
70	92.2	58.3	150.5
80	100	50	150
90	108.2	42.4	150.6
100	116.6	36.1	152.7
110	125.3	31.6	156.9
120	134.2	30	164.2

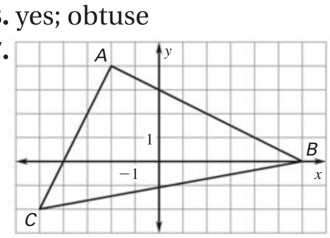
150 ft

c.



7.2 Skill Practice (pp. 444–445) **1.** hypotenuse **3.** right triangle **5.** not a right triangle **7.** right triangle **9.** right triangle **11.** right triangle **13.** right triangle **15.** yes; acute **17.** yes; obtuse **19.** yes; right **21.** no **23.** yes; obtuse

27.

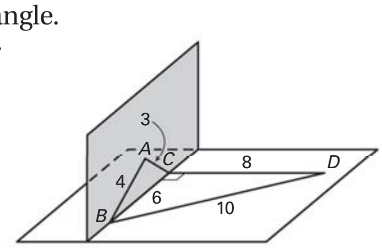


right

29. right **31.** < **33.** $8 < x < 12$

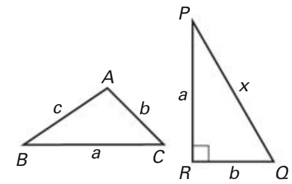
7.2 Problem Solving (pp. 445–447) **35.** Measure diagonally across the painting and it should be about 12.8 inches. **37. a.** 5 **b.** $3^2 + 4^2 = 5^2$ therefore $\triangle ABC$ is a right triangle.

c. *Sample:*



39. a. yes; $12^2 + 16^2 = 20^2$ **b.** no; $9^2 + 12^2 \neq 18^2$ **c.** No; if the car was not in an accident, the angles should form a right triangle.

41. Given: In $\triangle ABC$, $c^2 > a^2 + b^2$, where c is the length of the longest side.
Prove: $\triangle ABC$ is obtuse.



Statements	Reasons
1. In $\triangle ABC$, $c^2 > a^2 + b^2$ where c is the length of the longest side. In $\triangle PQR$, $\angle R$ is a right angle.	1. Given
2. $a^2 + b^2 = x^2$	2. Pythagorean Theorem
3. $c^2 > x^2$	3. Substitution
4. $c > x$	4. A property of square roots
5. $m\angle R = 90°$	5. Definition of a right angle
6. $m\angle C > m\angle R$	6. Converse of the Hinge Theorem
7. $m\angle C > 90°$	7. Substitution Property
8. $\angle C$ is an obtuse angle.	8. Definition of an obtuse angle
9. $\triangle ABC$ is an obtuse triangle.	9. Definition of an obtuse triangle

43. $\triangle ABC \sim \triangle DEC$, $\angle BAC$ is 90°, so $\angle EDC$ must also be 90°.

7.3 Skill Practice (pp. 453–454) **1.** similar **3.** $\triangle FHG \sim \triangle HEG \sim \triangle FEH$ **5.** about 53.7 ft **7.** about 6.7 ft **9.** $\triangle QSR \sim \triangle STR \sim \triangle QTS$; RQ **11.** *Sample answer:* The proportion must compare corresponding parts, $\frac{v}{z} = \frac{z}{w+v}$ **13.** about 6.7 **15.** about 45.6 **17.** about 6.3 **21.** 3 **23.** $x = 9$, $y = 15$, $z = 20$ **25.** right triangle; about 6.7 **27.** 25, 12

7.3 Problem Solving (pp. 455–456) **29.** about 1.1 ft **31.** 15 ft; no, but the values are very close **33. a.** $\overline{FH}$, $\overline{GF}$, $\overline{EF}$; each segment has a vertex as an endpoint and is perpendicular to the opposite side. **b.** $\sqrt{35}$ **c.** about 35.5

37.

Statements	Reasons
1. $\triangle ABC$ is a right triangle; $\overline{CD}$ is the altitude to $\overline{AB}$.	1. Given
2. $\triangle ABC \sim \triangle CBD$	2. Theorem 7.5
3. $\frac{AB}{CB} = \frac{BC}{BD}$	3. Definition of similar figures
4. $\triangle ABC \sim \triangle ACD$	4. Theorem 7.5
5. $\frac{AB}{AC} = \frac{AC}{AD}$	5. Definition of similar figures

7.4 Skill Practice (pp. 461–462) **1.** an isosceles right triangle **3.** $7\sqrt{2}$ **5.** 3 **7.** 2; 4 in. **9.** $x = 3, y = 6$

11.

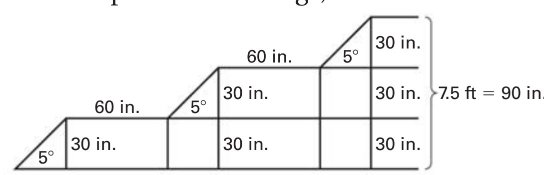
a	7	11	$5\sqrt{2}$	6	$\sqrt{5}$
b	7	11	$5\sqrt{2}$	6	$\sqrt{5}$
c	$7\sqrt{2}$	$11\sqrt{2}$	10	$6\sqrt{2}$	$\sqrt{10}$

13. $x = \frac{15}{2}\sqrt{3}, y = \frac{15}{2}$ **15.** $p = 12, q = 12\sqrt{3}$ **17.** $t = 4\sqrt{2}, u = 7$ **21.** The hypotenuse of a 45°-45°-90° triangle should be $x\sqrt{2}$, if $x = \sqrt{5}$, then the hypotenuse is $\sqrt{10}$. **23.** $f = \frac{20\sqrt{3}}{3}, g = \frac{10\sqrt{3}}{3}$ **25.** $x = 4, y = \frac{4\sqrt{3}}{3}$

7.4 Problem Solving (pp. 463–464) **27.** 5.5 ft **29.** *Sample answer:* Method 1. Use the Angle-Angle Similarity postulate, because by definition of an isosceles triangle, the base angles must be the same and in a right isosceles triangle, the angles are 45°. Method 2. Use the Side-Angle-Side Similarity Theorem, because the right angle is always congruent to another right angle and the ratio of sides of an isosceles triangle will always be the same. **31.** $10\sqrt{3}$ in. **33. a.** 45°-45°-90° for all triangles **b.** $\frac{3\sqrt{2}}{2}$ in. $\times \frac{3\sqrt{2}}{2}$ in. **c.** 1.5 in. × 1.5 in.

7.5 Skill Practice (pp. 469–470) **1.** the opposite leg, the adjacent leg **3.** $\frac{24}{7}$ or 3.4286, $\frac{7}{24}$ or 0.2917 **5.** $\frac{12}{5}$ or 2.4, $\frac{5}{12}$ or 0.4167 **7.** 7.6 **9.** 6; 6; they are the same. **11.** $4\sqrt{3}$; $4\sqrt{3}$; they are the same. **13.** Tangent is the ratio of the opposite and the adjacent side, not adjacent to hypotenuse; $\frac{80}{18}$. **15.** You need to know: that the triangle is a right triangle, which angle you will be applying the ratio to, and the lengths of the opposite side and the adjacent side to the angle. **19.** 15.5 **21.** 77.4 **23.** 60.6 **25.** 27.6 **27.** 60; 54 **29.** 82; 154.2

7.5 Problem Solving (pp. 471–472) **31.** 555 ft **33.** about 33.4 ft **35.** $\tan A = \frac{a}{b}$, $\tan B = \frac{b}{a}$; the tangent of one acute angle is the reciprocal of the other acute angle; complementary. **37. a.** 29 ft **b.** 3 ramps and 2 landings;

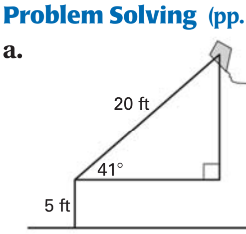


c. 96 ft

7.6 Skill Practice (pp. 477–478) **1.** the opposite leg, the hypotenuse **3.** $\frac{4}{5}$ or 0.8, $\frac{3}{5}$ or 0.6 **5.** $\frac{28}{53}$ or 0.5283, $\frac{45}{53}$ or 0.8491 **7.** $\frac{3}{5}$ or 0.6, $\frac{4}{5}$ or 0.8 **9.** $\frac{1}{2}$ or 0.5, $\frac{\sqrt{3}}{2}$ or 0.8660 **11.** $a = 14.9, b = 11.1$ **13.** $s = 17.7, r = 19.0$ **15.** $m = 6.7, n = 10.4$ **17.** The triangle must be a right triangle, and you need either an acute angle measure and the length of one side or the lengths of two sides of the triangle. **19.** 3.0 **21.** 20.2 **23.** 12; $\frac{2\sqrt{2}}{2}$ or 0.9428, $\frac{1}{3}$ or 0.3333 **25.** 3; $\frac{\sqrt{5}}{5}$ or 0.4472, $\frac{2\sqrt{5}}{5}$ or 0.8944 **27.** 33; $\frac{56}{65}$ or 0.8615, $\frac{33}{65}$ or 0.5077 **31.** about 18 cm

7.6 Problem Solving (pp. 479–480) **33.** about 36.9 ft

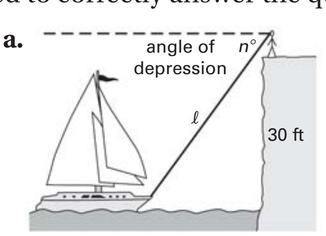
35. a.



b. About 18.1 ft; the height that the spool is off the ground has to be added.

37. Both; since different angles are used in each ratio, both the sine and cosine relationships can be used to correctly answer the question.

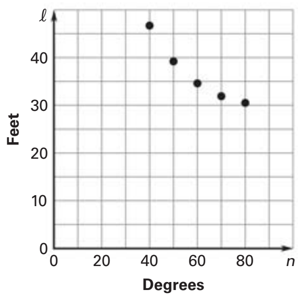
39. a.



b.

$n°$	40°	50°	60°	70°	80°
ℓ (ft)	46.7	39.2	34.6	31.9	30.5

c.



d. *Sample answer:* 60 ft

7.6 Problem Solving Workshop (p. 482) **1.** about 8.8 ft, about 18 ft **3.** The cosine ratio is the adjacent side over the hypotenuse, not opposite over adjacent; $\cos A = \frac{7}{25}$. **5.** $\cos 34° = \frac{x}{17}$, $\tan 34° = \frac{9.5}{x}$, $x^2 + 9.5^2 = 17^2$

7.7 Skill Practice (pp. 485–487) **1.** angles, sides **3.** 33.7° **5.** 74.1° **7.** 53.1° **11.** $N = 25°$, $NP \approx 21.4$, $NQ \approx 23.7$ **13.** $A \approx 36.9°$, $B \approx 53.1°$, $AC = 15$ **15.** $G \approx 29°$, $J \approx 61°$, $HJ = 7.7$ **17.** $D \approx 29.7°$, $E \approx 60.3°$, $ED \approx 534$ **19.** Since an angle was given, the $\sin^{-1}$ should not have been used; $\sin 36 = \frac{7}{WX}$. **21.** 30° **23.** 70.7° **25.** 45° **27.** 11° **31.** 45°; 60°

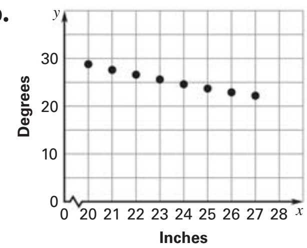
7.7 Problem Solving (pp. 487–489) **35.** about 59.7° **37.** $\tan^{-1}\frac{BC}{AC}$. *Sample answer:* The information needed to determine the measure of A was given if you used the tangent ratio, this will make the answer more accurate since no rounding has occurred.

39. a.

x (in.)	20	21	22	23
y (°)	28.8°	27.6°	26.6°	25.6°

x (in.)	24	25	26	27
y (°)	24.6°	23.7°	22.9°	22.2°

b.



c. *Sample answer:* The longer the rack, the closer to 20° the angle gets.

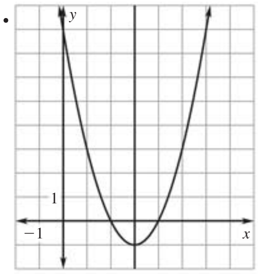
41. a. 38.4 ft **b.** about 71.2 ft **c.** about 48.7 ft **d.** About 61.7°, about 51.7°; neither; the sides are not the same, so the triangles are not congruent, and the angles are not the same, so the triangles are not similar. **e.** I used tangent because the height and the distance along the ground form a tangent relationship for the angle of elevation.

Extension (p. 491) **1.** $C = 66°$, $a = 4.4$, $c = 8.3$ **3.** $B = 81.8°$, $C = 47.2°$, $b = 22.9$ **5.** $A = 58.2°$, $B = 85.6°$, $C = 36.2°$ **7.** about 10 blocks

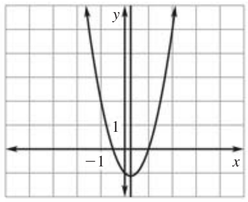
Chapter Review (pp. 494–497) **1.** $a^2 + b^2 = c^2$ **3.** *Sample answer:* The difference is your perspective on the situation. The angle of depression is the measure from your line of sight down, and the angle of elevation is the measure from your line of sight up, but if you construct the parallel lines in any situation, the angles are alternate interior angles and are congruent by Theorem 3.1. **5.** $2\sqrt{34}$ **7.** acute **9.** right **11.** right **13.** 13.5 **15.** $2\sqrt{10}$ **17.** 9 **19.** $6\sqrt{2}$ **21.** $16\sqrt{3}$ **23.** about 5.7 ft **25.** 9.3 **27.** $\frac{3}{5} = 0.6$, $\frac{4}{5} = 0.8$ **29.** $\frac{55}{73} = 0.7534$, $\frac{48}{73} = 0.6575$ **31.** $L = 53°$, $ML = 4.5$, $NL = 7.5$ **33.** 50°, 40°, 50°; about 6.4, about 8.4, about 13.1

Algebra Review (p. 499)

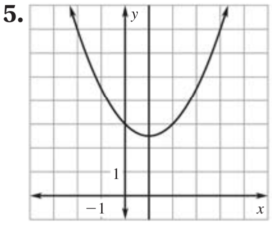
1.



3.



5.

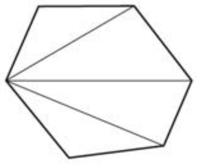


7. −2, 3
9. no solution
11. no solution
13. 0, 8
15. 2, 4
17. −5

Chapter 8

8.1 Skill Practice (pp. 510–511)

1. *Sample:*



3. 1260° **5.** 2520° **7.** quadrilateral **9.** 13-gon **11.** 117 **13.** $88\frac{1}{3}$ **15.** 66

17. The sum of the measures of the exterior angles of any convex n-gon is always 360°; the sum of the measures of the exterior angles of an octagon is the same as the sum of the measures of the exterior angles of a hexagon. **19.** 108°, 72° **21.** 176°, 4° **23.** The interior angle measures are the same in both pentagons and the ratio of corresponding sides would be the same. **25.** 40

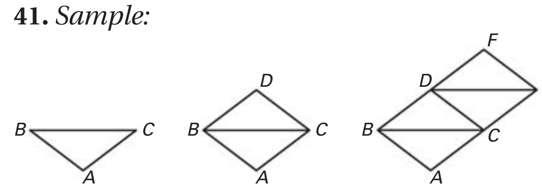
8.1 Problem Solving (pp. 512–513) **29.** 720° **31.** 144°; 36° **33.** In a pentagon draw all the diagonals from one vertex. Observe that the polygon is divided up into three triangles. Since the sum of the measures of the interior angles of each triangle is 180° the sum of the measures of the interior angles of the pentagon is $(5 - 2) \cdot 180° = 3 \cdot 180° = 540°$.
35. *Sample answer:* In a convex n-gon the sum of the measures of the n interior angles is $(n - 2) \cdot 180°$ using the Polygon Interior Angles Theorem. Since each of the n interior angles form a linear pair with their corresponding exterior angles you know that the sum of the measures of the n interior and exterior is angles $180°n$. Subtracting the sum of the interior angle measures from the sum of the measures of the linear pairs $(180°n - [(n - 2) \cdot 180°])$ you get 360°.
37. a.

Polygon	Number of sides	Number of triangles	Sum of measures of interior angles
Quadrilateral	4	2	$2 \cdot 180° = 360°$
Pentagon	5	3	$3 \cdot 180° = 540°$
Hexagon	6	4	$4 \cdot 180° = 720°$
Heptagon	7	5	$5 \cdot 180° = 900°$

b. $s(n) = (n - 2) \cdot 180°$; the table shows that the number of triangles is two less than the number of sides.

8.2 Skill Practice (pp. 518–519) **1.** A parallelogram is a quadrilateral with both pairs of opposite sides parallel; opposite sides are congruent, opposite angles are congruent, consecutive angles are supplementary, and the diagonals bisect each other. **3.** $x = 9, y = 15$ **5.** $a = 55$ **7.** $d = 126, z = 28$ **9.** 129° **11.** 61° **13.** $a = 3, b = 10$ **15.** $x = 4, y = 4$ **17.** $\overline{BC}$; opposite sides of a parallelogram are congruent. **19.** $\angle DAC$; alternate interior angles are congruent. **21.** 47°; consecutive angles of a parallelogram are supplementary and alternate interior angles are congruent. **23.** 120°; $\angle EJF$ and $\angle FJG$ are a linear pair. **25.** 35°; Triangle Sum Theorem **27.** 130°; sum of the measures of $\angle HGE$ and $\angle EGF$. **31.** 26°, 154° **33.** 20, 60°; $UV = TS = QR$ using the fact that opposite sides are congruent and the Transitive Property of Equality. $\angle TUS \cong \angle VSU$ using the Alternate Interior Angles Congruence Theorem and $m\angle TSU = 60°$ using the Triangle Sum Theorem. **35.** *Sample answer:* In a parallelogram opposite angles are congruent. $\angle A$ and $\angle C$ are opposite angles but not congruent.

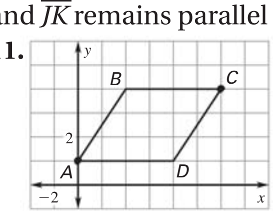
8.2 Problem Solving (pp. 520–521) **39. a.** 3 in. **b.** 70° **c.** It decreases; it gets longer; the sum of the measures of the interior angles always is 360°. As $m\angle Q$ increases so does $m\angle S$ therefore $m\angle P$ must decrease to maintain the sum of 180°. As $m\angle Q$ decreases $m\angle P$ increases moving Q farther away from S.
41. *Sample:*



Since $\triangle ABC \cong \triangle DCB$ you know $\angle ACB \cong \angle DBC$ and $\angle ABC \cong \angle DCB$. Using the Alternate Interior Angles Converse $\overline{BD} \parallel \overline{AC}$ and $\overline{AB} \parallel \overline{CD}$ thus making $ABDC$ a parallelogram; if two more triangles are positioned the same as the first, you can line up the pair of congruent sides and form a larger parallelogram because both pairs of alternate interior angles are congruent. Using the Alternate Interior Angles Converse, opposite sides are parallel. **43.** *Sample answer:* Given that $PQRS$ is a parallelogram you know that $\overline{QR} \parallel \overline{PS}$ with $\overline{QP}$ a transversal. By definition and the fact that $\angle Q$ and $\angle P$ are consecutive interior angles they are supplementary using the Consecutive Interior Angles Theorem. $x° + y° = 180°$ by definition of supplementary angles.

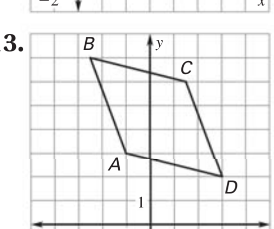
8.3 Skill Practice (pp. 526–527) **1.** The definition of a parallelogram is that it is a quadrilateral with opposite pairs of parallel sides. Since $\overline{AB}$, $\overline{CD}$ and $\overline{AD}$, $\overline{BC}$ are opposite pairs of parallel sides the quadrilateral $ABCD$ is a parallelogram. **3.** The congruent sides must be opposite one another. **5.** Theorem 8.7 **7.** Since both pairs of opposite sides of $JKLM$ always remain congruent, $JKLM$ is always a parallelogram and $\overline{JK}$ remains parallel to $\overline{ML}$. **9.** 8

11.



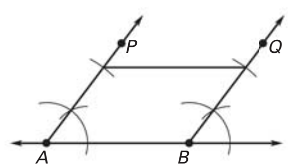
Sample answer: $AB = CD = 5$ and $BC = DA = 8$

13.



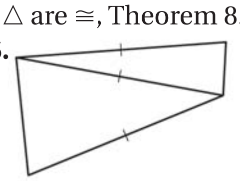
Sample answer: $AB = CD = 5$ and $BC = DA = \sqrt{65}$

15. *Sample answer:* Show $\triangle ADB \cong \triangle CBD$ using the SAS Congruence Postulate. This makes $\overline{AD} \cong \overline{CB}$ and $\overline{BA} \cong \overline{CD}$ using corresponding parts of congruent triangles are congruent. **17.** *Sample answer:* Show $\overline{AB} \parallel \overline{DC}$ by the Alternate Interior Angles Converse, and show $\overline{AD} \parallel \overline{BC}$ by the Corresponding Angles Converse. **19.** 114 **21.** 50
23. *PQRS* is a parallelogram if and only if $\angle P \cong \angle R$ and $\angle Q \cong \angle S$. **25.** $(-3, 2)$; since $\overline{DA}$ must be parallel and congruent to $\overline{BC}$ use the slope and length of $\overline{BC}$ to find point *D* by starting at point *A*. **27.** $(-5, -3)$; since $\overline{DA}$ must be parallel and congruent to $\overline{BC}$ use the slope and length of $\overline{BC}$ to find point *D* by starting at point *A*.
29. *Sample answer:* Draw a line passing through points *A* and *B*. At points *A* and *B* construct $\overrightarrow{AP}$ and $\overrightarrow{BQ}$ such that the angle each ray makes with the line is the same. Mark off congruent segments starting at *A* and *B* along $\overrightarrow{AP}$ and $\overrightarrow{BQ}$ respectively. Draw the line segment joining these two endpoints.



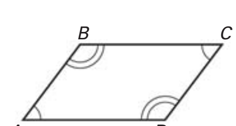
8.3 Problem Solving (pp. 528–529) **31. a.** *EFJK, FGHJ, EGHK*; in each case opposite pairs of sides are congruent. **b.** Since *EGHK* is a parallelogram, opposite sides are congruent. **33.** Alternate Interior Angles Congruence Theorem, Reflexive Property of Segment Congruence, Given, SAS, Corr. Parts of $\cong \triangle$ are $\cong$, Theorem 8.7

35.

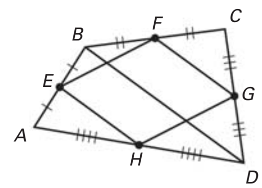


The opposite sides that are not marked in the given diagram are not necessarily the same length.

37. In a quadrilateral if consecutive angles are supplementary then the quadrilateral is a parallelogram; in *ABCD* you are given $\angle A$ and $\angle B$, $\angle C$ and $\angle B$ are supplementary which gives you $m\angle A = m\angle C$. Also $\angle B$ and $\angle C$, $\angle C$ and $\angle D$ are supplementary which give you $m\angle B = m\angle D$. So *ABCD* is a parallelogram by Theorem 8.8.



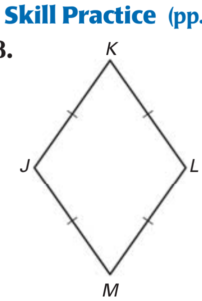
39. It is given that $\overline{KP} \cong \overline{MP}$ and $\overline{JP} \cong \overline{LP}$ by definition of segment bisector. $\angle KPL \cong \angle MPJ$ and $\angle KPJ \cong \angle MPL$ since they are vertical angles. $\triangle KPL \cong \triangle MPJ$ and $\triangle KPJ \cong \triangle MPL$ by the SAS Congruence Postulate. Using corresponding parts of congruent triangles are congruent, $\overline{KJ} \cong \overline{ML}$ and $\overline{JM} \cong \overline{LK}$. Using Theorem 8.7, *JKLM* is a parallelogram.
41. *Sample answer:* Consider the diagram.



$\overline{FG}$ is the midsegment of $\triangle CBD$ and therefore is parallel to $\overline{BD}$ and half of its length. $\overline{EH}$ is the midsegment of $\triangle ABD$ and therefore is parallel to $\overline{BD}$ and half of its length. This makes $\overline{EH}$ and $\overline{FG}$ both parallel and congruent. Using Theorem 8.9, *EFGH* is a parallelogram.

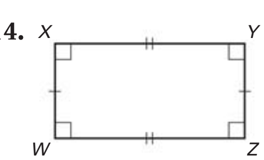
8.3 Problem Solving Workshop (p. 531) **1.** The slope of $\overline{AB}$ and $\overline{CD}$ is $\frac{2}{5}$ and the slope of $\overline{BC}$ and $\overline{DA}$ is -1. *ABCD* is a parallelogram by definition. **3.** No; the slope of the line segment joining Newton to Packard is $\frac{1}{3}$ while the slope of the line segment joining Riverdale to Quarry is $\frac{2}{7}$. **5.** $\overline{PQ}$ and $\overline{QR}$ are not opposite sides. $\overline{PQ}$ and $\overline{RS}$ are opposite sides, so they should be parallel and congruent. The slope of $\overline{PQ} = \frac{4-2}{3-2} = 2$. The slope of $\overline{RS} = \frac{5-4}{6-3} = \frac{1}{3}$. They are not parallel, so *PQRS* is not a parallelogram.

8.4 Skill Practice (pp. 537–539) **1.** square
3–8.



3. Sometimes; *JKLM* would need to be a square.
5. Always; in a rhombus all four sides are congruent.
7. Sometimes; diagonals are congruent if the rhombus is a square.

9–14.

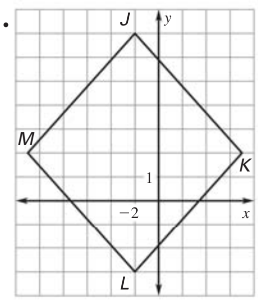


9. Always; in a rectangle all interior angles measure 90°. **11.** Sometimes; adjacent sides are congruent if the rectangle is a square. **13.** Sometimes; diagonals are perpendicular if the rectangle is a square.
15. Square; the quadrilateral has four congruent sides and angles. **17.** Rhombus. *Sample answer:* The fourth angle measure is 40°, meaning that both pairs of opposite sides are parallel. So the figure is a parallelogram with two consecutive sides congruent. But this is only possible if the remaining two sides are also congruent, so the quadrilateral is a rhombus.
19. rectangle, square **21.** rhombus, square
23. parallelogram, rectangle, rhombus, square
25. $7x - 4$ is not necessarily equal to $3x + 14$;

$(7x - 4) + (3x + 4) = 90, x = 9$. **27.** Rectangle; *JKLM* is a quadrilateral with four right angles; $x = 10, y = 15$. **29.** Parallelogram; *EFGH* is a quadrilateral with opposite pairs of sides congruent; $x = 13, y = 2$. **33.** 90° **35.** 16 **37.** 12 **39.** 112° **41.** 5 **43.** about 5.6

45. 45° **47.** 1 **49.** $\sqrt{2}$

51.



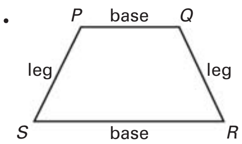
Rhombus; four congruent sides and opposite sides are parallel; $4\sqrt{106}$.

8.4 Problem Solving (pp. 539–540)

55. Measure the diagonals. If they are the same it is a square. **57.** If a quadrilateral is a rhombus, then it has four congruent sides; if a quadrilateral has four congruent sides, then it is a rhombus; the conditional statement is true since a quadrilateral is a parallelogram and a rhombus is a parallelogram with four congruent sides; the converse is true since a quadrilateral with four congruent sides is also a parallelogram with four congruent sides making it a rhombus. **59.** If a quadrilateral is a square, then it is a rhombus and a rectangle; if a quadrilateral is a rhombus and a rectangle, then it is a square; the conditional statement is true since a square is a parallelogram with four right angles and four congruent sides; the converse is true since a rhombus has four congruent sides and the rectangle has four right angles and thus a square follows. **61.** Since *WXYZ* is a rhombus the diagonals are perpendicular, making $\triangle WVX$, $\triangle WVZ$, $\triangle YVX$, and $\triangle YVZ$ right triangles. Since *WXYZ* is a rhombus $\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{ZW}$. Using Theorem 8.11 $\overline{WV} \cong \overline{YV}$ and $\overline{ZV} \cong \overline{XV}$. Now $\triangle WVX \cong \triangle WVZ \cong \triangle YVX \cong \triangle YVZ$. Using corresponding parts of congruent triangles are congruent, you now know $\angle WVZ \cong \angle WVX$ and $\angle YVZ \cong \angle YVX$ which implies $\overline{WY}$ bisects $\angle ZWX$ and $\angle XYZ$. Similarly $\angle VZW \cong \angle VZY$ and $\angle VXW \cong \angle VXY$. This implies $\overline{ZX}$ bisects $\angle WZY$ and $\angle YXW$. **63.** *Sample answer:* Let rectangle *ABCD* have vertices $(0, 0)$, $(a, 0)$, (a, b), and $(0, b)$ respectively. The diagonal $\overline{AC}$ has a length of $\sqrt{a^2 + b^2}$ and diagonal $\overline{BD}$ has a length of $\sqrt{a^2 + b^2}$. $AC = BD = \sqrt{a^2 + b^2}$.

8.5 Skill Practice (pp. 546–547)

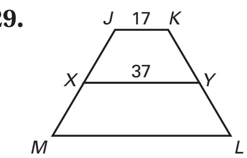
1.



3. trapezoid **5.** not a trapezoid **7.** 130°, 50°, 150° **9.** 118°, 62°, 62°

11. Trapezoid; $\overline{EF} \parallel \overline{HG}$ since they are both perpendicular to $\overline{EH}$. **13.** 14 **15.** 66.5 **17.** Only one pair of opposite angles in a kite is congruent. In this case $m\angle B = m\angle D = 120°$; $m\angle A + m\angle B + m\angle C + m\angle D = 360°$, $m\angle A + 120° + 50° + 120° = 360°$, so $m\angle A = 70°$. **19.** 80° **21.** $WX = XY = 3\sqrt{2}$, $YZ = ZW = \sqrt{34}$ **23.** $XY = YZ = 5\sqrt{5}$, $WX = WZ = \sqrt{461}$ **25.** 2 **27.** 2.3

29.

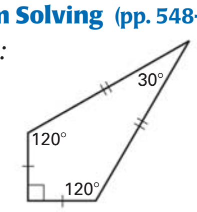


57

33. A kite or a general quadrilateral are the only quadrilaterals where a point on a line containing one of its sides can be found inside the figure.

8.5 Problem Solving (pp. 548–549)

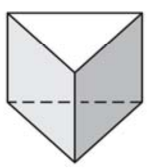
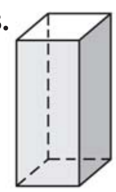
35. *Sample:*

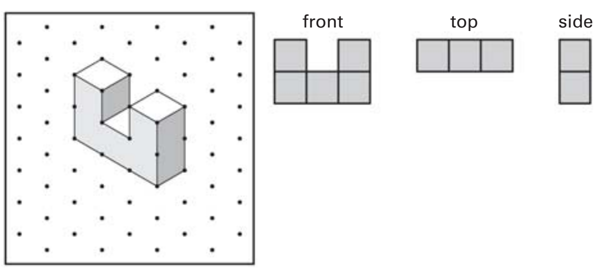


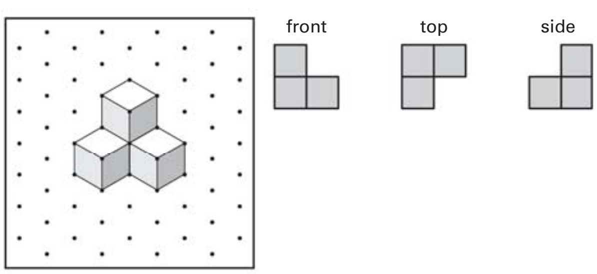
37. Since $\overline{BC} \parallel \overline{AE}$ and $\overline{AB} \parallel \overline{EC}$, *ABCE* is a parallelogram which makes $\overline{AB} \cong \overline{EC}$. Using the Transitive Property of Segment Congruence, $\overline{CE} \cong \overline{CD}$ making $\triangle ECD$ isosceles. Since $\triangle ECD$ is isosceles $\angle D \cong \angle CED$. $\angle A \cong \angle CED$ using the Corresponding Angles Congruence Postulate, therefore $\angle A \cong \angle D$ using the Transitive Property of Angle Congruence. $\angle CED$ and $\angle CEA$ form a linear pair and therefore are supplementary. $\angle A$ and $\angle ABC$, $\angle CEA$ and $\angle ECB$ are supplementary since they are consecutive pairs of angles in a parallelogram. Using the Congruent Supplements Theorem $\angle B \cong \angle C$ ($\angle ECB$). **39.** Given *JKLM* is an isosceles trapezoid with $\overline{KL} \parallel \overline{JM}$ and $\overline{JK} \cong \overline{LM}$. Since pairs of base angles are congruent in an isosceles trapezoid $\angle JKL \cong \angle MLK$. Using the Reflexive Property of Segment Congruence $\overline{KL} \cong \overline{KL}$. $\triangle JKL \cong \triangle MLK$ using the SAS Congruence Postulate. Using corresponding parts of congruent triangles are congruent, $\overline{JL} \cong \overline{KM}$.

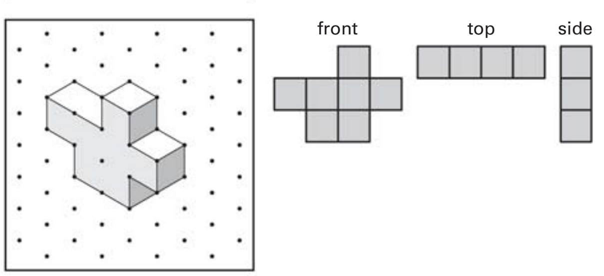
41. Given $ABCD$ is a kite with $\overline{AB} \cong \overline{CB}$ and $\overline{AD} \cong \overline{CD}$. Using the Reflexive Property of Segment Congruence, $\overline{BD} \cong \overline{BD}$ and $\overline{ED} \cong \overline{ED}$. Using the SSS Congruence Postulate, $\triangle BAD \cong \triangle BCD$. Using corresponding parts of congruent triangles are congruent, $\angle CDE \cong \angle ADE$. Using the SAS Congruence Postulate, $\triangle CDE \cong \triangle ADE$. Using corresponding parts of congruent triangles are congruent, $\angle CED \cong \angle AED$. Since $\angle CED$ and $\angle AED$ are congruent and form a linear pair, they are right angles. This makes $\overline{AC} \perp \overline{BD}$.

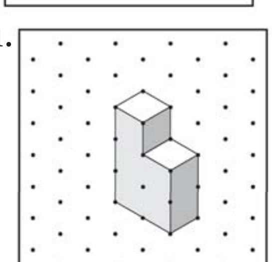
Extension (p. 551)

1. 
3. 

5. 

7. 

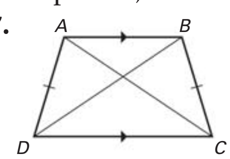
9. 

11. 

8.6 Skill Practice (pp. 554–555) **1.** isosceles trapezoid

	Property	Parallelogram	Rectangle	Rhombus	Square	Kite	Trapezoid
3.	All sides are ≅.			✘	✘		
5.	Both pair of opp. sides are ∥.	✘	✘	✘	✘		
7.	All ∡ are ≅.		✘		✘		
9.	Diagonals are ⊥.			✘	✘	✘	
11.	Diagonals bisect each other.	✘	✘	✘	✘		

15. Trapezoid; there is one pair of parallel sides.

17. 

isosceles trapezoid

$\overline{AC} \cong \overline{BD}$

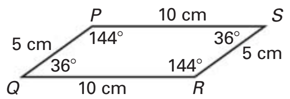
19. No; $m\angle F = 109°$ which is not congruent to $\angle E$. **21.** Kite; it has two pair of consecutive congruent sides. **23.** Rectangle; opposite sides are parallel with four right angles. **25. a.** rhombus, square, kite **b.** Parallelogram, rectangle, trapezoid; two consecutive pairs of sides are always congruent and one pair of opposite angles remain congruent. **27.** *Sample answer:* $m\angle B = 60°$ or $m\angle C = 120°$; then $\overline{AB} \parallel \overline{DC}$ and the base angles would be congruent. **29.** No; if $m\angle JKL = m\angle KJM = 90°$, $JKLM$ would be a rectangle. **31.** Yes; $JKLM$ has one pair of non-congruent parallel sides with congruent diagonals.

8.6 Problem Solving (pp. 556–557) **33.** trapezoid **35.** parallelogram **37.** Consecutive interior angles are supplementary making each interior angle 90°. **39. a.** Using the definition of a regular hexagon, $\overline{UV} \cong \overline{VQ} \cong \overline{RS} \cong \overline{ST}$ and $\angle V \cong \angle S$. Using the SAS Congruence Postulate, $\triangle QVU \cong \triangle RST$ and is isosceles. **b.** Using the definition of a regular hexagon, $\overline{QR} \cong \overline{RT}$. Using corresponding parts of congruent triangles are congruent, $\overline{QU} \cong \overline{RT}$. **c.** Since $\angle Q \cong \angle R \cong \angle T \cong \angle U$ and $\angle VUQ \cong \angle VQU \cong \angle STR \cong \angle SRT$, you know that $\angle UQR \cong \angle QRT \cong \angle RTU \cong \angle TUQ$ by the Angle Addition Postulate; 90°. **d.** Rectangle; there are 4 right angles and opposite sides are congruent.

Chapter Review (pp. 560–563) **1.** midsegment **3.** if the trapezoid has a pair of congruent base angles or if the diagonals are congruent **5.** A **7.** 24-gon; 165°

9. 82 **11.** 40°; the sum of the measures of the exterior angles is always 360°, and there are nine congruent external angles in a nonagon. **13.** $c = 6$, $d = 10$

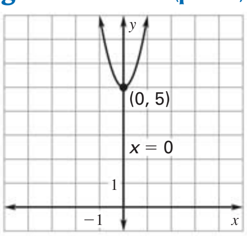
15.



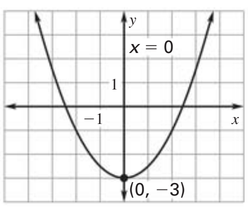
17. 100°, 80°; solve $5x + 4x = 180$ for x. **19.** 3
21. rectangle; 9, 5 **23.** 79°, 101°, 101° **25.** Rhombus; since all four sides are the same it is a rhombus. There are no known right angles. **27.** Parallelogram; since opposite pairs of sides are congruent it is a parallelogram. There are no known right angles.

Algebra Review (p. 565)

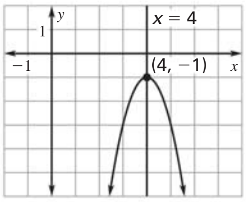
1.



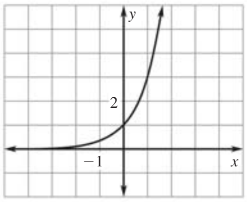
3.



5.



7.



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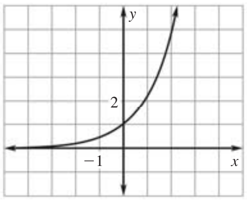

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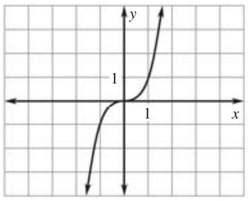

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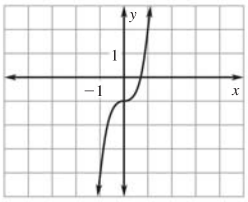

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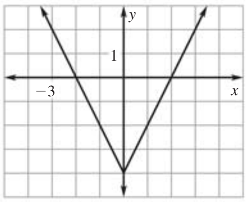
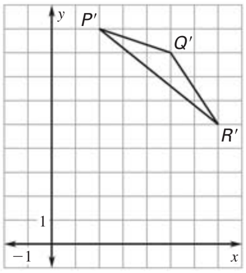


Chapter 9

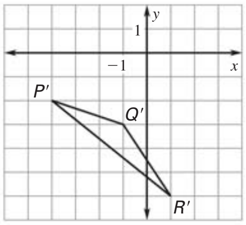
9.1 Skill Practice (pp. 576–577)

1. vector, direction
3. $A'(-6, 10)$ **5.** $C(5, -14)$

7.

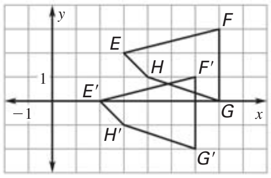


9.



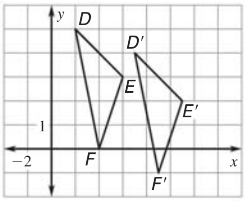
11. $(x, y) \rightarrow (x - 5, y + 2)$; $AB = A'B' = \sqrt{13}$, $AC = A'C' = 4$, and $BC = B'C' = \sqrt{5}$. $\triangle ABC \cong \triangle A'B'C'$ using the SSS Congruence Postulate.

13. The image should be 1 unit to the left instead of right and 2 units down instead of up.

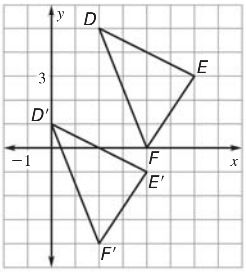


15. $\overrightarrow{CD}$, $\langle 7, -3 \rangle$ **17.** $\overrightarrow{JP}$, $\langle 0, 4 \rangle$ **19.** $\langle -1, 2 \rangle$ **21.** $\langle 0, -11 \rangle$
23. The vertical component is the distance from the ground up to the plane entrance.

25. $D'(7, 4)$, $E'(11, 2)$, $F'(9, -1)$



27. $D'(0, 1)$, $E'(4, -1)$, $F'(2, -4)$



29. $a = 35$, $b = 14$, $c = 5$ **31. a.** $Q'(-1, -5)$, $R'(-1, 2)$, $S'(2, 2)$, $T'(2, -5)$; 21, 21 **b.** The areas are the same; the area of an image and its preimage under a translation are the same.

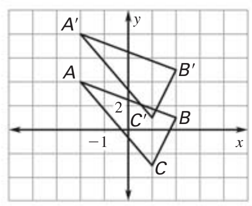
9.1 Problem Solving (pp. 578–579)

33. $(x, y) \rightarrow (x + 6, y)$, $(x, y) \rightarrow (x, y - 4)$, $(x, y) \rightarrow (x + 3, y - 4)$, $(x, y) \rightarrow (x + 6, y - 4)$ **35.** $\langle 1, 2 \rangle$ **37.** $\langle -4, -2 \rangle$ **39.** $\langle 3, 1 \rangle$
41. $\langle 22, 5 \rangle$; about 22.6 km **43. a.** 5 squares to the right followed by 4 squares down. **b.** $2\sqrt{41}$ mm **c.** about 0.523 mm/sec **45. a.** The graph is 4 units lower. **b.** The graph is 4 units to the right.

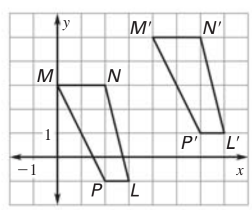
9.2 Skill Practice (pp. 584–585) **1.** elements **3.** $\begin{bmatrix} -1 & 2 & 6 \\ -2 & 2 & 1 \end{bmatrix}$ **5.** $\begin{array}{c} \begin{array}{cccc} B & C & D & E \end{array} \\ \begin{bmatrix} 2 & 6 & 5 & -1 \\ 2 & 1 & -1 & -2 \end{bmatrix} \end{array}$ **7.** $\begin{bmatrix} 12 & 7 \end{bmatrix}$

9. $\begin{bmatrix} 16 & 9 \\ 0 & 0 \\ -5 & -3 \end{bmatrix}$ **11.** $\begin{bmatrix} -13 & -4 \\ -12 & 16 \end{bmatrix}$

13. $\begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{bmatrix} -2 & 2 & 1 \\ 8 & 5 & 1 \end{bmatrix} \end{array}$



15. $\begin{array}{c} \begin{array}{cccc} L' & M' & N' & P' \end{array} \\ \begin{bmatrix} 7 & 4 & 6 & 6 \\ 1 & 5 & 5 & 1 \end{bmatrix} \end{array}$



19. $\begin{bmatrix} -6.9 \end{bmatrix}$ **21.** $\begin{bmatrix} -4 & 15.2 \\ -32.3 & -43.4 \end{bmatrix}$ **23.** $\begin{bmatrix} 38 \\ 36 \end{bmatrix}$

25. *Sample answer:* $\begin{bmatrix} -2 & 1 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}; \begin{bmatrix} 0 & -1 \\ 8 & 4 \end{bmatrix}$

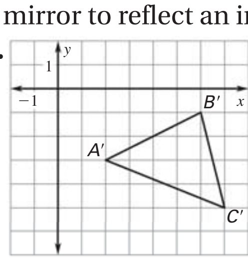
27. $\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} -7 & 0 & 0 & -7 \\ 3 & 3 & -1 & -1 \end{bmatrix} \end{array}$ **29.** $a = 8$, $b = -20$, $c = 20$, $m = 21$, $n = -1$, $v = -7$, $w = 12$; the sum of the corresponding elements on the left equals the corresponding elements on the right; $(21, -1)$, $(20, -9)$, $(-8, 13)$.

9.2 Problem Solving (pp. 586–587) **31.** Lab 1: \$840, Lab 2: \$970 **33. a.** $AB = BA$ **b.** $\begin{bmatrix} -3 & 15 \\ -14 & 30 \end{bmatrix}, \begin{bmatrix} 25 & -7 \\ 10 & 2 \end{bmatrix}$, $AB \neq BA$ **c.** Matrix multiplication is not commutative.

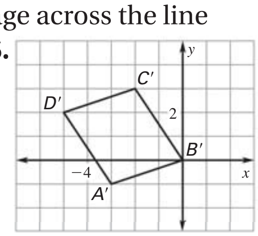
35. $\begin{bmatrix} 2 & 36 \\ 16 & 68 \end{bmatrix}, \begin{bmatrix} 2 & 36 \\ 16 & 68 \end{bmatrix}$; the Distributive Property holds for matrices.

9.3 Skill Practice (pp. 593–594) **1.** a line which acts like a mirror to reflect an image across the line

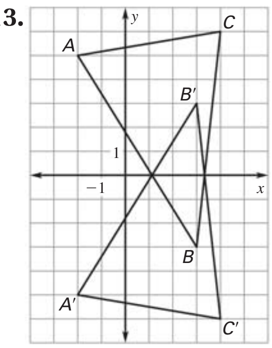
3.



5.



13.

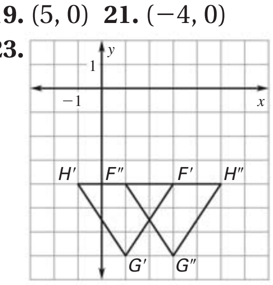


15. $\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 1 & 4 & 3 \\ 2 & 2 & -2 \end{bmatrix} \end{array}; \begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{bmatrix} -1 & -4 & -3 \\ 2 & 2 & -2 \end{bmatrix} \end{array}$

17. $\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} -4 & 3 & 2 \\ -2 & 1 & -3 \end{bmatrix} \end{array}; \begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{bmatrix} 4 & -3 & -2 \\ -2 & 1 & -3 \end{bmatrix} \end{array}$

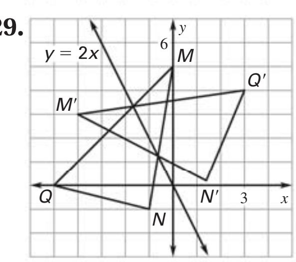
19. $(5, 0)$ **21.** $(-4, 0)$

23.



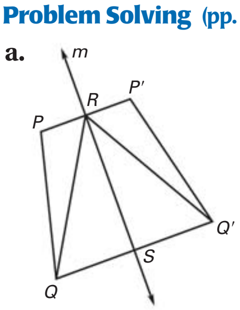
25. The order is reversed. **27.** $y = 3x - 4$

29.



9.3 Problem Solving (pp. 595–596) **31.** Case 4 **33.** Case 1

35. a.

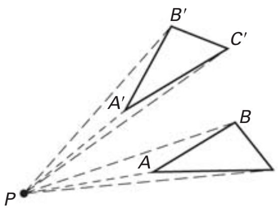


Given a reflection in m maps P to P' and Q to Q'. Using the definition of a line of reflection $\overline{QS} \cong \overline{Q'S}$ and $\angle QSR \cong \angle Q'SR$. Using the Reflexive Property of Segment Congruence, $\overline{RS} \cong \overline{RS}$. Using the SAS Congruence Postulate, $\triangle RSQ \cong \triangle RSQ'$.

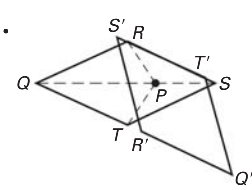
b. Using corresponding parts of congruent triangles are congruent, $\overline{RQ} \cong \overline{RQ'}$. Using the definition of a line of reflection $\overline{PR} \cong \overline{P'R}$. Since $\overline{PP'}$ and $\overline{QQ'}$ are both perpendicular to m, they are parallel. Using the Alternate Interior Angles Theorem, $\angle SQ'R \cong \angle P'RQ'$ and $\angle SQR \cong \angle PRQ$. Using corresponding parts of congruent triangles are congruent, $\angle SQ'R \cong \angle SQR$. Using the Transitive Property of Angle Congruence, $\angle P'RQ' \cong \angle PRQ$. $\triangle PRQ \cong \triangle P'RQ'$ using the SAS Congruence Postulate. Using corresponding parts of congruent triangles are congruent, $\overline{PQ} \cong \overline{P'Q'}$ which implies $PQ = P'Q'$. **37.** Given a reflection in m maps P to P' and Q to Q'. Also, P lies on m, and $\overline{PQ}$ is not perpendicular to m. Draw $\overline{Q'Q}$ intersecting m at point R. Using the definition of line of reflection m is the perpendicular bisector of $\overline{Q'Q}$ which implies $\overline{Q'R} \cong \overline{QR}$, $\angle Q'RP' \cong \angle QRP$, and P and P' are the same point. Using the Reflexive Property of Segment Congruence, $\overline{RP} \cong \overline{RP}$. Using the SAS Congruence Postulate, $\triangle Q'RP' \cong \triangle QRP$. Using corresponding parts of congruent triangles are congruent, $\overline{Q'P'} \cong \overline{QP}$ which implies $Q'P' = QP$. **39. a.** $(3, 5)$ **b.** $(0, 6)$; $(-1, 4)$ **c.** In every case point C bisects each line segment.

9.4 Skill Practice (pp. 602–603) **1.** a point which a figure is turned about during a rotation transformation **3.** Reflection; the horses are reflected across the edge of the stream which acts like a line of symmetry. **5.** Translation; the train moves horizontally from right to left. **7.** A

9.

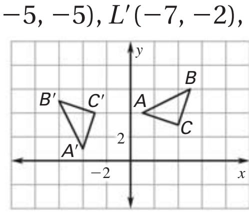


11.

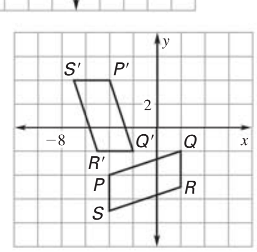


13. $J'(-1, -4)$, $K'(-5, -5)$, $L'(-7, -2)$, $M'(-2, -2)$

15. $\begin{array}{c} \begin{matrix} A' & B' & C' \end{matrix} \\ \begin{bmatrix} -4 & -6 & -3 \\ 1 & 5 & 4 \end{bmatrix} \end{array}$



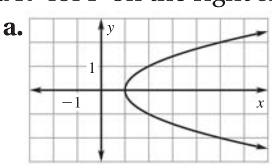
17. $\begin{array}{c} \begin{matrix} P' & Q' & R' & S' \end{matrix} \\ \begin{bmatrix} -4 & -2 & -5 & -7 \\ 4 & -2 & -2 & 4 \end{bmatrix} \end{array}$



19. The rotation matrix should be first; $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}$. **25.** $(-3, 2, 0)$

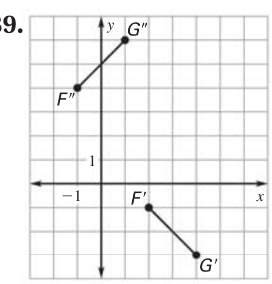
9.4 Problem Solving (pp. 604–605) **29.** 270°; the line segment joining A' to the center of rotation is perpendicular to the line segment joining A to the center of rotation. **31.** 120°; the line segment joining A' to the center of rotation is rotated $\frac{1}{3}$ of a circle from the line segment joining A to the center of rotation. **33.** a rotation about a point, Angle Addition Postulate, Transitive, Addition, $\triangle RPQ \cong \triangle R'PQ'$, Corr. Parts of $\cong \triangle$ are $\cong$, definition of segment congruence **35.** Given a rotation about P maps Q to Q' and R to R'. P and R are the same point. Using the definition of rotation about a point P, $PQ = PQ'$ and P, R, and R' are the same point. Substituting R for P on the left and R' for P on the right side, you get $RQ = R'Q'$.

37. a.



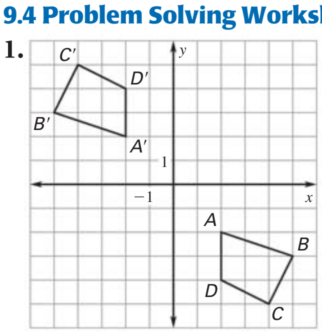
b. 270° **c.** No; the image does not pass the vertical line test.

39.



9.4 Problem Solving Workshop (p. 606)

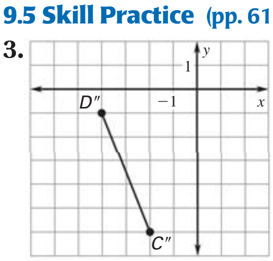
1.



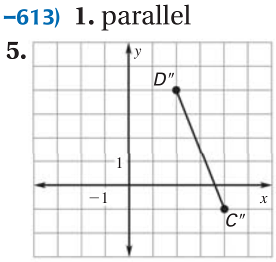
3. Since they are rotating in opposite directions they will each place you at 90° below your reference line. **5.** The x-coordinate is now -4; the y-coordinate is now 3.

9.5 Skill Practice (pp. 611–613) **1.** parallel

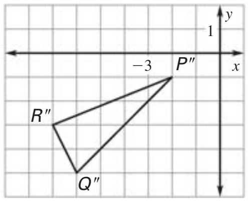
3.

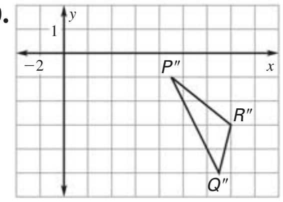


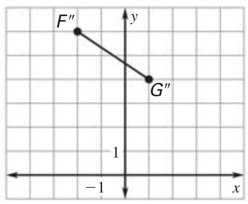
5.



SELECTED ANSWERS

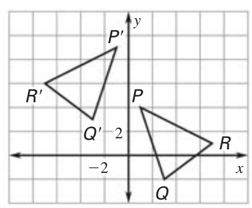
7. 

9. 

11. 

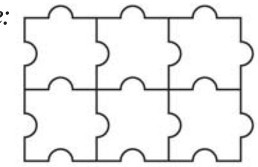
yes
13. $(x, y) \to (x + 5, y + 1)$ followed by a rotation of 180° about the origin.
15. $\triangle A''B''C''$
17. *Sample answer:* $\overline{AA'}$, $\overline{AA''}$

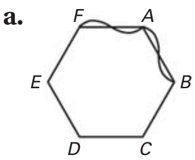
19. yes; definition of reflection of a point over a line
21. 30°
23. $\begin{array}{ccc} P' & Q' & R' \end{array}$
$\begin{bmatrix} -1 & -3 & -7 \\ 9 & 3 & 6 \end{bmatrix}$

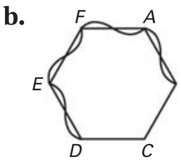


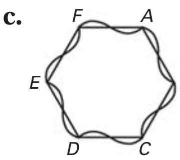
9.5 Problem Solving (pp. 613–615) **27.** *Sample answer:* $(x, y) \to (x + 9, y)$, reflected over a horizontal line that separates the left and right prints **31.** reflection **33.** translation **35.** Use the Rotation Theorem followed by the Reflection Theorem. **37.** Given a reflection in ℓ maps $\overline{JK}$ to $\overline{J'K'}$, a reflection in m maps $\overline{J'K'}$ to $\overline{J''K''}$, $\ell \parallel m$ and the distance between ℓ and m is d. Using the definition of reflection ℓ is the perpendicular bisector of $\overline{KK'}$ and m is perpendicular bisector of $\overline{K'K''}$. Using the Segment Addition Postulate, $KK' + K'K'' = KK''$. It follows that $\overline{KK'}$ is perpendicular to ℓ and m. Using the definition of reflection the distance from K to ℓ is the same as the distance from ℓ to K' and the distance from K' to m is the same as the distance from m to K''. Since the distance from ℓ to K' plus the distance from K' to m is d, it follows that $K'K'' = 2d$. **39. a.** translation and a rotation **b.** One transformation is not followed by the second. They are done simultaneously.

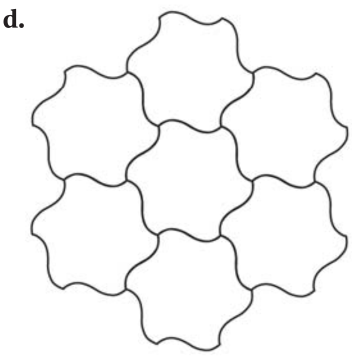
Extension (pp. 617–618) **1.** yes; regular **3.** yes; not regular **5. a.** 360°; the sum of the angle measures at any vertex is 360°. **b.** The sum of the measures of the interior angles is 360°.

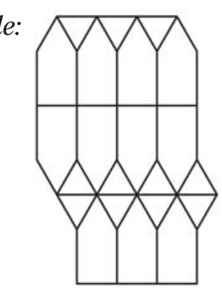
7. *Sample:* 

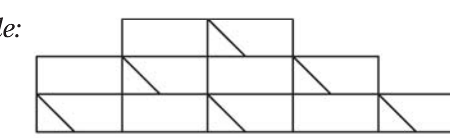
9. a. 

b. 

c. 

d. 

11. *Sample:* 

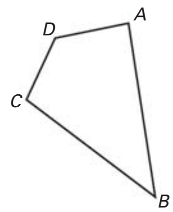
13. *Sample:* 

15. translation **17.** rotations

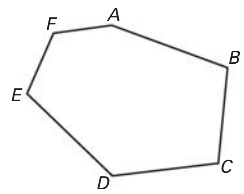
9.6 Skill Practice (pp. 621–623) **1.** If a figure has rotational symmetry it is the point about which the figure is rotated. **3.** 1 **5.** 1 **7.** yes; 72° or 144° about the center **9.** no **11.** Line symmetry, rotational symmetry; there are four lines of symmetry, two passing through the outer opposite pairs of leaves and two passing through the inner opposite pairs of leaves; 90° or 180° about the center. **15.** There is no rotational symmetry; the figure has 1 line of symmetry but no rotational symmetry.

SELECTED ANSWERS

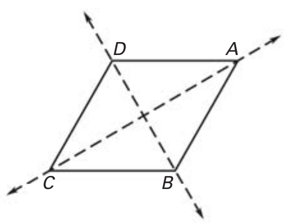
17. *Sample:*



19. *Sample:*



21. *Sample:*



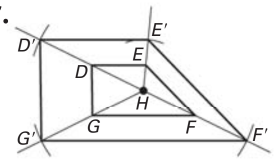
23. No; what's on the left and right of the first line would have to be the same as what's on the left and right of the second line which is not possible. **25.** 5

9.6 Problem Solving (pp. 623–624) **27.** no line symmetry, rotational symmetry of 180° about the center of the letter *O*. **29.** It has a line of symmetry passing horizontally through the center of each *O*, no rotational symmetry. **31.** 22.5° **33.** 15° **35. a.** line symmetry and rotational symmetry **b.** planes, *z*-axis

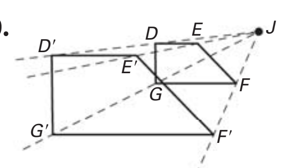
9.7 Skill Practice (pp. 629–630) **1.** a real number

3. $\frac{7}{3}$; enlargement; 8 **5.** $\frac{3}{2}$; enlargement; 10

7.

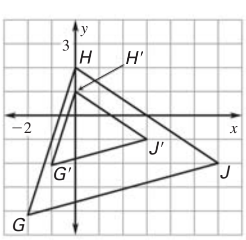


9.

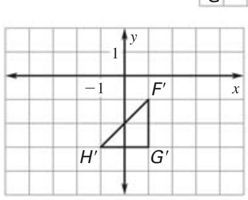


15. $\begin{bmatrix} 12 & 28 & 16 \\ 0 & 36 & -4 \end{bmatrix}$ **17.** $\begin{bmatrix} 0 & 27 & 18 \\ -9 & 63 & 0 \end{bmatrix}$

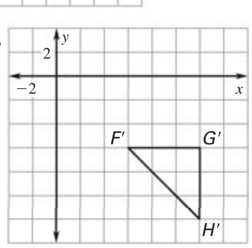
19. $\begin{array}{ccc} G' & H' & J' \end{array}$
$\begin{bmatrix} -1 & 0 & 3 \\ -2 & 1 & -1 \end{bmatrix}$



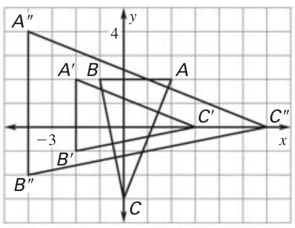
21.



23.



27. *Sample:*



No; the result is the same.

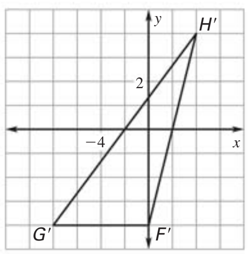
31. No; the ratio of the lengths of corresponding sides is not the same.

9.7 Problem Solving (pp. 631–632) **33.** 300 mm

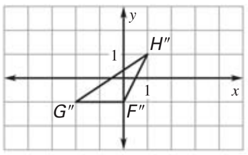
35. 940 mm **37. a.** $\frac{6}{1}$ **b.** 10.5 in.

39. a. $\begin{array}{c} \begin{array}{ccc} F & G & H \end{array} \\ \begin{bmatrix} 0 & 4 & -2 \\ 2 & 2 & -2 \end{bmatrix} \end{array}; \begin{array}{c} \begin{array}{ccc} F' & G' & H' \end{array} \\ \begin{bmatrix} 0 & -8 & 4 \\ -4 & -4 & 4 \end{bmatrix} \end{array}$

b.



c. $\begin{array}{c} \begin{array}{ccc} F'' & G'' & H'' \end{array} \\ \begin{bmatrix} 0 & -2 & 1 \\ -1 & -1 & 1 \end{bmatrix} \end{array}$



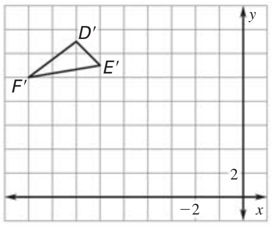
d. A reflection in both the *x*-axis and *y*-axis occurs as well as dilation. **41.** It's the center point of the dilation.

Chapter Review (pp. 636–639) **1.** isometry

3. Count the number of rows, *n*, and the number of columns, *m*. The dimensions are $n \times m$.

Sample answer: $\begin{bmatrix} 2 & 0 & 3 \\ -1 & 4 & 7 \end{bmatrix}$ is 2×3. **5.** A

7.



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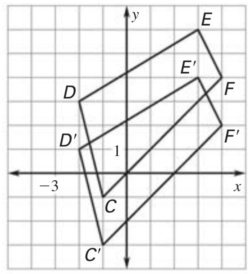
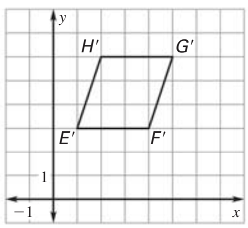
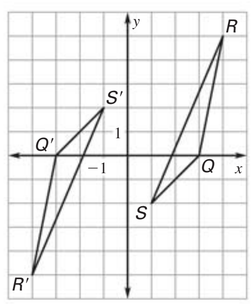
9. $\begin{array}{cccc} D' & E' & F' & G' \end{array}$ $\begin{bmatrix} -2 & 3 & 4 & -1 \\ 1 & 4 & 2 & -3 \end{bmatrix}$



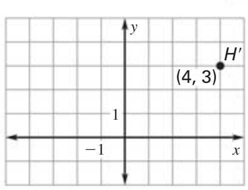
11.



13. $\begin{array}{ccc} Q' & R' & S' \end{array}$ $\begin{bmatrix} -3 & -4 & -1 \\ 0 & -5 & 2 \end{bmatrix}$

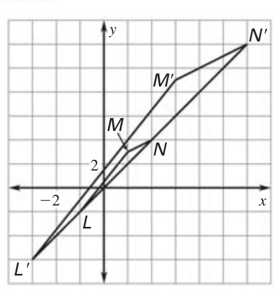


15.



17. line symmetry, no rotational symmetry; one **19.** line symmetry, rotational symmetry; two, 180° about the center

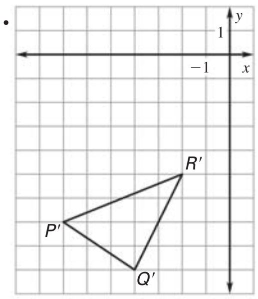
21. $\begin{array}{ccc} L' & M' & N' \end{array}$ $\begin{bmatrix} -3 & 3 & 6 \\ -6 & 9 & 12 \end{bmatrix}$



Algebra Review (p. 641) **1.** $x^2 + x - 6$ **3.** $x^2 - 16$ **5.** $49x^2 + 84x + 36$ **7.** $4x^2 - 1$ **9.** $2x^2 + 3xy + y^2$ **11.** 3, 4 **13.** $-2, -\frac{1}{4}$ **15.** $\frac{-1 \pm \sqrt{29}}{2}$ **17.** $\frac{-11 \pm \sqrt{105}}{2}$

Cumulative Review (pp. 646–647) **1.** neither **3.** $x = 4$ **5.** $y = \frac{1}{2}x - 2$ **7.** $\overline{QP} \cong \overline{SR}$ **9.** altitude **11.** median **13.** triangle; right **15.** not a triangle **17.** triangle; right **19.** Rectangle; the diagonals are congruent and they bisect each other; 5, 3.

21.

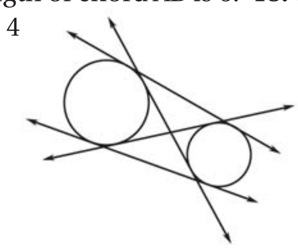


23. $\angle 4, \angle 5, \angle 7$
25. 132°
27. about 105 mi
29. about \$7.69
31. translations

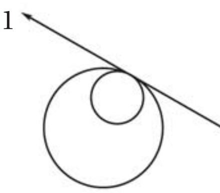
Chapter 10

10.1 Skill Practice (pp. 655–657) **1.** diameter **3.** G **5.** C **7.** F **9.** B **11.** $\overline{AB}$ is not a secant it is a chord; the length of chord $\overline{AB}$ is 6. **13.** 6, 12

15. 4



17. 1



19. not tangent; $9^2 + 15^2 \neq 18^2$ **21.** 10 **23.** 10.5 **25.** $\sqrt{2}$ **27.** external **31.** They will be parallel if they are tangent to opposite endpoints of the same diameter; lines perpendicular to the same line are parallel. **33.** No; no; no matter what the distance the external point is from the circle there will always be two tangents.

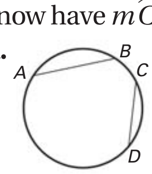
10.1 Problem Solving (pp. 657–658) **35.** radial spokes **37.** 14,426 mi **39. a.** Since R is exterior to $\odot Q$, $QR > QP$. **b.** Since $\overline{QR}$ is perpendicular to line m it must be the shortest distance from Q to line m, thus $QR < QP$. **c.** It was assumed $\overline{QP}$ was not perpendicular to line m but $\overline{QR}$ was perpendicular to line m. Since R is outside of $\odot Q$ you know that $QR > QP$ but Exercise 39b tells you that $QR < QP$ which is a contradiction. Therefore, line m is perpendicular to $\overline{QP}$. **41.** Given $\overline{SR}$ and $\overline{ST}$ are tangent to $\odot P$. Construct $\overline{PR}$, $\overline{PT}$, and $\overline{PS}$. Since $\overline{PR}$ and $\overline{PT}$ are radii of $\odot P$, $\overline{PR} \cong \overline{PT}$. With $\overline{PS} \cong \overline{PS}$, using the HL Congruence Theorem $\triangle RSP \cong \triangle TSP$. Using corresponding parts of congruent triangles are congruent, $\overline{SR} \cong \overline{ST}$.

10.2 Skill Practice (pp. 661–662) **1.** congruent **3.** minor arc; 70° **5.** minor arc; 135° **7.** minor arc; 115° **9.** major arc; 245° **13.** Not congruent; they are arcs of circles that are not congruent. **15.** You can tell that the circles are congruent since they have the same radius $\overline{CD}$. **19.** *Sample answer:* 15°, 185°

10.2 Problem Solving (p. 663) **23.** 18°

10.3 Skill Practice (pp. 667–668) **1.** *Sample answer:* Point Y bisects $\overset{\frown}{XZ}$ if $\overset{\frown}{XY} \cong \overset{\frown}{YZ}$. **3.** 75° **5.** 8 **7.** 5; use Theorem 10.5 and solve $5x - 6 = 2x + 9$. **9.** 5; use Theorem 10.6 and solve $18 = 5x - 7$. **11.** $\frac{7}{3}$; use Theorem 10.6 and solve $4x + 1 = x + 8$.
13. $\overline{JH}$ bisects $\overline{FG}$ and $\overset{\frown}{FG}$; Theorem 10.5. **17.** You don't know that $\overline{AC} \perp \overline{DB}$ therefore you can't show $\overset{\frown}{BC} \cong \overset{\frown}{CD}$.
19. Diameter; the two triangles are congruent using the SAS Congruence Postulate which makes $\overline{AB}$ the perpendicular bisector of $\overline{CD}$. Use Theorem 10.4.
21. Using the facts that $\triangle APB$ is equilateral which makes it equiangular and that $m\overset{\frown}{AC} = 30°$ you can conclude that $m\angle APD = m\angle BPD = 30°$. You now know that $m\overset{\frown}{BC} = 30°$ which makes $\overline{AC} \cong \overline{BC}$. $\triangle APD \cong \triangle BPD$ using the SAS Congruence Postulate since $\overline{BP} \cong \overline{AP}$ and $\overline{PD} \cong \overline{PD}$. Using corresponding parts of congruent triangles are congruent, $\overline{AD} \cong \overline{BD}$. Along with $\overline{DC} \cong \overline{DC}$ you have $\triangle ADC \cong \triangle BDC$ using the SSS Congruence Postulate. **23.** From the diagram $m\overset{\frown}{AC} = m\overset{\frown}{CB}$ and $m\overset{\frown}{AB} = x°$, so you know that $m\overset{\frown}{AC} + m\overset{\frown}{CB} + x° = 360°$. Replacing $m\overset{\frown}{CB}$ by $m\overset{\frown}{AC}$ and solving for $m\overset{\frown}{AC}$ you get $m\overset{\frown}{AC} = \frac{360° - x°}{2}$. This along with the fact that all arcs have integral measure implies that x is even.

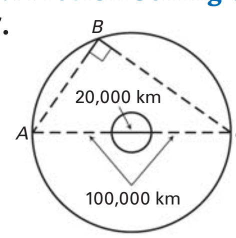
10.3 Problem Solving (pp. 669–670) **25.** $\overline{AB}$ should be congruent to $\overline{BC}$. **27.** Given $\overline{AB} \cong \overline{CD}$. Since $\overline{PA}$, $\overline{PB}$, $\overline{PC}$, and $\overline{PD}$ are radii of $\odot P$, they are congruent. Using the SSS Congruence Postulate, $\triangle PCD \cong \triangle PAB$. Using corresponding parts of congruent triangles are congruent, $\angle CPD \cong \angle APB$. With $m\angle CPD = m\angle APB$ and the fact they are both central angles you now have $m\overset{\frown}{CD} = m\overset{\frown}{AB}$ which leads to $\overset{\frown}{CD} \cong \overset{\frown}{AB}$.
29. a.



longer chord
b. The length of a chord in a circle increases as the distance from the center of the circle to the chord decreases.
c. Given radius r and real numbers a and b such that $r > a > b > 0$. Let a be the distance from one chord to the center of the circle and b be the distance from a second chord to the center of the circle. Using the Pythagorean Theorem the length of the chord a units away from the center is $2\sqrt{r^2 - a^2}$ and the length of the chord b units away from the center is $\sqrt{r^2 - b^2}$. Using properties of real numbers $\sqrt{r^2 - b^2} > \sqrt{r^2 - a^2}$. **31.** Given $\overline{QS}$ is perpendicular bisector of $\overline{RT}$ in $\odot L$. Suppose L is not on $\overline{QS}$. Since $\overline{LT}$ and $\overline{LR}$ are radii of the circle they are congruent. With $\overline{PL} \cong \overline{PL}$ you now have $\triangle RLP \cong \triangle TLP$ using the SSS Congruence Postulate. $\angle RPL$ and $\angle TPL$ are now congruent and they form a linear pair. This makes them right angles and leads to $\overline{QL}$ being perpendicular to $\overline{RT}$. Using the Perpendicular Postulate, L must be on $\overline{QS}$ and thus $\overline{QS}$ must be a diameter.

10.4 Skill Practice (pp. 676–677) **1.** inscribed **3.** 42° **5.** 10° **7.** 120° **9.** The measure of the arcs add up to 370°; change the measure of $\angle Q$ to 40° or change the measure of $\overset{\frown}{QS}$ to 90°. **11.** $\angle JMK$, $\angle JLK$ and $\angle LKM$, $\angle LJM$ **13.** $x = 100$, $y = 85$ **15.** $a = 20$, $b = 22$
17. a. 36°; 180° **b.** about 25.7°; 180° **c.** 20°; 180°
19. 90° **21.** Yes; opposite angles are 90° and thus are supplementary. **23.** No; opposite angles are not supplementary. **25.** Yes; opposite angles are supplementary.

10.4 Problem Solving (pp. 677–679)
27.



220,000 km

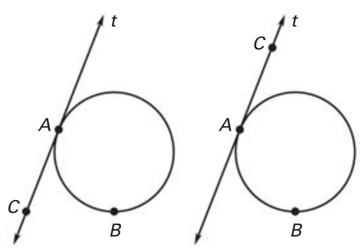
29. Double the length of the radius. **31.** Given $\angle B$ inscribed in $\odot Q$. Let $m\angle B = x°$. Point Q lies on $\overline{BC}$. Since all radii of a circle are congruent, $\overline{AQ} \cong \overline{BQ}$. Using the Base Angles Theorem, $\angle B \cong \angle A$ which implies $m\angle A = x°$. Using the Exterior Angles Theorem, $m\angle AQC = 2x°$ which implies $m\overset{\frown}{AC} = 2x°$. Solving for x, you get $\frac{1}{2}m\overset{\frown}{AC} = x°$. Substituting you get $\frac{1}{2}m\overset{\frown}{AC} = m\angle B$. **33.** Given: $\angle ABC$ is inscribed in $\odot Q$. Point Q is in the exterior of $\angle ABC$; Prove: $m\angle ABC = \frac{1}{2}m\overset{\frown}{AC}$; construct the diameter $\overline{BD}$ of $\odot Q$ and show $m\angle ABD = \frac{1}{2}m\overset{\frown}{AD}$ and $m\angle CBD = \frac{1}{2}m\overset{\frown}{CD}$. Use the Arc Addition Postulate and the Angle Addition Postulate to show $m\angle ABD - m\angle CBD = m\angle ABC$. Then use substitution to show $2m\angle ABC = m\overset{\frown}{AC}$.

SELECTED ANSWERS

35. Case 1: Given: $\odot D$ with inscribed $\triangle ABC$ where $\overline{AC}$ is a diameter of $\odot D$; Prove $\triangle ABC$ is a right triangle; let E be a point on $\widehat{AC}$. Show that $m\widehat{AEC} = 180°$ and then that $m\angle B = 90°$. Case 2: Given: $\odot D$ with inscribed $\triangle ABC$ with $\angle B$ a right angle; Prove: $\overline{AC}$ is a diameter of $\odot D$; using the Measure of an Inscribed Angle Theorem, show that $m\widehat{AC} = 180°$. **39.** yes

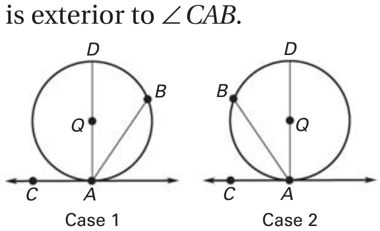
10.5 Skill Practice (pp. 683–684) **1.** outside **3.** 130° **5.** 130° **7.** 115 **9.** 90 **11.** 56 **15.** $m\angle LPJ \le 90°$; if $\overrightarrow{PL}$ is perpendicular to $\overline{KJ}$ at K, then $m\angle LPJ = 90°$, otherwise it would measure less than 90°. **17.** 120°, 100°, 140°

19. a.



b. $m\widehat{AB} = 2m\angle BAC$, $m\widehat{AB} = 2(180 - m\angle BAC)$
c. when $\overline{AB}$ is perpendicular to line t at point A

10.5 Problem Solving (pp. 685–686) **23.** 50° **25.** about 2.8° **27.** Given $\overleftrightarrow{CA}$ tangent to $\odot Q$ at A and diameter $\overline{AB}$. Using Theorem 10.1, $\overline{AB}$ is perpendicular to $\overleftrightarrow{CA}$. It follows that $m\angle CAB = 90°$. This is half of 180°, which is $m\widehat{AB}$; Case 1: the center of the circle is interior to $\angle CAB$, Case 2: the center of the circle is exterior to $\angle CAB$.



Construct diameter $\overline{AD}$. Case 1: Let B be a point on the left semicircle. Use Theorem 10.1 to show $m\angle CAB = 90°$. Use the Angle Addition Postulate and the Arc Addition Postulate to show that $m\angle CAD = \frac{1}{2}m\widehat{AB}$. Case 2: Let B be a point on the right semicircle. Prove similarly to Case 1.

10.6 Skill Practice (pp. 692–693) **1.** external segment **3.** 5 **5.** 4 **7.** 6 **9.** 12 **11.** 4 **13.** 5 **15.** 1 **17.** 18

10.6 Problem Solving (pp. 694–695)

21.

Statements	Reasons
1. Two intersecting chords in the same circle.	1. Given
2. Draw $\overline{AC}$ and $\overline{BD}$.	2. Two points determine a line.
3. $\angle ACD \cong \angle ABD$, $\angle CAB \cong \angle CDB$	3. Theorem 10.8
4. $\triangle ACE \sim \triangle DEB$	4. AA Similarity Postulate
5. $\frac{EA}{ED} = \frac{EC}{EB}$	5. If two triangles are similar, then the ratios of corresponding sides are equal.
6. $EA \cdot EB = EC \cdot ED$	6. Cross Products Property

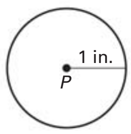
23. Given a secant segment containing the center of the circle and a tangent segment sharing an endpoint outside of a circle. Draw $\overline{AC}$ and $\overline{AD}$. $\angle ADC$ is inscribed, therefore $m\angle ADC = \frac{1}{2}m\widehat{AC}$. $\angle CAE$ is formed by a secant and a tangent, therefore $m\angle CAE = \frac{1}{2}m\widehat{AC}$. This implies $\angle ADC \cong \angle CAE$. $\angle E \cong \angle E$, therefore $\triangle AEC \sim \triangle DEC$ using the AA Similarity Postulate. Using corresponding sides of similar triangles are proportional, $\frac{EA}{EC} = \frac{ED}{EA}$. Cross multiplying you get $EA^2 = EC \cdot ED$. **25.** Given $\overline{EB}$ and $\overline{ED}$ are secant segments. Draw $\overline{AD}$ and $\overline{BC}$. Using the Measure of an Inscribed Angle Theorem, $m\angle B = \frac{1}{2}m\widehat{AC}$ and $m\angle D = \frac{1}{2}m\widehat{AC}$ which implies $m\angle B \cong m\angle D$. Using the Reflexive Property of Angle Congruence, $\angle E \cong \angle E$. Using the AA Similarity Postulate, $\triangle BCE \sim \triangle DAE$. Using corresponding sides of similar triangles are proportional, $\frac{EA}{EC} = \frac{ED}{EB}$. Cross multiplying you get $EA \cdot EB = EC \cdot ED$.
27. a. 60° **b.** Using the Vertical Angles Theorem, $\angle ACB \cong \angle FCE$. Since $m\angle CAB = 60°$ and $m\angle EFD = 60°$, then $\angle CAB \cong \angle EFD$. Using the AA Similarity Postulate, $\triangle ABC \sim \triangle FEC$. **c.** $\frac{y}{3} = \frac{x+10}{6}$; $y = \frac{x+10}{2}$
d. $y^2 = x(x+16)$ **e.** 2, 6 **f.** Since $\frac{CE}{CB} = \frac{2}{1}$, let $CE = 2x$ and $CB = x$. Using Theorem 10.14, $2x^2 = 60$ which implies $x = \sqrt{30}$ which implies $CE = 2\sqrt{30}$.

SELECTED ANSWERS

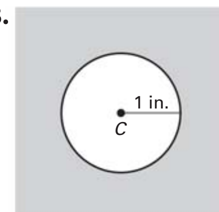
10.6 Problem Solving Workshop (p. 696) **1.** $2\sqrt{13}$ **3.** $\frac{24}{5}$

Extension (p. 698)

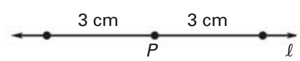
1.



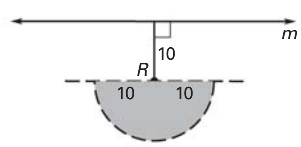
3.



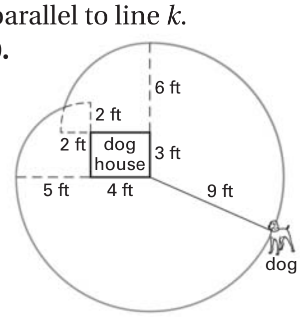
5. The locus of points consists of two points on line ℓ each 3 centimeters away from P.



7. The locus of points consists of a semicircle centered at R with a radius of 10 centimeters. The diameter bordering the semicircle is 10 centimeters from line k and parallel to line k.

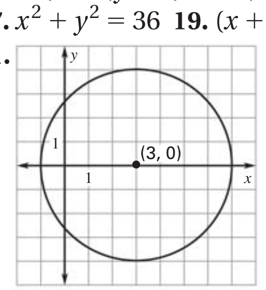


9.

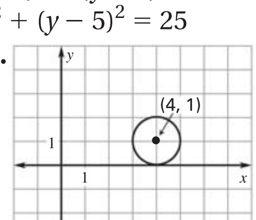


10.7 Skill Practice (pp. 702–703) **1.** center, radius **3.** $x^2 + y^2 = 4$ **5.** $x^2 + y^2 = 400$ **7.** $(x - 50)^2 + (y - 50)^2 = 100$ **9.** $x^2 + y^2 = 49$ **11.** $(x - 7)^2 + (y + 6)^2 = 64$ **13.** $(x - 3)^2 + (y + 5)^2 = 49$ **15.** If (h, k) is the center of a circle with a radius r, the equation of the circle should be $(x - h)^2 + (y - k)^2 = r^2$; $(x + 3)^2 + (y + 5)^2 = 9$. **17.** $x^2 + y^2 = 36$ **19.** $(x + 3)^2 + (y - 5)^2 = 25$

21.

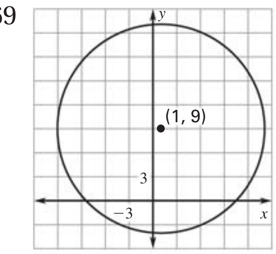


23.



27. circle; $x^2 + (y - 3)^2 = 4$ **29.** circle; $x^2 + (y + 2)^2 = 17$ **31.** secant **33.** secant

10.7 Problem Solving (pp. 703–705) **37.** $x^2 + y^2 = 5.76$, $x^2 + y^2 = 0.09$ **39.** $(x - 3)^2 + y^2 = 49$ **41.** The height (or width) always remains the same as the figure is rolled on its edge. **43. a.** $(1, 9)$, 13 **b.** $(x - 1)^2 + (y - 9)^2 = 169$



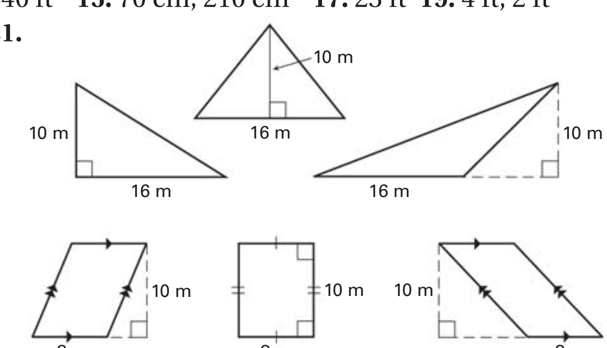
Chapter Review (pp. 708–711) **1.** diameter **3.** The measure of the central angle and the corresponding minor arc are the same. The measure of the major arc is 360° minus the measure of the minor arc. **5.** C **7.** 2 **9.** 12 **11.** 60° **13.** 80° **15.** 65° **17.** $c = 28$ **19.** $q = 100$, $r = 20$ **21.** 16 **23.** $10\frac{2}{3}$ ft **25.** $(x - 8)^2 + (y - 6)^2 = 36$ **27.** $x^2 + y^2 = 81$ **29.** $(x - 6)^2 + (y - 21)^2 = 16$ **31.** $(x - 10)^2 + (y - 7)^2 = 12.25$

Algebra Review (p. 713) **1.** $6x^2(3x^2 + 1)$ **3.** $3r(3r - 5s)$ **5.** $2t(4t^3 + 3t - 5)$ **7.** $y^3(5y^3 - 4y^2 + 2)$ **9.** $3x^2y(2x + 5y^2)$ **11.** $(y - 3)(y + 2)$ **13.** $(z - 4)^2$ **15.** $(5b - 1)(b - 3)$ **17.** $(5r - 9)(5r + 9)$ **19.** $(x + 3)(x + 7)$ **21.** $(y + 3)(y - 2)$ **23.** $(x - 7)(x + 7)$

Chapter 11

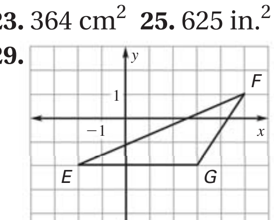
11.1 Skill Practice (pp. 723–724) **1.** bases, height **3.** 28 units^2 **5.** 225 units^2 **7.** 216 units^2 **9.** $A = 10(16) = 160$ units^2 or $A = 8(20) = 160$ units^2; the results are the same. **11.** 7 is not the base of the parallelogram; $A = bh = 3(4) = 12$ units^2. **13.** 30 ft, 240 ft^2 **15.** 70 cm, 210 cm^2 **17.** 23 ft **19.** 4 ft, 2 ft

21.



23. 364 cm^2 **25.** 625 in.^2 **27.** 52 in.^2

29. 7.5 units^2

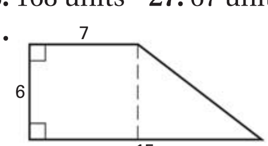


11.1 Problem Solving (pp. 725–726) **37.** 30 min; 86.4 min **39.** No; 2 inch square; the area of a square is side length squared, so $2^2 = 4$. **41.** 23 cm $\times$ 34 cm; 611 cm^2; 171 cm^2 **43.** Opposite pairs of sides are congruent making *XYZW* a parallelogram. The area of the parallelogram is *bh*, and since the parallelogram is made of two congruent triangles, the area of one triangle, $\triangle XYW$, is $\frac{1}{2}bh$. **45.** The base and the height are not necessarily side lengths of the parallelogram; yes; no; if the base and height represent a rectangle, then the perimeter is 20 ft^2, the greatest possible perimeter cannot be determined from the given data.

Extension (p. 728) **1.** Precision depends on the greatest possible error while accuracy depends on the relative error. *Sample answer:* Consider a target, if you are consistently hitting the same area, that is precision, if you hit the bull's eye, that is accuracy. **3.** 1 m; 0.5 m **5.** 0.0001 yd; 0.00005 yd **7.** about 1.8% **9.** about 0.04% **11.** This measurement is more accurate if you are measuring small items, if you are measuring large items, this would not be very accurate. **13.** 18.65 ft is more precise; 18.65 ft is more accurate. **15.** 3.5 ft is more precise; 35 in. is more accurate.

11.2 Skill Practice (pp. 733–734) **1.** height **3.** 95 units^2 **5.** 31 units^2 **7.** 1500 units^2 **9.** 189 units^2 **11.** 360 units^2 **13.** 13 is not the height of the trapezoid; $A = \frac{1}{2}(12)(14 + 19)$, $A = 198$ cm^2. **17.** 20 m **19.** 10.5 units^2 **21.** 10 units^2 **23.** 5 cm and 13 cm **25.** 168 units^2 **27.** 67 units^2 **29.** 42 units^2

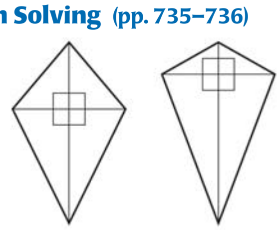
31.



38 units, 66 units^2

11.2 Problem Solving (pp. 735–736)

35. 20 mm^2;



37. a. right triangle and trapezoid **b.** 103,968 ft^2; 11,552 yd^2 **39.** If the kite in the activity were a rhombus, the results would be the same.

41. $A_{\triangle PSR} = \frac{1}{2}\left(\frac{1}{2}d_1\right)d_2$ and $A_{\triangle PQR} = \frac{1}{2}\left(\frac{1}{2}d_1\right)d_2$

$A_{\triangle PSR} = \frac{1}{4}d_1d_2$ and $A_{\triangle PQR} = \frac{1}{4}d_1d_2$

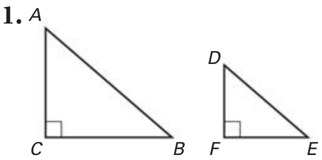
$A_{PQRS} = A_{\triangle PQR} + A_{\triangle PSR}$

$A_{PQRS} = \frac{1}{4}d_1d_2 + \frac{1}{4}d_1d_2$

$A_{PQRS} = \frac{1}{2}d_1d_2$

11.3 Skill Practice (pp. 740–741)

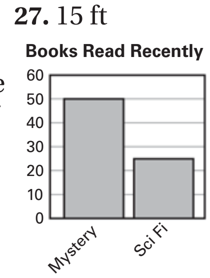
1.



$\triangle ABC \sim \triangle DEF$ tells you that the sides in the same position are proportional. *AB* is proportional to *DE* because the sides are both the hypotenuse of their respective triangle and are listed in the same order in the similarity statement. **3.** 6 : 11, 36 : 121 **5.** 1 : 3, 1 : 9; 18 ft^2 **7.** 7 : 9, 49 : 81; about 127 in.^{2} **9.** 7 : 4 **11.** 11 : 12 **13.** 8 cm **15.** The ratio of areas is 1 : 4, so the ratio of side lengths is 1 : 2; $ZY = 2(12) = 24$. **17.** 175 ft^2; 10 ft, 5.6 ft **19.** Sometimes; this is only true when the side length is 2. **21.** Sometimes; only when the octagons are also congruent will the perimeters be the same. **23.** AA Similarity Postulate; $\frac{10}{35} = \frac{2}{7}$ is the ratio of side lengths, so the ratio of areas is 4 : 49.

11.3 Problem Solving (pp. 742–743) **27.** 15 ft

31. There were twice as many mysteries read but the area of the mystery bar is 4 times the area of the science fiction bar giving the impression that 4 times as many mysteries were read.



33. a. $\triangle ACD \sim \triangle AEB$, $\triangle BCF \sim \triangle DEF$; AA Similarity Postulate **b.** *Sample answer:* 100 : 81 **c.** $\frac{10}{9} = \frac{20}{10 + x}$, $180 = 100 + 10x$, $x = 8$ OR $20(9) = (10 + x)(10)$, $180 = 100 + 10x$, $x = 8$

11.3 Problem Solving Workshop (p. 744) **1.** 18 in. **3.** $s\sqrt{2}$

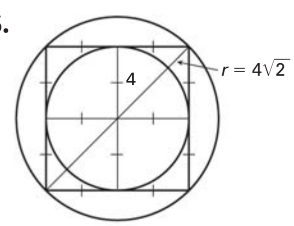
11.4 Skill Practice (pp. 749–751) **1.** arc length of $\widehat{AB}$, 360° **3.** about 37.70 in. **5.** about 10.03 ft **7.** 14 m **9.** about 31.42 units **11.** about 4.19 cm **13.** about 3.14 ft **15.** 300° **17.** 150° **19.** about 20.94 ft **21.** about 50° **23.** about 8.58 units **25.** about 21.42 units **27.** 6π **29.** $r = \frac{C}{2\pi}$; $d = \frac{C}{\pi}$; $r = 13$, $d = 26$ **31. a.** twice as large **b.** twice as large

11.4 Problem Solving (pp. 751–752) **35.** 21 feet 8 inches represents the circumference of the tree, so if you divide by π, you will get the diameter; about 7 ft. **37.** about 2186.55 in. **39.** 7.2°; 28,750 mi

Extension (p. 754) **1.** Equator and longitude lines; latitude lines; the equator and lines of longitude have the center of Earth as the center. Lines of latitude do not have the center of Earth as the center. **3.** If two lines intersect then their intersection is exactly 2 points. **5.** 4π

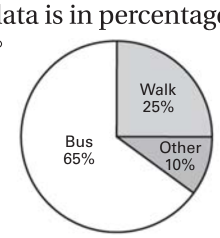
11.5 Skill Practice (pp. 758–759) **1.** sector **3.** 25π in.^{2}; 78.54 in.^{2} **5.** 132.25π cm^2; 415.48 cm^2 **7.** about 7 m **9.** 52 cm **11.** about 52.36 in.^{2} **13.** about 937.31 m^2 **15.** about 66.04 cm^2 **17.** about 7.73 m^2 **21.** about 57.23 in. **23.** about 66.24 in. **25.** about 27.44 in. **27.** about 33.51 ft^2 **29.** about 1361.88 cm^2 **31.** about 7.63 m **33.** For any two circles the ratio of their circumferences is equal to the ratio of their corresponding radii; for any two circles, if the length of their radii is in the ratio of $a:b$, then the ratio of their areas is $a^2:b^2$; all circles are similar, so you do not need to include similarity in the hypothesis.

35.



2 : 1

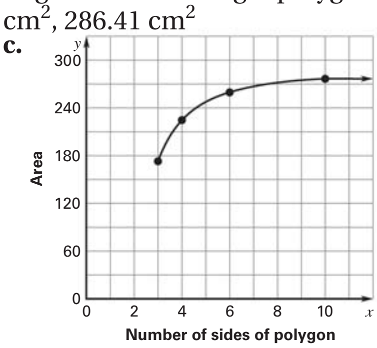
11.5 Problem Solving (pp. 760–761)
37. about 314.16 mi^2 **39. a.** The data is in percentages. **b.** bus: 234°, walk: 90°, other: 36°



c. bus: $\frac{13}{20}\pi r^2$, walk: $\frac{1}{4}\pi r^2$, other: $\frac{1}{10}\pi r^2$ **41. a.** old: about 370.53 mm, new: 681.88 mm; about 84%

11.6 Skill Practice (pp. 765–766) **1.** F **3.** 6.8 **5.** Divide 360° by the number of sides of the polygon. **7.** 20° **9.** 51.4° **11.** 22.5° **13.** 135° **15.** about 289.24 units^2 **17.** 7.5 is not the measure of a side length, it is the measure of the base of the triangle, it needs to be doubled to become the measure of the side length; $A = \frac{1}{2}a \cdot ns$, $A = \frac{1}{2}(13)(6)(15) = 585$ units^2. **19.** about 122.5 units, about 1131.8 units^2 **21.** 63 units, about 294.3 units^2 **23.** apothem, side length; special right triangles or trigonometry; about 392 units^2 **25.** side length; Pythagorean Theorem or trigonometry; about 204.9 units^2 **27.** about 79.6 units^2 **29.** about 1.4 units^2 **31.** True; since the radius is the same, the circle around the n-gons is the same but more and more of the circle is covered as the value of n increases. **33.** False; the radius can be equal to the side length as it is in a hexagon.

11.6 Problem Solving (pp. 767–768) **37.** 1.2 cm, about 4.8 cm^2; about 1.6 cm^2 **39.** 15.5 in.^{2}; 25.8 in.^{2} **41.** $\frac{360}{6} = 60$, so the central angle is 60°. All of the triangles are of the same side length, r, and therefore all six triangles have a vertex on the center with central angle 60° and side lengths r.
43. Because P is both the incenter and circumcenter of $\triangle ABC$ and letting E be the midpoint of $\overline{AB}$, you can show that $\overline{BD}$ and $\overline{CE}$ are both medians of $\triangle ABC$ and they intersect at P. By the Concurrency of Medians of a Triangle Theorem, $BP = \frac{2}{3}BD$ and $CP = \frac{2}{3}CE$. Using algebra, show that $2PD = CP$.
45. a. About 141.4 cm^2; square: about 225 cm^2, pentagon: about 247.7 cm^2, hexagon: about 259.9 cm^2, decagon: about 277 cm^2; the area is getting larger with each larger polygon. **b.** about 286.22 cm^2, 286.41 cm^2
c.



circle; about 286.5 cm^2

11.7 Skill Practice (pp. 774–775) **1.** 0, 1 **3.** $\frac{5}{8}$, 0.625, 62.5% **5.** $\frac{3}{8}$, 0.375, 37.5% **7.** $AD + DE = AE$, so $\frac{5}{8} + \frac{3}{8} = 1$ **9.** $\frac{1}{4}$ or 25% **11.** There is more than a semicircle in the rectangle, so you need to take the area of the rectangle minus the sum of the area of the semicircle and the area of a small rectangle located under the semicircle that has dimensions of 10×2;

$$\frac{10(7) - \left(\frac{1}{2}\pi(5)^2 + 10(2)\right)}{7(10)} = \frac{70 - (12.5\pi + 20)}{70} \approx 0.153 \text{ or}$$

about 15.3%. **13.** $\frac{43}{90}$ or about 47.8% **15.** The two triangles are similar by the AA Similarity Postulate and the ratio of sides is the same; $7:14$ or $1:2$, so the ratio of the areas is $1:4$. **17.** $\frac{2}{7}$ **19.** 1 **21.** $\frac{1}{9}$ or 11.1%; find the area of the whole figure, $\frac{1}{2}(14)(12) = 84$ which is the denominator of the fraction. The top triangle is similar to the whole figure by the AA Similarity Postulate, so use proportions to find the base of the small triangle to be $4\frac{2}{3}$. Since the height

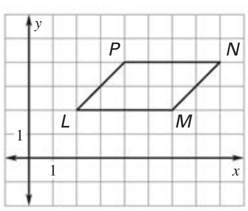
SELECTED ANSWERS

of the small triangle is 4, the area is $9\frac{1}{3}$, which is the numerator of the fraction. **25.** about 82.7% **27.** 100%, 50%

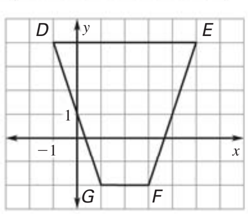
11.7 Problem Solving (pp. 776–777) **31. a.** $\frac{2}{5}$ or 40% **b.** $\frac{3}{5}$ or 60% **33.** $\frac{1}{6}$ or about 16.7%

35. The probability stays the same; the sector takes up the same percent of the area of the circle regardless of the length of the radius. *Sample answer:* Let the central angle be 90° and the radius be 2 units. The probability for that sector is $\frac{\frac{4\pi}{4}}{4\pi} = \frac{1}{4}$. Let the radius be doubled. The probability is $\frac{\frac{16\pi}{4}}{16\pi} = \frac{1}{4}$. **37. a.** $\frac{1}{81}$ or 1.2% **b.** about 2.4% **c.** about 45.4%

Chapter Review (pp. 780–783) **1.** two radii of a circle **3.** XZ **5.** 60 units^2 **7.** 448 units^2

9. 

8 units^2

11. 

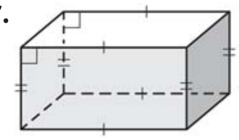
24 units^2

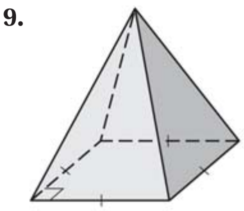
13. 10 : 13, 100 : 169, 152.1 cm^2 **15.** about 30 ft **17.** about 26.09 units **19.** about 17.72 in.^{2} **21.** about 39.76 in., about 119.29 in.^{2} **23.** $\frac{4}{7}$ **25.** about 76.09%

Algebra Review (p. 785) **1.** $d = \left(\frac{14.25}{1.5}\right)(2)$; 19 mi
3. $29.50 + 0.25m = 32.75$; 13 min
5. $18000(1 - 0.1)^5 = A$; \$10,628.82
7. $0 = -16t^2 + 47t + 6$; about 3.06 sec

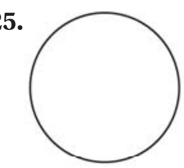
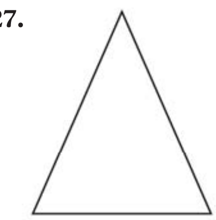
Chapter 12

12.1 Skill Practice (pp. 798–799) **1.** tetrahedron, 4 faces; hexahedron or cube, 6 faces; octahedron, 8 faces; dodecahedron, 12 faces; icosahedron, 20 faces
3. Polyhedron; pentagonal pyramid; the solid is formed by polygons and the base is a pentagon.
5. Not a polyhedron; the solid is not formed by polygons.

7. 

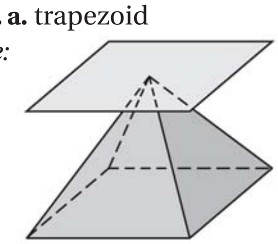
9. 

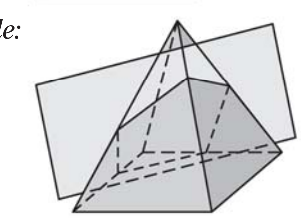
11. 8 **13.** 24 **15.** 4, 4, 6 **17.** 5, 6, 9 **19.** 8, 12, 18 **21.** A cube has six faces, and "hexa" means six. **23.** convex

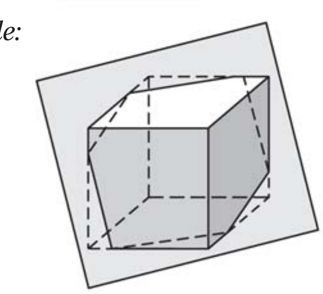
25.  circle **27.**  triangle

29. The concepts of edge and vertex are confused; the number of vertices is 4, and the number of edges is 6.

12.1 Problem Solving (pp. 800–801) **35.** 18, 12 **37.** square **39.** Tetrahedron; no; you cannot have a different number of faces because of Euler's Theorem. **41. a.** trapezoid

b. Yes. *Sample:*  **c.** square

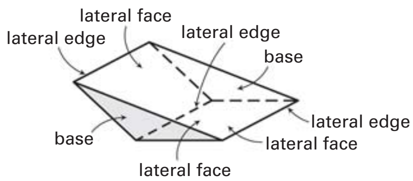
d. Yes. *Sample:*  **43.** no **45.** no

47. Yes. *Sample:* 

49. a. It will increase the number of faces by 1, the number of vertices by 2, and the number of edges by 3. **b.** It will increase the number of faces by 1, the number of vertices by 2 and the number of edges by 3. **c.** It will not change the number of faces, vertices, or edges. **d.** It will increase the number of faces by 3, the number of vertices by 6, and the number of edges by 9.

12.2 Skill Practice (pp. 806–808)

1.

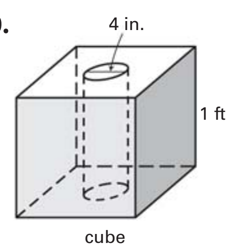


3. 150.80 in.^{2} **5.** 27,513.6 ft^2 **7.** 196.47 m^2 **9.** 14.07 in.^{2} **11.** 804.25 in.^{2} **13.** 9 yd **15.** 10.96 in. **19.** 1119.62 in.^{2}

12.2 Problem Solving (pp. 808–809)

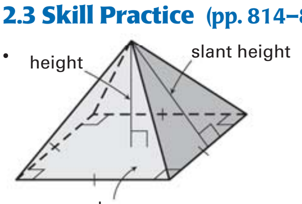
23. a. 360 in.^{2} **b.** There is overlap in some of the sides of the box. **c.** *Sample answer:* It is easier to wrap a present if you have some overlap of wrapping paper. **27. a.** 54 units^2 **b.** 52 units^2 **c.** When the red cubes are removed, inner faces of the cubes remaining replace the area of the red cubes that are lost. When the blue cubes are removed, there are still 2 faces of the blue cubes whose area is not replaced by inner faces of the remaining cubes. Therefore, the area of the solid after removing blue cubes is 2 units^2 less than the solid after removing red cubes.

29. 989.66 in.^{2}



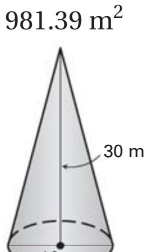
12.3 Skill Practice (pp. 814–815)

1.

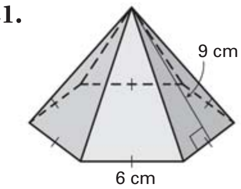


3. 40 cm^2 **5.** 580 ft^2 **7.** 672.5 mm^2

9. The height of the pyramid is used rather than the slant height; $S = 6^2 + \frac{1}{2}(24)(5) = 96$ ft^2. **11.** 12.95 in.^{2} **13.** 238.76 in.^{2} **15.** 226.73 ft^2 **19.** 981.39 m^2



21. 255.53 cm^2



23. 164.05 in.^{2} **25.** 27.71 cm^2

12.3 Problem Solving (pp. 816–817)

27. 96 in.^{2} **29.** square pyramid; 98.35 cm^2 **31. a.** Given: $\overline{AB} \perp \overline{AC}$; $\overline{DE} \perp \overline{DC}$

Prove: $\triangle ABC \sim \triangle DEC$

Statements	Reasons
1. $\overline{AB} \perp \overline{AC}$; $\overline{DE} \perp \overline{DC}$	1. Given
2. $\angle BAC$ and $\angle EDC$ are right angles.	2. Definition of perpendicular
3. $\angle BAC \cong \angle EDC$	3. Right angles are congruent.
4. $\overleftrightarrow{AB} \parallel \overleftrightarrow{DE}$	4. If two lines are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.
5. $\angle ABC \cong \angle DEC$	5. Corresponding Angles Postulate
6. $\triangle ABC \sim \triangle DEC$	6. AA Similarity Postulate

b. $5, \frac{3}{2}, \frac{5}{2}$ **c.** larger cone: 24π units^2, smaller cone: 6π units^2; the small cone has 25% of the surface area of the large cone. **33.** about 24.69 mi^2

12.4 Skill Practice (pp. 822–824)

1. cubic units **5.** 18 units^3 **7.** 175 in.^{3} **9.** 2630.55 cm^3 **11.** 314.16 in.^{3} **13.** The radius should be squared; $V = \pi r^2 h = \pi(4^2)(3) = 48\pi$ ft^3. **15.** 10 in. **17.** 8 in. **19.** 821.88 ft^3 **23.** 12.65 cm **25.** 2814.87 ft^3

12.4 Problem Solving (pp. 824–825)

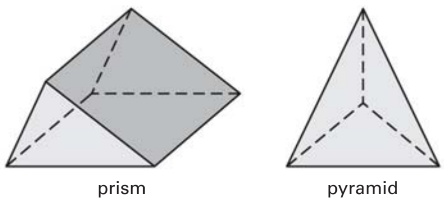
29. a. 720 in.^{3} **b.** 720 in.^{3} **c.** They are the same. **31.** 159.15 ft^3 **33. a.** 4500 in.^{3} **b.** 150 in.^{3} **c.** 10 rocks

12.4 Problem Solving Workshop (p. 827)

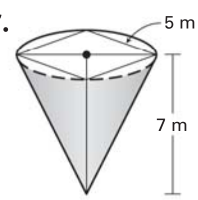
1. a. about 56.55 in.^{3} **b.** about 56.55 in.^{3} **3.** $r = \frac{R\sqrt{2}}{2}$ **5.** about 7.33 in.^{3}

12.5 Skill Practice (pp. 832–833)

1. A triangular prism is a solid with two bases that are triangles and parallelograms for the lateral faces while a triangular pyramid is a solid with a triangle for a base and triangles for lateral faces.

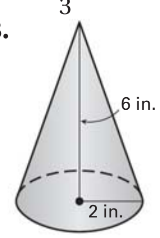


3. 50 cm^3 **5.** 13.33 $in.^3$ **7.** 6 $in.^3$ **9.** The slant height is used in the volume formula instead of the height; $V = \frac{1}{3}\pi(9^2)(12) = 324\pi \approx 1018\ ft^3$. **13.** 6 in. **15.** 3716.85 ft^3 **17.** 987.86 cm^3 **19.** 8.57cm **21.** 833.33 $in.^3$ **23.** 16.70 cm^3 **25.** 26.39 yd^3

27. 

about 91.63 m^3

12.5 Problem Solving (pp. 834–836) **29. a.** 201 $in.^3$ **b.** 13.4 $in.^3$ **31.** 3; since the cone and cylinder have the same radius and height, the volume of the cone will be $\frac{1}{3}$ the volume of the cylinder.

33. 

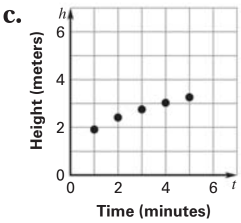
23.70 $in.^3$

35. a. The volume doubles. **b.** The volume is multiplied by 4. **c.** If you replace the height h by $2h$ in the volume formula, it will multiply the volume by 2. If you replace the side length s by $2s$ in the volume formula, it will multiply the volume by 4 because $(2s)^2 = 4s^2$. **37.** about 77.99 $in.^3$

39. a. $V_{\text{cone}} = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 \cdot h = \frac{\pi\left(\frac{1}{2}h\right)^2 \cdot h}{3} = \frac{\pi h^3}{12}$, where B is the area of the base of the cone, r is the radius, and h is the height

b.

Time (min)	Height h (m)
1	1.90
2	2.40
3	2.74
4	3.02
5	3.25

c. 

No; the points of the graph do not lie in a straight line.

41. a. $h_1 = \frac{r_1 h_2}{r_2 - r_1}$ **b.** $V = \frac{\pi r_2^2(h_1 + h_2)}{3} - \frac{\pi r_1^2 h_1}{3} = \frac{\pi r_2^2(h_1 + h_2)}{3} - \frac{\pi r_1^2 h_1}{3}$

12.6 Skill Practice (pp. 842–843) **1.** $S = 4\pi r^2$, $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere **3.** 201.06 ft^2 **5.** 1052.09 m^2 **7.** 4.8 in. **9.** about 144.76 $in.^2$ **11.** about 7359.37 cm^2 **13.** 268,082.57 mm^3 **15.** The radius should be cubed; $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(8)^3 = 682.67\pi \approx 2144.66\ ft^3$. **17.** 2.80 cm **19.** 6 ft **21.** 247.78 $in.^2$, 164.22 $in.^3$ **23.** 358.97 cm^2, 563.21 cm^3 **25.** 13 in.; $676\pi\ in.^2$; $\frac{8788}{3}\pi\ in.^3$ **27.** 21 m; 42π m; $1764\pi\ m^2$

12.6 Problem Solving (pp. 844–845) **31.** about 98,321,312 mi^2 **33. a.** 8.65 $in.^3$ **b.** 29.47 $in.^3$ **35. a.** about 80,925,856 mi^2, about 197,359,487 mi^2 **b.** about 41% **37.** $324\pi\ in.^2$, $972\pi\ in.^3$

12.7 Skill Practice (pp. 850–852) **1.** They are the same type of solid and corresponding linear measures have the same ratio. **3.** Not similar; the corresponding dimensions are not in the same ratio. **5.** Similar; each corresponding ratio is 3 : 4. **9.** about 166.67 m^2, about 127.21 m^3 **11.** The volumes are related by the third power; $\frac{500\pi}{\text{Volume of B}} = \frac{1^3}{4^3}$. **13.** 1 : 3 **15.** 4 : 3 **17.** 1 : 4 **19.** about 341.94 ft^2, about 502.65 ft^3 **21.** about 370.96 $in.^2$, about 73.58 $in.^3$ **23.** $r = 3$ ft, $h = 6$ ft; $r = 8$ ft; $h = 16$ ft

12.7 Problem Solving (pp. 852–853) **25.** about 8.04 fl oz **27.** 27 fl oz **29. a.** large orange: about 33.51 $in.^3$, small orange: about 17.16 $in.^3$ **b.** The ratio of the volumes is the cube of the ratio of diameters. **c.** large orange: 3.75 in., small orange: 2.95 in. **d.** The ratio of surface area multiplied by the ratio of the corresponding diameters equals the ratio of the volumes. **31. a.** 144 in. **b.** 3920.4 $in.^2$ **c.** 1.5 $in.^3$ **33.** About 11.5 kg; the ratio of the small snowball to the medium snowball is 5 : 7, so the ratio of their volumes is $5^3 : 7^3$. Solve $\frac{5^3}{7^3} = \frac{1.2}{x}$ to find the weight of the middle ball. Similarly, find the weight of the large ball.

Chapter Review (pp. 857–860) **1.** sphere **3.** 12 **5.** 36 **7.** 791.68 ft^2 **9.** 9 m **11.** 14.29 cm **13.** 11.34 m^3 **15.** 27.53 yd^3 **17.** 12 in.^2 **19.** 272.55 m^3 **21.** $1008\pi\ \text{m}^2$; $4320\pi\ \text{m}^3$

Cumulative Review (pp. 866–867) **1.** 75 **3.** 16 **5.** 4 **7.** Both pairs of opposite angles are congruent. **9.** The diagonals bisect each other. **11.** 45 **13.** about 36.35 in.^2 **15.** about 2.28 m^2 **17.** 131.05 in.^2, 80.67 in.^3 **19.** (4, 2) **21. a.** $(x+2)^2 + (y-4)^2 \leq 36$ **b.** (2, 0): yes, because it is a solution to the inequality; (3, 9): no, because it is not a solution to the inequality; (−6, −1): no, because it is not a solution to the inequality; (−6, 8): yes, because it is a solution to the inequality; (−7, 5): yes, because it is a solution to the inequality. **23. a.** 70.69 in.^2, 42.41 in.^3 **b.** about 25.45 in.^3

Skills Review Handbook

Operations with Rational Numbers (p. 869) **1.** 11 **3.** −15 **5.** −24 **7.** 0.3 **9.** 11.6 **11.** −4.9 **13.** −13.02 **15.** 29.2 **17.** $-\frac{13}{12}$ **19.** $\frac{6}{7}$ **21.** $-\frac{11}{12}$ **23.** $\frac{17}{18}$

Simplifying and Evaluating Expressions (p. 870) **1.** 33 **3.** −1 **5.** 36 **7.** 2.8 **9.** −6 **11.** $25x$ **13.** −36 **15.** −15 **17.** 15 **19.** 1 **21.** $-\frac{6}{5}$ **23.** $\frac{3}{4}$

Properties of Exponents (p. 871) **1.** 25 **3.** $\frac{1}{16}$ **5.** 78,125 **7.** 7^{32} **9.** a^4 **11.** $\frac{5a^5}{b^4}$ **13.** $\frac{81}{n^4}$ **15.** m^2 **17.** $16x^6y^2$ **19.** $\frac{b^2}{5a^3c}$ **21.** $8x$ **23.** $\frac{a^5}{7b^4c}$ **25.** $30x^3y$ **27.** $\frac{3a^{14}}{5b^2c^8}$

Using the Distributive Property (p. 872) **1.** $3x + 21$ **3.** $40n - 16$ **5.** $-x - 6$ **7.** $12x^2 - 8x + 16$ **9.** $-5x^2$ **11.** $2n + 5$ **13.** $5h^3 + 5h^2$ **15.** 10 **17.** $\frac{9}{10}a$ **19.** $3n + 4$ **21.** $2a^2 + 6a - 76$ **23.** $3x^2 - 10x + 5$ **25.** $4a^2 + 2ab - 1$

Binomial Products (p. 873) **1.** $a^2 - 11a + 18$ **3.** $t^2 + 3t - 40$ **5.** $25a^2 + 20a + 4$ **7.** $4c^2 + 13c - 12$ **9.** $z^2 - 16z + 64$ **11.** $2x^2 + 3x + 1$ **13.** $4x^2 - 9$ **15.** $6d^2 + d - 2$ **17.** $k^2 - 2.4k + 1.44$ **19.** $-z^2 + 36$ **21.** $5y^2 + 9y - 32$ **23.** $3x^2 - 17$

Radical Expressions (p. 874) **1.** ± 10 **3.** $\pm\frac{1}{2}$ **5.** no square roots **7.** ± 0.9 **9.** 11 **11.** $-3\sqrt{11}$ **13.** $2\sqrt{5}$ **15.** $3\sqrt{7}$ **17.** $4\sqrt{5}$ **19.** $210\sqrt{2}$ **21.** 137 **23.** 30 **25.** 8 **27.** $2\sqrt{6}$

Solving Linear Equations (p. 875) **1.** 31 **3.** −6 **5.** 39 **7.** 23.2 **9.** 18 **11.** 1 **13.** $\frac{7}{2}$ **15.** −1 **17.** 20 **19.** 16 **21.** −1 **23.** 7 **25.** 6.75 **27.** −0.82 **29.** −4 **31.** $\frac{5}{2}$ **33.** $-\frac{2}{5}$ **35.** $\frac{1}{2}$

Solving and Graphing Linear Inequalities (p. 876)

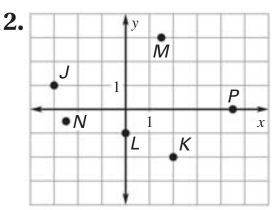
1. $x < 7$

3. $n \leq 4$

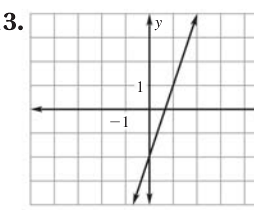
Solving Formulas (p. 877) **1.** $s = \frac{P}{4}$ **3.** $\ell = \frac{V}{wh}$ **5.** $b = \frac{2A}{h}$ **7.** $w = \frac{P}{2} - \ell$ **9.** $C = \frac{5}{9}(F - 32)$ **11.** $h = \frac{S - 2\pi r^2}{2\pi r}$ **13.** $y = -2x + 7$ **15.** $y = 3x + 2$ **17.** $y = \frac{5}{4}x$ **19.** $y = 62 - 15$

Graphing Points and Lines (p. 878) **1.** (3, 1) **3.** (0, 2) **5.** (3, −3)

7–12.



13.



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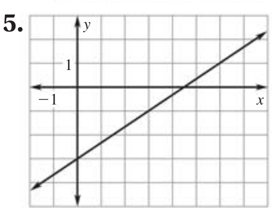
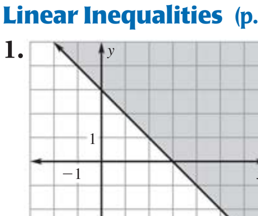


Slope and Intercepts of a Line (p. 879) **1.** $\frac{3}{2}$, x-intercept −2, y-intercept 3 **3.** 0, no x-intercept, y-intercept −2 **5.** x-intercept 3, y-intercept −15 **7.** x-intercept 3, y-intercept 3 **9.** x-intercept 2, y-intercept −6 **11.** x-intercept 0, y-intercept 0

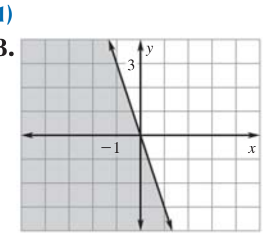
Systems of Linear Equations (p. 880) **1.** (2, 1) **3.** (4, −1) **5.** (6, −3) **7.** (−1, −4) **9.** (3, 2) **11.** (−1, −5) **13.** (−5, 1) **15.** (0.5, −2)

Linear Inequalities (p. 881)

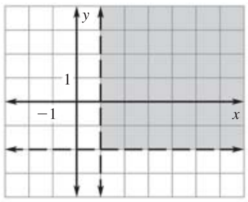
1.

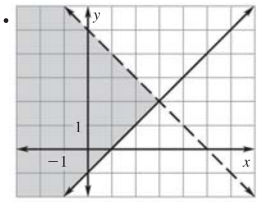


3.

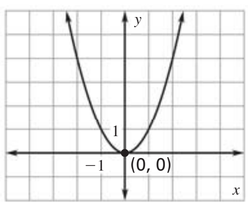


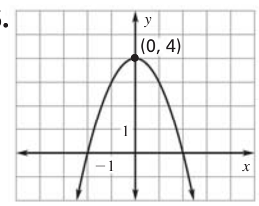
SELECTED ANSWERS

9. 

11. 

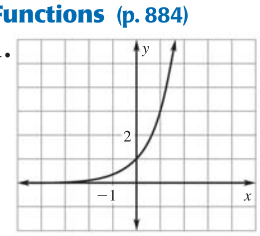
Quadratic Equations and Functions (p. 883) **1.** ± 12 **3.** -3 **5.** 0 **7.** -1 **9.** no real solutions **11.** $\pm\frac{\sqrt{5}}{3}$

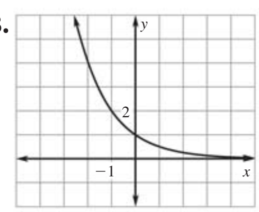
13. 

15. 

25. $-5, -1$ **27.** -3 **29.** $\frac{-7 \pm \sqrt{33}}{2}$ **31.** -2 **33.** $\frac{1 \pm \sqrt{31}}{5}$ **35.** no real solutions **37.** $\frac{1 \pm \sqrt{61}}{6}$ **39.** $\pm\sqrt{5}$

Functions (p. 884)

1. 

3. 

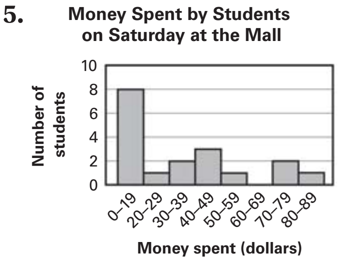
9. $y = x^2$ **11.** $y = 12x$; $72; 35 h

Problem Solving with Percents (p. 885) **1.** 24 questions **3.** yes **5.** 20% **7.** 500 residents **9.** about 50%

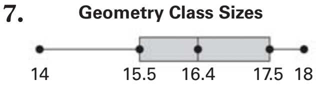
Converting Measurements and Rates (p. 886) **1.** 5 **3.** 3 **5.** 3.2 **7.** 160 **9.** 63,360 **11.** 576 **13.** 3,000,000 **15.** 6.5 **17.** 1020 **19.** 5104 **21.** 5280 **23.** 90,000,000

Mean, Median, and Mode (p. 887) **1.** The mean or the median best represent the given data because all of the values are close to these measures. **3.** The median or the mode best represent the data because all of the values are close to these measures. **5.** The median best represents the data because all of the values are close to this measure. **7.** The mean best represents the data because all of the values are close to this measure.

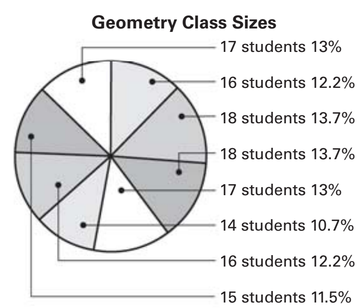
Displaying Data (p. 889) **1.** Line graph; this type of graph shows change over time and this is what the storeowner wants to evaluate. **3.** Histogram; this displays data in intervals.

5. 

The data is put into intervals; $0–$19.

7. 

The box-and-whisker plot shows how the class sizes relate to each other.



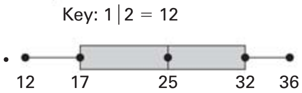
The circle graph shows how each class size contributes to the total number of students enrolled in Geometry.

9.

Stem	Leaves
1	2 3 5 5 6 8 8 9
2	4 5 5 6
3	0 0 2 2 3 5 5 6 7
4	
5	
6	
7	2

Key: 1 | 2 = 12

12, 72, 25.5, 18, 33

11. 

The data is more closely related to the mean and median in the new box-and-whisker plot than before dropping the two highest ages.

Sampling and Surveys (p. 890) **1.** Biased sample; the sample is unlikely to represent the entire population of students because only students at a soccer game are asked which day they prefer. **3.** Biased sample; the sample is biased because only people with e-mail can respond. **5.** The sample and the question are random.

Counting Methods (p. 892) **1.** 15 outfits **3.** 1,679,616 passwords **5.** 125,000 combinations **7.** 756 combinations **9.** 24 ways

Probability (p. 893) **1.** dependent; $\frac{33}{95} \approx 0.347$ or about 34.7% **3.** dependent; $\frac{1}{20} = 0.05$ or 5% **5.** dependent; $\frac{1}{8} = 0.125$ or 12.5%

Problem Solving Plan and Strategies (p. 895) **1.** $205 **3.** 4 **5.** 14 aspen and 7 birch, 16 aspen and 8 birch, or 18 aspen and 9 birch **7.** 24 pieces

Extra Practice

Chapter 1 (pp. 896–897) **1.** *Sample answer:* A, F, B; $\overleftrightarrow{AB}$ **3.** *Sample answer:* $\overrightarrow{FA}$, $\overrightarrow{FB}$ **5.** *Sample answer:* $\overleftrightarrow{AB}$ **7.** 43 **9.** 26 **11.** 28 **13.** $(3x - 7) + (3x - 1) = 16$; $x = 4$; $AB = 5$, $BC = 11$; not congruent **15.** $(4x - 5) + (2x - 7) = 54$; $x = 11$; $AB = 39$, $BC = 15$; not congruent **17.** $(3x - 7) + (2x + 5) = 108$; $x = 22$; $AB = 59$, $BC = 49$; not congruent **19.** $\left(-4\frac{1}{2}, 1\right)$ **21.** (1, 1) **23.** (5.1, −8.05) **25.** 10 **27.** 34 **29.** 20 **31.** 104° **33.** 88° **35.** adjacent angles **37.** vertical angles, supplementary **39.** *Sample answer:* $\angle ACE$, $\angle BCF$ **41.** polygon; concave **43.** Not a polygon; part of the figure is not a line segment. **45.** *DFHKB*, pentagon; *ABCDEFGHJK*, decagon **47.** 13 cm **49.** 11 m **51.** about 13.4 units, 4 units^2

Chapter 2 (pp. 898–899) **1.** Add 6 for the next number, then subtract 8 for the next number; 11. **3.** no pattern **5.** Each number is $\frac{1}{3}$ of the previous number; $\frac{1}{81}$. **7.** *Sample answer:* $-8 - (-5) = -3$ **9.** *Sample answer:* $m\angle A = 90°$ **11.** If-then form: if a figure is a square, then it is a four-sided regular polygon; Converse: if a figure is a four-sided regular polygon, then it is a square; Inverse: if a figure is not a square, then it is not a four-sided regular polygon; Contrapositive: if a figure is not a four-sided regular polygon, then it is not a square. **13.** true **15.** If two coplanar lines are not parallel, then they form congruent vertical angles. **17.** might **19.** true **21.** false **23.** true

25.

$4x + 15 = 39$	Write original equation.
$4x = 24$	Subtraction Property of Equality
$x = 6$	Division Property of Equality

27.

$2(-7x + 3) = -50$	Write original equation.
$-14x + 6 = -50$	Distributive Property
$-14x = -56$	Subtraction Property of Equality
$x = 4$	Division Property of Equality

29.

$13(2x - 3) - 20x = 3$	Write original equation.
$26x - 39 - 20x = 3$	Distributive Property
$6x - 39 = 3$	Simplify.
$6x = 42$	Addition Property of Equality
$x = 7$	Division Property of Equality

31. $m\angle JKL$, $m\angle ABC$; Transitive Property of Equality **33.** $m\angle XYZ$; Reflexive Property of Equality

21.

Statements	Reasons
1. $\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$	1. Given
2. $XY = YZ = ZX$	2. Definition of congruence for segments
3. Perimeter of $\triangle XYZ = XY + YZ + ZX$	3. Perimeter formula
4. Perimeter of $\triangle XYZ = XY + XY + XY$	4. Substitution
5. Perimeter of $\triangle XYZ = 3 \cdot XY$	5. Simplify.

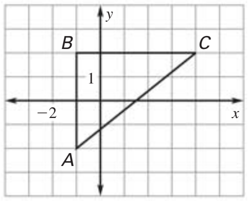
37. 23° **39.** 90°

41.

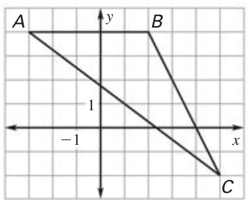
Statements	Reasons
1. $\angle UKV$ and $\angle VKW$ are complements.	1. Given
2. $m\angle UKV + m\angle VKW = 90°$	2. Definition of complementary angles
3. $\angle UKV \cong \angle XKY$, $\angle VKW \cong \angle YKZ$	3. Vertical angles are congruent.
4. $m\angle UKV = m\angle XKY$, $m\angle VKW = m\angle YKZ$	4. Definition of angle congruence
5. $m\angle YKZ + m\angle XKY = 90°$	5. Substitution
6. $\angle YKZ$ and $\angle XKY$ are complements.	6. Definition of complementary angles

Chapter 3 (pp. 900–901) **1.** corresponding **3.** consecutive interior **5.** corresponding **7.** $\angle HLM$ and $\angle MJC$ **9.** $\angle FKL$ and $\angle AML$ **11.** $\overleftrightarrow{BG}$ and $\overleftrightarrow{CF}$ **13.** 68°, 112°; $m\angle 1 = 68°$ because if two parallel lines are cut by a transversal, then the alternate interior angles are congruent, $m\angle 2 = 112°$ because it is a linear pair with $\angle 1$. **15.** 9, 1 **17.** 25, 19 **19.** Yes; if two lines are cut by a transversal so that a pair of consecutive interior angles are supplementary, then the lines are parallel. **21.** Yes; if two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel. **23.** Yes; if two lines are cut by a transversal so that a pair of consecutive interior angles are supplementary, then the lines are parallel. **25.** Neither; the slopes are not equal and they are not opposite reciprocals. **27.** Line 2 **29.** Line 1 **31.** $y = \frac{2}{3}x + 2$ **33.** $y = -2x$ **35.** $y = x + 10$ **37.** $y = \frac{2}{5}x + \frac{38}{5}$ **39.** 69° **41.** 73° **43.** 38° **45.** 1. Given; 2. $\angle ABC$ is a right angle.; 3. Definition of right angle; 4. $\overrightarrow{BD}$ bisects $\angle ABC$.; 5. Definition of angle bisector; 6. $m\angle ABD$, $m\angle DBC$; 7. Substitution Property of Equality; 8. $m\angle ABD$; 9. Simplify; 10. Division Property of Equality

Chapter 4 (pp. 902–903)

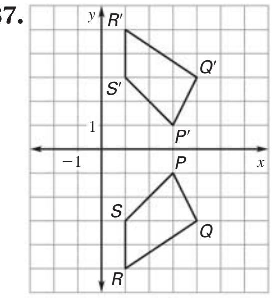
1. 

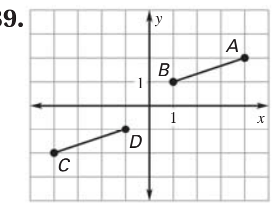
scalene; right triangle

3. 

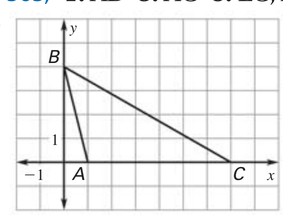
scalene; not a right triangle

5. 58; acute **7.** $\triangle DFG \cong \triangle FDE$; SAS Congruence Postulate or ASA Congruence Postulate
9. $STWX \cong UTWV$; all pairs of corresponding angles and sides are congruent. **11.** 7 **13.** No; a true congruence statement would be $\triangle JKM \cong \triangle LKM$.
15. congruent **17.** $\triangle XUV \cong \triangle VWX$; since $\overline{XV} \cong \overline{XV}$, with the givens you can use the HL Congruence Theorem. **19.** $\triangle HJL \cong \triangle KLJ$; use alternate interior angles to get $\angle HJL \cong \angle JLK$. Since $\overline{JL} \cong \overline{JL}$, with the given you can use the SAS Congruence Postulate.
21. yes; AAS Congruence Theorem **23.** Yes; use the ASA Congruence Postulate. **25.** State the givens from the diagram, and state that $\overline{AC} \cong \overline{AC}$ by the Reflexive Property of Congruence. Then use the SAS Congruence Postulate to prove $\triangle ABC \cong \triangle CDA$, and state $\angle 1 \cong \angle 2$ because corresponding parts of congruent triangles are congruent.
27. State the givens from the diagram and state that $\overline{SR} \cong \overline{SR}$ by the Reflexive Property of Congruence. Then use the Segment Addition Postulate to show that $\overline{PR} \cong \overline{US}$. Use the SAS Congruence Postulate to prove $\triangle QPR \cong \triangle TUS$, and state $\angle 1 \cong \angle 2$ because corresponding parts of congruent triangles are congruent. **29.** $AB = DE = \sqrt{26}$; $AC = DF = \sqrt{41}$; $BC = EF = \sqrt{17}$; $\triangle ABC \cong \triangle DEF$ by the SSS Congruence Postulate, and $\angle A \cong \angle D$ because corresponding parts of congruent triangles are congruent. **31.** $x = 6$, $y = 48$ **33.** $x = 2$
35. $x = 28$, $y = 29$

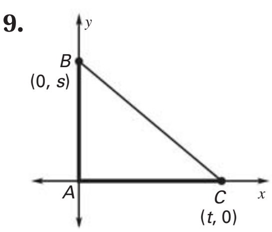
37. 

39. 

yes; 180°

Chapter 5 (pp. 904–905) **1.** $\overline{AB}$ **3.** $\overline{AC}$ **5.** LC, AL
7. *Sample answer:* 

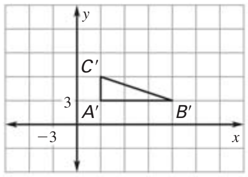
$A(1, 0)$, $B(0, 4)$, $C(7, 0)$

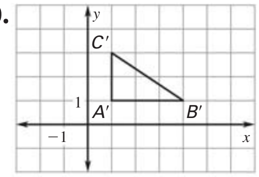
9. 

$A(0, 0)$, $B(0, s)$, $C(t, 0)$

11. 14 **13.** 12 **15.** 24 **17.** yes **19.** 15 **21.** No; there is not enough information. **23.** Yes; $x = 17$ by the Angle Bisector Theorem. **25.** 17 **27.** 8 **29.** angle bisector **31.** perpendicular bisector **33.** perpendicular bisector and angle bisector **35.** $\overline{JK}$, $\overline{LK}$, $\overline{JL}$, $\angle L$, $\angle J$, $\angle K$
37. 1 in. $< \ell <$ 17 in. **39.** 6 in. $< \ell <$ 12 in.
41. 2 ft $< \ell <$ 10 ft **43.** $>$ **45.** $>$ **47.** $=$ **49.** $>$ **51.** $<$

Chapter 6 (pp. 906–907) **1.** 20°, 60°, 100° **3.** 36°, 54°, 90°
5. 4 **7.** 10 **9.** −10 **11.** 10 **13.** 6 **15.** 12 **17.** $\frac{y}{9}$ **19.** 4
21. similar; $RQPN \sim STUV$, 11 : 20 **23.** 3 : 1
25. $\triangle PQR$: 90, $\triangle LMN$: 30 **27.** angle bisector, 7
29. not similar **31.** Similar; $\triangle JKL \sim \triangle NPM$; since $\overline{JK} \parallel \overline{NP}$ and $\overline{KL} \parallel \overline{PM}$, $\angle J \cong \angle PNM$ and $\angle L \cong \angle PMN$ by the Corresponding Angles Postulate. Then the triangles are similar by the AA Similarity Postulate.
33. Since $\frac{KH}{TS} = \frac{KJ}{TR} = \frac{HJ}{SR} = \frac{3}{5}$, $\triangle KHJ \sim \triangle TSR$ by the SSS Similarity Theorem. **35.** $x = 3$, $y = 8.4$

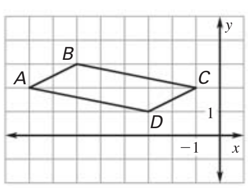
37. 

39. 

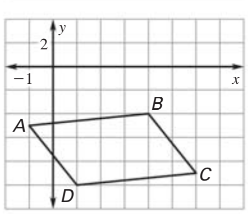
41. enlargement; 1 : 3

Chapter 7 (pp. 908–909) **1.** 50 **3.** 60 **5.** 240 ft^2 **7.** right triangle **9.** not a right triangle **11.** right triangle **13.** triangle; acute **15.** not a triangle **17.** triangle; acute **19.** $\triangle ADB \sim \triangle BDC \sim \triangle ABC$; DB **21.** $\triangle PSQ \sim \triangle QSR \sim \triangle PQR$; RP **23.** 2 **25.** 4.8 **27.** 9.7 **29.** $g = 9$, $h = 9\sqrt{3}$ **31.** $m = 5\sqrt{3}$, $n = 10$ **33.** $v = 20$, $w = 10$ **35.** $\frac{3}{5}$, 0.6; $\frac{5}{3}$, 1.6667 **37.** 6.1 **39.** 16.5 **41.** $x = 12.8$, $y = 15.1$ **43.** $x = 7.5$, $y = 7.7$ **45.** $x = 16.0$, $y = 16.5$ **47.** $GH = 9.2$, $m\angle G = 49.4°$, $m\angle H = 40.6°$

Chapter 8 (pp. 910–911) **1.** 112 **3.** 117 **5.** 68 **7.** 120°, 60° **9.** about 158.8°, about 21.2° **11.** $a = 5$, $b = 5$ **13.** $a = 117°$, $b = 63°$ **15.** $a = 7$, $b = 3$ **17.** $\angle XYV$ **19.** YV **21.** ZX

23. 

$\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$

25. 

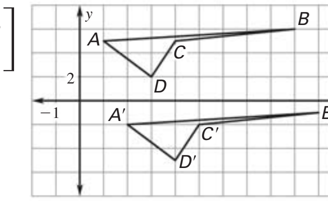
$\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$

27. Show $\angle QPR \cong \angle SRP$ making $\angle SPQ \cong \angle QRS$. You now have opposite pairs of angles congruent which makes the quadrilateral a parallelogram. **29.** Square; since the quadrilateral is both a rectangle and rhombus it is a square. **31.** Rectangle; since the quadrilateral is a parallelogram with congruent diagonals it is a rectangle. **33.** 90° **35.** 25 **37.** 0.4 **39.** 98° **41.** Parallelogram; the diagonals bisect one another. **43.** Rhombus; it is a parallelogram with perpendicular diagonals. **45.** Isosceles trapezoid; it has one pair of parallel opposite sides and congruent base angles. **47.** Kite; it has consecutive pairs of congruent sides and perpendicular diagonals. **49.** Trapezoid; it has one pair of parallel sides.

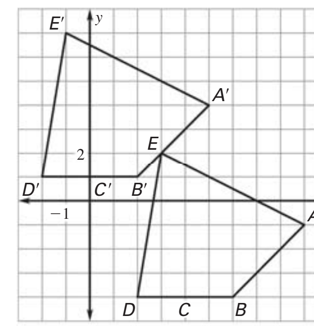
Chapter 9 (pp. 912–913) **1.** $(x, y) \rightarrow (x + 4, y - 2)$; $AB = A'B'$, $BC = B'C'$, $AC = A'C'$ **3.** $\langle -10, 7 \rangle$

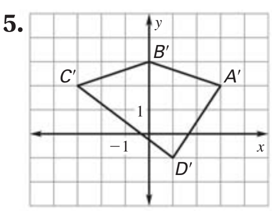
5. $\langle -4, 5 \rangle$ **7.** $\begin{bmatrix} 5 \\ 11 \end{bmatrix}$ **9.** $\begin{bmatrix} -4 & -31 \\ 64 & 67 \end{bmatrix}$

11. $\begin{bmatrix} 2 & 10 & 5 & 4 \\ -2 & -1 & -2 & -5 \end{bmatrix}$



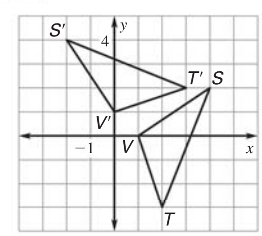
13. $\begin{bmatrix} 5 & 2 & 0 & -2 & -1 \\ 4 & 1 & 1 & 1 & 7 \end{bmatrix}$

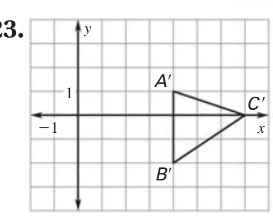


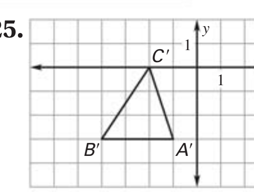
15. 

17. $A'(1, 2)$, $B'(2, -4)$, $C'(0, -1)$ **19.** $A'(-1, 2)$, $B'(-1, 5)$, $C'(2, 6)$, $D'(3, 3)$, $E'(1, -1)$

21. $\begin{array}{c} \begin{matrix} S' & T' & V' \end{matrix} \\ \begin{bmatrix} -2 & 3 & 0 \\ 4 & 2 & 1 \end{bmatrix} \end{array}$

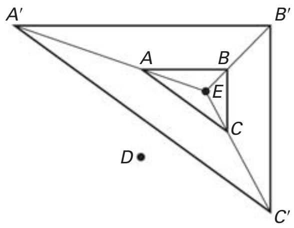


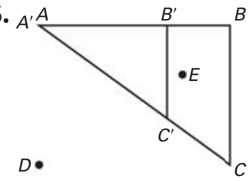
23. 

25. 

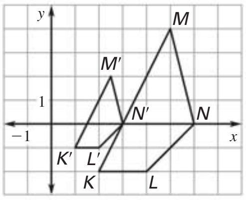
27. 88° **29.** Line symmetry, rotational symmetry; the figure has two lines of symmetry, one line passing horizontally through the center of the circle and the other passing vertically through the center of the circle; it has rotational symmetry of 180°. **31.** Line symmetry, no rotational symmetry; the figure has one line of symmetry passing vertically through the center of the rectangle; it does not have rotational symmetry.

SELECTED ANSWERS

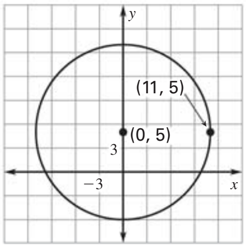
33.


35.


39.
$$\begin{array}{cccc} K' & L' & M' & N' \\ \end{array}$$
$$\begin{bmatrix} 1 & 2 & \frac{5}{2} & 3 \\ -1 & -1 & 2 & 0 \end{bmatrix}$$



Chapter 10 **(pp. 914–915)** **1.** *Sample answer:* $\overline{KF}$ **3.** *Sample answer:* $\overleftrightarrow{CD}$ **5.** *Sample answer: K* **7.** $\overline{GH}$ **9.** $\frac{8}{3}$ **11.** 12 **13.** 4 **15.** minor arc; 30° **17.** minor arc; 105° **19.** minor arc; 105° **21.** 310° **23.** 130° **25.** 115° **27.** 45° **29.** $\overparen{AB} \cong \overparen{DE}$ using Theorem 10.3. **31.** $x = 90°$, $y = 50°$ **33.** $x = 25, y = 22$ **35.** $x = 7, y = 14$ **37.** 45 **39.** 55 **41.** 3 **43.** 2 **45.** 2 **47.** 3 **49.** $x^2 + (y + 2)^2 = 16$ **51.** $(x - m)^2 + (y - n)^2 = h^2 + k^2$

53.


Chapter 11 **(pp. 916–917)** **1.** 143 units^2 **3.** 56.25 units^2 **5.** 60 cm, 150 cm^2 **7.** 5 **9.** 0.8 **11.** 22 units^2 **13.** 70 units^2 **15.** 72 units^2 **17.** 13.5 units^2 **19.** 10 : 9 **21.** $2\sqrt{2} : 1$ **23.** 14 m **25.** about 15.71 units **27.** about 28.27 units **29.** about 4.71 m **31.** about 2.09 in. **33.** 9π in.^{2}; 28.27 in.^{2} **35.** 100π ft^2; 314.16 ft^2 **37.** about 9.82 in.^{2} **39.** about 42.76 ft^2 **41.** 45° **43.** 18° **45.** 54 units, $81\sqrt{3}$ units^2 **47.** 27 units, about 52.61 units^2 **49.** about 58.7% **51.** 30% **53.** 3.75%

Chapter 12 **(pp. 918–919)** **1.** Polyhedron; pentagonal prism; it is a solid bounded by polygons. **3.** Polyhedron; triangular pyramid; it is a solid bounded by polygons. **5.** 6 faces **7.** 156.65 cm^2 **9.** 163.36 cm^2 **11.** 4285.13 in.^{2} **13.** 10 in. **15.** 14 ft **17.** 16.73 cm^2 **19.** 103.67 in.^{2} **21.** 678.58 yd^2 **23.** 1960 cm^3 **25.** 2 cm **27.** 5.00 in. **29.** 173.21 ft^3 **31.** 6107.26 in.^{3} **33.** 12.66 ft^3 **35.** 40.72 in.^{2}, 24.43 in.^{3} **37.** 589.65 cm^2, 1346.36 cm^3 **39.** 3848.45 mm^2, 22,449.30 mm^3 **41.** 1661.90 ft^2, 6370.63 ft^3 **43.** 216 ft^2, 216 ft^3 **45.** 1 : 3